Evaluation of Network Topologies

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Abstract. The most fundamental question when designing any network is how to connect the network elements together. The network topology defines such crucial parameters as latency, throughput, ease of routing, and fault tolerance. In this paper, we present our evaluation of various network topologies, including stars, rings, n-dimensional meshes, hex meshes, and chordal rings. We derive upper limits on the performance of each; this analytical model is general enough to apply to many different types of networks, including Fibre Channel, ATM, and even Ethernet networks. We then compare the different topologies and discuss where each particular topology is appropriate.

Key Words. Networks, topology, graph theory, Fibre Channel, performance analysis, analytic modelling.

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1 Introduction

The first consideration in cascading Fibre Channel switches [AC92, GR92] to form fabrics is the topology to use. This note considers both a theoretical upper bound on fabric performance over the range of topologies and a practical evaluation of most current popular interconnect topologies.

We define the following fixed parameters:

- N is the total number of nodes in the network. A node may be a computer, a disk, a set of disks, or any other device.
- c is the number of ports on each switch. For simplicity, we assume the switches are all the same. Each port can connect either to a node or to some other port on another switch, but not both.

Our goal is to build a network with N nodes out of switches with c ports. We shall assume uniform traffic from each node, and we shall assume each node requires one port. We determine a lower bound on the number of switches required to construct such a network without reducing the bandwidth available from each node to every other node.

For the purpose of analysis, we also define the following variables:

- a is the number of ports on each switch connected to nodes.
- b is the number of ports on each switch connected to other switch ports through trunk links.
- p is the maximum over the minimum path length from one node to any other node.
- g is the amount of traffic generated by each node as a fraction of the port bandwidth. This might be related to the maximum throughput a particular interface element can support, or it might be derived from the traffic that an application generates.
- S is the number of switches in the network.

If there is some amount of traffic that is known to be local to a switch over and above that generated by a uniform random traffic distribution, that amount can be trivially subtracted from g.

2 Moore Graphs

Our first analysis considered only the average throughput and bandwidth requirements of ports and trunk links. This is equivalent to ignoring transient link contention. It is also equivalent to assuming the nodes have infinite buffering to smooth out the transient port contention. We shall later use simulation to investigate the performance in the presence of link contention and finite buffering.

For the theoretical investigations, we assume each switch has the same number of trunk ports and the same number of terminal ports. It is not clear whether this assumption is valid for optimal topologies, but in our practical investigation of common networks we have found few networks that do better than this theoretical upper bound on performance. Therefore, we shall retain this assumption; it simplifies things greatly.

In general, the topology is defined by p and b, and we will be investigating the maximum attainable performance of the best possible topology for given values of p and b. From p, b, and g we can calculate S, a, c, and N.

The number of switches is N/a; since N is fixed, we would like to maximize a to minimize the number of switches. Our approach is to calculate the best possible relationship among these variables for any topology. Since we have three parameters (N, c, and g), we require four additional equations. The first is trivially

$$N = Sa \tag{1}$$

We also have a trivial relationship on the number of ports:

$$a + b = c \tag{2}$$

Both of these equations are valid for arbitrary topologies.

Now let us imagine what the best possible topology looks like. Such a topology will have the maximum attainable throughput, minimum average path length, and maximum number of nodes. To maximize the number of nodes for a given path length, we want to eliminate redundant paths of short length; if there are multiple shortest paths to a single destination, then the number of nodes reachable in that distance is less than it could be. If we assume the best possible topology, then

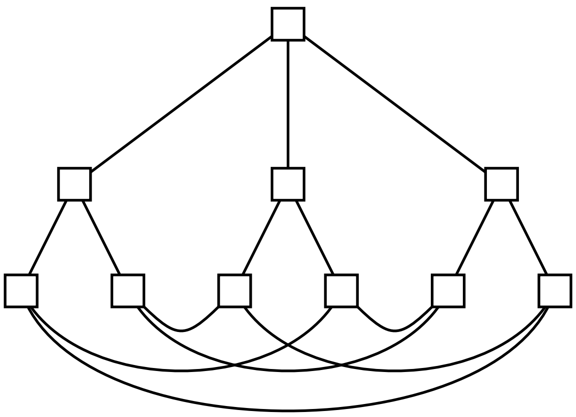


Figure 1: A completely populated spanning tree, with $b = 3$ and $p = 2$, with additional links connecting the leaves.

the single-source minimum-distance spanning tree has a branching factor of b at the root and $b - 1$

at every other node, as illustrated in figure 1. If the tree is fully populated, the number of nodes is

$$S = \left(1 + b \sum_{0 \le i < p} (b-1)^i\right)$$

The geometric sum term is simplified to

$$\frac{(b-1)^p - 1}{b-2}$$

for $b > 2$, which simplifies our equation to

$$S = \frac{b(b-1)^p - 2}{b-2}. \tag{3}$$

This is called the Moore limit [HR90, HS93]. This equation is only valid for a hypothetical 'optimal' topology that may or may not exist. Graphs that satisfy the Moore limits are called Moore graphs. We also refer to them as the perfect or optimal topology.

Moore graphs have a number of interesting properties. The smallest cycle is of length $2p + 1$, meaning that there are no redundant minimal paths between any pair of nodes. This minimal cycle information is useful in two ways.

First, a long minimal cycle has negative implications for fault tolerance. If a link connecting two nodes breaks, then the shortest path between those nodes is of length $2p$. In practice, however, p should normally stay small enough where the effect is not that great.

Secondly, knowing what the minimal cycle is helps when constructing Moore graphs (or other graphs with long minimal cycles). For instance, no mesh can be a Moore graph with $p > 1$, since any mesh has many cycles of length four. Most other graph construction techniques yield graphs with specific cycle lengths; these cycle lengths can provide insight into the performance of the graph.

For $b = 1$, the largest network has only two switches; we shall only concern ourselves here with larger networks. For $b = 2$, the network is simply a line or ring network. If $p = 1$, then the network is a complete graph (all nodes connected to all other nodes).

Now let us consider bandwidth requirements. The bandwidth for link traffic on a switch must be the average path length times the bandwidth for a single switch element, and thus the number of link ports and node ports must satisfy this relation:

$$b = g\bar{p}a \tag{4}$$

This equation is valid for arbitrary topologies.

The above equation underscores the importance of the average path length. It is obvious that the average path length is closely tied to the average packet latency; this equation shows that the average path length is also directly related to the attainable throughput. In particular, as the average path length increases, so does the number of ports per switch that need to be assigned to trunk links rather than terminal nodes, so the cost for switches per terminal node rises with the average path length.

When $b > 2$, then most nodes are at distance p. In general, the sum of path lengths from the root to any other node is

$$b \sum_{0 \le i < p} (i+1)(b-1)^i$$

We can rewrite this as

$$pb \sum_{0 \le i < p} (b-1)^i - b \sum_{0 \le i < p} (p-i-1)(b-1)^i$$

The first term is a geometric sum we can solve directly. The second is a triangular sum that we can rewrite as a double sum:

$$pb \left(\frac{(b-1)^p - 1}{b-2} \right) - b \sum_{0 \le i < p-1} \sum_{0 \le j \le i} (b-1)^j$$

This simplifies to

$$pb \left(\frac{(b-1)^p - 1}{b-2} \right) - b \sum_{0 \le i < p-1} \frac{(b-1)^{i+1} - 1}{b-2}$$

which can be written as

$$pb \left(\frac{(b-1)^p - 1}{b-2} \right) - \frac{b}{b-2} \left(1 - p + \sum_{0 \le i < p-1} (b-1)^{i+1} \right)$$

and further simplified to

$$pb \left(\frac{(b-1)^p - 1}{b-2} \right) + \frac{b(p-1)}{b-2} - \frac{b}{b-2} \frac{(b-1)\left((b-1)^{p-1} - 1\right)}{b-2}$$

or

$$\frac{b}{b-2} \left((b-1)^p (p - \frac{1}{b-2}) + \frac{1}{b-2} \right)$$

The average path length is this value divided by the number of switches, which yields

$$\bar{p} = \frac{b(b-1)^p (p - \frac{1}{b-2}) + \frac{b}{b-2}}{(b(b-1)^p - 2)} \tag{5}$$

This yields our fourth equation relating the original variables:

$$a = g \frac{(b(b-1)^p - 2)}{(b-1)^p (p - \frac{1}{b-2}) + \frac{1}{b-2}} \tag{6}$$

The variables b and p determine the 'topology'. From this we can calculate how many nodes a particular switch in that topology can support, and from this, the total number of ports required on each switch and the total number of nodes in the switch.

We can characterize the cost efficiency of a particular topology as the percentage of total ports that are used for nodes; this is simply a/c. Let us consider some special cases. When $p = 0$, then we can only have a single switch, so $a = c = N$ is arbitrary and the switch efficiency is 100%.

When $p = 1$, then a perfect topology exists; it is simply a complete graph of $b + 1$ switches, with $a = (b + 1)/g$ and thus $N = (b + 1)^2/g$. The cost efficiency is about $1/(g + 1)$ as b increases. In general, our analysis only holds if $b > 2$, but for this case it also holds for $b = 2$ (a simple triangle) and $b = 1$ (two switches connected point to point.)

When $b = 2$, then a perfect topology exists; it is simply a ring of S switches. (More precisely, it is only a Moore graph when S is odd, since any ring with an even number of nodes has a redundant minimal path across the diameter.) In this case, $a = 8S/(g(S^2 - 1))$ which is asymptotically $8/(gS)$, and the cost efficiency is about $1/(1 + gS/4)$. Except for these two cases ($p = 1$ and $b = 2$), there is

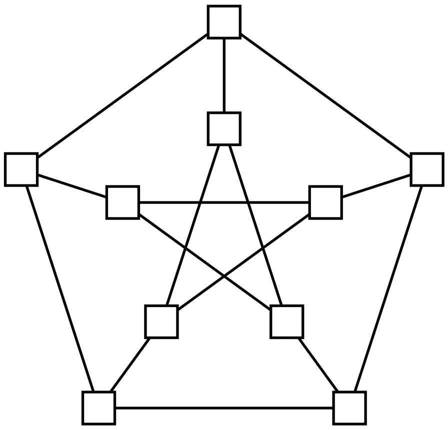


Figure 2: The Petersen graph, with ten nodes, $b = 3$ and $p = 2$. This graph is isomorphic to that in the previous figure.

only two or three remaining Moore topologies. One is the Petersen graph, shown in figure 2, with 10 switches, $p = 2$, and $b = 3$. Another is a graph with $p = 2$ and $b = 7$; the number of switches is 50. Finally, there may be a Moore graph with $p = 2$ and $b = 57$, with 3250 switches. No other Moore graphs exist.

The equations derived above are useful despite the fact that the Moore graphs do not exist for most values of b and p. They provide an upper bound on the performance of a network given a particular c, S and N, or a lower bound on the number of c-port switches needed to support a traffic of g on a network of N terminal nodes.

3 Spanning Tree Analysis

Moore graphs get very large quickly as b and p increase. Between the discrete solutions to the equations given above are regions where we can approximate graphs of the same N, b, and a values with fractional p values. By simply rewriting equation (3), we find that

$$p = \log\left(\frac{2 + (b-2) * S}{b}\right) / \log(b-1) \tag{7}$$

We can then substitute this p directly into equation (4) to obtain the value $\bar{p}_{Moore}$.

Unfortunately, the continuous region between the discrete solutions is not a good approximation of attainable performance. In this section we discuss a spanning tree analysis which is more accurate than using the continuous Moore equations, and that provides a larger number of optimal topologies. The Moore graph assumes that we can build a fully-populated spanning tree from any node, and

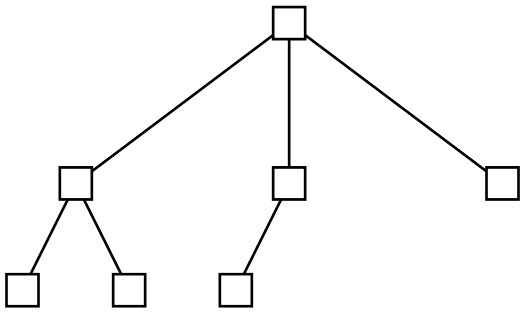


Figure 3: An incompletely populated spanning tree. Each level is fully populated except for the last.

calculates the average path length (and other parameters) from this assumption. The idea behind the spanning tree analysis is to build the optimal partial spanning tree from a given node, but without fully populating the lowest level of the tree. Figure 3 gives a sample tree. Then, we calculate the average path length and other parameters from this spanning tree. The calculation is similar to the previous calculation. Specifically, for a given S and b, we calculate p through equation (7). The floor of this value is the number of levels of the tree that are fully populated, and their average path length can be calculated by equation (5). The remaining nodes are at distance $\lfloor p \rfloor + 1$, and we can calculate how many there are by subtracting from S the Moore limit for b, $\lfloor p \rfloor$. The final value is $\bar{p}_s$, the spanning tree average path length. No graph with branching factor b and S nodes can have a shorter average path length.

This gives us a tighter lower bound on the average path length, as shown by figure 4. This graph shows the ratio of the spanning tree average path length to the Moore average path length against the number of switches for a given branching factor. The spanning tree analysis fully coincides with the Moore equations at those points in which the spanning tree is fully populated. The graph shows that the difference in path length for the spanning tree analysis and the Moore analysis can differ significantly.

The spanning tree analysis allows us to derive, for a given number of nodes S and branch factor b, the values $\bar{p}_s$ and thence, with a given c, the maximum attainable g (using equation (4)). In our analysis of specific topologies in the next section, we will compare the actual average path length and attainable throughput against these spanning tree numbers to determine how 'good' a particular topology is.

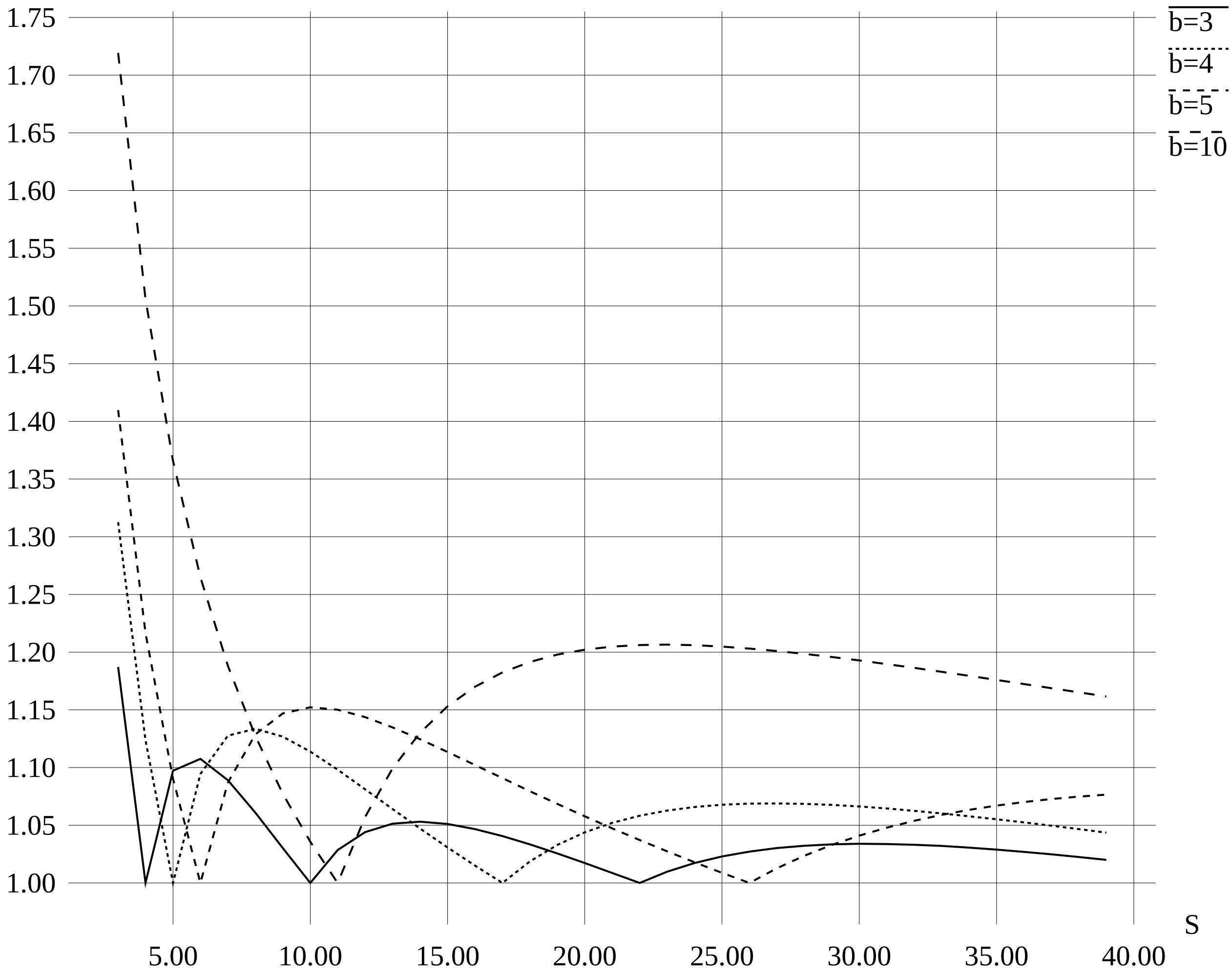


Figure 4: The vertical axis shows the ratio of the spanning tree average path length to the Moore average path length; the horizontal axis is the number of switches, and separate lines are given for branching factors of 3, 4, and 10.

The minimal cycle length of a spanning-tree optimal graph is $2p + 1$, where p is the lowest fully-populated level. There can be no redundant paths between the root node and any node at this level or higher in the tree, just as with Moore graphs. For a given minimal cycle size, however, spanning-tree optimal graphs can be larger, since the last level is potentially larger (for $b \geq 3$) than the previous levels combined.

In order for a graph to be spanning-tree optimal, it must also be link-balanced. This means that the traffic on each trunk link must be identical, given the simple routing procedure given below. In this report, we do not consider explicit traffic balancing. We shall later investigate traffic balancing and how it can improve the performance of unbalanced network; normally, our simple routing scheme that we use in this report yields a fairly well-balanced traffic distribution. Certainly, any graph for which the edges are isomorphic will be balanced by symmetry, and this is true for a large percentage of the graphs we consider here.

We have been able to find a number of graphs that are spanning tree optimal. All Moore graphs mentioned in the previous section are spanning tree optimal; these include the rings, complete graphs, and Petersen graph. In addition, the chordal ring generated by the parameters $(14, 5)$ is optimal, as is the E-3 network and the 13-mesh network; each of these will be described below. The Tutte-Coxeter graph with node-degree 3 and 30 nodes is spanning-tree optimal. Another graph with node-degree 4 and 20 nodes is spanning-tree optimal; we have been unable to find this graph in any references. The complement of the product of any two complete graphs with more than three nodes is spanning-tree optimal, as is the complement of the Petersen graph. The Cartesian product of any two equivalent complete graphs is a spanning-tree optimal topology. In addition, any spanning-tree optimal topology with a maximum path length of two describes a family of spanning-tree optimal topologies when extended by the partite operation, which is described in the section on graph algebras.

We have performed an exhaustive search for spanning-tree optimal topologies for graphs with up to twelve nodes, and no other spanning-tree optimal topologies exist other than the ones mentioned above. However, we do not believe that the above set is exhaustive in general.

We have also been able to find some graphs that would be spanning-tree optimal, except that they are not edge-isomorphic, and also not link-balanced. One example is the product of two non-equivalent complete graphs. For some examples of these graphs, it is possible that careful routing might balance the link traffic in such a way that they become link-balanced; this is not true of the above example.

Finally, there are some values of S and b for which no possible spanning-tree optimal graphs exist. Since the number of nodes in a graph with an even degree is always odd, if both S and b are odd, no such graph can exist.

Table 3 summarizes the known spanning-tree optimal graphs.

4 Graph Algebras

Many simple networks such as the ring, star, and complete graphs exist. However, each of these networks has limits either on the size to which they can be extended, or the performance which can be maintained as they are made larger. In this section, we present three algebraic operations on graphs that produce new graphs that often perform well. In particular, we will show how to use these operations to generate new families of graphs that are spanning-tree optimal.

The first operation is a generalization of the k-dimensional mesh, the second operation is a generalization of the star network, and the third operation is the graph complement operation.

	3	4	5					10					15					20					25					30
2	C	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R
3		C	i	A	i	$\frac{11}{12}$	i	P	i	$\frac{21}{22}$	i	H	i	$\frac{11}{12}$	i	$\frac{39}{43}$	i	$\frac{15}{17}$	i	m	i	$\frac{59}{60}$	i	$\frac{67}{69}$	i	U	i	T
4			C	A	$\frac{12}{13}$	A	X	A	$\frac{6}{7}$	$\frac{9}{10}$	M	$\frac{11}{14}$	$\frac{6}{7}$	e	m	e	$\frac{51}{52}$	U	$\frac{20}{21}$	$\frac{43}{46}$								
5				C	i	$\frac{18}{19}$	i	A	i	$\frac{51}{55}$	i	$\frac{9}{10}$	i		i		i		i		i		i	m	i		i	
6					C	A	A	Q	$\frac{35}{37}$	A	U	$\frac{40}{41}$	A	X			E	A										
7						C	i	$\frac{33}{34}$	i	$\frac{900}{931}$	i	A	i		i		i		i		i		i		i		i	
8							C	A	$\frac{42}{43}$	A	$\frac{30}{31}$		N	A		A		A					X	A				
9								C	i	A	i	$\frac{476}{499}$	i	N	i	A	i		i		i		i		i		i	A
10									C	A	$\frac{63}{64}$	$\frac{576}{593}$	A			N		A					A					
11										C	i	$\frac{75}{76}$	i		i		i		i	A	i		i		i		i	
12											C	A	A	A		A		A				A		A	A			A
13												C	i	$\frac{102}{103}$	i		i		i		i		i	A	i		i	
14													C	A	$\frac{117}{118}$				A			N				A		
15														C	i	A	i	A	i		i	N	i		i		i	A

Table 1: A table of the known spanning-tree optimal topologies. The horizontal axis is the number of switches; the vertical axis is the branching factor. Key: i—impossible because of odd degree/odd size; m—impossible because of impossible Moore graph; e—impossible through exhaustive search; C—complete; R—ring; H—chordal ring; P—Petersen; E—E-3; X—product; A—artite of other spanning-tree optimal graph; Q—complement of Petersen; N—complement of product of complete graphs; M—M13; T—the (3,8)-cage (Tutte-Coxeter); U—found but structure unclassified; space—unknown; fraction: best possible throughput graph?.

