Optical Power and Wavelength for Single-Source Simulation of EDFA WDM Gain: Theoretical Basis

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HPL-96-21
February, 1996

WDM, gain, measurements, EDFA, optical fiber

This paper presents a theory, based on a homogeneous model for the erbium-doped fiber amplifier, that yields the optimum optical source power and wavelength to simulate multiple WDM channels for the purpose of measuring optical gain.

1. Introduction

Testing the gain of an EDFA is easily accomplished at a single wavelength using a single laser source and appropriate optical measuring equipment. For multichannel measurements such as those required for WDM applications, the costs of an n-channel system may become prohibitive in terms of the number of sources required and the overall test system complexity. A different approach would be to simulate the effects of n channels using a single laser, and then probe the gain profile with a wideband noise source such as a fiber-coupled EE-LED. The purpose of this document is to present a theoretical basis for this alternative method and suggest an algorithm for its implementation.

2. Theory

The EDFA is a multilevel system comprising a ground state and excited states. The most important states are labeled with populations n_1 and n_2, which indicate the number of ions residing in that particular state. The various transitions are shown in Fig. 1. The pump from the ground state to the excited state II decays rapidly to the excited state I. This decay is assumed to be rapid such that the system can be treated as a two-level system. The transitions are as follows: (a) the desired amplification transition, resulting in decay of the ion energy to the ground state; (b) spontaneous decay, resulting in emission of a noise photon; (c) absorption of a signal photon; and (d) emission of a photon at the pump wavelength.

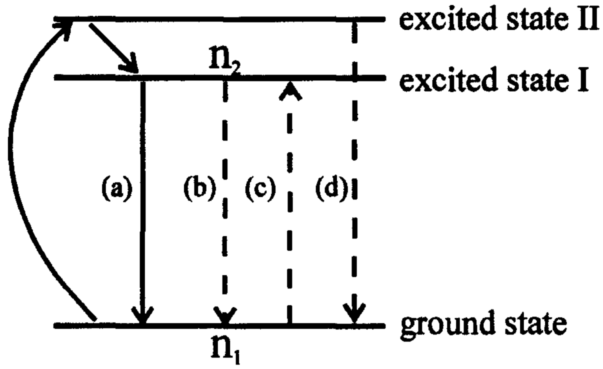

Figure 1. Energy diagram for erbium ion showing signal, pump and spontaneous emission transitions.

The rate equation for a two-level representation of this system is composed of transitions induced by the various signals, pump beams and loss due to spontaneous emission. Upconversion processes for the signal are not included; it is assumed that the differences in the excited state populations between the WDM experiment and the single-source method are small due to the proximity of the signal wavelengths. Spectral hole burning, (SHB), is not accounted for which may give rise to small differences between the single-source method and the multisource method. Note that for the case of very closely spaced WDM channels, the SHB due to the single source will be the same as that due to the n channels. These approximations are implicit in the standard two-level formulation for the upper state decay rate:

$$\frac{dn_2}{dt} = \sum_k \frac{I_k \sigma_{a,k} n_1}{h \nu_k} - \sum_k \frac{I_k \sigma_{e,k} n_2}{h \nu_k} + \frac{n_2}{\tau} \qquad (1)$$

where h is Planck's constant, ν_k, is the optical frequency of the kth beam, and σ is the ionic absorption or emission cross-section designated by the subscript a or e respectively. The kth input beam intensities are given by I_k, and the spontaneous lifetime is designated by τ. The amplifier is considered to be a reservoir of ions; no propagation effects are included. This simplification is reasonable since the propagation effects impacting the WDM channels should affect the single-source and probe signals similarly. Note that in this model, amplified spontaneous emission (ASE) is not included; the impact of ASE on the measurement will be determined at a latter time. Since the ASE is present in both cases (i.e., the single-source experiment and the multichannel one), it should affect both experiments in the same way. The EDFA should be in the same state for both types of measurements — that is have the excited state population the same for the case of the single-source channel (experiment #2) as for the WDM system (experiment #1):

$$n_2^{\#1} = n_2^{\#2} \qquad (2)$$

If this is achieved with the single-source, then the ASE generation and the pump absorption for the two experiments will be the same, within limits determined by the validity of the assumptions on SHB. From equation (1) the steady-state population for $n_2^{\#1}$ is found:

$$n_2^{\#1} = n_t \frac{\sum_k \frac{I_k \sigma_{a,k}}{h \nu_k}}{\sum_k \frac{I_k \sigma_{a,k}}{h \nu_k} + \sum_k \frac{I_k \sigma_{e,k}}{h \nu_k} + \frac{1}{\tau}} \qquad (3)$$

