HEWLETT PACKARD

Neidenoff's Noise Equivalent

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Abstract

This is an analysis of a claim [1] that the conventional 'Noise Factor' is a flawed metric for the level of noise in an electronic circuit and that an alternative definition is urgently required.

This report concludes that, while it could be argued that the existing metric encourages misinterpretation of noise theory, it is not actually wrong and is sufficient if it is properly understood and used. If there is indeed an actual flaw in the conventional 'Noise Factor', this author (gjp) has failed to comprehend it.

However, Neidenoff's method does identify and isolate the critical contributor to noise in circuits, and hence could arguably improve understanding of designing low noise circuits.

1 Introduction

The publication[1] [1] is a resume of Neidenoff's objections to the conventional metric of noise in circuits and contains his definition of a metric which is claimed to be more useful. Unfortunately the english of the publication is poor and it is consequently very hard to understand[2].

This report reproduces the mathematics of Neidenoff's method and compares its predictions with the conventional method. An attempt is made to summarise his objections to the conventional method, but it is by no means certain that they have been understood.

This report does not address the many controversial opinions and arguments which also appear in [1], since direct argument based on the mathematics is sufficient for a comparison of the two methods.

2 Analysis

Neidenoff's concern is that designing a circuit for lowest conventional noise factor may be taken to mean designing the circuit that has lowest noise when driven by a 50 ohm source and matched to that source by having a 50 ohm input impedence. That is not necessarily the same as designing the circuit that has lowest noise, which might be a circuit that has a different input impedence. He reminds us that the conventional F number is specified with the amplifier driven by the thermal noise from a 50 ohm resistor purely because an F number depends on the ratio of input noise to a circuit and the internal noise in a circuit. It is therefore impossible to compare F numbers unless they have all been measured with sources of the same impedence, or at least corrected to compensate for a source of other impedence.

Older descriptions of noise theory tended to introduce the topic with power matching between source and amplifier. Neidenoff appears to be concerned that this may be misunderstood to imply that low noise circuits should be designed to have the same input impedence as used by the conventional F number, which restricts the design of low noise circuits to a subset of the possible design space. He is also concerned over *noise matching*, where tuning the source resistance is used to improve the noise performance of the driven circuit.

In comparison, Neidenoff's metric (the *noise equivalent*) is dependent *only* on the characteristics of the amplifier and can be quoted independent of any value of noise input. It therefore eliminates the possible misconception identified by him.

[1] The publication was received by Thomas Harbach in the Intellectual Property Section at HP Boeblingham, and then passed to HPlabs Bristol for comment.

[2] Every care has been taken, but misunderstandings may have arisen.

Neidenoff therefore emphasises that the proper way to design low noise circuits is to design for low 'noise equivalent', not low (conventional) noise factor.

Note that the author (gjp) could not reproduce the equation for the F number of an amplifier *while it is in a cascade*, ie. the equation for F_μ on page 45 of [1].

3 Neidenoff's method

Neidenoff's method is derive a figure of merit for each amplifier that dependent on the characteristics of that amplifier *only*. This is Weq, and is the noise power introduced in a gain stage divided by the gain of that stage (equation 3)

$$Weq_x = \frac{L_x}{G_x}$$

The use of Weq enables the simple calculation of signal-to-noise ratios for single stages or cascades with arbitary input impedences. The method predicts that the initial stages of a cascade are the most critical in determining the noise perfromance of the cascade.