4.1 Cartesian Product

The first operation is the Cartesian product operation. This operation maps pairs of rings into two-dimensional meshes, and a ring and a mesh into a three-dimensional mesh, etc. Given two graphs G_1 and G_2, $G_1 \times G_2$ generates a new graph G such that:

- For each node n_i in G_1 and node n_j in G_2, G has a node (n_i, n_j).
- For each edge $n_i \leftrightarrow n_j$ in G_1 and each node n_k in G_2, G has an edge $(n_i, n_k) \leftrightarrow (n_j, n_k)$.
- For each edge $n_i \leftrightarrow n_j$ in G_2 and each node n_k in G_1, G has an edge $(n_i, n_k) \leftrightarrow (n_j, n_k)$.

The resultant graph can be characterized in terms of the input graphs. The total number of nodes is the product of the numbers of nodes in the original graphs. If both input graphs are regular, so is the output graph; the degree is the sum of the degrees of the original graphs. The average path length of the resultant graph is the sum of the average path lengths of the input graphs. The maximum path length of the resultant graph is the sum of the maximum path lengths of the input graphs.

Because any such resultant graph has cycles of length four, such graphs can only be spanning-tree optimal up to a p of one. In such a spanning-tree optimal graph, the maximum path length is two. Thus, in order to generate a spanning-tree optimal graph, the two input graphs must have a maximum path length of one; only the complete graphs satisfy this criteria.

The resultant graph G is link-balanced if and only if the two input graphs are equal and link-balanced. Thus, we have identified a new class of spanning-tree optimal graphs—the product of two complete graphs. We write this ${K_n}^2$, where K_n is the complete graph with n nodes. Of course, ${K_3}^2$ is a three by three wrapped mesh; the three-node ring and three-node complete graph are identical.

Despite the fact that product graphs are only spanning-tree optimal if the input graphs are equal complete graphs, product graphs often generate networks with reasonably good performance. Our analysis of discrete topologies will support this.

4.2 Partite Operation

The second operation is the partite operation. This operation is called 'partite' because it was inspired by bipartite graphs such as star networks and Clos networks.

The partite operation takes as input a graph G_1 and a vector of positive integers v, where the number of nodes in G_1 equals the length of the vector v. The graph is constructed by replicating node n_i from G_1, and all of its edges, v_i times. Construction proceeds as follows.

- For each node n_i in G_1 and value $1 \leq j \leq v_i$ in G_2, G has a node (n_i, j).
- For each edge $n_i \leftrightarrow n_j$ in G_1, and for each value $1 \leq k \leq v_i$ and $1 \leq m \leq v_j$, G has an edge $(n_i, k) \leftrightarrow (n_j, m)$.

Consider, for instance, using a simple two-node graph with one edge linking the nodes for G_1. If v is $\{1, c\}$, then G is a star network with a single hub and c satellite nodes. If v is $\{2, c\}$, then G is a star network with two hubs, no connection between hubs and each satellite connected to both hubs. In general, $v = \{a, b\}$ applied to G_1 generates a bipartite graph with a nodes on one side and b nodes on the other—known in graph theory as a $K_{a,b}$ graph.

If we apply the partite operation to a graph with n nodes, the resultant graph has n classes of nodes, with connections between the classes but no connections between members of each class.

The partite operation has some useful properties. If the input graph is link-balanced and v is uniform (all elements have the same value), then the resultant graph is also link-balanced. If the maximum path length of G_1 is greater than one, for any v, the resultant graph has a maximum path length equal to the maximum path length of G_1. If there are not duplicate edges in G_1, G will also not have duplicate edges. Thus, if G_1 is spanning-tree optimal with a maximum path length of one or two, and v is uniform, then G will be spanning-tree optimal with a maximum path length of two. This allows us to take a large number of spanning-tree optimal graphs, including all of the complete graphs, rings up to size five, the Petersen graph, the 13-node wrapped mesh, and the E-3 hexagonal mesh, and from each generate a full family of spanning-tree optimal graphs. In general, if the input graph has n nodes and each node has a degree of b, and v is uniformally k, then the resultant graph will have kn nodes and each node will have a degree of kb.

When a non-uniform v is used in the partite operation, the resultant graphs are generally irregular, but they often perform quite nicely. We shall investigate the performance of a number of such graphs when we consider specific topologies.

4.3 Complement Operation

The graph complement operation builds a graph that has the same number of nodes as the original graph, but a different set of edges. This operation is less obviously useful than the previous two, but it generates some spanning-tree optimal graphs so we include it here.

Given a graph G_1, the negate operation generates a graph G such that

- The set of nodes is the same.
- For each pair of nodes n_i, n_j such that G_1 does not contain an edge $n_i \leftrightarrow n_j$, G includes an edge $n_i \leftrightarrow n_j$. G has no other edges.

Complement graphs may be unconnected; for instance, the complement of any complete graph has no edges. In addition, complement graphs may have an extremely high number of edges, and vertices with a large degree. Thus, given a fixed number of ports on a switch, a complement graph may not be implementable.

Nonetheless, certain complemented graphs are interesting. The complement of the Petersen graph is a spanning-tree optimal graph with ten nodes and a vertex degree of six. In addition, the complement of the product of two complete graphs of size three or greater is also a spanning-tree optimal graph.

4.4 Recursively Building Networks

Another algebraic technique for constructing larger networks is to construct a network with N terminal nodes out of S switches with c ports each, and then use this resulting network as a primitive switch with N ports, replicating it and connecting them in the fashion of larger switches. Unfortunately, doing this breaks the implicit balance between terminal ports that we are assuming; those ports used for trunk links will often get higher traffic than those ports used for terminal nodes.

Nonetheless, we considered this style of network construction as well. The resulting networks were almost always worse than networks constructed out of the fundamental c-port switches themselves. The exception to this case was when very large numbers of switches with very low number of ports were connected together; in this case, the primitive network types and network operations were too restricted by the low number of ports to easily build large networks. Because of the lower performance of these recursively built networks, and because they violate our balanced traffic assumption, we shall not include these topologies here.

5 Analysis of Specific Topologies

Over the past few decades, many hundreds of papers have been published on various specific interconnect topologies [AK89, ODH94, SS88, Sen89]. Some work has attempted a broad characterization and comparison of their performance[AJ75]. In this section of this report, we evaluate and compare specific topologies for networks of up to twenty-seven switches. We consider ring networks, star networks, complete graphs, two and three dimensional mesh networks, chordal ring networks, and a few special-purpose graphs such as Petersen, the 13-node mesh, and the E3 hexagonal mesh. We allow trunk links to have arbitrary multiplicity, and we lift the requirement that the nodes be symmetrical. We consider the use of switches with four, eight, or sixteen ports.

Any solution that contains multiple links between the same pair of nodes, but is not a complete graph, can almost certainly be improved by moving one of the links to a different node. The redundant paths implied by the trunk multiplicity increase the average path length and thus the number of required trunks, and therefore the overall throughput is decreased. We still include these cases for completeness.

5.1 Analyzing a particular topology

The analysis of a topology proceeds as follows. We shall describe the abstract view of the evaluation. The actual calculations are performed more efficiently than described here, although the results are precisely the same. The inputs are the type of topology to use (one of ring, star, complete, 2d, 3d, chordal ring, or special), the number of switches, the number of ports per switch, and the average number of trunk links per switch. From this information, the appropriate network is constructed and the number of terminal ports is calculated.

Next, an adjacency matrix indexed by terminal ports is constructed. Floyd's algorithm is run over the matrix to calculate the shortest paths between all pairs of terminal ports. The average path length is then calculated as the sum of all of the entries of the matrix divided by the size of the matrix.

Finally, we calculate g. Because the input network might not be fully symmetrical, we must determine what ports are the bottleneck in the performance of the network and use them to determine g. To perform this calculation, we must make some assumptions about the routing strategy used by the fabric when there are multiple minimal-length paths.

A poor approximation would have been to give each path equal weight. This would penalize sets of paths with a high degree of interconnectivity, because the total number of paths is exponential in the number of links in such a case.

Instead, we pretend that each switch is a perfectly naive router, choosing at each step each port on some minimal path to the destination with equal probability. Since this procedure depends on the direction of the path (from source to destination, or from destination to source), and since the switches are probably slightly more intelligent, we average the results from each direction. While in some extremely unsymmetrical cases this might yield poor results, for the topologies we considered, this procedure spread traffic across the paths uniformly.

We assumed each node was talking to each other node at full bandwidth, and using the algorithm described above, calculated the portion of traffic that each link would be used for. We summed the traffic for all pairs of terminal nodes. We then located the port that had the highest traffic rate; this value was the amount we had to scale traffic down by to use all links at 100% or less. Thus, g was simply the reciprocal of this number.

Thus, given a particular network graph described by the number of switches, number of ports per switch, average number of trunk links per switch, and topology, we calculated the number of terminal ports, average path length, and maximum traffic supported.

Now we will describe the various networks topologies we considered, giving the additional detail required for each.

5.2 Ring topologies

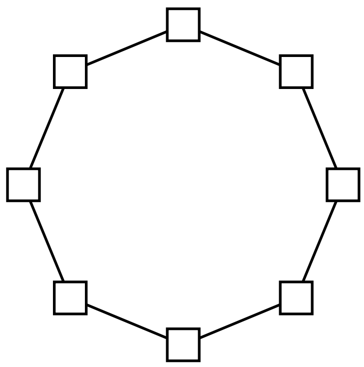


Figure 5: A ring network.

The first type of network is the ring network, as shown in figure 5. In this case, the number of trunk links per switch must be a multiple of two, since there are symmetrical connections to the neighboring switches. The ring network is a moore graph.

5.3 Star topologies

The next type of network is the star network, as illustrated by figure 6. In this case, there is a central switch with connections to satellite switches; the satellite switches are only connected to the central switch. The average number of trunk links is approximately twice the number of trunk links connecting a satellite to a switch. We fully populated the central switch as well as the satellite switches with terminal nodes. We did not consider hierarchical star networks with a primary and

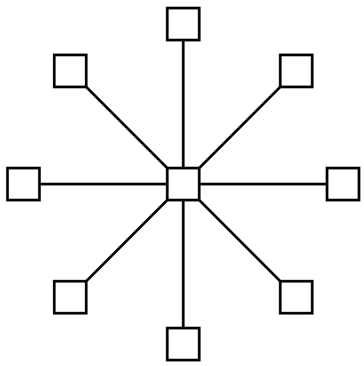


Figure 6: A star network.

secondary level of satellite switches. The maximum number of switches is equal to one more than the maximum number of ports.

5.4 Complete topologies

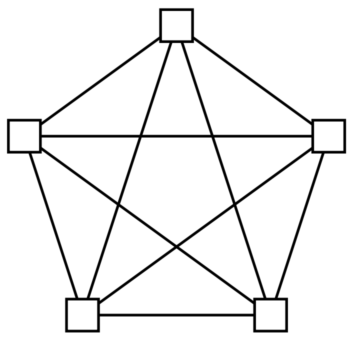


Figure 7: A complete network.

The next type of network is the complete graph, with each switch connected to each other switch. An example is shown in figure 7. The maximum number of switches is equal to the maximum number of ports; this leaves a single port for a terminal node per switch.

5.5 Ring-product topologies

The next two types of networks are the k-dimensional networks generated by products of rings. A two-dimensional example is shown in figure 8. In general, the number of switches must not be prime. We considered just products of rings separately because they are such common network topologies; in a later section we considered general product graphs.

Twisting the wrap connections can decrease the average path length and increase the throughput very slightly in some cases; we did not consider these special topologies since the difference was so slight.

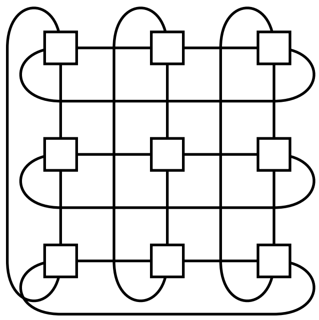


Figure 8: A mesh network.

If one of the factors was two, then the wrap-around connections are isomorphic to the unwrapped connection, so the number of trunk links in that direction per switch could be odd. In all other cases, because of symmetry, the number of trunk links had to be even.

5.6 Chordal ring topologies

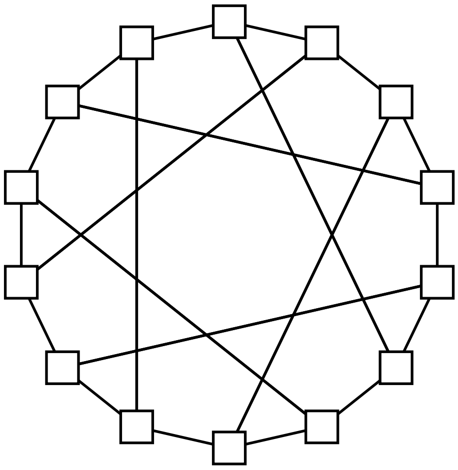


Figure 9: A chordal ring network. This example has fourteen nodes and a chord length of five.

The next type of graph is the chordal ring, an example of which is shown in figure 9. This is a special ring graph where connections that jumped over a certain number of neighboring nodes were also included. A chordal ring must have an even number of switches, and the chordal length is always odd. Topologically, a chordal ring is very similar to a two-dimensional mesh. Using the results of a 1981 paper by Arden and Lee, we only considered the optimal chord length. We performed our own analysis and found that the chords required less bandwidth than the direct neighbor connections; therefore, we allowed three or five or more trunk links per node, always insuring that the number of trunks connecting neighbors was more than or equal to one third of the trunk links, while the chord connections got the remaining links.

5.7 Special topologies

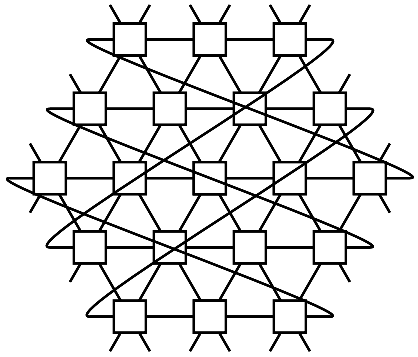


Figure 10: The E3 network. Only one set of wrap links are shown for clarity.

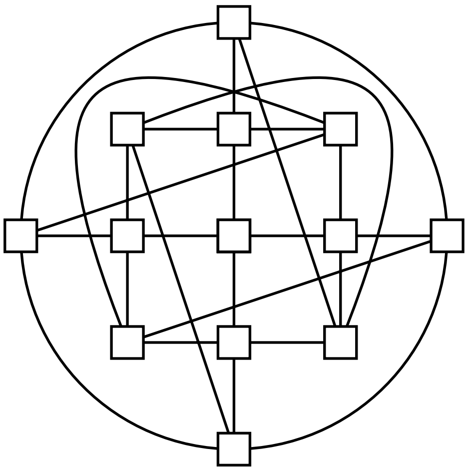


Figure 11: The mesh-13 network.

We also considered some specific networks of particular interest that were not covered by the previous network types. One such network is the Petersen graph, which is a ten-node Moore graph with connections from each switch to three neighbors. The second is the E3 hexagonal interconnect with nineteen nodes and connections from each switch to six neighbors. The E3 network is pictured in figure 10. The third is a 13-node spanning-tree optimal mesh network, with the wrap connections twisted in a specific way. That network is pictured in figure 11. We also considered a spanning-tree optimal graph with twenty nodes and vertex degree of four which we found through exhaustive search.

5.8 Algebraic topologies

All of the graphs we considered so far, except the k-dimensional product topologies for $k > 1$ and the star networks, we call *primitive* graphs. We omit the meshes and stars because those are generated from rings and complete graphs from our algebraic operations. We also considered networks that were generated by our algebraic operations of Cartesian product and artite. Because each operation generates a larger network, often much larger, we only considered product graphs of any number of the more primitive topologies. We also considered applying the artite operation to any of these resulting graphs with a uniform vector v. We considered the application of the artite operation to any primitive or product topology with a vector v that was uniform except for the first value. Finally, we considered the application of the complement operation at any point in the generation of these graphs. The results are summarized in chart and picture form in the last half of this report.

5.9 The bump operation

All of the topologies we have considered have a maximum of one edge between any pair of nodes. One method for increasing the throughput of a given network is to simply increase the number of trunk links between a given pair of nodes. The bump operation provides this capability.

The bump operation takes a specific network topology, locates the set of links that are utilized the heaviest, and increases their multiplicity by one. For each graph constructed so far, we applied the bump operation to yield a new graph which was also considered. We continued to apply the bump operation until the g value attained unity, or we ran out of ports on each switch.

6 Comparison of Topologies

The combinations of parameters we considered was huge:

- The number of switches varied from two to twenty-seven.
- The number of ports per switch was four, eight, or sixteen.
- The number of trunk links varied from one to the number of ports per switch.

In all, there were thousands of network graphs considered. To present the results from an analysis involving so many graphs, we defined a partial order on network graphs.

We defined a network graph G to be better than another network graph G' if $c = c'$, $S \leq S'$, $N \geq N'$, and $g \geq g'$, and at least one of the inequalities was strict. That is, if one graph had fewer switches, more nodes, and a better performance than another graph, then there would be no reason to even consider the second graph.

Because this is a partial order, there were still many graphs left, each one appropriate for a particular number of switches, number of nodes, and traffic rate. We collected and illustrated this information in a series of colored two- and three-dimensional charts.

Using these charts it is easy to determine the 'best' network that, for instance, supports 100 terminal nodes at a traffic rate of 30% with the smallest number of 16-port switches. In this case, a ten-switch Petersen network, with doubled trunk links, would be the best; it would support a maximum of 100 nodes at a traffic rate of 40% and an average path length of 1.5 hops. If only nine switches could be afforded but 100 terminal nodes were required, the best traffic rate would be 25%, supplied by a three by three wrapped mesh with single links in all four directions; the average path length would then be 1.33 hops. If we could only afford nine switches but were unwilling to compromise on the traffic rate, then the best network would be the same three by three network, but populated to only 90 terminal nodes.

If we needed to support 101 terminal nodes at 30% traffic, on the other hand, then we would require a minimum of twelve switches in a two-dimensional three by four mesh, with doubled trunk links in the longer dimension. If we could not afford twelve switches but needed to support 101 terminal nodes, the best we could do would be the nine switch three by three mesh; there are no better solutions with ten or eleven switches among the topologies we considered.

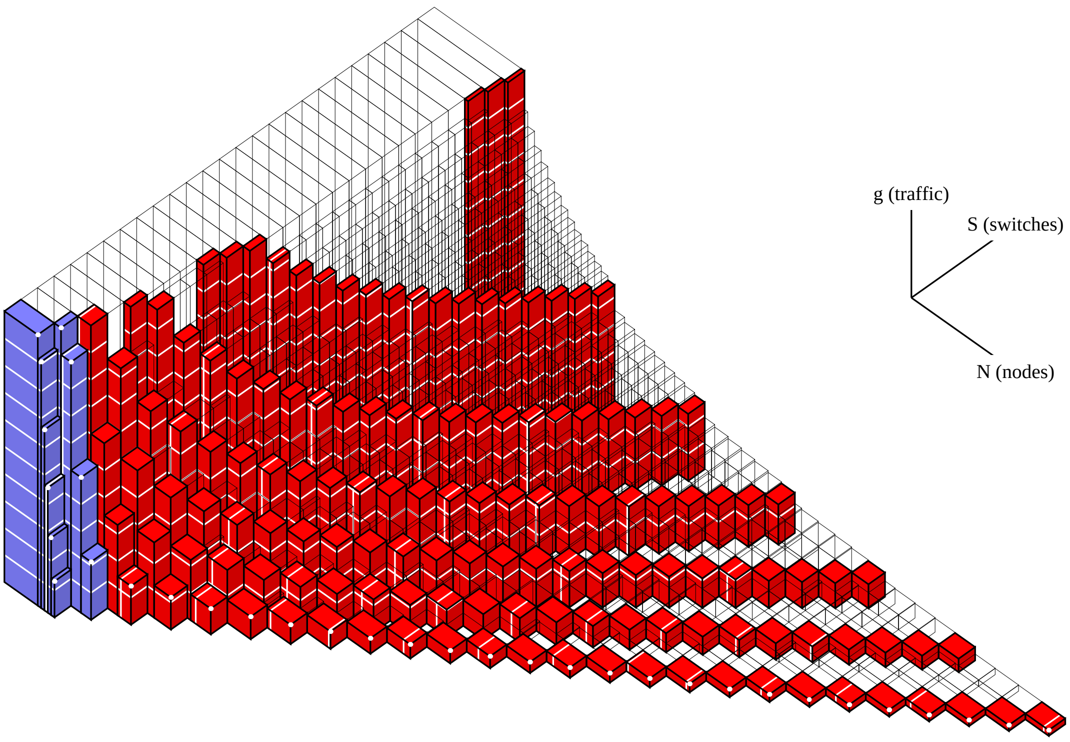
7 Charts for 16-Port Switches

This section provides the numerical results of our topological analysis, along with illustrating charts.

