The First Notre Dame Workshop on Mathematical Methods in Nonlinear Optics

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The first Notre Dame workshop on Mathematical methods in nonlinear optics was held April 18-21 1996, at the University of Notre Dame. It was sponsored by the University of Notre Dame, BRIMS, Hewlett Packard Research Lab and Center for Nonlinear Studies, Los Alamos National Lab and National Science Foundation (NSF). The workshop met in conjunction with the University of Notre Dame Symposium on Current and Future Directions in Applied Mathematics along with a variety of other workshops on various topics in Applied Mathematics.
The workshop assembled forty one leading mathematical scientists in nonlinear optics and related areas to discuss relevant advanced mathematical techniques and theory for these areas. During the workshop Bill Kath and Yuji Kodama provided tutorial reviews of Mathematical models and techniques used in the area of nonlinear fiber communications. These talks were extremely well received for their emphasis on the origins of models and issues in this area. Excellent contributions were made by experimental groups working with both AT&T and German Telekom on the development of communication systems. These talks provided direct links to questions important in applications for the mathematical participants and engendered lively discussions. New ideas in dynamical systems were strongly represented. Also, several other important new areas in nonlinear optics such as pattern formation and the analysis of systems that combine continuous and discrete properties were discussed by leaders in these fields. Many of the participants have made new contacts which allow them to learn and contribute to new areas of applied mathematics. We believe the meeting made a positive impact on the research directions of many of the participants.

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1 Review Talks

Bill Kath of Northwestern University started off the workshop on Thursday with a thorough review of the origin of the nonlinear Schrödinger equation and its use in modeling the propagation of light in optical fibers. During his talk he described inverse scattering and the bound state solutions of importance in the soliton-soliton interaction process. He described

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several forms of perturbations and the soliton perturbation theories available to deal with them. He also described the coupled NLS and its use. In the final segment of his talk he reviewed concepts of guiding center solitons and Gordon-Haus jitter. The sliding filter was described as a control for the jitter, and he showed how long-haul communications devices are closely connected to ring devices.

On Friday Yuji Kodama of Ohio State University and author of *Solitons in Optical Communications* gave a review of the modeling and analysis of communications devices. He began by reviewing the origins of NLS as a model of the propagation of light in optical fibers. He then reviewed the inverse scattering method and contrasted the solutions of the self-focusing NLS and the defocusing NLS. He continued by describing issues connected with the use of solitons as information carriers including the guiding center concept and the use of Lie transform methods, the effects of noise and the use of dispersion management and filters to control errors. The non-return to zero (NRZ) systems were then treated by using a form of WKB or semi-classical analysis of nonlinear integrable equations. Kodama explained that these techniques provide the key to understanding the dynamics of dispersion managed NRZ systems with net positive average dispersion.

2 Focus Groups

Sessions were held in four focus areas which were chosen and organized by several of the participants. This was very effective in promoting discussion and interactions both formally and informally. The time was split among four focus groups as described below.

The title of the first session was Long Haul Optical Communications and was organized by Bill Kath and Nathan Kutz. It began with a talk by Stephan Evangelides of AT&T who described current NRZ systems. He pointed out that there is a real need for analysis of these systems, especially for bit error rate estimation and system optimization. He also discussed wavelength division multiplexing of these systems. Issues such as polarization mode dispersion, dispersion maps, spectral spreading, pulse distortion, and noise in these systems were all discussed as areas requiring further analytical treatment.

Nathan Kutz described a perturbation analysis of dispersion managed systems with zero average dispersion. His reduced average equations were able to capture short distance propagation accurately. He also discussed the plane-wave stability of the reduced equations. Yuji Kodama also participated in this focus group by describing control techniques and a Whitham type theory for NRZ systems with positive (defocusing) average dispersion. He extended his theory to WDM systems and the coupled NLS. The session ended with a talk by Katya Golovchenko. She described experimental results for a return to zero dispersion managed system with net averaged dispersion in the focusing regime.

It was clear from this session that many new systems in this area of nonlinear optics call for some of the most advanced analysis of near-integrable nonlinear evolution equations. There is much work to be done on the theory of dispersion managed systems. Semi-classical theory and modulation theory for n-phase waves of NLS previously developed in applied mathematics are just beginning to be applied to problems in theoretical nonlinear optics. Also advanced tools for control and the analysis of multiplicative noise in pde's seem to be emerging as areas of basic importance in theoretical nonlinear optics. The speakers seemed to

agree that continued work on the coupled NLS equations should lead to better understanding of emerging systems and that control of wave trains without the use of sliding filters appears to be a key element in applications to communications.

The workshop continued with a focus group organized by Alejandro Aceves and Darryl Holm called Applications of Hamiltonian and Soliton Theory in Single and Coupled Optical Fiber Systems. Aceves and Michael Weinstein discussed the theory of dispersive waveguides with linear nearest-neighbor coupling. Ideas of localization and steering of light on an array of fibers were reviewed. Mathematical proofs were also given which formalize the stability and localization ideas. This was followed by a discussion by Ben Luce of the persistence of an NLS wave train under perturbations. Franko Küppers described efforts at German Telekom to optimize the use of existing (already installed) fiber systems. He pointed out that the use of sliding filters presented several technical problems which preclude their use in presently installed systems. He showed an experimental system using new semiconductor amplifiers that transmits at 40Gb/sec and requires only one filter at the end to remove jitter and noise. He also described polarization control experiments in which trapping seems to have occurred to some benefit. Sergey Burtsev described the control of solitons using sliding guiding frequency filters. His work is based on singular perturbation methods with numerical diagnostics of the nonlinear spectral content.

Ildar Gabitov gave us an update on the European deployment of soliton communications systems. He introduced an averaged equation which appears to give reasonable results for dispersion managed systems with net negative (focusing) averaged dispersion. Taras Lakoba described the existence of several families of solutions of a pair of coupled waveguides which included polarization effects.

The main theme of the third focus group was similarity solutions and near-integrable theory of pde's and it was called Analytic and Numerical Techniques for Nonlinear Waves. Annalisa Calini discussed homoclinic chaos in the discrete NLS using diagnostic information from spectral codes. George Haller discussed the homoclinic jumping in nonlinear evolution equations using recently developed ideas about Melnikov type methods for pde's.

Friday afternoon Joceline Lega and Nick Ercolani lead a discussion on patterns called Patterns in Optical, Biological and Chemical Systems. Lega gave a review of experimental observations of patterns in various physical systems and discussed the basic analysis of pattern forming systems. Mary Silber described the use of ideas of symmetry and hetereoclinic cycles in pattern forming systems. The session ended with a review of state of the art experiments and theories of patterns in optical systems.

3 Reflections

The workshop promoted discussions and inquiry on advanced mathematical techniques and theory which are suggested by or may be useful in the analysis of problems in nonlinear optics. Theoretical nonlinear optics has only begun to take advantage of work on integrable and near-integrable evolution equations. As this is achieved we expect many advances to be made. Conversely, nonlinear optics continues to be a important source of rich mathematical problems. Certainly the workshop did not cover all the important areas in theoretical nonlinear optics where applied mathematics is needed. For example, one important area that

was left out from the discussions was the analysis and control of chaotic behavior in optical science.

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