The following expressions are derived for the application of an arbitary signal containing noise input N_{x-1} to a cascade of amplifiers (m+1 stages, from x....x+m):

- the noise figure for the first stage (stage x) is (equation 4)

  $$F_x = 1 + \frac{Weq_x}{N_{x-1}}$$

  He reminds us that the conventional F number is a special case of the generic F number of a single stage (above), and is obtained when the noise input is thermal noise from a 50 ohm resistor at 290K. Hence (equation 6):

  $$Weq_x = (F_{conventional} - 1) * 8.0 * 10^{-19} V^2/Hz$$

- the noise figure for a cascade from stage x to stage x+m is (equation 9)

  $$F^{tot}_{x:x+m} = 1 + \frac{Weq^{tot}_{x:x+m}}{N_{x-1}}$$

  where (equation 10)

  $$Weq^{tot}_{x:x+m} = \sum_{i=m}^{i=1} \frac{Weq_{x+i}}{\prod_{j=i-1}^{j=0} G_{x+j}} + Weq_x$$

  So that for stage x+1 (two stages)

$$Weq^{tot}_{x:x+1} = \frac{Weq_{x+1}}{G_x} + Weq_x$$

and for stage x+2 (three stages)

$$Weq^{tot}_{x:x+2} = \frac{Weq_{x+2}}{G_{x+1}G_x} + \frac{Weq_{x+1}}{G_x} + Weq_x$$

and so on.

- The total F number of a cascade is the product of individual F numbers of that cascade (equation 8), provided those individual F numbers are measured with an input noise equal to the actual noise from the previous stage. Those individual F numbers are not those that would be obtained by independently testing each stage with the same noise source (as is done to determine the conventional noise factor).

  The F number of a stage x+m *while it is connected in a cascade starting with stage x* is (equation 12):

$$F_{x+m} = 1 + \frac{\frac{Weq_{x+m}}{\prod_{i=0}^{i=m-1} G_{x+i}}}{N_{x-1} + Weq^{total}_{x:x+m-1}}$$

4 The conventional method

The conventional method (see [2] for a fuller description) is to consider an amplifier driven by a standardised noise source.

The F number of each amplifier is (logically) obtained by driving an amplifier by a fixed (50 ohm) resistor at 290K. These can then be used to obtain the overall F number of a cascade driven by the same (50 ohm) resistor at 290K. The equations predict that earlier stages have the most effect on noise performance.

5 Conclusion

Not all of [1] could be properly understood (due to the quality of the english), many of the symbols used in the mathematics are very confusing, and one equation could not be reproduced. Hence it is felt that Neidenoff's argument must be presented much more concisely and clearly if it is to reach a wider audience - at the moment only the really determined will struggle right to the end of the booklet.

However, Neidenoff's method emphasises the independence of low noise design from any particular input impedence and may encourage a wider design freedom.

The author (gjp) has not found any reason to believe that the conventional method is wrong, but accepts that it might mislead some designers and cause them to unnecessarily restrict their design freedom. It is unknown whether the misconceptions identified by Neiderhoff are widely held, and whether they have affected the practical implementation of low noise circuits. However, it is worth noting that one of the most important uses of low noise circuits is the front-end of radio receivers, which are characterised with (standard) 50 ohm input impedences because loading affects the transfer characteristics of amplifiers at rf frequencies. So the 'misunderstanding' that low noise circuits must be designed with 50 ohm impedences is actually true for this important class of low noise circuits.

No fundamental contradictions were found between the predictions of the two methods.

6 References

[1] A.Neidenoff *'Lies, Damm Lies & The Noise Factor'*, IBN-verlag 1995.

[2] L.W.Couch II *'Digital and Analog Communication Systems'*, third Edition, Macmillan.

7 Appendix

7.1 Algebraic symbols

[1] uses a large number of symbols, many of which are written in very similar forms, and this tended to confuse the author (gjp). This report therefore uses a much smaller set of symbols, and writes those symbols in distinct forms which are different from that used in [1].

All descriptions involving noise must deal with powers rather than voltages, and hence the key figure of merit is the square of the signal-to-noise ratio, rather than the signal-to-noise ratio itself. Individual factors in the following mathematics therefore have the dimension of volts-squared, but for convenience are referred to as 'powers' of one sort or another.

Any resistances R are assumed to be at temperature T and K is Boltzmanns constant.

The following sections consider a cascade of amplification stages, amplification stage x having input signal power S_{x-1}, input noise power N_{x-1}, power gain G_x, internal noise power L_x, output signal power S_x and output noise power N_x.

