

Influence of Optical Source Characteristics Upon Measurement of Polarization-Mode Dispersion of Highly Mode-Coupled Fibers

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optical, fiber optics, incoherent, lightwave components A new theoretical analysis of the interferometric dispersion polarization-mode (PMD) measurement predicts, in the limit of large PMD, a relationship to the highly mode-coupled principal states model which is different from the previously published relation [4]. Computer simulation confirms this prediction, and shows the ratio of the mean differential group delay (DGD) to interferometric PMD to be a function of the the bandwidth and spectral shape of the source used for interferometry. Jones matrix eigenanalysis and wavelength scanning with extrema counting are shown to measure the mean DGD independently of the optical source spectrum, while interferometrically measured PMD is shown to be influenced by optical source characteristics as well as by characteristics of the fiber to be measured.

Internal Accession Date Only

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Polarization-mode dispersion (PMD) is now widely recognized as a fundamental specification of single-mode optical fiber owing to its effects upon the linearity of analog transmission links and upon bit rate limitation in high-speed digital links. Reflecting the complex phenomenon of PMD and the different techniques used for its measurement, two different definitions of PMD have been proposed. An aim of this letter is to present results of computer simulations of fibers in the long-length regime, and to discuss, for each definition of PMD, the implications of these results. As these simulations imply relations between the different measurement methods which are different from those previously predicted by theory, a second aim of this letter is to present a new theoretical analysis of the interferometric measurement which predicts results consistant with the simulation.

Jones matrix eigenanalysis (JME) [1] and wavelength scanning with extrema counting (WSEC) [2] are rigorously based upon the principal states model of PMD [3]. This model accounts for coherent addition of transmitted optical spectral components to arrive at a three-dimensional, wavelength-dependent polarization dispersion vector Ω . Each orthogonal component of Ω is normally distributed with zero mean and variance σ_{Ω}^2 . JME and WSEC yield the mean DGD $\langle \tau \rangle$, which in turn determines all statistical properties of this model. Accordingly, one proposed definition of fiber PMD in the long-length regime is simply the mean DGD. This mean can be interpreted as an average over a large wavelength range, or as an average at a single wavelength over a large range of environmental fluctuation.

A second proposed definition of PMD is based upon the time-dependent intensity I(t) of the output of a long fiber in response to an input pulse of duration much shorter than the fiber PMD [4]. Based upon modal dispersion in multimode fibers and upon the central limit theorem, we might expect I(t) to be a Gaussian pulse of variance σ_I^2 multiplied by a random amplitude caused by coherent addition of randomly phased

components of the Gaussian. The proposed second-moment definition is

$$\tau_I = 2 \,\sigma_I = 2 \left(\frac{\int t^2 I(t) \,dt}{\int I(t) \,dt} - \left(\frac{\int t \,I(t) \,dt}{\int I(t) \,dt} \right)^2 \right)^{1/2} . \tag{1}$$

This definition enjoys the advantage of a direct connection to pulse spreading, but it suffers the disadvantage that no widely used PMD measurement technique measures τ_I , directly or indirectly. Interferometric measurement of PMD (Fig. 1) yields data in the time domain [5] by measurement of mutual coherence by means of a field autocorrelator. Interferometric PMD measurement does not yield an intensity autocorrelation as asserted in [4]. In principle σ_I and τ_I can be measured using an intensity autocorrelator [6], e.g. by introducing a frequency-doubling crystal into the autocorrelator of Fig. 1. However, as a field autocorrelation is not sufficient to determine the width of a short pulse [7], the field autocorrelator used in PMD measurements cannot measure the width of an intensity envelope.

As neither interferometric measurement, JME, nor WSEC has been simply related to the pulse distortion caused by PMD, choice of a measurement method must be based upon other considerations. In this letter we present simulations of fibers in the long-length regime which show that the result of interferometric PMD measurement depends upon the characteristics of the optical source in addition to the PMD to be measured. In contrast, we show that the result of JME and WSEC is independent of the characteristics of the optical source. As these simulations imply relations between the different measurement methods which are new, we begin by presenting a new theoretical analysis of the interferometric measurement which confirms the simulation results in the limit of large PMD.