Each table shows, in order, the number of switches, the minimum and maximum number of terminal nodes supported by that topology for which no better solution exists, the average path length, the maximum supportable traffic, the ratio of the average path length to the spanning tree average path length, and the ratio of the supportable traffic to the spanning tree supportable traffic, and the topology used. The topology used is in prefix functional notation except for products, which are in infix notation. The letter 'r' means ring, the letter 'c' means complete graph, the letters 'ch' mean chordal, the letter 'b' means bump, the letter 'x' is infix product, the letter 'a' means partite, the letter 'p' means Petersen, the letter 'e' means E-3, and the letter 'm' means mesh-13.

The pictures are colored three-dimensional charts. The vertical dimension is the attainable throughput. The dimension to the right and out of the page is for increasing terminal nodes, and the dimension to the right and into the page is for increasing numbers of switches. Horizontal white lines cutting through the shapes are graduations of 10% traffic; vertical white lines are graduations of 25 terminal nodes. If a topology has a small white dot in the corner, that indicates that it is a spanning-tree optimal topology; if it has a large white dot in the corner, that indicates that it performs better than a spanning-tree optimal topology. Wire frame shapes indicate that for that region of the graph, the best known solution uses fewer switches.

7.1 Ring topologies



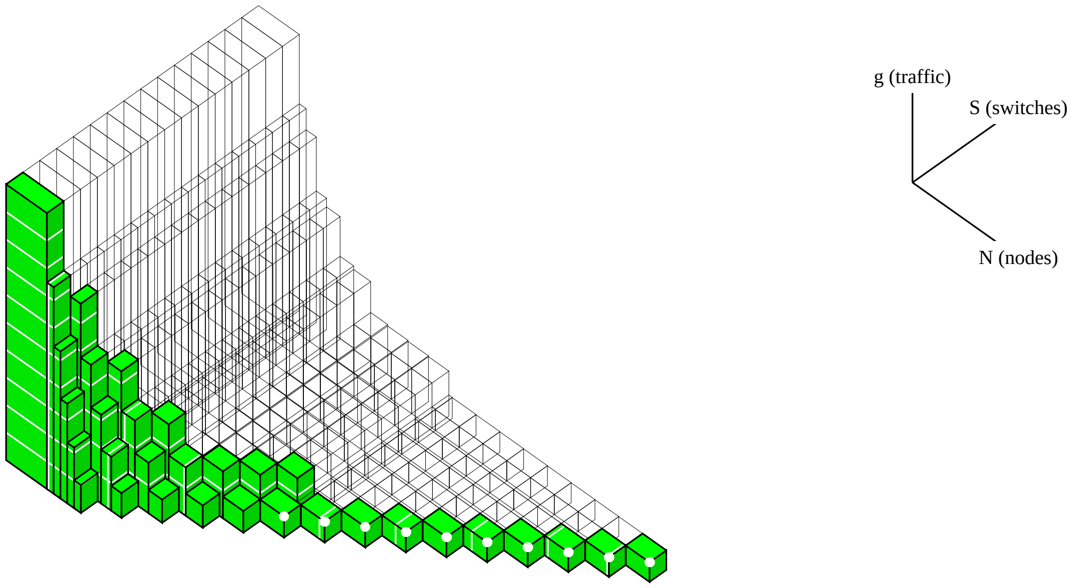
Ring topologies, switches with 16 ports.

S	min	max	$\bar{p}$	g	$\bar{p}/\bar{p}_s$	g/g_s	top
2	1	20	0.5	1	1	1	b5c2
2	21	22	0.5	0.909091	1	1	b4c2
2	23	24	0.5	0.666667	1	1	b3c2
2	25	26	0.5	0.461538	1	1	b2c2
2	27	28	0.5	0.285714	1	1	b1c2
2	29	30	0.5	0.133333	1	1	c2
3	21	24	0.666667	1	1	1	b3c3
3	25	30	0.666667	0.9	1	1	b2c3
3	31	36	0.666667	0.5	1	1	b1c3
3	37	42	0.666667	0.214286	1	1	c3
4	25	32	1	1	1.33333	0.75	b3r4
4	33	40	1	0.6	1.33333	0.75	b2r4
4	41	48	1	0.333333	1.33333	0.75	b1r4
4	49	56	1	0.142857	1	1	r4
5	33	40	1.2	0.833333	1.5	0.666667	b3r5
5	41	50	1.2	0.5	1.5	0.666667	b2r5
5	51	60	1.2	0.277778	1.5	0.666667	b1r5
5	61	70	1.2	0.119048	1	1	r5
6	33	36	1.5	1	1.8	0.555556	b4r6
6	41	48	1.5	0.666667	1.8	0.555556	b3r6
6	51	60	1.5	0.4	1.8	0.555556	b2r6
6	61	72	1.5	0.222222	1.5	0.666667	b1r6
6	73	84	1.5	0.0952381	1	1	r6
7	37	42	1.71429	0.972222	2	0.5	b4r7
7	49	56	1.71429	0.583333	2	0.5	b3r7
7	61	70	1.71429	0.35	2	0.5	b2r7
7	73	84	1.71429	0.194444	1.5	0.666667	b1r7
7	85	98	1.71429	0.0833333	1	1	r7

8	43	48	2	0.833333	2.28571	0.4375	b4r8
8	57	64	2	0.5	2.28571	0.4375	b3r8
8	71	80	2	0.3	2	0.5	b2r8
8	85	96	2	0.166667	1.6	0.625	b1r8
8	99	112	2	0.0714286	1	1	r8
9	49	54	2.22222	0.75	2.5	0.4	b4r9
9	65	72	2.22222	0.45	2.5	0.4	b3r9
9	81	90	2.22222	0.27	2	0.5	b2r9
9	97	108	2.22222	0.15	1.66667	0.6	b1r9
9	113	126	2.22222	0.0642857	1	1	r9
10	37	40	2.5	1	2.77778	0.36	b5r10
10	55	60	2.5	0.666667	2.77778	0.36	b4r10
10	73	80	2.5	0.4	2.5	0.4	b3r10
10	91	100	2.5	0.24	2.08333	0.48	b2r10
10	109	120	2.5	0.133333	1.78571	0.56	b1r10
10	127	140	2.5	0.0571429	1	1	r10
11	41	44	2.72727	1	3	0.333333	b5r11
11	61	66	2.72727	0.611111	3	0.333333	b4r11
11	81	88	2.72727	0.366667	2.5	0.4	b3r11
11	101	110	2.72727	0.22	2.14286	0.466667	b2r11
11	121	132	2.72727	0.122222	1.875	0.533333	b1r11
11	141	154	2.72727	0.052381	1	1	r11
12	45	48	3	1	3.27273	0.305556	b5r12
12	67	72	3	0.555556	3	0.333333	b4r12
12	89	96	3	0.333333	2.57143	0.388889	b3r12
12	111	120	3	0.2	2.25	0.444444	b2r12
12	133	144	3	0.111111	2	0.5	b1r12
12	155	168	3	0.047619	1	1	r12
13	49	52	3.23077	0.928571	3.5	0.285714	b5r13
13	73	78	3.23077	0.515873	3	0.333333	b4r13
13	97	104	3.23077	0.309524	2.625	0.380952	b3r13
13	121	130	3.23077	0.185714	2.33333	0.428571	b2r13
13	145	156	3.23077	0.103175	2.1	0.47619	b1r13
13	169	182	3.23077	0.0442177	1	1	r13
14	53	56	3.5	0.857143	3.5	0.285714	b5r14
14	79	84	3.5	0.47619	3.0625	0.326531	b4r14
14	105	112	3.5	0.285714	2.72222	0.367347	b3r14
14	131	140	3.5	0.171429	2.45	0.408163	b2r14
14	157	168	3.5	0.0952381	2.22727	0.44898	b1r14
14	183	196	3.5	0.0408163	1	1	r14
15	57	60	3.73333	0.803571	3.5	0.285714	b5r15
15	85	90	3.73333	0.446429	3.11111	0.321429	b4r15
15	113	120	3.73333	0.267857	2.8	0.357143	b3r15
15	141	150	3.73333	0.160714	2.54545	0.392857	b2r15
15	169	180	3.73333	0.0892857	2.33333	0.428571	b1r15
15	197	210	3.73333	0.0382653	1	1	r15
16	61	64	4	0.75	3.55556	0.28125	b5r16
16	91	96	4	0.416667	3.2	0.3125	b4r16
16	121	128	4	0.25	2.90909	0.34375	b3r16
16	151	160	4	0.15	2.66667	0.375	b2r16
16	181	192	4	0.0833333	2.46154	0.40625	b1r16
16	211	224	4	0.0357143	1	1	r16
17	65	68	4.23529	0.708333	3.6	0.277778	b5r17
17	97	102	4.23529	0.393519	3.27273	0.305556	b4r17
17	129	136	4.23529	0.236111	3	0.333333	b3r17
17	161	170	4.23529	0.141667	2.76923	0.361111	b2r17
17	193	204	4.23529	0.0787037	2.57143	0.388889	b1r17
17	225	238	4.23529	0.0337302	1	1	r17
18	69	72	4.5	0.666667	3.68182	0.271605	b5r18
18	103	108	4.5	0.37037	3.375	0.296296	b4r18
18	137	144	4.5	0.222222	3.11538	0.320988	b3r18
18	171	180	4.5	0.133333	2.89286	0.345679	b2r18
18	205	216	4.5	0.0740741	2.6129	0.382716	b1r18
18	239	252	4.5	0.031746	1	1	r18

19	73	76	4.73684	0.633333	3.75	0.266667	b5r19
19	109	114	4.73684	0.351852	3.46154	0.288889	b4r19
19	145	152	4.73684	0.211111	3.21429	0.311111	b3r19
19	181	190	4.73684	0.126667	3	0.333333	b2r19
19	217	228	4.73684	0.0703704	2.64706	0.377778	b1r19
19	253	266	4.73684	0.0301587	1	1	r19
20	77	80	5	0.6	3.84615	0.26	b5r20
20	115	120	5	0.333333	3.57143	0.28	b4r20
20	153	160	5	0.2	3.33333	0.3	b3r20
20	191	200	5	0.12	3.125	0.32	b2r20
20	229	240	5	0.0666667	2.7027	0.37	b1r20
20	267	280	5	0.0285714	1	1	r20
21	81	84	5.2381	0.572727	3.92857	0.254545	b5r21
21	121	126	5.2381	0.318182	3.66667	0.272727	b4r21
21	161	168	5.2381	0.190909	3.4375	0.290909	b3r21
21	201	210	5.2381	0.114545	3.23529	0.309091	b2r21
21	241	252	5.2381	0.0636364	2.75	0.363636	b1r21
21	281	294	5.2381	0.0272727	1	1	r21
22	85	88	5.5	0.545455	4.03333	0.247934	b5r22
22	127	132	5.5	0.30303	3.78125	0.264463	b4r22
22	169	176	5.5	0.181818	3.55882	0.280992	b3r22
22	211	220	5.5	0.109091	3.36111	0.297521	b2r22
22	253	264	5.5	0.0606061	2.81395	0.355372	b1r22
22	295	308	5.5	0.025974	1	1	r22
23	89	92	5.73913	0.522727	4.125	0.242424	b5r23
23	133	138	5.73913	0.290404	3.88235	0.257576	b4r23
23	177	184	5.73913	0.174242	3.66667	0.272727	b3r23
23	221	230	5.73913	0.104545	3.47368	0.287879	b2r23
23	265	276	5.73913	0.0580808	2.86957	0.348485	b1r23
23	309	322	5.73913	0.0248918	1	1	r23
24	93	96	6	0.5	4.23529	0.236111	b5r24
24	139	144	6	0.277778	4	0.25	b4r24
24	185	192	6	0.166667	3.78947	0.263889	b3r24
24	231	240	6	0.1	3.6	0.277778	b2r24
24	277	288	6	0.0555556	2.93878	0.340278	b1r24
24	323	336	6	0.0238095	1	1	r24
25	49	50	6.24	1	4.58824	0.217949	b6r25
25	97	100	6.24	0.480769	4.33333	0.230769	b5r25
25	145	150	6.24	0.267094	4.10526	0.24359	b4r25
25	193	200	6.24	0.160256	3.9	0.25641	b3r25
25	241	250	6.24	0.0961538	3.71429	0.269231	b2r25
25	289	300	6.24	0.0534188	3	0.333333	b1r25
25	337	350	6.24	0.0228938	1	1	r25
26	51	52	6.5	1	4.69444	0.213018	b6r26
26	101	104	6.5	0.461538	4.44737	0.224852	b5r26
26	151	156	6.5	0.25641	4.225	0.236686	b4r26
26	201	208	6.5	0.153846	4.02381	0.248521	b3r26
26	251	260	6.5	0.0923077	3.84091	0.260355	b2r26
26	301	312	6.5	0.0512821	3.07273	0.325444	b1r26
26	351	364	6.5	0.021978	1	1	r26
27	53	54	6.74074	1	4.78947	0.208791	b6r27
27	105	108	6.74074	0.445055	4.55	0.21978	b5r27
27	157	162	6.74074	0.247253	4.33333	0.230769	b4r27
27	209	216	6.74074	0.148352	4.13636	0.241758	b3r27
27	261	270	6.74074	0.089011	3.95652	0.252747	b2r27
27	313	324	6.74074	0.0494505	3.13793	0.318681	b1r27
27	365	378	6.74074	0.0211931	1	1	r27

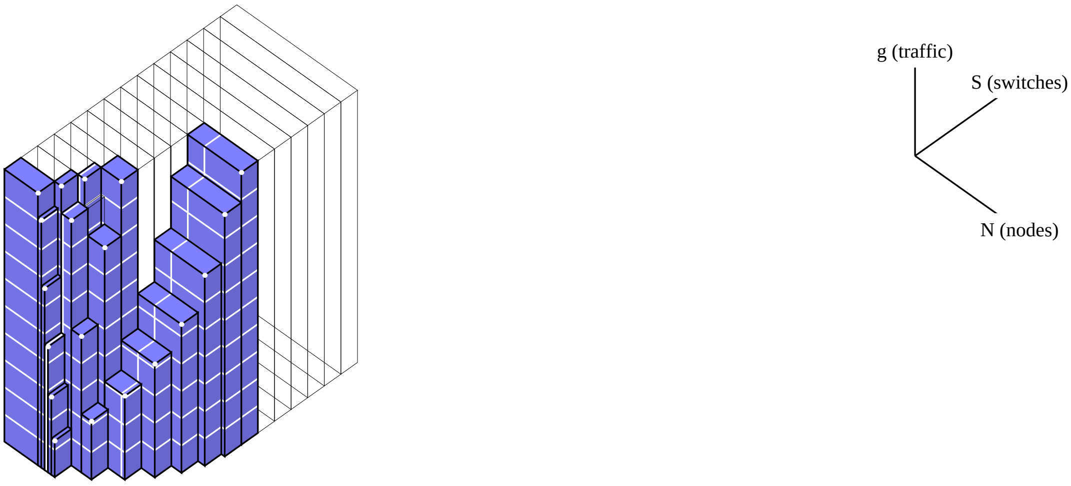
7.2 Star topologies



Star topologies, switches with 16 ports.

S	min	max	$\bar{p}$	g	$\bar{p}/\bar{p}_s$	g/g_s	top
3	1	24	0.972222	1	1.45833	0.685714	b5a1,2c2
3	25	28	0.954082	0.748663	1.43112	0.698752	b4a1,2c2
3	29	32	0.9375	0.533333	1.40625	0.711111	b3a1,2c2
3	33	36	0.92284	0.361204	1.38426	0.722408	b2a1,2c2
3	37	40	0.91	0.21978	1.365	0.732601	b1a1,2c2
3	41	44	0.89876	0.101149	1.34814	0.494508	a1,2c2
4	29	34	1.31315	0.671937	1.75087	0.571146	b4a1,3c2
4	35	40	1.26	0.47619	1.68	0.595238	b3a1,3c2
4	41	46	1.21645	0.321678	1.62193	0.61655	b2a1,3c2
4	47	52	1.18047	0.195489	1.57396	0.635338	b1a1,3c2
4	53	58	1.15042	0.0899225	1.15042	0.651938	a1,3c2
5	41	48	1.5	0.444444	1.875	0.533333	b3a1,4c2
5	49	56	1.42602	0.300537	1.78253	0.561002	b2a1,4c2
5	57	64	1.36719	0.182857	1.42415	0.702171	b1a1,4c2
5	65	72	1.31944	0.0842105	1.09954	0.727579	a1,4c2
6	57	66	1.58173	0.287373	1.89807	0.526851	b2a1,5c2
6	67	76	1.50277	0.175115	1.35249	0.739375	b1a1,5c2
6	77	86	1.43997	0.0807512	0.959978	0.868075	a1,5c2
7	77	88	1.60537	0.169884	1.31105	0.762745	b1a1,6c2
7	89	100	1.53	0.0784314	0.8925	0.960384	a1,6c2
8	89	100	1.6856	0.166113	1.28427	0.778654	b1a1,7c2
8	101	114	1.59972	0.0767677	0.799861	1.09394	a1,7c2
9	101	112	1.75	0.163265	1.26562	0.790123	b1a1,8c2
9	115	128	1.65527	0.0755162	0.744873	1.19334	a1,8c2
10	129	142	1.70056	0.0745407	0.680222	1.3231	a1,9c2
11	143	156	1.73817	0.0737589	0.637327	1.42641	a1,10c2
12	157	170	1.7699	0.0731183	0.589965	1.55376	a1,11c2
13	171	184	1.79702	0.0725838	0.556221	1.65955	a1,12c2
14	185	198	1.82048	0.0721311	0.520136	1.78525	a1,13c2
15	199	212	1.84096	0.0717428	0.493114	1.89273	a1,14c2
16	213	226	1.85899	0.071406	0.464749	2.01722	a1,15c2
17	227	240	1.875	0.0711111	0.442708	2.12595	a1,16c2

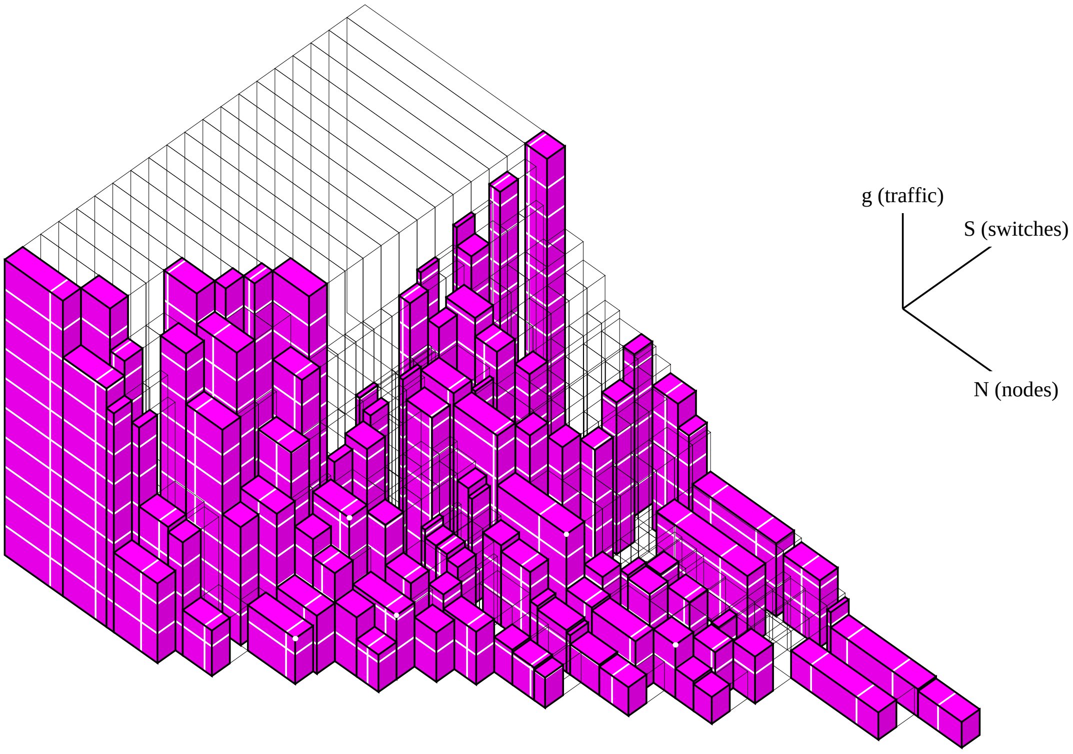
7.3 Complete topologies



Complete topologies, switches with 16 ports.