Clearly

$$S_x = G_x S_{x-1} \tag{1}$$

$$N_x = G_x N_{x-1} + L_x \tag{2}$$

7.2 The mathematics of Neidenoff's method

7.2.1 Internal equivalent noise power

Neiderhoff introduces a factor (written here as Weq_x) as the factor representing the noise introduced inside amplification stage x. Page 25 of [1] defines the output signal-to-noise power ratio

$$(\frac{S}{N})_{out} = \frac{S_{in}}{N_{in}+Weq}$$

In terms of the symbols defined in this report, and using equation 1,

$$(\frac{S}{N})_x = \frac{S_{x-1}}{N_{x-1}+Weq_x} = \frac{G_x S_{x-1}}{G_x N_{x-1}+G_x Weq_x} = \frac{S_x}{G_x N_{x-1}+G_x Weq_x}$$

Clearly the divisor must be N_x and hence (using equation 2)

$$N_x = G_x N_{x-1} + L_x \equiv G_x N_{x-1} + G_x Weq$$

So

$$Weq_x \equiv \frac{L_x}{G_x} \tag{3}$$

7.2.2 Single stage noise factor

The noise factor F_x of stage x is by definition

$$F_x = \frac{(\frac{S}{N})_{in}}{(\frac{S}{N})_{out}} = \frac{S_{x-1}}{N_{x-1}} \frac{N_x}{S_x}$$

Equations 1, 2 and 3 give

$$F_x = \frac{S_{x-1}}{N_{x-1}} \frac{G_x N_{x-1} + L_x}{G_x S_{x-1}} = \frac{S_{x-1}}{N_{x-1}} \frac{N_{x-1} + \frac{L_x}{G_x}}{S_{x-1}}$$

$$F_x = 1 + \frac{Weq_x}{N_{x-1}} \tag{4}$$

Clearly, this may be written as

$$Weq_x = (F_x - 1) N_{x-1} \tag{5}$$

Note that small values of noise inside a circuit (small Weq) is consistant with a small F.

Conventional F numbers are generated using a 50 ohm thermal-noise source at 290K as the input. Hence when F is the conventional noise factor, the noise input is the thermal noise of a 50 ohm resistor at 290 Kelvin:

$$N_{x-1}^{conventional} = 4KTR \equiv 4 * 1.38 * 10^{-23} * 290 * 50 = 8.0 * 10^{-19} V^2/Hz$$

and hence

$$Weq_x = (F_{conventional} - 1) * 8.0 * 10^{-19} V^2/Hz \tag{6}$$

7.2.3 Two stage cascade noise factor

By definition

$$F^{tot}_{x:x+1} = \frac{(\frac{S_{x-1}}{N_{x-1}})}{(\frac{S_{x+1}}{N_{x+1}})}$$

and hence (using generalisations of equations 1, 2)

$$F^{tot}_{x:x+1} = \frac{S_{x-1}}{N_{x-1}}\frac{N_{x+1}}{S_{x+1}} = \frac{S_{x-1}}{N_{x-1}}\frac{G_{x+1}G_xN_{x-1}+G_{x+1}L_x+L_{x+1}}{G_{x+1}G_xS_{x-1}} = \frac{G_{x+1}G_xN_{x-1}+G_{x+1}L_x+L_{x+1}}{G_{x+1}G_xN_{x-1}}$$

$$= 1 + \frac{\frac{L_x}{G_x}+\frac{L_{x+1}}{G_{x+1}G_x}}{N_{x-1}}$$

Using generalisations of equation 3 gives

$$F^{tot}_{x:x+1} = 1 + \frac{\frac{Weq_{x+1}}{G_x} + Weq_x}{N_{x-1}} \tag{7}$$

7.2.4 Multistage cascade noise factor

Clearly (from the definition of F number), the overall F number of a cascade can be written