The transmission $T(\omega)$ of Fig. 1 will be treated as an ergodic random process in order to develop new theory which builds upon some of the results arrived at in [2]. The true transmission $\tilde{T}(\omega)$ through a fiber and polarizer was shown in [2] to be uniformly distributed over the interval (0,1). $\mathcal{E}(t)$, the envelope of the a.c. photocurrent p(t), includes a spike at t=0 which must be ignored when calculating the variance $\sigma_{\mathcal{E}}^2$. The undesired spike is eliminated by working with a mean-shifted transmission $T(\omega)$ which has a mean value of zero: $T(\omega) = \tilde{T}(\omega) - 1/2$. $T(\omega)$ is uniformly distributed over the interval (-1/2, 1/2), as shown in Fig. 2a. Analysis is facilitated by working with the squared magnitude of the a.c. photocurrent $h(t) = \mathcal{E}^2(t) = |p(t)|^2$ rather than the envelope $\mathcal{E}(t)$. $\mathcal{E}(t)$ is a Gaussian multiplied by a random amplitude, so $\sigma_{\mathcal{E}}^2 = 2\sigma_h^2$ if we assume the random amplitude is a stationary process.

The Fourier transform relationship between the envelope of the interferometric photocurrent and the transmitted power spectrum is illustrated in Fig. 2. The broad shape of $\mathbf{E}(t)$ determines $\sigma_{\mathbf{E}}$ and the fine structure of $T(\omega)$. The fine structure of $\mathbf{E}(t)$ determines the broad shape of $T(\omega)$, i.e. the power spectrum of the source. We obtain results valid for large PMD by considering a rectangular power spectrum of width $\Delta \omega$ much larger than any variations in $T(\omega)$. According to the definition of h(t), its transform is given for small ω by

$$H(\omega) = \int_{\Delta\omega} T(\eta) T(\eta + \omega) d\eta = \Delta\omega R_{TT}(\omega)$$
(2)

where $R_{TT}(\omega)$ is the autocorrelation of $T(\omega)$, i.e. the expected value of $T(\eta)T(\eta + \omega)$. Since h(t) is an even function, we can obtain σ_h^2 , the variance of h, from the following Fourier transform relation:

$$\sigma_h^2 = \frac{\int h(t) t^2 dt}{\int h(t) dt} = -\frac{H''(0)}{H(0)}$$
(3)

where H'(0) is the second derivative of $H(\omega)$ evaluated at $\omega = 0$. Evaluation of H(0) is based upon the probability density function of T, which is uniform over (-1/2, 1/2):

$$H(0) = \Delta \omega \langle T^2 \rangle = \frac{\Delta \omega}{12}$$
(4)

A property of the derivatives of a stationary random process [8] is $R_{TT}^{\prime\prime}(0) = -\langle T^{\prime 2} \rangle$. Integrating equation (26) of [2] yields the probability density function of T', which in turn allows calculation of $H^{\prime\prime}(0)$:

$$f_{T'}(T') = \frac{1}{\sigma_{\Omega}} \sqrt{\frac{\pi}{2}} \left[1 - \operatorname{Erf}\left(\sqrt{2} \, \frac{|T'|}{\sigma_{\Omega}}\right) \right] \qquad -\infty < T' < \infty$$
(5)

$$\Rightarrow H'(0) = -\Delta\omega \langle T'^2 \rangle = -\frac{\Delta\omega}{6}\sigma_{\Omega}^2$$
(6)

The rms DGD is related to the polarization dispersion vector by $\langle \tau^2 \rangle^{1/2} = \sqrt{3} \sigma_{\Omega}$ [3]. This, in combination with (3), (4), and (6), yields the relations of the interferometric photocurrent envelope to the rms DGD and the mean DGD in the limit of high PMD or large optical source bandwidth:

$$\langle \tau^2 \rangle^{1/2} = \frac{\sqrt{3}}{2} \sigma_{\varepsilon} \approx 0.866 \sigma_{\varepsilon}$$
 (7)

$$\langle \tau \rangle = \sqrt{\frac{2}{\pi}} \sigma_{\varepsilon} \approx 0.798 \sigma_{\varepsilon}$$
 (8)

The characteristic width σ_{ϵ} of the interferometric photocurrent envelope is substantially different from the rms DGD, in contradiction of the equality predicted by the theory of [4]. Relation (8) is confirmed by the simulation in the limit of high PMD.

Highly mode-coupled fibers were simulated using concatenations of waveplates, as described in [2]. The frequency-dependent Jones matrix of a fiber was found by calculating the product of 2000 frequency-dependent waveplate matrices at 19.5-GHz intervals over a frequency range of 185 to 225 THz, or approximately 1333 to 1622 nm. Each waveplate angle was uniformly distributed over the interval $(-\pi, \pi)$, and each waveplate delay was uniformly distributed over the interval $(0, \tau_{max})$. Different fibers were simulated with τ_{max} ranging between 0.45 and 4.95 fs.