S	min	max	$\bar{p}$	g	$\bar{p}/\bar{p}_s$	g/g_s	top
2	1	20	0.5	1	1	1	b5c2
2	21	22	0.5	0.909091	1	1	b4c2
2	23	24	0.5	0.666667	1	1	b3c2
2	25	26	0.5	0.461538	1	1	b2c2
2	27	28	0.5	0.285714	1	1	b1c2
2	29	30	0.5	0.133333	1	1	c2
3	21	24	0.666667	1	1	1	b3c3
3	25	30	0.666667	0.9	1	1	b2c3
3	31	36	0.666667	0.5	1	1	b1c3
3	37	42	0.666667	0.214286	1	1	c3
4	25	28	0.75	1	1	1	b2c4
4	31	40	0.75	0.8	1	1	b1c4
4	41	52	0.75	0.307692	1	1	c4
5	29	40	0.8	1	1	1	b1c5
5	41	60	0.8	0.416667	1	1	c5
6	41	66	0.833333	0.545455	1	1	c6
7	41	70	0.857143	0.7	1	1	c7
8	41	72	0.875	0.888889	1	1	c8
9	41	72	0.888889	1	1	1	c9

7.4 Special topologies



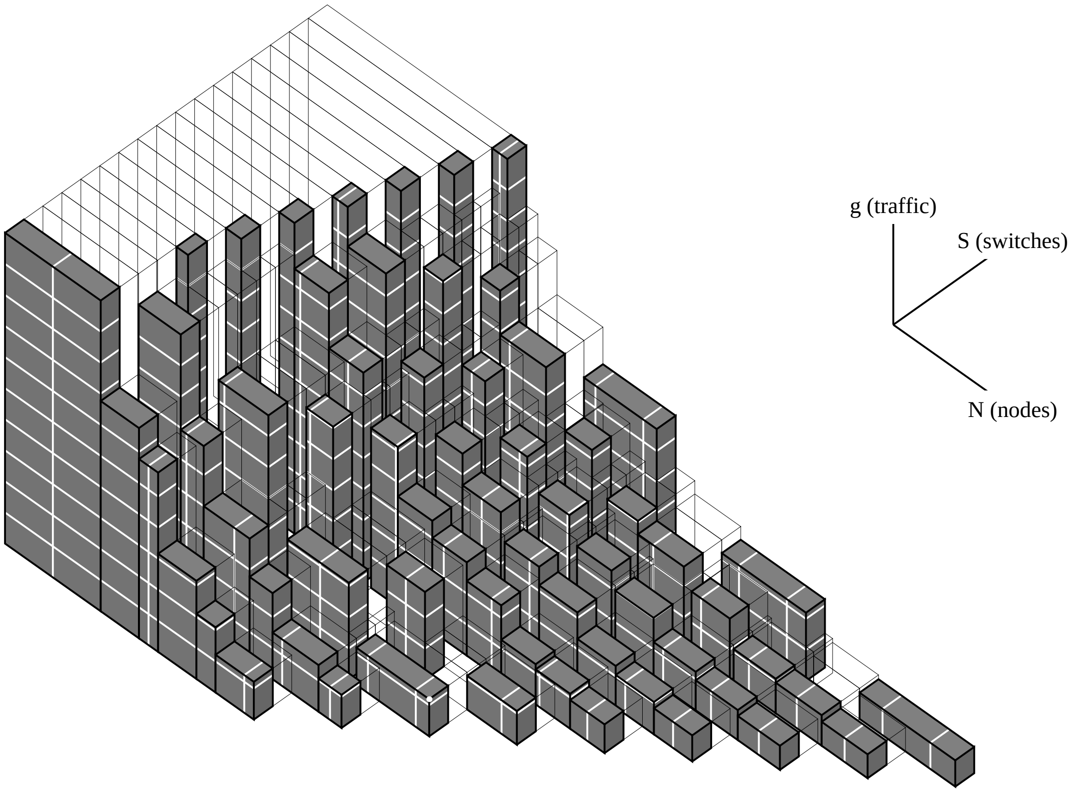
Special topologies, switches with 16 ports.

S	min	max	$\bar{p}$	g	$\bar{p}/\bar{p}_s$	g/g_s	top
7	1	32	1.14453	1	1.33529	0.736887	b3s7,4
7	33	56	1.14286	0.807692	1.33333	0.692308	b2s7,4
7	57	60	1.14333	0.740741	1.33389	0.732601	b1s7,4
7	61	84	1.14286	0.269231	1	0.923077	s7,4
8	33	48	1.125	1	1.28571	0.736842	b2s8,5
8	49	56	1.375	0.857143	1.57143	0.583333	b4s8,3
8	61	64	1.375	0.666667	1.57143	0.583333	b3s8,3
8	65	80	1.375	0.4	1.375	0.666667	b2s8,3
8	81	88	1.125	0.382775	1	0.947368	s8,5
8	89	104	1.375	0.153846	1	0.916667	s8,3
10	57	70	1.5	0.857143	1.66667	0.6	b2p
10	71	90	1.1	0.686275	1	0.970588	s10,7
10	91	100	1.5	0.4	1.25	0.8	b1p
10	105	130	1.5	0.153846	1	1	p
11	49	66	1.45455	1	1.6	0.6	b3s11,4
11	67	88	1.09091	0.895349	1	0.976744	s11,8
11	91	110	1.27273	0.445946	1	0.945946	s11,6
11	111	132	1.45455	0.196429	1	0.857143	s11,4
12	67	72	1.41667	1	1.41667	0.654545	b2s12,5
12	91	108	1.25	0.601504	1	0.966702	s12,7
12	111	120	1.41667	0.36	1.0625	0.8	b1s12,5
12	121	132	1.41667	0.297521	1	0.927273	s12,5
12	133	144	1.5	0.2	1	0.9	s12,4
12	145	156	1.75	0.125874	1	0.954545	s12,3
13	73	78	1.07692	1	1	0.984375	s13,10
13	89	104	1.23077	0.78629	1	0.967742	s13,8

13	111	130	1.38462	0.433333	1	1	s13,6
13	133	156	1.53846	0.216667	1	1	m13
14	79	98	1.21429	1	1	0.953908	s14,9
14	109	112	1.57143	0.5	1.22222	0.642857	b3s14,4
14	131	140	1.42857	0.409756	1	0.97561	s14,6
14	141	154	1.5	0.272727	1	0.9	s14,5
14	157	168	1.57143	0.166667	1	0.785714	s14,4
15	109	120	1.6	0.535714	1.2	0.714286	b4s15,4
15	155	162	1.59968	0.226573	1.05242	0.715275	b1s15,4
15	163	180	1.6	0.178571	1	0.857143	s15,4
16	105	106	2.0607	0.605714	1.5986	0.551768	b13s16,3
16	109	112	2.0625	0.571429	1.57143	0.583333	b12s16,3
16	141	142	2.06209	0.315556	1.44234	0.561954	b9s16,3
16	143	148	2.06209	0.287658	1.41907	0.572819	b8s16,3
16	149	154	2.06165	0.285185	1.39625	0.635769	b7s16,3
16	155	160	2.0625	0.266667	1.375	0.666667	b6s16,3
16	181	190	2.06227	0.118159	1.27522	0.550095	b3s16,3
16	191	202	2.062	0.110202	1.08616	0.782605	b1s16,3
16	203	208	2.0625	0.102564	1	0.916667	s16,3
18	109	110	2.16446	0.597826	1.61587	0.494871	b11s18,3
18	113	126	2.16667	0.538206	1.56	0.581395	b10s18,3
18	141	146	2.16673	0.386755	1.49366	0.576837	b8s18,3
18	147	148	2.16654	0.359223	1.4872	0.553217	b7s18,3
18	155	164	2.16568	0.271973	1.43787	0.541781	b6s18,3
18	165	180	2.16667	0.251163	1.39286	0.651163	b5s18,3
18	181	184	2.16647	0.153206	1.38176	0.424989	b4s18,3
18	185	200	2.1667	0.141643	1.33462	0.522617	b3s18,3
18	201	202	2.1666	0.133158	1.32449	0.511622	b2s18,3
18	203	218	2.16611	0.10322	1.22268	0.569492	b1s18,3
18	219	234	2.16667	0.0966011	1	0.906977	s18,3
19	99	104	1.79105	0.786885	1.33589	0.548596	b6s19,4
19	111	128	1.78882	0.587156	1.27119	0.600907	b5s19,4
19	129	152	1.78947	0.548077	1.21429	0.807692	b4s19,4
19	153	190	1.57895	0.38	1	1	e3
19	191	204	1.78922	0.192453	1.106	0.635126	b1s19,4
19	205	228	1.78947	0.182692	1	0.980769	s19,4
20	99	100	2.248	0.831025	1.66519	0.509947	b11s20,3
20	105	110	2.24793	0.6875	1.63486	0.495164	b10s20,3
20	129	130	2.24852	0.559541	1.57791	0.545552	b9s20,3
20	153	160	1.85	0.540541	1.23333	0.810811	b1s20,4
20	191	200	2.25	0.235294	1.40625	0.627451	b4s20,3
20	229	240	1.85	0.18018	1	1	s20,4
20	241	250	2.2496	0.0998801	1.09737	0.731265	b1s20,3
20	251	260	2.25	0.0904977	1	0.882353	s20,3
21	105	120	1.905	0.720721	1.34632	0.566553	b8s21,4
21	121	132	1.90702	0.658354	1.32232	0.614358	b7s21,4
21	161	168	1.90476	0.492188	1.25	0.75	b5s21,4
21	201	204	1.90484	0.215873	1.18649	0.535611	b3s21,4
21	205	216	1.90561	0.204352	1.16718	0.600544	b2s21,4
21	241	252	1.90476	0.164062	1	0.9375	s21,4
22	99	100	1.9548	0.896861	1.40792	0.494138	b16s22,4
22	101	108	1.95439	0.833977	1.39107	0.518623	b15s22,4
22	133	140	1.95306	0.574359	1.32764	0.557969	b12s22,4
22	169	176	1.95455	0.47482	1.26471	0.733813	b9s22,4
22	205	212	1.95461	0.210631	1.20667	0.516655	b4s22,4
22	217	228	1.95399	0.185328	1.18216	0.563247	b3s22,4
22	241	244	1.95472	0.170351	1.14538	0.65682	b2s22,4
22	253	264	1.95455	0.158273	1	0.928058	s22,4
24	99	104	2.45932	0.948328	1.71914	0.503893	b5s24,3
24	161	168	2.45833	0.514286	1.59459	0.616667	b4s24,3
24	191	240	2.45833	0.24	1.475	0.666667	b2s24,3
24	265	312	2.45833	0.0923077	1	0.983333	s24,3
26	99	110	2.57769	1	1.75303	0.530187	b8s26,3
26	153	158	2.57803	0.547344	1.6725	0.516677	b7s26,3

26	169	182	2.57692	0.484472	1.63415	0.594203	b6s26,3
26	183	188	2.57718	0.403433	1.62516	0.527525	b5s26,3
26	191	236	2.57742	0.24712	1.55566	0.536809	b4s26,3
26	241	260	2.57692	0.226087	1.52273	0.637681	b3s26,3
26	265	266	2.57705	0.144487	1.51486	0.435882	b2s26,3
26	267	314	2.5772	0.0933968	1.1982	0.618414	b1s26,3
26	315	338	2.57692	0.0869565	1	0.971014	s26,3

7.5 Chordal ring topologies

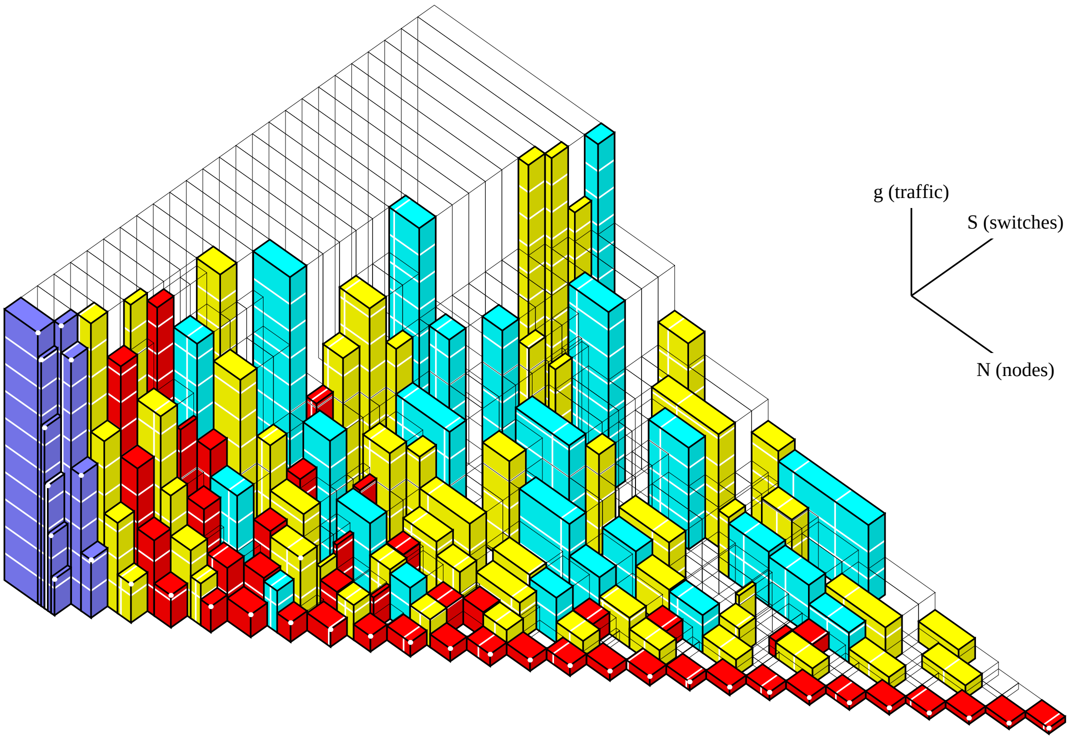


Chordal ring topologies, switches with 16 ports.

S	min	max	$\bar{p}$	g	$\bar{p}/\bar{p}_s$	g/g_s	top
10	1	50	1.7	1	1.88889	0.516746	b5ch10,3
10	51	70	1.7	0.676692	1.88889	0.473684	b4ch10,3
10	71	80	1.7	0.576923	1.7	0.576923	b3ch10,3
10	81	100	1.7	0.315789	1.41667	0.631579	b2ch10,3
10	101	110	1.7	0.20979	1.30769	0.6	b1ch10,3
10	111	130	1.7	0.121457	1.13333	0.789474	ch10,3
12	51	72	1.83333	0.9	1.83333	0.54	b5ch12,5
12	73	84	1.83333	0.593407	1.69231	0.5	b4ch12,5
12	85	108	1.83333	0.4	1.46667	0.642857	b3ch12,5
12	109	120	1.83333	0.276923	1.375	0.615385	b2ch12,5
12	121	144	1.83333	0.15	1.22222	0.675	b1ch12,5
12	145	156	1.83333	0.106509	1.04762	0.807692	ch12,5
14	51	56	1.92857	1	1.92857	0.518519	b3ch14,5
14	73	98	1.92857	0.666667	1.58824	0.62963	b2ch14,5

14	109	140	1.92857	0.311111	1.35	0.740741	b1ch14,5
14	145	182	1.92857	0.119658	1	1	ch14,5
16	57	64	2.125	1	1.88889	0.529412	b3ch16,5
16	99	112	2.125	0.605042	1.61905	0.617647	b2ch16,5
16	141	160	2.125	0.282353	1.41667	0.705882	b1ch16,5
16	183	208	2.125	0.108597	1.0303	0.970588	ch16,5
18	65	72	2.27778	1	1.86364	0.488889	b9ch18,5
18	73	90	2.27778	0.852632	1.78261	0.495215	b8ch18,5
18	91	108	2.27778	0.675	1.70833	0.54	b7ch18,5
18	113	126	2.27778	0.514286	1.64	0.555556	b6ch18,5
18	127	144	2.27778	0.355263	1.57692	0.513158	b5ch18,5
18	145	162	2.27778	0.3	1.51852	0.578571	b4ch18,5
18	163	180	2.27778	0.24	1.46429	0.622222	b3ch18,5
18	181	198	2.27778	0.129187	1.41379	0.457895	b2ch18,5
18	199	216	2.27778	0.1125	1.32258	0.58125	b1ch18,5
18	217	234	2.27778	0.0923077	1.05128	0.866667	ch18,5
20	73	80	2.4	1	1.84615	0.47561	b9ch20,5
20	81	100	2.4	0.872727	1.77778	0.535537	b8ch20,5
20	109	120	2.4	0.625	1.71429	0.525	b7ch20,5
20	127	140	2.4	0.467532	1.65517	0.527273	b6ch20,5
20	141	160	2.4	0.365854	1.6	0.54878	b5ch20,5
20	163	180	2.4	0.277778	1.54839	0.553571	b4ch20,5
20	181	200	2.4	0.218182	1.5	0.581818	b3ch20,5
20	201	220	2.4	0.133038	1.45455	0.482927	b2ch20,5
20	221	240	2.4	0.104167	1.2973	0.578125	b1ch20,5
20	241	260	2.4	0.0839161	1.06667	0.818182	ch20,5
22	81	88	2.59091	1	1.9	0.463918	b9ch22,5
22	101	110	2.59091	0.806107	1.83871	0.516308	b8ch22,5
22	121	132	2.59091	0.578947	1.78125	0.505263	b7ch22,5
22	141	154	2.59091	0.431843	1.72727	0.503817	b6ch22,5
22	161	176	2.59091	0.340206	1.67647	0.525773	b5ch22,5
22	181	198	2.59091	0.25731	1.62857	0.526316	b4ch22,5
22	201	220	2.59091	0.201527	1.58333	0.549618	b3ch22,5
22	221	242	2.59091	0.123711	1.54054	0.457732	b2ch22,5
22	243	264	2.59091	0.0964912	1.32558	0.565789	b1ch22,5
22	265	286	2.59091	0.0775103	1.11765	0.778626	ch22,5
24	89	96	2.66667	1	1.88235	0.472222	b9ch24,7
24	111	120	2.66667	0.732203	1.82857	0.485362	b8ch24,7
24	121	144	2.66667	0.590164	1.77778	0.531148	b7ch24,7
24	155	168	2.66667	0.428571	1.72973	0.513889	b6ch24,7
24	177	192	2.66667	0.305085	1.68421	0.483051	b5ch24,7
24	193	216	2.66667	0.262295	1.64103	0.548009	b4ch24,7
24	221	240	2.66667	0.2	1.6	0.555556	b3ch24,7
24	243	264	2.66667	0.11094	1.56098	0.416949	b2ch24,7
24	265	288	2.66667	0.0983607	1.30612	0.602459	b1ch24,7
24	289	312	2.66667	0.0769231	1.08475	0.819444	ch24,7
26	97	104	2.73077	1	1.86842	0.535211	b3ch26,7
26	145	182	2.73077	0.470825	1.73171	0.577465	b2ch26,7
26	217	260	2.73077	0.219718	1.61364	0.619718	b1ch26,7
26	289	338	2.73077	0.084507	1.0597	0.943662	ch26,7

7.6 Mesh topologies

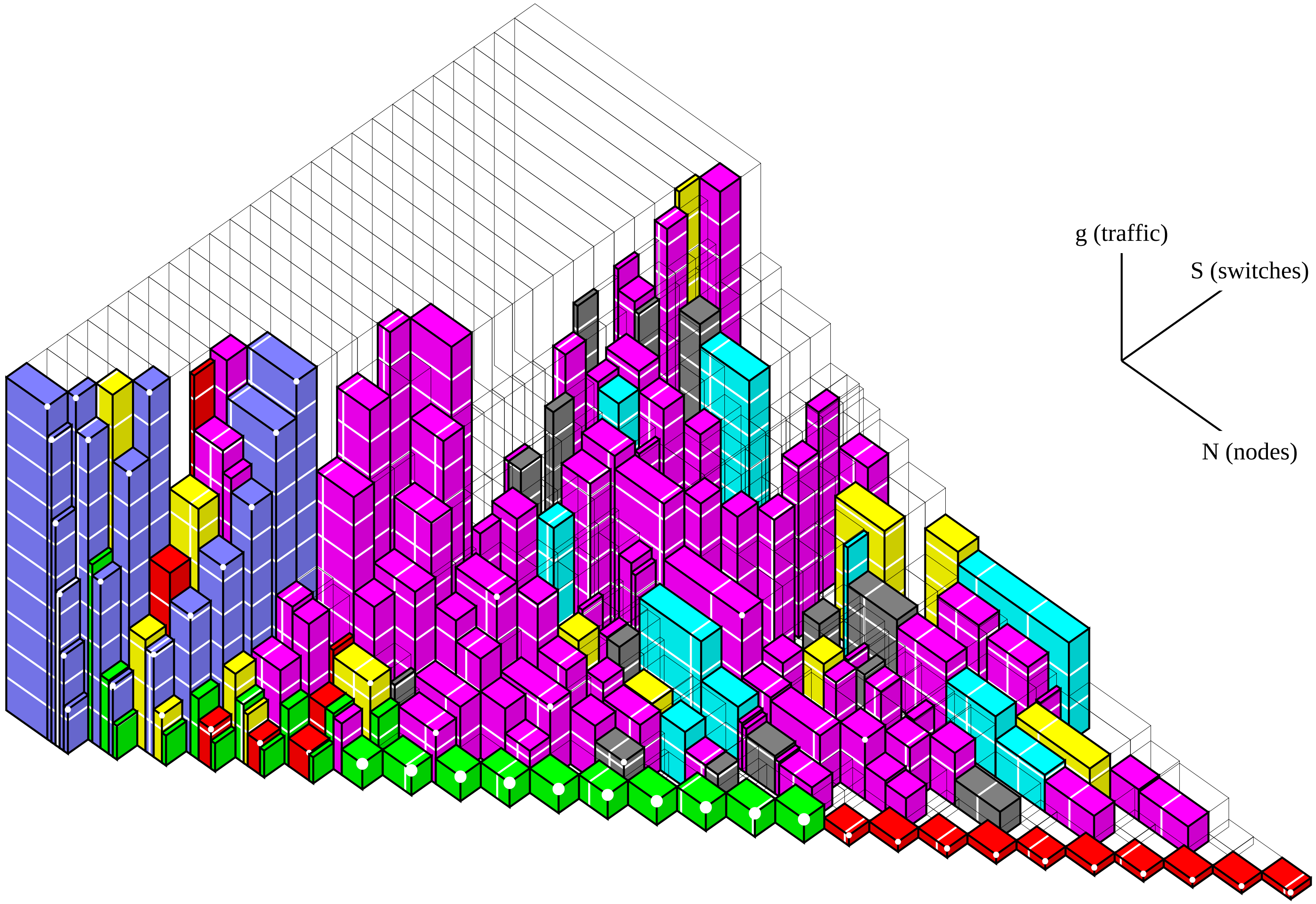


Mesh topologies, switches with 16 ports.

