

Timing Analysis of Digital Circuits and the Theory of Min-Max Functions

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Recent progress in the timing analysis of digital circuits has involved the use of maximum and minimum timing constraints. We put forward the theory of min-max functions, a non-linear generalization of max-plus algebra, as the correct framework in which to study problems with mixed constraints. We state several new results which enable us to (a) refine and extend the work of Burns on timing metrics for asynchronous circuits and (b) explain, on the basis of general theory, the observations of Szymanski and Shenoy on the verification of clock schedules for synchronous circuits. The work described here was done as part of project STETSON, a joint project between HP Labs and Stanford University on asynchronous hardware design.

1 Introduction

In the last few years considerable progress has been made in the timing analysis of both asynchronous and synchronous digital circuits. Burns, in his thesis, [3], introduced a timing metric for asynchronous circuits arising from Martin's synthesis method, [7], and developed efficient methods for calculating it. On the synchronous front, Sakallah, Mudge and Olukotun (SMO), [9], formulated an elegant mathematical model for latch-controlled circuits with multiple clock phases. Several authors have extended the SMO formulation and we shall be particularly concerned with the work of Szymanski and Shenoy, [11], who have carefully studied the computational aspects of verifying clock schedules in the SMO formulation.

In this paper we describe the mathematical theory of min-max functions and apply it to the problems treated in [3, 11]. The new results which we present include the following:

- conditions for steady-state behaviour (Theorem 2.1);
- a formula for the timing metric (Theorem 2.3);
- all of the above for circuits with mixed maximum and minimum timing constraints (as opposed to the circuits considered in [3] which have only maximum timing constraints);
- a simple geometric explanation, on the basis of general theory, for the observations in [11].

The work described here was undertaken as part of project STETSON, a joint project between Hewlett-Packard Laboratories and Stanford University on asynchronous hardware design.

2 Timing metrics for event-rule systems

We begin this section by introducing min-max functions. We then study an example of an event-rule system and use this to motivate the concept of cycle time for a min-max function. This corresponds to the timing metric studied in [3]. We then use new results in the theory of min-max functions to calculate the cycle time of our example.

The real numbers will be denoted by \mathbf{R} . We shall use the infix operators \vee and \wedge to denote maximum (least upper bound) and minimum (greatest lower bound), respectively, of real numbers.

Definition 2.1 *A min-max expression, f , is a term in the grammar:*

$$f := x, y, \dots \mid f + a \mid f \wedge f \mid f \vee f$$

where x, y, \dots are variables and $a \in \mathbf{R}$.

$x + 5 \vee y - 1$ and $(x + 3 \vee x + 2) \wedge x + 3.14159$ are both min-max expressions, the former of two variables and the latter of one. (We assume that $+$ has higher binding than \wedge or \vee .) However, $2x$ and $x + y$ are not min-max expressions. Expressions can be simplified by using

the associativity and commutativity of \wedge and \vee as well as the fact that addition distributes over both \wedge and \vee :

$$h + (a \wedge b) = h + a \wedge h + b, \quad h + (a \vee b) = h + a \vee h + b. \quad (1)$$

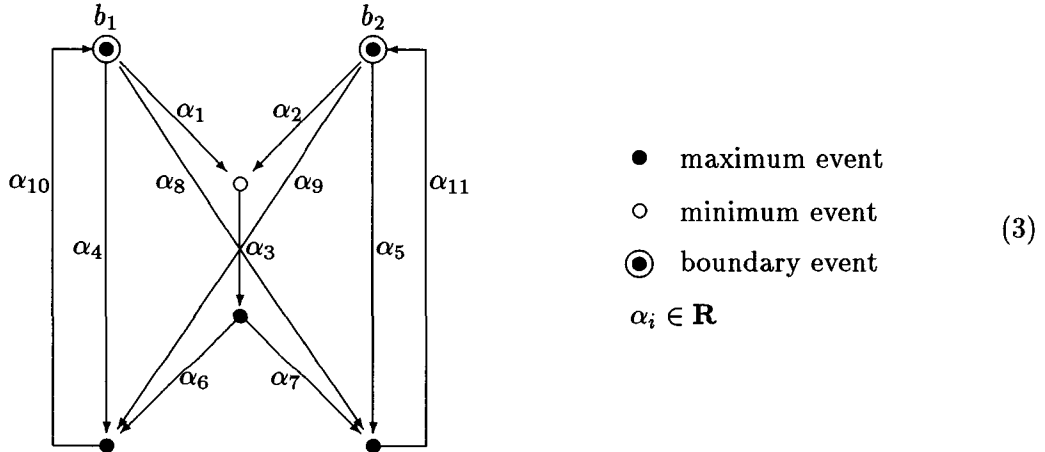
Definition 2.2 A min-max function of dimension d is a function $F : \mathbf{R}^d \rightarrow \mathbf{R}^d$ such that each component F_i is a min-max expression in the d variables x_1, \dots, x_d .