$$F^{tot}_{x:x+m} = \frac{(\frac{S_{x-1}}{N_{x-1}})}{(\frac{S_{x+m}}{N_{x+m}})} = \frac{(\frac{S_{x+m-1}}{N_{x+m-1}})}{(\frac{S_{x+m}}{N_{x+m}})}\frac{(\frac{S_{x+m-2}}{N_{x+m-2}})}{(\frac{S_{x+m-1}}{N_{x+m-1}})}\cdots\frac{(\frac{S_x}{N_x})}{(\frac{S_{x+1}}{N_{x+1}})}\frac{(\frac{S_{x-1}}{N_{x-1}})}{(\frac{S_x}{N_x})}$$

$$F^{tot}_{x:x+m} = \prod_{i=x+m}^{i=x} F_i \tag{8}$$

but note that each individual stage has a different noise input (the actual noise output from the previous stage) and hence the F number of each stage is measured under different (unique) conditions.

However, observe that the noise figure for a two stage cascade can be written in the same form as the expression for the noise figure for a single stage: From equation 7

$$F^{tot}_{x:x+1} = 1 + \frac{Weq^{tot}_{x:x+1}}{N_{x-1}}$$

where

$$Weq^{tot}_{x:x+1} = \frac{Weq_{x+1}}{G_x} + Weq_x$$

Clearly then, the noise figure for a cascade from stage x to stage x+m is

$$F^{tot}_{x:x+m} = 1 + \frac{Weq^{tot}_{x:x+m}}{N_{x-1}} \tag{9}$$

where

$$Weq^{tot}_{x:x+m} = \sum_{i=m}^{i=1} \frac{Weq_{x+i}}{\prod_{j=i-1}^{j=0} G_{x+j}} + Weq_x \tag{10}$$

7.2.5 General stage noise factor

By definition, the noise factor of stage x+1 is

$$F_{x+1} = \frac{(\frac{S_x}{N_x})}{(\frac{S_{x+1}}{N_{x+1}})}$$

and hence (using equations 1, 2)

$$F_{x+1} = \frac{S_x}{N_x}\frac{N_{x+1}}{S_{x+1}} = \frac{G_x S_{x-1}}{G_x N_{x-1} + L_x}\frac{G_{x+1}G_x N_{x-1} + G_{x+1}L_x + L_{x+1}}{G_{x+1}G_x S_{x-1}} = \frac{G_x N_{x-1} + L_x + \frac{L_{x+1}}{G_{x+1}}}{G_x N_{x-1} + L_x}$$

$$= 1 + \frac{\frac{L_{x+1}}{G_{x+1}}}{G_x N_{x-1} + L_x} = 1 + \frac{\frac{L_{x+1}}{G_{x+1}G_x}}{N_{x-1} + \frac{L_x}{G_x}}$$

Using equation 3 gives

$$F_{x+1} = 1 + \frac{\frac{Weq_{x+1}}{G_x}}{N_{x-1} + Weq_x} \tag{11}$$

If stage x is considered as the cascade stage y driven by stage z, from equation 10

$$Weq_x \equiv Weq^{tot}_{z:y} = \frac{Weq_y}{G_z} + Weq_z$$

and clearly

$$G_x \equiv G_z G_y$$

So equation 11 becomes

$$F_{x+1} = 1 + \frac{\frac{Weq_{x+1}}{G_x}}{N_{x-1}+Weq_x} = 1 + \frac{\frac{Weq_{x+1}}{G_z G_y}}{N_{x-1}+\frac{Weq_y}{G_z}+Weq_z}$$

Relabelling stage z as x, stage y as x+1, stage x+1 as x+2 gives

$$F_{x+2} = 1 + \frac{\frac{Weq_{x+2}}{G_x G_{x+1}}}{N_{x-1}+\frac{Weq_{x+1}}{G_x}+Weq_x}$$

Rewriting using equation 10 gives

$$F_{x+2} = 1 + \frac{\frac{Weq_{x+2}}{\prod_{i=0}^{i=1} G_{x+i}}}{N_{x-1}+Weq_{x:x+1}^{total}}$$

Hence in general

$$F_{x+m} = 1 + \frac{\frac{Weq_{x+m}}{\prod_{i=0}^{i=m-1} G_{x+i}}}{N_{x-1}+Weq_{x:x+m-1}^{total}} \tag{12}$$

This equation does not agree with the equation labelled F_μ at the bottom of page 45 of [1].