S	min	max	$\bar{p}$	g	$\bar{p}/\bar{p}_s$	g/g_s	top
2	1	20	0.5	1	1	1	b5c2
2	21	22	0.5	0.909091	1	1	b4c2
2	23	24	0.5	0.666667	1	1	b3c2
2	25	26	0.5	0.461538	1	1	b2c2
2	27	28	0.5	0.285714	1	1	b1c2
2	29	30	0.5	0.133333	1	1	c2
3	21	24	0.666667	1	1	1	b3c3
3	25	30	0.666667	0.9	1	1	b2c3
3	31	36	0.666667	0.5	1	1	b1c3
3	37	42	0.666667	0.214286	1	1	c3
4	25	32	1	1	1.33333	0.75	b3c2xc2
4	33	40	1	0.6	1.33333	0.75	b2c2xc2
4	41	48	1	0.333333	1.33333	0.75	b1c2xc2
4	49	56	1	0.142857	1	1	c2xc2
5	33	40	1.2	0.833333	1.5	0.666667	b3r5
5	41	50	1.2	0.5	1.5	0.666667	b2r5
5	51	60	1.2	0.277778	1.5	0.666667	b1r5
5	61	70	1.2	0.119048	1	1	r5
6	33	36	1.16667	1	1.4	0.666667	b4c2xc3
6	41	54	1.16667	0.666667	1.4	0.714286	b3c2xc3
6	55	60	1.16667	0.4	1.4	0.555556	b2c2xc3
6	61	72	1.16667	0.25	1.16667	0.75	b1c2xc3
6	73	78	1.16667	0.153846	1	0.777778	c2xc3
6	79	84	1.5	0.0952381	1	1	r6
7	37	42	1.71429	0.972222	2	0.5	b4r7
7	55	56	1.71429	0.583333	2	0.5	b3r7
7	61	70	1.71429	0.35	2	0.5	b2r7
7	73	84	1.71429	0.194444	1.5	0.666667	b1r7

7	85	98	1.71429	0.0833333	1	1	r7
8	43	56	1.5	0.857143	1.71429	0.583333	b2c2xc2xc2
8	57	64	2	0.5	2.28571	0.4375	b3r8
8	65	80	1.5	0.4	1.5	0.666667	b1c2xc2xc2
8	85	96	2	0.166667	1.6	0.625	b1r8
8	97	104	1.5	0.153846	1.09091	0.916667	c2xc2xc2
8	105	112	2	0.0714286	1	1	r8
9	57	72	1.33333	0.75	1.5	0.666667	b1c3xc3
9	81	90	2.22222	0.27	2	0.5	b2r9
9	91	108	1.33333	0.25	1	1	c3xc3
9	113	126	2.22222	0.0642857	1	1	r9
10	37	50	1.7	1	1.88889	0.490909	b5c2xr5
10	73	80	1.7	0.5	1.7	0.5	b3c2xr5
10	81	100	1.7	0.333333	1.41667	0.666667	b2c2xr5
10	109	110	1.7	0.181818	1.30769	0.52	b1c2xr5
10	111	120	2.5	0.133333	1.78571	0.56	b1r10
10	121	130	1.7	0.128205	1.13333	0.833333	c2xr5
10	131	140	2.5	0.0571429	1	1	r10
11	81	88	2.72727	0.366667	2.5	0.4	b3r11
11	109	110	2.72727	0.22	2.14286	0.466667	b2r11
11	131	132	2.72727	0.122222	1.875	0.533333	b1r11
11	141	154	2.72727	0.052381	1	1	r11
12	51	72	1.66667	1	1.66667	0.6	b3c2xc2xc3
12	81	96	1.66667	0.5	1.42857	0.583333	b2c2xc2xc3
12	101	120	1.66667	0.3	1.25	0.666667	b1c2xc2xc3
12	121	132	2	0.181818	1.41176	0.566667	b1c2xr6
12	133	144	1.66667	0.166667	1.11111	0.75	c2xc2xc3
12	145	156	2	0.102564	1.14286	0.777778	c2xr6
12	157	168	3	0.047619	1	1	r12
13	73	78	3.23077	0.515873	3	0.333333	b4r13
13	101	104	3.23077	0.309524	2.625	0.380952	b3r13
13	121	130	3.23077	0.185714	2.33333	0.428571	b2r13
13	145	156	3.23077	0.103175	2.1	0.47619	b1r13
13	169	182	3.23077	0.0442177	1	1	r13
14	73	84	2.21429	0.666667	1.9375	0.457143	b4c2xr7
14	97	112	2.21429	0.4375	1.72222	0.5625	b3c2xr7
14	121	140	2.21429	0.233333	1.55	0.555556	b2c2xr7
14	141	154	2.21429	0.181818	1.47619	0.6	b1c2xr7
14	157	168	3.5	0.0952381	2.22727	0.44898	b1r14
14	169	182	2.21429	0.0897436	1.14815	0.75	c2xr7
14	183	196	3.5	0.0408163	1	1	r14
15	73	90	1.86667	0.833333	1.55556	0.6	b3c3xr5
15	113	120	1.86667	0.416667	1.4	0.555556	b2c3xr5
15	121	150	1.86667	0.3	1.27273	0.733333	b1c3xr5
15	155	180	1.86667	0.138889	1.16667	0.666667	c3xr5
15	197	210	3.73333	0.0382653	1	1	r15
16	91	96	2.5	0.666667	2	0.5	b3c2xr8
16	97	128	2	0.5	1.45455	0.6875	b1c2xc2xc2xc2
16	155	176	2.5	0.181818	1.6	0.625	b1c2xr8
16	177	192	2	0.166667	1.23077	0.8125	c2xc2xc2xc2
16	193	208	2.5	0.0769231	1.21212	0.6875	c2xr8
16	211	224	4	0.0357143	1	1	r16
17	193	204	4.23529	0.0787037	2.57143	0.388889	b1r17
17	225	238	4.23529	0.0337302	1	1	r17
18	73	90	1.83333	1	1.43478	0.69697	b3c2xc3xc3
18	97	108	1.83333	0.666667	1.375	0.533333	b2c2xc3xc3
18	129	144	2.16667	0.375	1.5	0.541667	b2c3xr6
18	151	180	1.83333	0.3	1.17857	0.777778	b1c2xc3xc3
18	181	198	1.83333	0.181818	1.13793	0.644444	c2xc3xc3
18	199	216	2.16667	0.111111	1.25806	0.574074	c3xr6
18	217	234	2.72222	0.0692308	1.25641	0.65	c2xr9
18	239	252	4.5	0.031746	1	1	r18
19	217	228	4.73684	0.0703704	2.64706	0.377778	b1r19
19	253	266	4.73684	0.0301587	1	1	r19

20	109	120	2.2	0.666667	1.57143	0.56	b3c2xc2xr5
20	129	160	2.2	0.416667	1.46667	0.625	b2c2xc2xr5
20	181	200	2.2	0.2	1.375	0.533333	b1c2xc2xr5
20	201	220	3	0.145455	1.81818	0.528	b1c2xr10
20	221	240	2.2	0.138889	1.18919	0.770833	c2xc2xr5
20	241	260	3	0.0615385	1.33333	0.6	c2xr10
20	267	280	5	0.0285714	1	1	r20
21	121	126	2.38095	0.583333	1.66667	0.5	b3c3xr7
21	161	168	2.38095	0.375	1.5625	0.571429	b2c3xr7
21	181	210	2.38095	0.233333	1.47059	0.62963	b1c3xr7
21	241	252	2.38095	0.0972222	1.25	0.555556	c3xr7
21	281	294	5.2381	0.0272727	1	1	r21
22	129	132	3.22727	0.488889	2.21875	0.426667	b4c2xr11
22	241	242	3.22727	0.133333	1.91892	0.493333	b1c2xr11
22	261	264	5.5	0.0606061	2.81395	0.355372	b1r22
22	265	286	3.22727	0.0564103	1.39216	0.566667	c2xr11
22	295	308	5.5	0.025974	1	1	r22
23	265	276	5.73913	0.0580808	2.86957	0.348485	b1r23
23	309	322	5.73913	0.0248918	1	1	r23
24	91	96	2.66667	1	1.88235	0.472222	b3c3xr8
24	121	144	2.16667	0.666667	1.44444	0.6	b2c2xc2xc2xc3
24	169	192	2.16667	0.375	1.36842	0.59375	b1c2xc2xc2xc3
24	211	216	3.5	0.222222	2.15385	0.464286	b2c2xr12
24	217	240	2.5	0.2	1.5	0.555556	b1c2xc2xr6
24	241	264	2.16667	0.181818	1.26829	0.683333	c2xc2xc2xc3
24	265	288	2.5	0.111111	1.22449	0.680556	c2xc2xr6
24	289	312	3.5	0.0512821	1.42373	0.546296	c2xr12
24	323	336	6	0.0238095	1	1	r24
25	97	100	2.4	1	1.66667	0.6	b2r5xr5
25	161	200	2.4	0.416667	1.5	0.666667	b1r5xr5
25	265	300	2.4	0.138889	1.15385	0.866667	r5xr5
25	337	350	6.24	0.0228938	1	1	r25
26	101	104	3.73077	0.77381	2.55263	0.376984	b5c2xr13
26	217	234	3.73077	0.206349	2.25581	0.438776	b2c2xr13
26	313	338	3.73077	0.047619	1.44776	0.531746	c2xr13
26	351	364	6.5	0.021978	1	1	r26
27	101	108	2	1	1.35	0.740741	b1c3xc3xc3
27	145	162	2.88889	0.5	1.85714	0.466667	b3c3xr9
27	201	216	2.88889	0.3375	1.77273	0.55	b2c3xr9
27	217	270	2	0.3	1.17391	0.851852	c3xc3xc3
27	301	324	2.88889	0.075	1.34483	0.483333	c3xr9
27	365	378	6.74074	0.0211931	1	1	r27



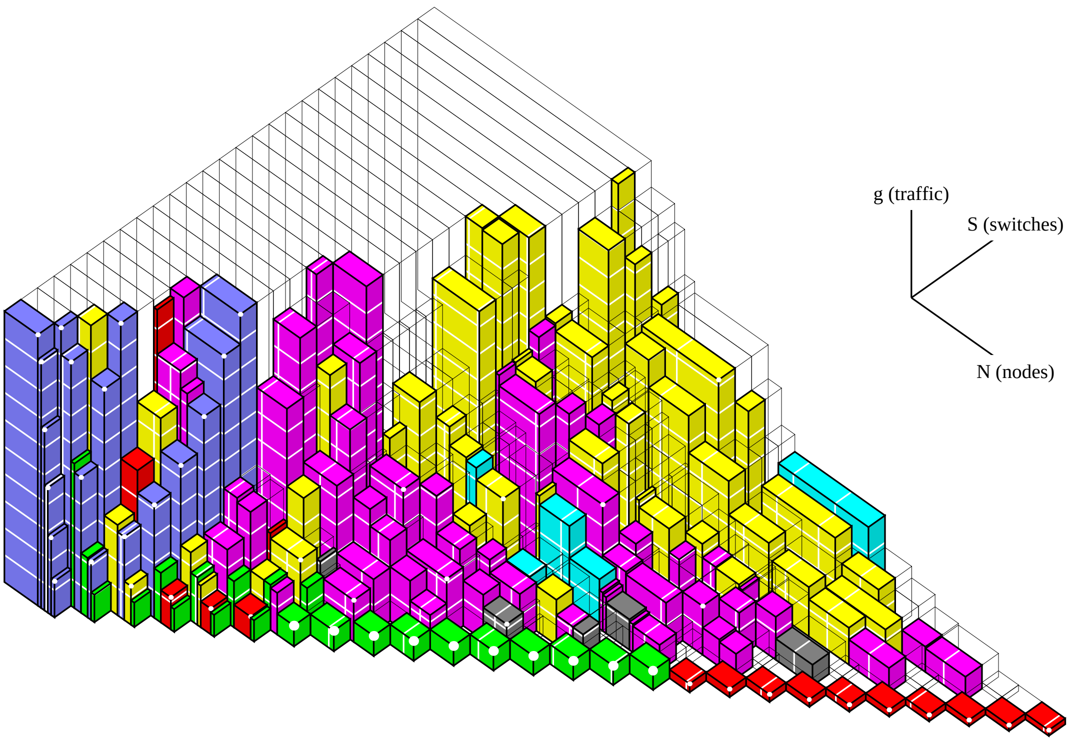
Primitive topologies, switches with 16 ports.

S	min	max	$\bar{p}$	g	$\bar{p}/\bar{p}_s$	g/g_s	top
2	1	20	0.5	1	1	1	b5c2
2	21	22	0.5	0.909091	1	1	b4c2
2	23	24	0.5	0.666667	1	1	b3c2
2	25	26	0.5	0.461538	1	1	b2c2
2	27	28	0.5	0.285714	1	1	b1c2
2	29	30	0.5	0.133333	1	1	c2
3	21	24	0.666667	1	1	1	b3c3
3	25	30	0.666667	0.9	1	1	b2c3
3	31	32	0.9375	0.533333	1.40625	0.711111	b3a1,2c2
3	33	36	0.666667	0.5	1	1	b1c3
3	37	40	0.91	0.21978	1.365	0.732601	b1a1,2c2
3	41	42	0.666667	0.214286	1	1	c3
3	43	44	0.89876	0.101149	1.34814	0.494508	a1,2c2
4	25	32	1	1	1.33333	0.75	b3c2xc2
4	33	40	0.75	0.8	1	1	b1c4
4	41	48	1	0.333333	1.33333	0.75	b1c2xc2
4	49	52	0.75	0.307692	1	1	c4
4	53	56	1	0.142857	1	1	c2xc2
4	57	58	1.15042	0.0899225	1.15042	0.651938	a1,3c2
5	33	40	0.8	1	1	1	b1c5
5	41	50	1.2	0.5	1.5	0.666667	b2r5
5	51	60	0.8	0.416667	1	1	c5
5	61	64	1.36719	0.182857	1.42415	0.702171	b1a1,4c2
5	65	70	1.2	0.119048	1	1	r5
5	71	72	1.31944	0.0842105	1.09954	0.727579	a1,4c2
6	41	54	1.16667	0.666667	1.4	0.714286	b3c2xc3
6	55	66	0.833333	0.545455	1	1	c6
6	67	72	1.16667	0.25	1.16667	0.75	b1c2xc3
6	73	76	1.50277	0.175115	1.35249	0.739375	b1a1,5c2
6	77	78	1.16667	0.153846	1	0.777778	c2xc3
6	79	84	1.5	0.0952381	1	1	r6
6	85	86	1.43997	0.0807512	0.959978	0.868075	a1,5c2
7	41	42	1.71429	0.972222	2	0.5	b4r7
7	43	56	1.14286	0.807692	1.33333	0.692308	b2s7,4
7	57	60	1.14333	0.740741	1.33389	0.732601	b1s7,4
7	61	70	0.857143	0.7	1	1	c7
7	71	84	1.14286	0.269231	1	0.923077	s7,4
7	85	88	1.60537	0.169884	1.31105	0.762745	b1a1,6c2
7	89	98	1.71429	0.0833333	1	1	r7
7	99	100	1.53	0.0784314	0.8925	0.960384	a1,6c2
8	41	48	1.125	1	1.28571	0.736842	b2s8,5
8	49	72	0.875	0.888889	1	1	c8
8	73	80	1.375	0.4	1.375	0.666667	b2s8,3
8	81	88	1.125	0.382775	1	0.947368	s8,5
8	89	96	2	0.166667	1.6	0.625	b1r8
8	97	100	1.6856	0.166113	1.28427	0.778654	b1a1,7c2
8	101	104	1.375	0.153846	1	0.916667	s8,3
8	105	114	1.59972	0.0767677	0.799861	1.09394	a1,7c2
9	49	72	0.888889	1	1	1	c9
9	89	90	2.22222	0.27	2	0.5	b2r9
9	91	108	1.33333	0.25	1	1	c3xc3
9	109	112	1.75	0.163265	1.26562	0.790123	b1a1,8c2
9	115	128	1.65527	0.0755162	0.744873	1.19334	a1,8c2
10	73	90	1.1	0.686275	1	0.970588	s10,7
10	91	100	1.5	0.4	1.25	0.8	b1p
10	109	110	1.7	0.20979	1.30769	0.6	b1ch10,3
10	113	130	1.5	0.153846	1	1	p
10	131	142	1.70056	0.0745407	0.680222	1.3231	a1,9c2
11	73	88	1.09091	0.895349	1	0.976744	s11,8
11	91	110	1.27273	0.445946	1	0.945946	s11,6
11	111	132	1.45455	0.196429	1	0.857143	s11,4
11	143	156	1.73817	0.0737589	0.637327	1.42641	a1,10c2

12	91	108	1.25	0.601504	1	0.966702	s12,7
12	111	120	1.41667	0.36	1.0625	0.8	b1s12,5
12	121	132	1.41667	0.297521	1	0.927273	s12,5
12	133	144	1.5	0.2	1	0.9	s12,4
12	145	156	1.75	0.125874	1	0.954545	s12,3
12	157	170	1.7699	0.0731183	0.589965	1.55376	a1,11c2
13	73	78	1.07692	1	1	0.984375	s13,10
13	89	104	1.23077	0.78629	1	0.967742	s13,8
13	111	130	1.38462	0.433333	1	1	s13,6
13	133	156	1.53846	0.216667	1	1	m13
13	171	184	1.79702	0.0725838	0.556221	1.65955	a1,12c2
14	79	98	1.21429	1	1	0.953908	s14,9
14	109	112	1.57143	0.5	1.22222	0.642857	b3s14,4
14	131	140	1.42857	0.409756	1	0.97561	s14,6
14	141	154	1.5	0.272727	1	0.9	s14,5
14	157	168	1.57143	0.166667	1	0.785714	s14,4
14	169	182	1.92857	0.119658	1	1	ch14,5
14	185	198	1.82048	0.0721311	0.520136	1.78525	a1,13c2
15	109	120	1.6	0.535714	1.2	0.714286	b4s15,4
15	141	150	1.86667	0.3	1.27273	0.733333	b1c3xr5
15	155	162	1.59968	0.226573	1.05242	0.715275	b1s15,4
15	163	180	1.6	0.178571	1	0.857143	s15,4
15	199	212	1.84096	0.0717428	0.493114	1.89273	a1,14c2
16	105	106	2.0607	0.605714	1.5986	0.551768	b13s16,3
16	107	112	2.125	0.605042	1.61905	0.617647	b2ch16,5
16	121	128	2	0.5	1.45455	0.6875	b1c2xc2xc2xc2
16	141	142	2.06209	0.315556	1.44234	0.561954	b9s16,3
16	151	154	2.06165	0.285185	1.39625	0.635769	b7s16,3
16	155	160	2.125	0.282353	1.41667	0.705882	b1ch16,5
16	163	176	2.5	0.181818	1.6	0.625	b1c2xr8
16	181	192	2	0.166667	1.23077	0.8125	c2xc2xc2xc2
16	193	202	2.062	0.110202	1.08616	0.782605	b1s16,3
16	203	208	2.125	0.108597	1.0303	0.970588	ch16,5
16	213	226	1.85899	0.071406	0.464749	2.01722	a1,15c2
17	227	240	1.875	0.0711111	0.442708	2.12595	a1,16c2
18	105	108	2.27778	0.675	1.70833	0.54	b7ch18,5
18	113	126	2.16667	0.538206	1.56	0.581395	b10s18,3
18	141	146	2.16673	0.386755	1.49366	0.576837	b8s18,3
18	147	148	2.16654	0.359223	1.4872	0.553217	b7s18,3
18	151	180	1.83333	0.3	1.17857	0.777778	b1c2xc3xc3
18	181	198	1.83333	0.181818	1.13793	0.644444	c2xc3xc3
18	199	200	2.1667	0.141643	1.33462	0.522617	b3s18,3
18	201	202	2.1666	0.133158	1.32449	0.511622	b2s18,3
18	203	216	2.27778	0.1125	1.32258	0.58125	b1ch18,5
18	217	218	2.16611	0.10322	1.22268	0.569492	b1s18,3
18	219	234	2.16667	0.0966011	1	0.906977	s18,3
18	241	252	4.5	0.031746	1	1	r18
19	99	104	1.79105	0.786885	1.33589	0.548596	b6s19,4
19	113	128	1.78882	0.587156	1.27119	0.600907	b5s19,4
19	129	152	1.78947	0.548077	1.21429	0.807692	b4s19,4
19	153	190	1.57895	0.38	1	1	e3
19	191	204	1.78922	0.192453	1.106	0.635126	b1s19,4
19	205	228	1.78947	0.182692	1	0.980769	s19,4
19	253	266	4.73684	0.0301587	1	1	r19
20	99	100	2.4	0.872727	1.77778	0.535537	b8ch20,5
20	105	110	2.24793	0.6875	1.63486	0.495164	b10s20,3
20	111	120	2.2	0.666667	1.57143	0.56	b3c2xc2xr5
20	129	130	2.24852	0.559541	1.57791	0.545552	b9s20,3
20	153	160	1.85	0.540541	1.23333	0.810811	b1s20,4
20	191	200	2.25	0.235294	1.40625	0.627451	b4s20,3
20	229	240	1.85	0.18018	1	1	s20,4
20	241	250	2.2496	0.0998801	1.09737	0.731265	b1s20,3
20	251	260	2.25	0.0904977	1	0.882353	s20,3
20	267	280	5	0.0285714	1	1	r20