The total number of ions, n_t, is equal to the sum of the ions in the ground state and in the excited states. The steady-state population for experiment #2, the single-source method, is found from simplification of equation (3) for just two light beams corresponding to the pump and single-source beams:

$$n_2^{\#2} = n_t \frac{\sum_{k=2} \frac{I_k \sigma_{a,k}}{h \nu_k}}{\sum_{k=2} \frac{I_k \sigma_{a,k}}{h \nu_k} + \sum_{k=2} \frac{I_k \sigma_{e,k}}{h \nu_k} + \frac{1}{\tau}} \qquad (4)$$

or:

$$n_2^{\#2} = \frac{n_t \left[\frac{I_s \sigma_{a,s}}{h \nu_s} + \frac{I_p \sigma_{a,p}}{h \nu_p} \right]}{\frac{I_s}{h \nu_s}\left(\sigma_{a,s} + \sigma_{e,s}\right) + \frac{I_p}{h \nu_p}\left(\sigma_{a,p} + \sigma_{e,p}\right) + \frac{1}{\tau}} \tag{5}$$

Equating the excited state populations in the two experiments places conditions on the second experiment to achieve the best simulation of WDM performance:

$$\frac{\sum_k \frac{I_k \sigma_{a,k}}{h \nu_k}}{\sum_k \frac{I_k \sigma_{a,k}}{h \nu_k} + \sum_k \frac{I_k \sigma_{e,k}}{h \nu_k} + \frac{1}{\tau}} = \frac{\frac{I_s \sigma_{a,s}}{h \nu_s} + \frac{I_p \sigma_{a,p}}{h \nu_p}}{\frac{I_s}{h \nu_s}\left(\sigma_{a,s} + \sigma_{e,s}\right) + \frac{I_p}{h \nu_p}\left(\sigma_{a,p} + \sigma_{e,p}\right) + \frac{1}{\tau}} \tag{6}$$

Equation (6) can be considered satisfied if the numerators on each side of the equality are equal, and the denominators on each side of the equality are set equal as well. The physical insight here is the requirement that the emission and absorption rates for the ensemble of optical beams are to be equal in the two experiments. Thus, equating numerators yields:

$$\sum_n \frac{I_n \sigma_{a,n}}{h \nu_n} = \frac{I_s \sigma_{a,s}}{h \nu_s} \tag{7}$$

where the replacement of the subscript k with n indicates that the pump beam is no longer part of the ensemble of beams. Likewise the denominators of equation (6) are equated:

$$\sum_n \frac{I_n \sigma_{a,n}}{h \nu_n} + \sum_n \frac{I_n \sigma_{e,n}}{h \nu_n} = \frac{I_s}{h \nu_s}\left(\sigma_{a,s} + \sigma_{e,s}\right) \tag{8}$$

Note that if equation (7) is satisfied, then equation (7) can be subtracted from equation (8), yielding:

$$\sum_n \frac{I_n \sigma_{e,n}}{h \nu_n} = \frac{I_s \sigma_{e,s}}{h \nu_s} \tag{9}$$

In arriving at equations (7) and (8) the pumping rates were assumed to be the same. The conditions are thus summarized below:

$$\frac{I_s\,\sigma_{a,s}(\lambda)}{h\,\nu_s} = \sum_n \frac{I_n\,\sigma_{a,n}(\lambda)}{h\,\nu_n} \tag{10a}$$

$$\frac{I_s\,\sigma_{e,s}(\lambda)}{h\,\nu_s} = \sum_n \frac{I_n\,\sigma_{e,n}(\lambda)}{h\,\nu_n} \tag{10b}$$

Thus a constraint has been placed on the experiments that the total absorption rates and the total emission rates must be equal.

3. Determining the Single-Source Power

Proper settings of the single-source power and wavelength will satisfy the requirements set by equation (10). Assuming that the absorption rates are satisfied, subtracting equation (10b) from (10a) yields a condition for the single-source power in terms of measurable parameters:

$$\frac{I_s}{h\nu_s}\left(n_2\sigma_{e,s} - n_1\sigma_{a,s}\right) = \sum_n \frac{I_n}{h\nu_n}\left(n_2\sigma_{e,n} - n_1\sigma_{a,n}\right) \tag{11}$$

which is rewritten (after integration) as:

$$P_s = \frac{1}{\lambda_s G_s} \sum_n P_n \lambda_n G_n \tag{12}$$

where P_S and P_n are the input single-source and WDM input channel powers, respectively. Thus, this equation provides a basis for equating the gain-power product of the single source to the individual gain-products of the WDM channels.