7.3 The mathematics of the conventional method

7.3.1 Single stage noise factor

By definition, the noise factor F_x of stage x driven by a signal S_{ref} containing noise N_{ref} is

$$F_x = \frac{(\frac{S}{N})_{in}}{(\frac{S}{N})_{out}} = \frac{S_{ref}}{N_{ref}}\frac{N_x}{S_x} = \frac{S_{ref}}{N_{ref}}\frac{N_x}{G_x S_{ref}}$$

$$F_x = \frac{N_x}{G_x N_{ref}} \tag{13}$$

using equation 2 gives

$$F_x = \frac{G_x N_{ref}+L_x}{G_x N_{ref}}$$

$$F_x = 1 + \frac{L_x}{G_x N_{ref}} \tag{14}$$

Similarly, the noise factor F_y of stage y driven by a signal S_{ref} containing noise N_{ref} is

$$F_y = 1 + \frac{L_y}{G_y N_{ref}} \tag{15}$$

and the noise factor F_z of stage z driven by a signal S_{ref} containing noise N_{ref} is

$$F_z = 1 + \frac{L_z}{G_z N_{ref}} \tag{16}$$

7.3.2 Two stage cascade noise factor

A cascade of two amplifiers (stage y followed by stage z) may clearly be considered as a single stage (stage x) with total power gain

$$G_x \equiv G_z G_y$$

and total noise output

$$N_x \equiv L_z + G_z L_y + G_z G_y N_{ref}.$$

Using equation 13 gives

$$F_x = \frac{N_x}{G_x N_{ref}} \equiv \frac{L_z + G_z L_y + G_z G_y N_{ref}}{G_z G_y N_{ref}} = 1 + \frac{L_z}{G_z G_y N_{ref}} + \frac{L_y}{G_y N_{ref}}$$

Using equations 15 and 16 gives

$$F_x = 1 + \frac{(F_z - 1) G_z N_{ref}}{G_z G_y N_{ref}} + \frac{(F_y - 1) G_y N_{ref}}{G_y N_{ref}} == 1 + \frac{(F_z - 1)}{G_y} + (F_y - 1) = \frac{(F_z - 1)}{G_y} + F_y$$

Generalising to stages x, x+1 gives

$$F^{tot}_{x:x+1} = \frac{(F_{x+1} - 1)}{G_x} + F_x \tag{17}$$

7.3.3 Multistage cascade noise factor

From equation 17, it is trivial to show that the noise figure for a cascade from stage x to stage x+m is

$$F^{tot}_{x:x+m} = \sum_{i=m}^{i=1} \frac{F_{x+i} - 1}{\prod_{j=i-1}^{j=0} G_{x+j}} + F_x \tag{18}$$

7.4 Predictions

Clearly the two methods predict that a cascade of amplifiers behave in the same way. *If the noise input N_{x-1} is fixed to be that noise specific to the conventional noise factor (N_{ref}), equations from Neidenoff's method can be mixed with equations from the conventional method.* With that proviso, equation 4 in equation 18 gives

$$F^{tot}_{x:x+m} = \sum_{i=m}^{i=1} \frac{1+\frac{Weq_{x+i}}{N_{ref}}-1}{\prod_{j=i-1}^{j=0} G_{x+j}} + 1 + \frac{Weq_x}{N_{ref}} = 1 + \frac{\sum_{i=m}^{i=1} \frac{Weq_{x+i}}{\prod_{j=i-1}^{j=0} G_{x+j}} + Weq_x}{N_{ref}}$$

Using equation 10 gives

$$F^{tot}_{x:x+m} = 1 + \frac{Weq^{tot}_{x:x+m}}{N_{ref}}$$

which is the same as equation 9.

This demonstrates that Neidenoff's method is (in one sense) a superset of the conventional method.