21	105	120	1.905	0.720721	1.34632	0.566553	b8s21,4
21	121	132	1.90702	0.658354	1.32232	0.614358	b7s21,4
21	161	168	1.90476	0.492188	1.25	0.75	b5s21,4
21	201	210	2.38095	0.233333	1.47059	0.62963	b1c3xr7
21	211	216	1.90561	0.204352	1.16718	0.600544	b2s21,4
21	241	252	1.90476	0.164062	1	0.9375	s21,4
21	281	294	5.2381	0.0272727	1	1	r21
22	99	100	1.9548	0.896861	1.40792	0.494138	b16s22,4
22	101	108	1.95439	0.833977	1.39107	0.518623	b15s22,4
22	109	110	2.59091	0.806107	1.83871	0.516308	b8ch22,5
22	133	140	1.95306	0.574359	1.32764	0.557969	b12s22,4
22	169	176	1.95455	0.47482	1.26471	0.733813	b9s22,4
22	191	198	2.59091	0.25731	1.62857	0.526316	b4ch22,5
22	211	212	1.95461	0.210631	1.20667	0.516655	b4s22,4
22	217	220	2.59091	0.201527	1.58333	0.549618	b3ch22,5
22	221	228	1.95399	0.185328	1.18216	0.563247	b3s22,4
22	241	244	1.95472	0.170351	1.14538	0.65682	b2s22,4
22	253	264	1.95455	0.158273	1	0.928058	s22,4
22	265	286	2.59091	0.0775103	1.11765	0.778626	ch22,5
22	295	308	5.5	0.025974	1	1	r22
23	309	322	5.73913	0.0248918	1	1	r23
24	99	104	2.45932	0.948328	1.71914	0.503893	b5s24,3
24	111	120	2.66667	0.732203	1.82857	0.485362	b8ch24,7
24	121	144	2.16667	0.666667	1.44444	0.6	b2c2xc2xc2xc3
24	161	168	2.45833	0.514286	1.59459	0.616667	b4s24,3
24	191	192	2.16667	0.375	1.36842	0.59375	b1c2xc2xc2xc3
24	193	216	2.66667	0.262295	1.64103	0.548009	b4ch24,7
24	217	240	2.45833	0.24	1.475	0.666667	b2s24,3
24	241	264	2.16667	0.181818	1.26829	0.683333	c2xc2xc2xc3
24	265	288	2.5	0.111111	1.22449	0.680556	c2xc2xr6
24	289	312	2.45833	0.0923077	1	0.983333	s24,3
24	323	336	6	0.0238095	1	1	r24
25	99	100	2.4	1	1.66667	0.6	b2r5xr5
25	177	200	2.4	0.416667	1.5	0.666667	b1r5xr5
25	265	300	2.4	0.138889	1.15385	0.866667	r5xr5
25	337	350	6.24	0.0228938	1	1	r25
26	101	110	2.57769	1	1.75303	0.530187	b8s26,3
26	153	158	2.57803	0.547344	1.6725	0.516677	b7s26,3
26	169	182	2.57692	0.484472	1.63415	0.594203	b6s26,3
26	217	236	2.57742	0.24712	1.55566	0.536809	b4s26,3
26	241	260	2.57692	0.226087	1.52273	0.637681	b3s26,3
26	265	266	2.57705	0.144487	1.51486	0.435882	b2s26,3
26	301	314	2.5772	0.0933968	1.1982	0.618414	b1s26,3
26	315	338	2.57692	0.0869565	1	0.971014	s26,3
26	351	364	6.5	0.021978	1	1	r26
27	201	216	2.88889	0.3375	1.77273	0.55	b2c3xr9
27	217	270	2	0.3	1.17391	0.851852	c3xc3xc3
27	365	378	6.74074	0.0211931	1	1	r27

7.8 Product topologies



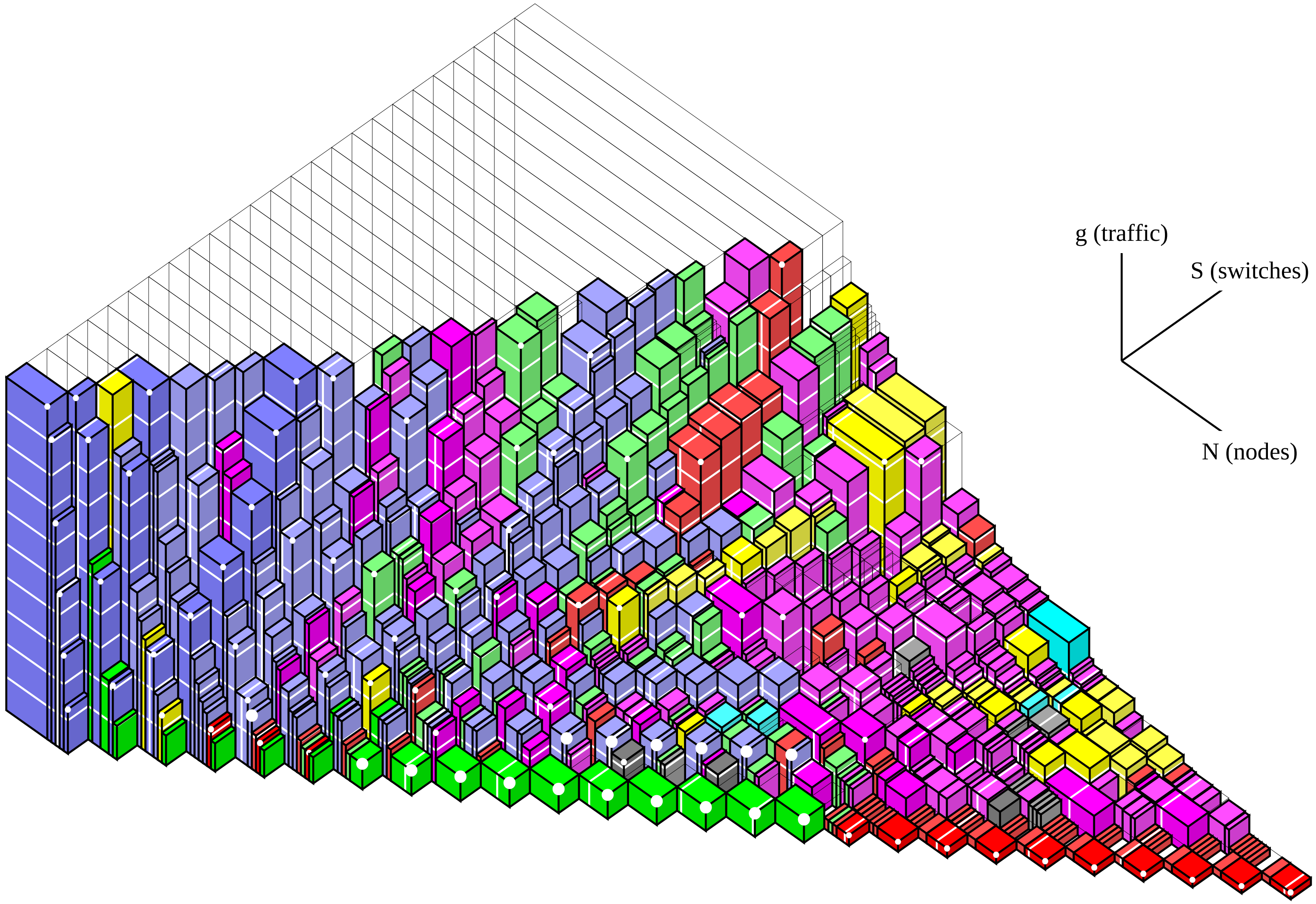
Product topologies, switches with 16 ports.

S	min	max	$\bar{p}$	g	$\bar{p}/\bar{p}_s$	g/g_s	top
2	1	20	0.5	1	1	1	b5c2
2	21	22	0.5	0.909091	1	1	b4c2
2	23	24	0.5	0.666667	1	1	b3c2
2	25	26	0.5	0.461538	1	1	b2c2
2	27	28	0.5	0.285714	1	1	b1c2
2	29	30	0.5	0.133333	1	1	c2
3	21	24	0.666667	1	1	1	b3c3
3	25	30	0.666667	0.9	1	1	b2c3
3	31	32	0.9375	0.533333	1.40625	0.711111	b3a1,2c2
3	33	36	0.666667	0.5	1	1	b1c3
3	37	40	0.91	0.21978	1.365	0.732601	b1a1,2c2
3	41	42	0.666667	0.214286	1	1	c3
3	43	44	0.89876	0.101149	1.34814	0.494508	a1,2c2
4	25	32	1	1	1.33333	0.75	b3c2xc2
4	33	40	0.75	0.8	1	1	b1c4
4	41	48	1	0.333333	1.33333	0.75	b1c2xc2
4	49	52	0.75	0.307692	1	1	c4
4	53	56	1	0.142857	1	1	c2xc2
4	57	58	1.15042	0.0899225	1.15042	0.651938	a1,3c2
5	33	40	0.8	1	1	1	b1c5
5	41	50	1.2	0.5	1.5	0.666667	b2r5
5	51	60	0.8	0.416667	1	1	c5
5	61	64	1.36719	0.182857	1.42415	0.702171	b1a1,4c2
5	65	70	1.2	0.119048	1	1	r5
5	71	72	1.31944	0.0842105	1.09954	0.727579	a1,4c2
6	41	54	1.16667	0.666667	1.4	0.714286	b3c2xc3
6	55	66	0.833333	0.545455	1	1	c6
6	67	72	1.16667	0.25	1.16667	0.75	b1c2xc3

6	73	76	1.50277	0.175115	1.35249	0.739375	b1a1,5c2
6	77	78	1.16667	0.153846	1	0.777778	c2xc3
6	79	84	1.5	0.0952381	1	1	r6
6	85	86	1.43997	0.0807512	0.959978	0.868075	a1,5c2
7	41	42	1.71429	0.972222	2	0.5	b4r7
7	43	56	1.14286	0.807692	1.33333	0.692308	b2s7,4
7	57	60	1.14333	0.740741	1.33389	0.732601	b1s7,4
7	61	70	0.857143	0.7	1	1	c7
7	71	84	1.14286	0.269231	1	0.923077	s7,4
7	85	88	1.60537	0.169884	1.31105	0.762745	b1a1,6c2
7	89	98	1.71429	0.0833333	1	1	r7
7	99	100	1.53	0.0784314	0.8925	0.960384	a1,6c2
8	41	48	1.125	1	1.28571	0.736842	b2s8,5
8	49	72	0.875	0.888889	1	1	c8
8	73	80	1.375	0.4	1.375	0.666667	b2s8,3
8	81	88	1.125	0.382775	1	0.947368	s8,5
8	89	96	1.25	0.166667	1	0.625	c2xc4
8	97	100	1.6856	0.166113	1.28427	0.778654	b1a1,7c2
8	101	104	1.375	0.153846	1	0.916667	s8,3
8	105	114	1.59972	0.0767677	0.799861	1.09394	a1,7c2
9	49	72	0.888889	1	1	1	c9
9	89	90	2.22222	0.27	2	0.5	b2r9
9	91	108	1.33333	0.25	1	1	c3xc3
9	109	112	1.75	0.163265	1.26562	0.790123	b1a1,8c2
9	115	128	1.65527	0.0755162	0.744873	1.19334	a1,8c2
10	73	90	1.1	0.686275	1	0.970588	s10,7
10	91	100	1.3	0.4	1.08333	0.8	b1c2xc5
10	109	110	1.7	0.20979	1.30769	0.6	b1ch10,3
10	113	130	1.5	0.153846	1	1	p
10	131	142	1.70056	0.0745407	0.680222	1.3231	a1,9c2
11	73	88	1.09091	0.895349	1	0.976744	s11,8
11	91	110	1.27273	0.445946	1	0.945946	s11,6
11	111	132	1.45455	0.196429	1	0.857143	s11,4
11	143	156	1.73817	0.0737589	0.637327	1.42641	a1,10c2
12	89	96	1.33333	0.75	1.14286	0.875	b2c2xc6
12	97	108	1.25	0.601504	1	0.966702	s12,7
12	111	120	1.41667	0.36	1.0625	0.8	b1s12,5
12	121	132	1.41667	0.297521	1	0.927273	s12,5
12	133	144	1.5	0.2	1	0.9	s12,4
12	145	156	1.75	0.125874	1	0.954545	s12,3
12	157	170	1.7699	0.0731183	0.589965	1.55376	a1,11c2
13	73	78	1.07692	1	1	0.984375	s13,10
13	89	104	1.23077	0.78629	1	0.967742	s13,8
13	111	130	1.38462	0.433333	1	1	s13,6
13	133	156	1.53846	0.216667	1	1	m13
13	171	184	1.79702	0.0725838	0.556221	1.65955	a1,12c2
14	79	98	1.21429	1	1	0.953908	s14,9
14	109	112	1.35714	0.5	1.05556	0.642857	b1c2xc7
14	131	140	1.42857	0.409756	1	0.97561	s14,6
14	141	154	1.5	0.272727	1	0.9	s14,5
14	157	168	1.57143	0.166667	1	0.785714	s14,4
14	169	182	1.92857	0.119658	1	1	ch14,5
14	185	198	1.82048	0.0721311	0.520136	1.78525	a1,13c2
15	105	120	1.46667	0.625	1.1	0.833333	b1c3xc5
15	141	150	1.46667	0.3	1	0.733333	c3xc5
15	155	162	1.59968	0.226573	1.05242	0.715275	b1s15,4
15	163	180	1.6	0.178571	1	0.857143	s15,4
15	199	212	1.84096	0.0717428	0.493114	1.89273	a1,14c2
16	121	128	1.625	0.526316	1.18182	0.723684	b4c2xs8,5
16	129	138	1.62539	0.456651	1.14945	0.75518	b3c2xs8,5
16	139	144	1.75	0.444444	1.21739	0.821429	b1c2xc2xc4
16	145	160	1.5	0.4	1	1	c4xc4
16	163	176	1.75	0.181818	1.12	0.625	c2xc2xc4
16	181	192	1.875	0.166667	1.15385	0.8125	c2xs8,3

16	193	202	2.062	0.110202	1.08616	0.782605	b1s16,3
16	203	208	2.125	0.108597	1.0303	0.970588	ch16,5
16	213	226	1.85899	0.071406	0.464749	2.01722	a1,15c2
17	227	240	1.875	0.0711111	0.442708	2.12595	a1,16c2
18	99	126	1.5	0.857143	1.08	0.925926	b1c3xc6
18	161	162	1.5	0.333333	1	0.642857	c3xc6
18	163	180	1.83333	0.3	1.17857	0.777778	b1c2xc3xc3
18	181	198	1.83333	0.181818	1.13793	0.644444	c2xc3xc3
18	199	200	2.1667	0.141643	1.33462	0.522617	b3s18,3
18	201	202	2.1666	0.133158	1.32449	0.511622	b2s18,3
18	203	216	2.27778	0.1125	1.32258	0.58125	b1ch18,5
18	217	218	2.16611	0.10322	1.22268	0.569492	b1s18,3
18	219	234	2.16667	0.0966011	1	0.906977	s18,3
18	241	252	4.5	0.031746	1	1	r18
19	127	128	1.78882	0.587156	1.27119	0.600907	b5s19,4
19	129	152	1.78947	0.548077	1.21429	0.807692	b4s19,4
19	161	190	1.57895	0.38	1	1	e3
19	191	204	1.78922	0.192453	1.106	0.635126	b1s19,4
19	205	228	1.78947	0.182692	1	0.980769	s19,4
19	253	266	4.73684	0.0301587	1	1	r19
20	99	108	1.60082	1	1.16848	0.725017	b5c2xs10,7
20	109	120	1.6	0.992908	1.14286	0.834043	b4c2xs10,7
20	127	128	1.60059	0.591224	1.12717	0.559692	b3c2xs10,7
20	129	140	1.95	0.571429	1.34483	0.644444	b2c4xr5
20	153	160	1.85	0.540541	1.23333	0.810811	b1s20,4
20	161	180	1.55	0.444444	1	0.885714	c4xc5
20	191	200	2.25	0.235294	1.40625	0.627451	b4s20,3
20	229	240	1.85	0.18018	1	1	s20,4
20	241	250	2.2496	0.0998801	1.09737	0.731265	b1s20,3
20	251	260	2.25	0.0904977	1	0.882353	s20,3
20	267	280	5	0.0285714	1	1	r20
21	109	126	1.52381	1	1.06667	0.857143	b1c3xc7
21	127	132	1.90702	0.658354	1.32232	0.614358	b7s21,4
21	161	168	1.90476	0.492188	1.25	0.75	b5s21,4
21	191	192	1.8112	0.318197	1.14761	0.669584	b1c3xs7,4
21	193	210	1.80952	0.291667	1.11765	0.787037	c3xs7,4
21	211	216	1.90561	0.204352	1.16718	0.600544	b2s21,4
21	241	252	1.90476	0.164062	1	0.9375	s21,4
21	281	294	5.2381	0.0272727	1	1	r21
22	127	132	1.95455	0.666667	1.34375	0.581818	b3c2xs11,4
22	133	154	1.77273	0.637066	1.18182	0.743243	b4c2xs11,6
22	161	166	1.77391	0.529224	1.16338	0.720188	b3c2xs11,6
22	169	176	1.77273	0.487342	1.14706	0.753165	b2c2xs11,6
22	211	220	1.95455	0.235714	1.19444	0.642857	b1c2xs11,4
22	221	228	1.95399	0.185328	1.18216	0.563247	b3s22,4
22	229	242	1.95455	0.181818	1.16216	0.672727	c2xs11,4
22	243	244	1.95472	0.170351	1.14538	0.65682	b2s22,4
22	253	264	1.95455	0.158273	1	0.928058	s22,4
22	265	286	2.59091	0.0775103	1.11765	0.778626	ch22,5
22	295	308	5.5	0.025974	1	1	r22
23	309	322	5.73913	0.0248918	1	1	r23
24	127	144	1.75	0.902256	1.16667	0.81203	b6c2xs12,7
24	155	168	1.79167	0.601504	1.16216	0.721248	b2c3xs8,5
24	169	192	1.58333	0.5	1	0.791667	c4xc6
24	193	216	1.91667	0.363636	1.17949	0.75974	b1c2xs12,5
24	217	240	2	0.24	1.2	0.666667	b1c2xs12,4
24	243	264	2	0.181818	1.17073	0.683333	c2xs12,4
24	265	288	2.25	0.136364	1.10204	0.835227	c2xs12,3
24	289	312	2.45833	0.0923077	1	0.983333	s24,3
24	323	336	6	0.0238095	1	1	r24
25	145	150	2	0.833333	1.31579	0.76	b2r5xc5
25	155	200	1.6	0.625	1	1	c5xc5
25	265	300	2.4	0.138889	1.15385	0.866667	r5xr5
25	337	350	6.24	0.0228938	1	1	r25

26	127	130	1.73077	1	1.15385	0.806367	b6c2xs13,8
26	151	156	2.03846	0.666667	1.325	0.615385	b2c2xm13
26	201	208	1.88462	0.5	1.16667	0.807692	b1c2xs13,6
26	217	260	2.03846	0.26	1.20455	0.733333	b1c2xm13
26	265	286	2.03846	0.181818	1.17778	0.692308	c2xm13
26	301	314	2.5772	0.0933968	1.1982	0.618414	b1s26,3
26	315	338	2.57692	0.0869565	1	0.971014	s26,3
26	351	364	6.5	0.021978	1	1	r26
27	217	270	2	0.3	1.17391	0.851852	c3xc3xc3
27	365	378	6.74074	0.0211931	1	1	r27



All topologies, switches with 16 ports.