4. Determining the Single-Source Wavelength

The single-source wavelength can be determined depending on the information available about the amplifier. If the absorption cross-section of the amplifier is known, the single-source wavelength can be calculated from the absorption cross-section, the WDM channel powers and the WDM gains. Generally, aluminum co-doped silica fiber amplifiers have similar cross-section shapes. Without any knowledge of the absorption cross-section, the single-source wavelength is calculated on the basis of the weighted wavelength:

$$\lambda_{sat} = \frac{\sum_n P_n \lambda_n}{\sum_n P_n} \qquad (13)$$

If the absorption cross-section varied linearly across the wavelength of interest and the channel powers were equal, then the optimum wavelength would be in the center of the band.

In general, the condition on the single-source wavelength is found from equation (10a) using $\nu = c/\lambda$:

$$\lambda_s = \sum_n \frac{I_n}{I_s} \frac{\sigma_{a,n}}{\sigma_{a,s}} \lambda_n \qquad (14)$$

or

$$\lambda_s = \sum_n \frac{P_n}{P_s} \frac{\sigma_{a,n}(\lambda)}{\sigma_{a,s}(\lambda)} \lambda_n \qquad (15)$$

It is clear from equation (14) that if the WDM channel powers were equal and the absorption had no wavelength dependence, the optimum wavelength would be in the middle of the band:

$$\lambda_s = \sum_n \frac{P_n}{P_s} \lambda_n = \frac{1}{n} \sum_n \lambda_n \qquad (16)$$

However, intuitively this isn't clear, since if the absorption cross-sections were equal, than what difference should the wavelength make on the absorption rate? This dilemma is resolved by noting that the conversion of optical power to photons requires for equal absorption rate that the single-source wavelength be in the middle of the band. This is because the shorter wavelengths result in fewer photons per watt of optical power while the longer wavelengths result in more photons. So the weighting function takes into account the effect of the variation of the number of photons per watt of optical power. If this dependence didn't exist, then the single-source wavelength would be anywhere in the WDM channel.

In general, the absorption cross-section is not constant, as shown in Figure 2 [1].

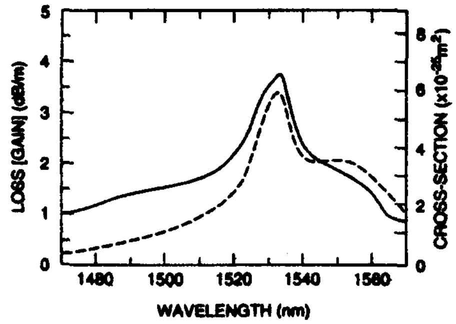

Figure 2. Absorption (——), emission (----) cross-sections for Al:Ge fiber. (with permission after ref [1], ©1991 IEEE)

Fortunately, the absorption cross-section is a fairly simple function in the region around 1.55 µm, and a simple relation should model well its behavior. To solve equation (14), the absorption cross-section versus wavelength is required. The absolute value of the result is not critical since it is in the form of a ratio. For example, from Figure 2, the shape in the important region of 1550 nm ±10 nm can be approximated as:

$$\sigma_a(\lambda) = \frac{4.0 - 0.1(\lambda - 1540)}{10^{25}} \qquad (17)$$

This implies that the ratio of the WDM channel power absorption to the single-source absorption can vary approximately as:

$$\frac{1}{2} \leq \frac{\sigma_{a,n}}{\sigma_{a,s}} \leq 2 \qquad (18)$$

This approximation will permit an estimate of the optimum wavelength for the single-source. Fortunately, there are two factors that reduce the impact of the absorption rate problem: the absorption cross-sections are lower than the emission cross-sections in the 1550-nm band, and, more importantly, the absorption rate depends on the level of the ground state population, which should be small if the amplifier is sufficiently pumped.

5. Summary

The conditions for the wavelength and power were derived for a fixed-wavelength source to simulate the effects on an EDFA of an n-channel WDM system for the purposes of measuring WDM gain in a homogeneously broadened environment. Since both absorption and emission rates are required to be satisfied, resulting in a wavelength requirement for the single-source, this method is differentiated from the variable-wavelength single-source technique where the signal is tuned to each channel wavelength and gain measurements are made at that wavelength. This latter method cannot strictly satisfy requirements on both net gain and signal absorption rates. Simple relations for the single-source strength and the single-source wavelength were given. The method can be easily used to simulate eight channels using a smaller number of source channels. For example 3 channels representing 8 should provide a better match to the gain profile if SHB becomes significant.

Reference

[1] C. R. Giles, E. Desurvire, "Modeling erbium-doped fiber amplifiers:, J. Lightwave Technol., vol 9, no.2, pp. 271-283, 1991.