An important property of min-max expressions is that, for any $h \in \mathbf{R}$, $f(x_1+h, x_2+h, \dots, x_d+h) = f(x_1, x_2, \dots, x_d) + h$, [6, Lemma 5.1]. Hence, if F is a min-max function,

$$F(\vec{x} + \vec{c}(h)) = F(\vec{x}) + \vec{c}(h), \quad (2)$$

where $\vec{c}(h)$ is a convenient notation for the constant vector (h, h, \dots, h) .

Min-max functions provide an abstract mathematical setting in which to study problems with maximum and minimum timing constraints. In this section we shall use them to study event-rule systems. The formalism of event-rule systems was introduced in [3] to specify the timing relationships between the events in an asynchronous circuit. Event-rule systems can be generated at various levels in the Martin synthesis procedure, [7], for asynchronous circuits; see [3, Chapter 4] for a thorough discussion. We shall use the following example to motivate the theory of min-max functions.



(Strictly speaking, event-rule systems such as this are more general than those dealt with in [3] and correspond to the timed {AND, OR} automata of [6].) We have adopted a graphical representation for the event-rule system which is convenient for our purposes here. Each annotated edge in the graph represents an event-rule as defined [3, §2.2]. The annotation specifies a time delay, usually a positive real number. A node in the graph is an event which can occur repeatedly: for example, a high-going transition on a wire. If a is an event then a_i will denote the i -th occurrence of the corresponding event. The events are either maximum events, which are required to wait for all preceding events to occur, or minimum events, which are only required to wait for the first preceding event. Certain events are designated as boundary events. If the time at which the boundary events first occur is specified, the information in the graph enables us to calculate the time at which each occurrence of each event takes place. (The boundary events are analogous to the boundary conditions in a difference or differential equation.) The boundary events in this example are denoted b^1 and b^2 .

If $\vec{v} \in \mathbf{R}^2$ is a vector of initial time values, $\vec{v} = (v_1, v_2)$, then under suitable conditions on the event-rule system, [6, Proposition 3.1], [3, Lemma 2.1], there exists a unique “timing simulation”, t_v , such that

- $t_v(b_1^1) = v_1, \quad t_v(b_1^2) = v_2$;
- $t_v(a_i) = \text{OP}\{t_v(b_{i-\epsilon}) + \alpha \mid b \xrightarrow{\alpha} a \text{ is an edge from } b \text{ to } a\}$, where OP is maximum or minimum depending on whether a is a maximum event or a minimum event respectively, and $\epsilon = 1$ if a is a boundary event and $\epsilon = 0$ otherwise.

The timing simulation defines the temporal behaviour of the event-rule system¹. However, it is unacceptable to have to compute t_v in its entirety to understand the time behaviour of the system. It is essential to find a compact way of summarizing the information in t_v : to find, in effect, an asynchronous replacement for the clock rate. The following limit has been used by many authors in different contexts:

$$\lim_{i \rightarrow \infty} \frac{t_{(0,0)}(a_i)}{i}. \quad (4)$$

This can be thought of as the asymptotic average time to the next occurrence of event a . Its reciprocal is the asymptotic average number of occurrences per unit time of event a . This is a reasonable measure of the performance of the asynchronous circuit at the event a . In this paper we shall be concerned with systems where (4) is independent of a and is a property only of the system.

If the event-rule system has only maximum events it is shown in [3] using linear programming techniques that, under suitable conditions, (4) can be calculated as the maximum cycle mean of a certain graph and its value is independent of a . This is the timing metric studied in [3].

We shall extend the results of [3] in two respects. Firstly, we shall consider systems such as (3) with both maximum and minimum events. For such systems, it is suggested that “*simulation must proceed until the circuit has reached a steady-state in order to determine a performance metric . . .*”, [3, page 79]. No analytical methods are presented in [3] to deal with such systems and the quoted remark begs the question of whether such a steady-state is ever reached. This brings us to our second extension. The limit (4) gives only asymptotic information. Since the temporal behaviour of an event rule system is deterministic we could reasonably ask to know more about the finite temporal behaviour of the system. Does its behaviour eventually reach a steady-state or does it jitter for ever around the asymptotic average? Are there some initial values which are particularly good, so that the system stabilizes immediately, or particularly bad, so that it always jitters? Is there an upper bound on how long the system may jitter?

In the remainder of this section we shall use the theory of min-max functions to answer some of these questions and to show that simulation is unnecessary to determine the performance of systems with mixed constraints.