S	min	max	$\bar{p}$	g	$\bar{p}/\bar{p}_s$	g/g_s	top
2	1	20	0.5	1	1	1	b5c2
2	21	22	0.5	0.909091	1	1	b4c2
2	23	24	0.5	0.666667	1	1	b3c2
2	25	26	0.5	0.461538	1	1	b2c2
2	27	28	0.5	0.285714	1	1	b1c2
2	29	30	0.5	0.133333	1	1	c2
3	21	24	0.666667	1	1	1	b3c3
3	25	30	0.666667	0.9	1	1	b2c3
3	31	32	0.9375	0.533333	1.40625	0.711111	b3a1,2c2
3	33	36	0.666667	0.5	1	1	b1c3
3	37	40	0.91	0.21978	1.365	0.732601	b1a1,2c2
3	41	42	0.666667	0.214286	1	1	c3
3	43	44	0.89876	0.101149	1.34814	0.494508	a1,2c2
4	25	32	1	1	1.33333	0.75	b3c2xc2
4	33	36	0.901235	0.830769	1.20165	0.801099	b3a2,1c3
4	37	40	0.75	0.8	1	1	b1c4
4	41	44	0.896694	0.458333	1.19559	0.75625	b2a2,1c3
4	45	46	0.885633	0.380165	1.18084	0.72865	b1a2,1c3
4	47	48	1	0.333333	1.33333	0.75	b1c2xc2
4	49	52	0.75	0.307692	1	1	c4
4	53	54	0.884088	0.192857	1.01039	0.91125	a2,1c3
4	55	56	1	0.142857	1	1	c2xc2
4	57	58	1.15042	0.0899225	1.15042	0.651938	a1,3c2
5	25	40	0.8	1	1	1	b1c5
5	41	42	1.12925	0.7875	1.41156	0.696316	b2a3,1c3
5	43	44	0.900826	0.776471	1.12603	0.759216	b2a2,1c4
5	45	48	0.972222	0.576	1.21528	0.6912	b2a1,2c3
5	49	50	0.88	0.5	1.1	0.666667	b1a2,1c4
5	51	54	1.09259	0.428571	1.36574	0.712088	b1a3,1c3
5	55	60	0.8	0.416667	1	1	c5
5	61	62	0.887617	0.291994	1.00866	0.885065	a2,1c4
5	63	64	0.964844	0.227219	1.00505	0.872521	a1,2c3
5	65	66	1.06887	0.181319	1.02776	0.88898	a3,1c3
5	67	68	1.12716	0.156562	1.0064	0.993645	a2,3c2
5	69	70	1.2	0.119048	1	1	r5
5	71	72	1.31944	0.0842105	1.09954	0.727579	a1,4c2
6	41	48	1	1	1.2	0.833333	b1a2c3
6	49	54	1.03704	0.736364	1.24444	0.788961	b1a3,1c4
6	55	66	0.833333	0.545455	1	1	c6
6	67	68	0.895329	0.404762	1.00724	0.873772	a2,1c5
6	69	72	1	0.333333	1	1	a2c3
6	73	78	1.16667	0.197802	1	1	a3c2
6	79	80	1.245	0.160643	0.974348	1.02633	a2,4c2
6	81	82	1.28316	0.108466	0.923879	0.882359	a2,1r5
6	83	84	1.5	0.0952381	1	1	r6
6	85	86	1.43997	0.0807512	0.959978	0.868075	a1,5c2
7	49	52	0.998521	1	1.16494	0.788338	b1a3,1c5
7	53	56	1.14286	0.807692	1.33333	0.692308	b2s7,4
7	57	60	1.14333	0.740741	1.33389	0.732601	b1s7,4
7	61	70	0.857143	0.7	1	1	c7
7	71	72	0.903549	0.536513	1.00623	0.86718	a2,1c6
7	73	76	0.982687	0.448642	1.00316	0.927804	a1,2c4
7	77	80	1.0675	0.35057	1.00591	0.930083	a3,2c3
7	81	82	1.11362	0.284722	1.01051	0.857653	a1,3c3
7	83	84	1.14286	0.269231	1	0.923077	s7,4
7	85	88	1.23037	0.221662	1.0048	0.995219	a3,4c2
7	89	90	1.33827	0.169173	1.05767	0.875682	a5,1c3
7	91	92	1.34735	0.161347	1.00031	0.999692	a2,5c2
7	93	94	1.36442	0.101732	0.928562	0.780634	a3,1r5
7	95	96	1.55599	0.0846561	0.977481	0.808552	a2,1-(c2xc3)
7	97	98	1.71429	0.0833333	1	1	r7
7	99	100	1.53	0.0784314	0.8925	0.960384	a1,6c2

8	53	56	1.16327	1	1.32945	0.747277	b1a4,2c3
8	57	72	0.875	0.888889	1	1	c8
8	73	74	0.911249	0.69375	1.00552	0.861567	a2,1c7
8	75	80	1	0.6	1	1	a2c4
8	81	86	1.09816	0.394858	1.00403	0.884317	a2,3c3
8	87	88	1.125	0.382775	1	0.947368	s8,5
8	89	92	1.19305	0.287141	1.00468	0.871393	a2,1s7,4
8	93	96	1.25	0.266667	1	1	a4c2
8	97	98	1.30175	0.235162	1.016	0.984253	a3,5c2
8	99	100	1.6856	0.166113	1.28427	0.778654	b1a1,7c2
8	101	102	1.43656	0.165584	1.06907	0.872901	a6,1c3
8	103	104	1.43491	0.160825	1.04357	0.958247	a2,6c2
8	105	106	1.43859	0.0970696	0.939488	0.716164	a4,1r5
8	107	108	1.60974	0.0779221	0.95392	0.710065	a3,1-(c2xc3)
8	109	114	1.59972	0.0767677	0.799861	1.09394	a1,7c2
9	57	72	0.888889	1	1	1	c9
9	73	74	0.918188	0.88547	1.00504	0.855174	a2,1c8
9	75	80	0.99	0.768176	1.00238	0.948365	a1,2c5
9	81	84	1.04252	0.622661	1.00528	0.904012	a3,2c4
9	85	90	1.11111	0.54	1	1	a3c3
9	91	94	1.16614	0.422282	1.00486	0.921305	a2,1s8,5
9	95	96	1.23438	0.364228	1.0415	0.863354	a5,2c3
9	97	104	1.2892	0.298336	1.00409	0.995927	a4,5c2
9	105	108	1.33333	0.25	1	1	c3xc3
9	109	112	1.75	0.163265	1.26562	0.790123	b1a1,8c2
9	113	114	1.51801	0.162857	1.07858	0.870984	a7,1c3
9	115	116	1.50981	0.159874	1.05426	0.948529	a2,7c2
9	117	118	1.50474	0.0936508	0.982935	0.650664	a5,1r5
9	119	128	1.65527	0.0755162	0.744873	1.19334	a1,8c2
10	73	80	1	1	1	1	a2c5
10	81	88	1.08781	0.691358	1.00723	0.912593	a1,3c4
10	89	90	1.1	0.686275	1	0.970588	s10,7
10	91	94	1.14554	0.576182	1.00486	0.93551	a4,3c3
10	95	100	1.2	0.5	1	1	-(p)
10	101	104	1.31953	0.366197	1.06413	0.8433	a6,2c3
10	105	110	1.3	0.34965	1	1	a5c2
10	111	112	1.33929	0.32	1.01461	0.9856	a4,6c2
10	113	114	1.34364	0.261268	1.00272	0.867637	-(a2,1-(c3xc3))
10	115	116	1.36415	0.258736	1.00305	0.927686	a2,1-(c3xc3)
10	117	118	1.44226	0.246788	1.04511	0.956833	a3,7c2
10	119	120	1.4	0.238095	1	1	a2r5
10	121	124	1.5757	0.162099	1.09424	0.804009	a3,1-(c2xc4)
10	125	126	1.58642	0.160714	1.08659	0.869559	a8,1c3
10	127	128	1.57422	0.158809	1.06366	0.940149	a2,8c2
10	129	130	1.5	0.153846	1	1	p
10	131	142	1.70056	0.0745407	0.680222	1.3231	a1,9c2
11	81	86	1.07788	0.895833	1.00326	0.919689	a2,3c4
11	87	88	1.09091	0.895349	1	0.976744	s11,8
11	89	92	1.12925	0.724409	1.0047	0.891756	a2,1s10,7
11	93	96	1.16146	0.638177	1.00383	0.886063	a3,4c3
11	97	98	1.19617	0.602459	1.01927	0.888305	a5,3c3
11	99	102	1.20992	0.519878	1.00274	0.864644	-(a2,1p)
11	103	104	1.228	0.516899	1.00397	0.913235	a2,1-(p)
11	105	106	1.28693	0.466373	1.03813	0.875482	a4,1s8,5
11	107	110	1.27273	0.445946	1	0.945946	s11,6
11	111	116	1.32729	0.389698	1.00376	0.996252	a5,6c2
11	117	120	1.395	0.334528	1.02924	0.971593	a4,7c2
11	121	122	1.53776	0.258292	1.1209	0.800571	a8,1c4
11	123	124	1.40375	0.257796	1.01103	0.85353	a3,1-(c3xc3)
11	125	128	1.50684	0.248866	1.06004	0.943359	a3,8c2
11	129	132	1.45455	0.196429	1	0.857143	s11,4
11	133	134	1.61762	0.160543	1.0874	0.761964	a4,1-(c2xc4)
11	135	138	1.64461	0.158986	1.0585	0.897072	a9,1c3
11	139	140	1.63	0.157756	1.02724	0.973483	a2,9c2

11	141	142	1.61516	0.0889724	0.997111	0.601916	a7,1r5
11	143	156	1.73817	0.0737589	0.637327	1.42641	a1,10c2
12	81	84	1.08333	1	1	0.962963	-(r12)
12	85	88	1.11674	0.95393	1.00506	0.896857	a2,1s11,8
12	89	96	1.16667	0.857143	1	1	a4c3
12	97	102	1.25894	0.62111	1.04188	0.850575	a6,3c3
12	103	108	1.25	0.601504	1	0.966702	s12,7
12	109	114	1.29686	0.501466	1.00402	0.946678	a2,1s11,6
12	115	120	1.33333	0.45	1	1	-(c3xc4)
12	121	122	1.36617	0.419986	1.01406	0.986134	a5,7c2
12	123	128	1.45312	0.344086	1.04625	0.955795	a4,8c2
12	129	132	1.5	0.30303	1.05882	0.944444	-(c2xc6)
12	133	138	1.56679	0.249749	1.07437	0.930776	a3,9c2
12	139	140	1.49959	0.201149	1.01859	0.797291	b1-(s12,7)
12	141	144	1.5	0.2	1	0.9	s12,4
12	145	150	1.69467	0.157563	1.04287	0.914428	a10,1c3
12	151	152	1.67867	0.156766	1.0072	0.992849	a2,10c2
12	153	156	1.75	0.125874	1	0.954545	s12,3
12	157	170	1.7699	0.0731183	0.589965	1.55376	a1,11c2
13	85	88	1.14979	1	1.01206	0.897747	a1,4c4
13	89	96	1.18924	0.920128	1.0049	0.93335	a5,4c3
13	97	104	1.23077	0.78629	1	0.967742	s13,8
13	105	110	1.27107	0.641988	1.00379	0.912474	a2,1s12,7
13	111	112	1.30867	0.569354	1.02392	0.848977	a1,6c3
13	113	118	1.3341	0.544855	1.0156	0.938397	a3,1s11,6
13	119	124	1.35406	0.500288	1.00367	0.996347	a6,7c2
13	125	128	1.41309	0.442294	1.02936	0.971477	a5,8c2
13	129	130	1.38462	0.433333	1	1	s13,6
13	131	136	1.51168	0.350215	1.06447	0.939432	a4,9c2
13	137	138	1.43552	0.319938	1.00249	0.903183	a2,1s12,5
13	139	142	1.66812	0.254025	1.14599	0.795553	a10,1c4
13	143	148	1.62217	0.249916	1.08788	0.919216	a3,10c2
13	149	156	1.53846	0.216667	1	1	m13
13	157	162	1.73815	0.156371	1.03432	0.925431	a11,1c3
13	163	164	1.72145	0.155853	0.996316	1.0037	a2,11c2
13	165	166	1.76579	0.113673	0.994726	0.797534	a2,1s12,3
13	171	184	1.79702	0.0725838	0.556221	1.65955	a1,12c2
14	89	98	1.21429	1	1	0.953908	s14,9
14	99	104	1.25	0.822785	1.0041	0.887712	a2,1s13,8
14	105	112	1.28571	0.724138	1	0.931034	a2s7,4
14	113	122	1.38175	0.577743	1.03368	0.923718	a4,1s11,6
14	123	126	1.35714	0.573099	1	1	a7c2
14	127	128	1.38672	0.540845	1.01417	0.98603	a6,8c2
14	129	134	1.46547	0.458311	1.04829	0.953932	a5,9c2
14	135	140	1.42857	0.409756	1	0.97561	s14,6
14	141	144	1.56944	0.353982	1.08314	0.923244	a4,10c2
14	145	146	1.46275	0.319778	1.00244	0.873407	a2,3r5
14	147	154	1.5	0.272727	1	0.9	s14,5
14	155	158	1.67321	0.249653	1.1005	0.908678	a3,11c2
14	159	162	1.54344	0.201493	1.0017	0.81121	-(a2,1-(m13))
14	163	164	1.55354	0.198068	1.00162	0.839699	a2,1m13
14	165	168	1.85714	0.179487	1.18182	0.846154	a2r7
14	169	174	1.77626	0.155357	1.03002	0.932333	a12,1c3
14	175	176	1.7593	0.155021	0.990869	1.00922	a2,12c2
14	177	182	1.92857	0.119658	1	1	ch14,5
14	185	198	1.82048	0.0721311	0.520136	1.78525	a1,13c2
15	99	100	1.25	1	1.00446	0.989364	a1,2s8,5
15	101	104	1.28402	0.862832	1.01727	0.83283	a3,1s13,8
15	105	112	1.30293	0.78935	1.00397	0.896351	a3,2s7,4
15	113	120	1.33333	0.75	1	1	-(c3xc5)
15	121	128	1.37402	0.636816	1.00375	0.996263	a7,8c2
15	129	132	1.42906	0.57253	1.03057	0.970333	a6,9c2
15	133	140	1.52143	0.469484	1.06975	0.934794	a5,10c2
15	141	144	1.51524	0.410034	1.05225	0.885673	a2,1-(c2xc7)

15	145	150	1.46667	0.409091	1	1	a3r5
15	151	152	1.62569	0.356124	1.10175	0.907647	a4,11c2
15	153	156	1.62147	0.298991	1.0858	0.829201	a1,2-(c2xc4)
15	157	158	1.51418	0.276304	1.00796	0.79977	b1a2,1s14,5
15	159	160	1.51391	0.261296	1.00185	0.789693	a2,1s14,5
15	161	162	1.77229	0.250903	1.16598	0.792083	a12,1c4
15	163	168	1.72024	0.249135	1.11222	0.8991	a3,12c2
15	169	172	1.57382	0.183761	1.006	0.727164	a3,1m13
15	173	176	1.58445	0.18107	1.00141	0.787856	a2,1s14,4
15	177	180	1.6	0.178571	1	0.857143	s15,4
15	181	186	1.80992	0.154485	1.02836	0.936523	a13,1c3
15	187	188	1.79301	0.154264	0.98879	1.01134	a2,13c2
15	189	190	1.93163	0.112139	1.0348	0.795436	-(a2,1-(ch14,5))
15	191	192	1.93989	0.110493	1.01036	0.848586	a2,1ch14,5
15	199	212	1.84096	0.0717428	0.493114	1.89273	a1,14c2
16	101	112	1.3125	0.979592	1	1	-(c4xc4)
16	113	120	1.34833	0.779221	1.00341	0.923892	a2,1-(c3xc5)
16	121	128	1.375	0.727273	1	1	a8c2
16	129	130	1.40343	0.690615	1.01491	0.985308	a7,9c2
16	131	136	1.47924	0.596491	1.0519	0.950658	a6,10c2
16	137	144	1.5	0.518519	1.04348	0.958333	-(c2xc8)
16	145	146	1.57956	0.476982	1.09289	0.915007	a5,11c2
16	147	148	1.53944	0.418868	1.0594	0.8341	a3,1-(c2xc7)
16	149	154	1.48001	0.416431	1.00234	0.928357	a4,3r5
16	155	160	1.5	0.4	1	1	c4xc4
16	161	162	1.53574	0.27871	1.01852	0.724247	b2a3,1s14,5
16	163	164	1.53495	0.27748	1.01275	0.749686	b1a3,1s14,5
16	165	168	1.71429	0.257353	1.11953	0.752319	a7,1-(c2xc5)
16	169	172	1.81679	0.249637	1.17449	0.790703	a13,1c4
16	173	178	1.7636	0.24847	1.12309	0.8904	a3,13c2
16	179	184	1.60208	0.196581	1.00523	0.800659	a3,1s14,4
16	185	186	1.60365	0.174157	1.00131	0.74114	-(a2,1-(s15,4))
16	187	188	1.61131	0.172161	1.0012	0.766022	a2,1s15,4
16	189	192	1.875	0.166667	1.15385	0.8125	c2xs8,3
16	193	198	1.83986	0.153727	1.0284	0.938884	a14,1c3
16	199	200	1.8232	0.153576	0.988854	1.01127	a2,14c2
16	201	202	2.062	0.110202	1.08616	0.782605	b1s16,3
16	203	208	2.125	0.108597	1.0303	0.970588	ch16,5
16	213	226	1.85899	0.071406	0.464749	2.01722	a1,15c2
17	101	110	1.32678	1	1.00377	0.900246	a2,1-(c4xc4)
17	113	120	1.37556	0.810811	1.01412	0.868251	a3,1-(c3xc5)
17	121	128	1.38965	0.809557	1.00402	0.995995	a8,9c2
17	129	132	1.44421	0.734382	1.03311	0.967947	a7,10c2
17	133	134	1.56739	0.630633	1.1157	0.860262	a7,1s11,6
17	135	140	1.53571	0.613953	1.07724	0.928301	a6,11c2
17	141	144	1.4429	0.563443	1.0024	0.912427	a1,2-(c3xc3)
17	145	146	1.51295	0.528165	1.04603	0.885178	a2,1-(c2xc8)
17	147	152	1.63885	0.481724	1.11705	0.895218	a5,12c2
17	153	158	1.50136	0.420213	1.00905	0.866547	a5,3r5
17	159	162	1.50412	0.415385	1.00159	0.918681	-(a2,1-(c4xc4))
17	163	164	1.5113	0.413098	1.00176	0.946372	a2,1c4xc4
17	165	168	1.73214	0.357388	1.1377	0.878964	a4,13c2
17	169	172	1.64008	0.279221	1.06753	0.737838	b2a5,1-(s13,8)
17	173	176	1.7194	0.268675	1.10916	0.76357	a2,3-(c2xc3)
17	177	182	1.85714	0.248521	1.18219	0.789495	a14,1c4
17	183	188	1.80364	0.247725	1.13316	0.882489	a3,14c2
17	189	192	1.62305	0.183908	1.01091	0.708651	a4,1s14,4
17	193	200	2.0094	0.171556	1.23033	0.778301	a2,1c2xc2xc2xc2
17	201	204	1.93637	0.153224	1.17565	0.757105	b3a12,1-(c2xc3)
17	205	210	1.86667	0.153061	1.02952	0.939996	a15,1c3
17	211	212	1.85039	0.152951	0.990302	1.00979	a2,15c2
17	213	216	2.1271	0.101727	1.07471	0.776604	-(a2,1-(ch16,5))
17	217	218	2.071	0.100215	1.01789	0.823138	a2,1s16,3
17	227	240	1.875	0.0711111	0.442708	2.12595	a1,16c2

18	113	126	1.38889	0.925714	1	1	a9c2
18	127	128	1.41797	0.881543	1.01642	0.983845	a8,10c2
18	129	134	1.49532	0.768566	1.05783	0.945335	a7,11c2
18	135	144	1.44444	0.692308	1	1	a2-(c3xc3)
18	145	148	1.53561	0.536788	1.05411	0.826672	a3,1-(c2xc8)
18	149	158	1.69853	0.484411	1.14175	0.87585	a5,13c2
18	159	162	1.50892	0.426316	1.00594	0.82218	-(a3,1-(c4xc4))
18	163	168	1.52891	0.413793	1.00684	0.879693	a3,1c4xc4
18	169	176	1.78202	0.357101	1.15475	0.865987	a4,14c2
18	177	180	1.72222	0.348387	1.10714	0.903226	a3-(c2xc3)
18	181	184	1.7949	0.254229	1.14478	0.705225	a9,1-(c2xc5)
18	185	186	1.80385	0.248037	1.14598	0.711957	b2a9,1p
18	189	198	1.84068	0.246944	1.14249	0.875281	a3,15c2
18	199	200	1.88795	0.17417	1.16292	0.642629	b5a7,1s12,3
18	201	202	2.0051	0.174138	1.22576	0.669079	-(a3,1-(c2xc2xc2xc2))
18	203	208	2.02145	0.171636	1.20839	0.746511	a3,1c2xc2xc2xc2
18	213	216	1.95653	0.152758	1.13605	0.78925	b3a13,1-(c2xc3)
18	217	222	1.86807	0.152473	0.998771	0.959242	b1a14,1r5
18	223	224	1.875	0.152381	0.976688	1.02387	a2,16c2
18	225	234	2.16667	0.0966011	1	0.906977	s18,3
18	241	242	4.24267	0.032988	1.3243	0.555989	b2a2,1r17
18	243	244	4.23613	0.0322751	1.22327	0.619802	b1-(a2,1-(r17))
18	245	246	4.24099	0.0318323	1.13937	0.693995	b1a2,1r17
18	247	252	4.5	0.031746	1	1	r18
19	111	124	1.40245	1	1.00453	0.995491	a9,10c2
19	125	128	1.45996	0.941806	1.03749	0.963863	a8,11c2
19	129	136	1.5558	0.793995	1.08845	0.918734	a7,12c2
19	137	144	1.45525	0.72	1.00257	0.940587	a3,2-(c3xc3)
19	145	148	1.66271	0.633937	1.13681	0.879651	a6,13c2
19	149	152	1.78947	0.548077	1.21429	0.807692	b4s19,4
19	153	160	1.49937	0.542899	1.00236	0.902325	a3,4r5
19	161	164	1.75803	0.485577	1.16663	0.857166	a5,14c2
19	165	166	1.56336	0.421462	1.03365	0.766784	a7,3r5
19	169	172	1.55192	0.409719	1.01493	0.816342	a4,1c4xc4
19	173	190	1.57895	0.38	1	1	e3
19	191	192	1.68441	0.268531	1.06306	0.729402	b3a5,1s15,4
19	193	196	1.83642	0.247163	1.15095	0.715699	b2a10,1p
19	199	208	1.875	0.246154	1.15115	0.868698	a3,16c2
19	209	228	1.78947	0.182692	1	0.980769	s19,4
19	229	230	2.04802	0.152116	1.11682	0.867008	b3a13,1r7
19	231	236	1.96775	0.120408	1.00051	0.821883	b1a14,1-(c2xc3)
19	237	238	2.16884	0.0968661	1.07845	0.70248	b1-(a2,1-(s18,3))
19	239	240	2.17417	0.0966573	1.05778	0.745011	b1a2,1s18,3
19	241	242	2.16829	0.0930769	1.03266	0.76283	-(a2,1-(s18,3))
19	243	244	2.17327	0.092006	1.01363	0.802211	a2,1s18,3
19	245	246	2.73865	0.0361552	1.25146	0.335581	b1a9,1r11
19	247	248	3.48745	0.0337415	1.48814	0.35018	b2a6,1r14
19	249	250	3.72774	0.0323499	1.42857	0.390807	b2a5,1r15
19	253	254	4.24217	0.0310665	1.35046	0.49575	b2a3,1r17
19	255	256	4.50653	0.0310454	1.32265	0.564148	b2a2,1r18
19	257	266	4.73684	0.0301587	1	1	r19
20	125	128	1.51562	0.989691	1.06734	0.936907	a8,12c2
20	129	140	1.5	0.857143	1.03448	0.966667	-(c2xc10)
20	141	144	1.47531	0.75	1.01049	0.895909	a4,2-(c3xc3)
20	145	160	1.5	0.666667	1	1	a4r5
20	161	170	1.81696	0.485622	1.19145	0.839316	a5,15c2
20	171	180	1.55	0.444444	1	0.885714	c4xc5
20	181	182	1.5887	0.393939	1.02167	0.80789	b1a2,1e3
20	191	200	1.6	0.375	1	1	a2p
20	209	218	1.82783	0.229958	1.11115	0.808482	b1a11,1p
20	219	220	1.7936	0.185263	1.08703	0.672505	b4-(a2,1-(s19,4))
20	221	222	1.79929	0.183244	1.07742	0.693223	b4a2,1s19,4
20	229	240	1.85	0.18018	1	1	s20,4
20	241	244	2.21439	0.114797	1.14735	0.711315	b3a13,1r8