The simulated data were analyzed using three methods: WSEC, interferometry, and JME. The set of simulated data was augmented for WSEC and interferometry by simulating results with the output polarizer P2 of Fig. 1 selecting 0-degree, 45-degree, and circular polarizations. JME and WSEC yielded identical values of $\langle \tau \rangle$ when a mode-coupling factor k=0.814 was used. This mode coupling factor is essentially the same as the k=0.824 arrived at in [2]. In order to investigate dependence on the spectral width of the source, each fiber was first analyzed at 2048 frequencies using a spectral window of width W_0 encompassing the full 40-THz simulation range. The data were then analyzed in two groups of 1024 frequencies each, using a window of width $W_0/2$. Similarly, the data were subdivided and analyzed using windows of widths $W_0/4$, $W_0/8$, $W_0/16$ and $W_0/32$.

As the frequency range is subdivided to simulate optical source spectra of successively smaller bandwidth, both JME and WSEC lead to the same mean DGD no matter how many times the spectrum is subdivided, as long as all subdivisions are included in the mean. This source independence is a direct result of each measurement's specification. The measured power spectrum through the output polarizer is normalized by the source spectrum to obtain $T(\omega)$ for WSEC [2]. Similarly, matrices for JME are measured one at a time, each at a separate wavelength, in a manner independent of the shape of the source spectrum [1]. Measurement of mean DGD, whether by WSEC or JME, is therefore based strictly upon the properties of the fiber to be tested, independently of the optical source characteristics.

Interferometric second moments were obtained directly by applying the formula $\sigma_{\varepsilon}^2 = \int \mathcal{E}(t) t^2 dt / \int \mathcal{E}(t) dt$, integrating only over ranges of t for which $\mathcal{E}(t)$ remained above 10⁻³ times its peak value to minimize the (already small) effects of round-off noise. In the absence of PMD, the optical source spectrum itself results in a characteristic interferometric envelope of variance σ_S^2 . As the simulated spectrum was subdivided, a second-moment source width σ_S was found for each spectral window, and the source width was subtracted from each measurement by calculating $\tilde{\sigma}_{\varepsilon} = (\sigma_{\varepsilon}^2 - \sigma_S^2)^{1/2}$. For each of over 10⁵ simulated fibers, the mean DGD $\langle \tau \rangle$ and

source-subtracted interferometric PMD $\tilde{\sigma}_{E}$ were calculated. A scatterplot was then constructed of the correction factor $\langle \tau \rangle / \tilde{\sigma}_{E}$ versus the dimensionless product $\tilde{\sigma}_{E} \Delta \omega_{FWHM}$, where $\Delta \omega_{FWHM}$ is the full-width at half-maximum of the source spectrum in rad/s. A running average of the scatter was found by calculating the mean value of $\langle \tau \rangle / \tilde{\sigma}_{E}$ for each interval $2^{m/2} < \tilde{\sigma}_{E} \Delta \omega_{FWHM} < 2^{(m+1)/2}$, where *m* is an integer index. Traces connecting the means of these intervals are shown in Fig. 3, with different traces indicating the results of optical source spectra of slightly different shape. At large $\tilde{\sigma}_{E} \Delta \omega_{FWHM}$ these traces confirm (8), while at smaller values the correction factor is not constant, but must be specified as a function of $\tilde{\sigma}_{E} \Delta \omega_{FWHM}$. Moreover, this function depends upon the shape of the optical source spectrum. As $\langle \tau \rangle$ is independent of the source spectrum, the dependence of $\langle \tau \rangle / \tilde{\sigma}_{E}$ on the source spectrum indicates a dependence of $\tilde{\sigma}_{E}$ on the source spectrum. The second-moment measurement of PMD is dependent upon both the device to be measured and the source used for measurement.

In conclusion, the PMD measured by interferometry is not related by a simple constant to the mean DGD. In the limit of large PMD or large source bandwidth, the two measured results are related by the ratio given by (8). This ratio is predicted theoretically and confirmed by simulation. A second-moment definition of PMD results in measurements which depend upon the spectral shape and bandwidth of the optical source, especially for measurement of PMD less than approximately 0.3 ps. A mean DGD definition of PMD results in measurements which depend only upon the device to be measured, independent of the optical source.

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