First we must translate (3) into a min-max function. The idea is that the function expresses the relationship between the initial values of the boundary events and the times of their next occurrence. Furthermore, this functional relationship can be iterated.

¹The timing simulation as defined in [3, Lemma 2.1] is actually $t_{(0,0)}$ as defined here. Behaviour under initial values other than $(0, 0)$ is not considered in [3].

Proposition 2.1 (Adapted from [6, Theorem 5.1]) *If b^1, \dots, b^d are the boundary events of an event-rule system and $\vec{v} \in \mathbf{R}^d$ is a vector of initial values, then, under the conditions of Theorem 5.1 of [6], there exists a min-max function F of d variables, such that, for any $k \geq 0$,*

$$F^k(v_1, \dots, v_d) = (t_v(b_{k+1}^1), \dots, t_v(b_{k+1}^d)),$$

It is not difficult to show that (3) gives rise to the following min-max function of dimension 2.

$$\begin{aligned} F_1(x_1, x_2) &= (x_1 + \beta_1 \vee x_2 + \beta_2) \wedge (x_1 + \beta_3 \vee x_2 + \beta_4) \\ F_2(x_1, x_2) &= (x_1 + \delta_1 \vee x_2 + \delta_2) \wedge (x_1 + \delta_3 \vee x_2 + \delta_4) \\ \beta_1 &= \alpha_4 + \alpha_{10} \vee (\alpha_1 + \alpha_3 + \alpha_6 + \alpha_{10}) \\ \beta_2 &= \alpha_9 + \alpha_{10} \\ \beta_3 &= \alpha_4 + \alpha_{10} \\ \beta_4 &= \alpha_9 + \alpha_{10} \vee (\alpha_2 + \alpha_3 + \alpha_6 + \alpha_{10}) \\ \delta_1 &= \alpha_8 + \alpha_{11} \vee (\alpha_1 + \alpha_3 + \alpha_7 + \alpha_{11}) \\ \delta_2 &= \alpha_5 + \alpha_{11} \\ \delta_3 &= \alpha_8 + \alpha_{11} \\ \delta_4 &= \alpha_5 + \alpha_{11} \vee (\alpha_2 + \alpha_3 + \alpha_7 + \alpha_{11}) \end{aligned} \tag{5}$$

In view of Proposition 2.1 the limit, (4), which we want to calculate is given by the vector $\lim_{i \rightarrow \infty} F^i(\vec{v})/i$ whose j -th component is (4) for the boundary event b^j . For certain initial values \vec{v} , this limit can be reduced to an arithmetic calculation.

- F is eventually periodic (EP) at \vec{v} with period $k \geq 1$ if, and only if, for some $s \geq 0$, $F^{k+s}(\vec{v}) = F^s(\vec{v}) + \vec{c}(h)$ for some $h \in \mathbf{R}$, and k is the least integer with this property for any s . If $k = 1$ and $s = 0$ then \vec{v} is a fixed point of F .

If F is EP at \vec{v} , then by using (2) it is easy to see that $F^{k+t}(\vec{v}) = F^t(\vec{v}) + \vec{c}(h)$ for all $t \geq s$. Hence eventual periodicity captures the idea of a steady state behaviour. It follows easily from this that

$$\lim_{i \rightarrow \infty} \frac{F^i(\vec{v})}{i} = \vec{c}\left(\frac{h}{k}\right).$$

Hence, h/k is the limit we want to calculate and this limit is independent of which component of F (ie: which boundary event) we consider. We call it the cycle time of F at \vec{v} .

Proposition 2.2 (Adapted from [5, Theorem 5.1].) *Let F be any min-max function. The cycle time of F is independent of the initial value \vec{v} for all eventually periodic points \vec{v} .*

In the light of this we shall use $\chi(F)$ to denote the cycle time of F . It is important to understand that this is only defined if F is eventually periodic somewhere. It is possible for a function not to be eventually periodic anywhere, in which case the cycle time is undefined. It is an interesting problem to study $\lim_{i \rightarrow \infty} F^i(\vec{v})/i$ in such cases but this falls outside the scope of the present paper; see [4] for a detailed discussion. It turns out that, at least in dimension 2, eventual periodicity is an all-or-nothing phenomenon.

Theorem 2.1 ([6, Theorem 6.2]) *Let F be a min-max function of dimension 2. The following statements hold.*

1. F is EP everywhere $\Leftrightarrow F$ is EP somewhere $\Leftrightarrow F$ has a fixed point.
2. Whenever F is EP its period is at most 2.