20	245	246	2.04591	0.100408	1.03853	0.657565	b2a14,1r7
20	247	250	2.2496	0.0998801	1.09737	0.731265	b1s20,3
20	251	260	2.25	0.0904977	1	0.882353	s20,3
20	261	262	3.7129	0.0311905	1.47045	0.35576	b2a6,1r15
20	263	264	4.50706	0.0303658	1.60966	0.400828	b3a3,1r18
20	267	270	4.74266	0.0294889	1.30832	0.577244	b2a2,1r19
20	271	280	5	0.0285714	1	1	r20
21	125	128	1.58496	1	1.10596	0.90419	a8,13c2
21	129	138	1.51092	0.86382	1.03788	0.876463	a2,1-(c2xc10)
21	141	144	1.50463	0.782609	1.02398	0.862467	a5,2-(c3xc3)
21	145	160	1.50937	0.688666	1.00246	0.942639	a5,4r5
21	161	168	1.90476	0.492188	1.25	0.75	b5s21,4
21	169	176	1.76653	0.488889	1.14565	0.829227	b1a1,4-(c2xc3)
21	177	182	1.55778	0.459596	1.00143	0.844914	a2,1c4xc5
21	183	186	1.60354	0.393235	1.02487	0.762929	b1a3,1e3
21	201	204	1.60755	0.372101	1.00132	0.923234	a3,2p
21	205	210	1.90476	0.315	1.17647	0.85	a3r7
21	211	212	1.97655	0.240615	1.2174	0.6679	b10a10,1s12,3
21	219	228	1.85549	0.22619	1.11786	0.792607	b1a12,1p
21	229	234	1.85905	0.180974	1.09312	0.706083	b2a2,1s20,4
21	241	246	1.8566	0.169711	1.01082	0.852017	b1a2,1s20,4
21	247	252	1.90476	0.164062	1	0.9375	s21,4
21	253	254	2.25649	0.101763	1.16252	0.611849	b3a2,1s20,3
21	255	258	2.25744	0.0976903	1.12109	0.650655	b2a2,1s20,3
21	259	266	2.25502	0.0909091	1.0446	0.745743	b1a2,1s20,3
21	267	270	2.25528	0.0899101	1.01075	0.820701	a2,1s20,3
21	271	272	3.45475	0.0319024	1.52354	0.30745	b2a8,1r14
21	273	274	3.69535	0.0302028	1.52304	0.323856	b2a7,1r15
21	275	276	4.49895	0.0292063	1.66167	0.363748	b3a4,1r18
21	277	278	4.74277	0.0287785	1.58692	0.412251	b3a3,1r19
21	281	284	5.00521	0.0279418	1.30609	0.584813	b2a2,1r20
21	285	294	5.2381	0.0272727	1	1	r21
22	129	132	1.5	1	1.03125	0.969697	-(c2xc11)
22	133	136	1.53439	0.869543	1.04893	0.800874	a3,1-(c2xc10)
22	141	142	1.78506	0.828471	1.21004	0.826417	a7,15c2
22	143	144	1.54321	0.818182	1.04318	0.837947	a6,2-(c3xc3)
22	145	160	1.5275	0.705467	1.00999	0.889122	a6,4r5
22	161	176	1.54545	0.559322	1	0.864407	a2s11,4
22	177	184	1.57089	0.475452	1.0057	0.813377	a3,1c4xc5
22	187	190	1.62283	0.392562	1.03077	0.724864	b1a4,1e3
22	205	208	1.61982	0.368359	1.00512	0.857475	a4,2p
22	209	216	1.76698	0.320475	1.0853	0.828685	b3a8,1s15,4
22	217	220	2.16174	0.252632	1.32106	0.688995	b1a1,3r8
22	221	224	2.15119	0.230255	1.30801	0.662697	b11a7,1s16,3
22	229	230	1.83078	0.199134	1.10486	0.622075	b6a4,1s19,4
22	231	232	1.87444	0.197812	1.1284	0.635286	b4a3,1s20,4
22	233	234	1.99587	0.19697	1.19851	0.650464	b7a11,1s12,3
22	235	236	1.90886	0.185413	1.14343	0.629739	b3a2,1s21,4
22	237	238	1.98333	0.184746	1.1851	0.645485	b6a11,1s12,3
22	239	242	1.95455	0.181818	1.16216	0.672727	c2xs11,4
22	243	244	1.95472	0.170351	1.14538	0.65682	b2s22,4
22	245	252	1.91188	0.169916	1.05875	0.773217	b1a2,1s21,4
22	253	260	1.91041	0.166934	1.00286	0.898703	a2,1s21,4
22	261	264	1.95455	0.158273	1	0.928058	s22,4
22	265	268	2.26732	0.0915613	1.12207	0.590284	b2a3,1s20,3
22	271	272	2.26487	0.0871795	1.08534	0.618542	b1a3,1s20,3
22	273	280	2.26102	0.0859819	1.01893	0.741979	a3,1s20,3
22	281	286	2.59091	0.0775103	1.11765	0.778626	ch22,5
22	287	288	4.48471	0.0282187	1.71997	0.331103	b3a5,1r18
22	289	290	4.73427	0.027619	1.63437	0.374212	b3a4,1r19
22	295	298	5.24278	0.0266738	1.29333	0.596707	b2a2,1r21
22	299	308	5.5	0.025974	1	1	r22
23	133	144	1.50077	0.912034	1.0075	0.873365	b1a1,2s12,5
23	145	148	1.58492	0.89384	1.05861	0.900261	a1,2-(c2xc6)

23	149	160	1.55438	0.718074	1.02272	0.839511	a7,4r5
23	161	170	1.66706	0.646442	1.08338	0.854048	a2,3-(c2xc4)
23	177	184	1.58176	0.51172	1.01057	0.800954	b1a1,2s12,4
23	185	186	1.58897	0.482866	1.01273	0.774267	a4,1c4xc5
23	187	192	1.74957	0.461697	1.10708	0.795969	a3,4-(c2xc3)
23	193	194	1.64598	0.391919	1.03905	0.692209	b1a5,1e3
23	209	212	1.63626	0.364192	1.01119	0.800865	a5,2p
23	217	224	1.79524	0.315715	1.0941	0.805833	b2a9,1s15,4
23	225	232	1.91454	0.250901	1.15615	0.708762	b1a1,2ch12,5
23	233	234	2.16575	0.227553	1.30488	0.659526	b10a8,1s16,3
23	235	236	1.84994	0.209593	1.11206	0.623365	b7a5,1s19,4
23	237	238	2.1723	0.205172	1.30289	0.626276	b9a8,1s16,3
23	239	240	1.88948	0.201566	1.1307	0.63156	b4a4,1s20,4
23	241	244	1.95949	0.186341	1.16731	0.615507	b1a2,1c2xs11,4
23	245	248	1.96013	0.184389	1.16246	0.642563	a2,1c2xs11,4
23	249	252	1.92423	0.173148	1.13607	0.637108	b2a3,1s21,4
23	253	254	1.96249	0.169482	1.1484	0.645306	b9a2,1s22,4
23	255	260	1.92101	0.168132	1.07422	0.72383	b1a3,1s21,4
23	265	268	1.91802	0.156909	1.01261	0.796515	a3,1s21,4
23	269	270	1.95997	0.150106	1.0205	0.794282	b1a2,1s22,4
23	271	272	1.95967	0.14997	1.00647	0.827337	a2,1s22,4
23	273	274	2.2762	0.0995278	1.15336	0.572548	b2a4,1s20,3
23	275	278	2.27615	0.0909388	1.11904	0.571358	b1a4,1s20,3
23	281	282	2.59567	0.0826775	1.23481	0.569885	b7a2,1ch22,5
23	283	284	2.59621	0.0816583	1.21539	0.589742	b6a2,1ch22,5
23	285	290	2.26709	0.0811189	1.01291	0.675027	a4,1s20,3
23	291	292	2.59594	0.0745431	1.14247	0.650767	b2a2,1ch22,5
23	293	294	2.59542	0.0734985	1.12539	0.673439	b1a2,1ch22,5
23	295	296	2.59491	0.0700352	1.10881	0.673815	a2,1ch22,5
23	297	298	3.65524	0.0286098	1.53951	0.289179	b2a9,1r15
23	299	300	4.46587	0.0273673	1.76039	0.306296	b3a6,1r18
23	301	302	4.71957	0.0266314	1.66889	0.344614	b3a5,1r19
23	303	304	4.99602	0.0260989	1.60175	0.386673	b3a4,1r20
23	309	312	5.50423	0.0254113	1.28497	0.606454	b2a2,1r22
23	313	322	5.73913	0.0248918	1	1	r23
24	133	144	1.5	1	1	0.931034	a2s12,5
24	145	154	1.5215	0.896246	1.00273	0.910561	a4,5r5
24	155	168	1.54167	0.771429	1	0.925	a3s8,3
24	171	176	1.58058	0.578947	1.01609	0.762033	b1a2-(s12,7)
24	177	192	1.58333	0.571429	1	0.904762	a2s12,4
24	195	198	1.67248	0.391304	1.0494	0.663876	b1a6,1e3
24	213	216	1.91667	0.363636	1.17949	0.75974	b1c2xs12,5
24	217	220	1.85723	0.332382	1.13805	0.727649	b13a6,1s19,4
24	225	240	1.83333	0.313043	1.1	0.869565	a2s12,3
24	241	244	2.1856	0.231426	1.30592	0.675038	b10a9,1s16,3
24	245	248	2.18669	0.211981	1.30117	0.649625	b9a9,1s16,3
24	249	250	1.96746	0.187579	1.1683	0.589342	b1a3,1c2xs11,4
24	251	254	1.96835	0.186499	1.16403	0.616174	a3,1c2xs11,4
24	255	256	2	0.183908	1.18033	0.623244	b1c2x-(s12,7)
24	257	264	2	0.181818	1.17073	0.683333	c2xs12,4
24	265	268	1.93167	0.161252	1.09512	0.657131	b1a4,1s21,4
24	269	270	2.10088	0.160269	1.17258	0.680089	b1a11,1ch14,5
24	273	276	1.92722	0.148627	1.02785	0.71217	a4,1s21,4
24	277	278	1.96721	0.143379	1.03386	0.715505	b1a3,1s22,4
24	279	280	1.96653	0.140845	1.01863	0.732063	a3,1s22,4
24	281	288	2.25	0.136364	1.10204	0.835227	c2xs12,3
24	289	312	2.45833	0.0923077	1	0.983333	s24,3
24	313	314	4.70015	0.02578	1.70699	0.318416	b3a6,1r19
24	315	316	5.50433	0.0253327	1.80552	0.35889	b4a3,1r22
24	323	326	5.74297	0.0243574	1.26936	0.619403	b2a2,1r23
24	327	336	6	0.0238095	1	1	r24
25	145	150	1.52	1	1	1	a5r5
25	155	166	1.67419	0.789891	1.0832	0.866077	a4,3-(c2xc4)
25	171	172	1.75514	0.635478	1.12856	0.745556	b1a7,2p

25	173	200	1.6	0.625	1	1	c5xc5
25	201	208	1.61501	0.431919	1.00137	0.754649	a1,2m13
25	209	216	1.92172	0.374031	1.18216	0.713769	b6a2,1c2xs12,5
25	217	218	1.92227	0.372172	1.18018	0.726101	b5a2,1c2xs12,5
25	219	220	1.67975	0.355447	1.02926	0.708998	a7,2p
25	221	226	1.88014	0.325036	1.14531	0.693039	b14a7,1s19,4
25	227	228	1.877	0.32337	1.14117	0.705049	b13a7,1s19,4
25	229	232	1.87623	0.322334	1.13628	0.734996	b12a7,1s19,4
25	241	244	1.83855	0.305837	1.10067	0.799054	a3,2s12,3
25	245	246	2.1972	0.242604	1.31286	0.648579	b12a10,1s16,3
25	247	250	2.19834	0.231696	1.30853	0.648749	b11a10,1s16,3
25	251	254	2.19952	0.229553	1.30427	0.673479	b10a10,1s16,3
25	255	258	2.20053	0.209642	1.29993	0.644787	b9a10,1s16,3
25	259	260	1.95639	0.189386	1.15353	0.596512	b3a5,1s21,4
25	261	262	2.00492	0.188275	1.17992	0.607378	b2a2,1c2x-(s12,7)
25	263	266	2.00424	0.185576	1.17509	0.628313	b1a2,1c2xs12,4
25	267	270	2.17125	0.18519	1.26825	0.65848	a2,1c2xc2xc2xc3
25	271	280	2.11592	0.1566	1.18076	0.654795	b1a12,1ch14,5
25	281	284	1.93771	0.141636	1.04764	0.641374	a5,1s21,4
25	285	300	2.4	0.138889	1.15385	0.866667	r5xr5
25	301	304	2.46414	0.0926406	1.1459	0.630845	b5a2,1s24,3
25	313	316	2.46311	0.0870723	1.04298	0.773561	b2a2,1s24,3
25	317	320	2.46223	0.0842031	1.01243	0.819128	b1a2,1s24,3
25	321	322	2.46185	0.0833981	0.99783	0.849418	a2,1s24,3
25	323	324	4.41876	0.025974	1.76581	0.277094	b3a8,1r18
25	325	326	4.67714	0.0250384	1.75253	0.29438	b3a7,1r19
25	327	328	5.49558	0.0244813	1.85261	0.33083	b4a4,1r22
25	329	330	5.74296	0.0242504	1.75948	0.373152	b4a3,1r23
25	337	340	6.00349	0.0233068	1.26336	0.627606	b2a2,1r24
25	341	350	6.24	0.0228938	1	1	r25
26	155	164	1.69021	0.807038	1.09025	0.814239	a5,3-(c2xc4)
26	171	200	1.6054	0.641026	1.00115	0.951775	a2,1c5xc5
26	201	208	1.61538	0.619048	1	1	a2m13
26	209	212	1.92978	0.388278	1.19027	0.654203	b6a3,1c2xs12,5
26	213	220	1.93198	0.379679	1.18299	0.695993	b5a3,1c2xs12,5
26	221	224	1.70599	0.351097	1.04084	0.671378	a8,2p
26	225	228	1.95591	0.334311	1.18902	0.666938	b9a6,1s21,4
26	229	238	1.92716	0.327502	1.1611	0.726804	b8a7,1s20,4
26	239	244	1.89536	0.314879	1.13587	0.745362	b11a8,1s19,4
26	245	248	1.84703	0.298316	1.103	0.737426	a4,2s12,3
26	249	260	2.03846	0.26	1.20455	0.733333	b1c2xm13
26	261	264	1.94069	0.211488	1.14277	0.623793	b4a7,1s20,4
26	265	266	2.27658	0.201515	1.33824	0.607924	b7a9,1s18,3
26	267	268	1.9721	0.19269	1.15724	0.594619	b3a6,1s21,4
26	269	272	2.01076	0.187726	1.17584	0.606377	b1a3,1c2x-(s12,7)
26	273	276	2.17785	0.186354	1.26916	0.630422	a3,1c2xc2xc2xc3
26	277	286	2.03846	0.181818	1.17778	0.692308	c2x-(s13,8)
26	287	288	2.26321	0.142279	1.28566	0.563539	b5a3,1c2xs12,3
26	289	300	2.4052	0.140358	1.24116	0.703435	b1a2,1r5xr5
26	301	308	2.40369	0.127907	1.16899	0.750048	a2,1r5xr5
26	309	312	3.30769	0.100775	1.56364	0.639535	a2r13
26	313	314	2.5772	0.0933968	1.1982	0.618414	b1s26,3
26	315	322	2.46958	0.090538	1.07706	0.711122	b3a3,1s24,3
26	323	338	2.57692	0.0869565	1	0.971014	s26,3
26	339	340	5.48021	0.0237397	1.90371	0.305729	b4a5,1r22
26	341	342	5.73424	0.023399	1.80295	0.343941	b4a4,1r23
26	343	344	6.00338	0.0231806	1.724	0.385665	b4a3,1r24
26	351	354	6.24319	0.0224164	1.25086	0.638816	b2a2,1r25
26	355	364	6.5	0.021978	1	1	r26
27	155	162	1.77778	0.833333	1.14286	0.777778	b1c3x-(r9)
27	169	172	1.57207	0.744446	1.00178	0.772835	a1,2s14,5
27	173	176	1.62758	0.699029	1.03354	0.756802	b1a3,2m13
27	177	200	1.6152	0.657895	1.00468	0.911798	a3,1c5xc5
27	209	216	1.6418	0.45	1.00747	0.733333	b1a1,2s14,4

27	217	224	1.92825	0.406744	1.17533	0.718638	a3,4r7
27	225	228	1.94498	0.35602	1.18158	0.654986	b5a4,1c2xs12,5
27	229	232	1.9835	0.344815	1.20097	0.660606	b9a7,1s21,4
27	233	240	1.97063	0.328392	1.1853	0.682461	b8a7,1s21,4
27	241	246	1.94577	0.317629	1.16458	0.701879	b8a8,1s20,4
27	247	250	1.91965	0.308514	1.14519	0.710373	b12a9,1s19,4
27	251	270	2	0.3	1.17391	0.851852	c3xc3xc3
27	271	272	1.95894	0.214469	1.14612	0.623166	b4a8,1s20,4
27	273	276	2.28807	0.199062	1.33015	0.60582	b7a10,1s18,3
27	277	278	2.01359	0.188626	1.16686	0.587594	b13a6,1s22,4
27	279	284	2.04151	0.18653	1.17186	0.623565	b1a2,1c2xm13
27	285	292	2.04246	0.18413	1.15782	0.677475	a2,1c2xm13
27	293	296	2.25963	0.147444	1.27301	0.569621	b4a10,1s18,3
27	301	308	2.41141	0.139278	1.2503	0.66722	b1a3,1r5xr5
27	309	316	2.40827	0.119426	1.18145	0.663159	a3,1r5xr5
27	317	320	3.31055	0.0962638	1.58151	0.575734	a3,2r13
27	321	322	2.58298	0.093295	1.21798	0.579167	b5a2,1s26,3
27	323	330	2.47636	0.089708	1.0981	0.654515	b4a4,1s24,3
27	331	334	2.58175	0.0872101	1.10973	0.691489	b3a2,1s26,3
27	339	346	2.58019	0.0808364	1.01564	0.826224	b1a2,1s26,3
27	347	348	2.57983	0.0780444	1.00144	0.832932	a2,1s26,3
27	349	350	4.62377	0.0238095	1.77034	0.265427	b3a9,1r19
27	351	352	5.45961	0.023088	1.96255	0.282605	b4a6,1r22
27	353	354	5.71892	0.0226575	1.85293	0.317376	b4a5,1r23
27	355	356	5.99467	0.022345	1.76784	0.354925	b4a4,1r24
27	357	358	6.243	0.0222748	1.68936	0.39823	b4a3,1r25
27	365	368	6.50292	0.021528	1.24622	0.645929	b2a2,1r26
27	369	378	6.74074	0.0211931	1	1	r27

8 Conclusion and Remaining Questions

This report has developed a simple analytical model that allows the comparison of different network topologies. Based on this model and a large set of common topologies, we have identified those that perform the best. We found that the primary parameter affecting both latency and throughput was path length; those topologies with the lowest average path length also tended to have the best performance characteristics.

This paper raises as many questions as it attempts to answer. Certainly the assumption of regular topologies needs to be questioned. Are there irregular graphs that beat spanning-tree optimal regular graphs? In a few cases, the answer is 'yes'. The only such examples we have been able to find so far are some star networks which beat rings for large rings.

Are there any more spanning-tree optimal regular graphs? Undoubtedly, the answer is yes; finding them and adding them to our table of optimal networks would improve the accuracy of our comparisons.

If the assumption of regularity in our analytical model is lifted, a broader range of graphs, with a fractional average degree, can be considered. How can the analytical model be extended to more accurately deal with these graphs? Some initial results show that, with a slightly more complex analytical model, it is possible to calculate to a fair degree of accuracy the optimal throughput for any number of trunk links and any number of switches.

How important is traffic balancing? Again, initial results indicate that traffic balancing can bring some networks closer to optimal performance. For instance, the closest to a spanning-tree optimal graph using the simple routing in this report for seven switches and a degree (b) of 4 yields a throughput of 12/13ths of optimal. Traffic balancing brings this to a full 100% of optimal.

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