Before calculating the cycle time of our example we shall need to know whether F has an eventually periodic point. Equivalently, by Theorem 2.1, we need to know whether F has a fixed point. This is a hard problem in general (see [4] for more results on this) but for functions of dimension 2 there is a simple test.

If F is a min-max function of dimension 2, consider the “auxiliary function” $H(x) = F_1(x, 0) - F_2(x, 0)$. It is not difficult to see that H is piecewise-nice, [5, Definition 3.1]. That is, its graph is connected and composed of finitely many straight lines of slope -1 , 0 or $+1$, [5, Corollary 3.1]. Let $\text{char}_+(F), \text{char}_-(F) \in \{-1, 0, +1\}$ denote, respectively, the slope of $H(x)$ as $x \rightarrow +\infty$ and the slope of $H(x)$ as $x \rightarrow -\infty$. (The slope of the graph of $y = x$ is $+1$ as $x \rightarrow +\infty$ and $+1$ as $x \rightarrow -\infty$.) The characteristic of F is then defined to be the pair $\text{char}(F) = [\text{char}_+(F), \text{char}_-(F)]$. Suppose that $\text{char}_\pm(F) < +1$. It is then easy to see that the graph of H must intersect the main diagonal so that H has a fixed point in the usual sense: $H(a) = a$. But then $F(a, 0) = (a, 0) + \vec{c}(h)$ for some $h \in \mathbf{R}$. Hence F is EP at $(a, 0)$, [5, Proposition 4.1]. It only remains to point out that $\text{char}(F)$ can be easily calculated from the syntactic structure of F as shown in [5, Lemma 4.1].

If we carry out this calculation for our example (3), we find that its characteristic is $[0, 0]$. We conclude that $\chi(F)$ is defined. It remains to calculate it. If F is a max-only function of dimension d , we can always simplify it into canonical form:

$$F_i(x_1, \dots, x_d) = A_{i1} + x_1 \vee \dots \vee A_{id} + x_d \quad (6)$$

where $A_{ij} \in \mathbf{R} \cup \{-\infty\}$. The value $A_{ij} = -\infty$ is used whenever the corresponding term $A_{ij} + x_j$ does not appear. This representation is unique, [5, §2]. The values A_{ij} form a $d \times d$ matrix, A , in max-plus algebra, [1, Chapter 3]. We recall that this is the algebra on $\mathbf{R} \cup \{-\infty\}$ in which the operations $+$ and \times are re-defined so that $+$ is \vee and \times is $+$. We can then rewrite (6) as a vector equation:

$$F(\vec{x}) = A\vec{x}^T.$$

It follows that a fixed point of F is the same thing as a real eigenvector of A . (A real vector, $\vec{x} \in (\mathbf{R} \cup \{-\infty\})^d$, is one for which $x_i \neq -\infty$ for any i .) Moreover, the corresponding eigenvalue is equal to the cycle time of F . The following is one of the fundamental results in max-plus algebra.

Theorem 2.2 (Adapted from [1, Theorem 3.23].) *If A is any square matrix in max-plus algebra then the eigenvalue of any real eigenvector of A is given by the maximum cycle mean of the precedence graph of A .*

The precedence graph of A , [1, Definition 2.8], has one node corresponding to each row of the matrix and has an edge from node j to node i if, and only if, $A_{ij} \neq -\infty$. In this case the edge

is annotated with the number A_{ij} . The cycle mean of a circuit in this graph is the sum of the annotations on the edges in the circuit divided by the number of edges. The maximum cycle mean over all circuits in the graph—it is sufficient to consider only circuits with no repeated nodes—is denoted $\mu(A)$. The reader should have no difficulty in showing that the matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

has $\mu(A) = a \vee (b + c)/2 \vee d$.

Max-plus algebra is a highly developed theory, [1], which seems to be largely unknown to those working in timing analysis. (Compare [3, Theorem 2.8] with Theorem 2.2 above.) It is a powerful tool for studying systems with only maximum constraints and is an essential foundation for the deeper results in the theory of min-max functions.

We have shown that if F is a max-only function and A is the corresponding matrix in max-plus algebra then, provided $\chi(F)$ is defined, $\chi(F) = \mu(A)$. Now suppose that F is an arbitrary min-max function of dimension d . We can also write it in canonical form (similar to conjunctive normal form in propositional calculus) as shown below:

$$F_i(x_1, \dots, x_d) = (A_{i1}^1 + x_1 \vee \dots \vee A_{id}^1 + x_d) \wedge \dots \wedge (A_{i1}^{l(i)} + x_1 \vee \dots \vee A_{id}^{l(i)} + x_d).$$

Here $l(i)$, for $1 \leq i \leq d$, is a count of the number of conjunctions in the expression F_i . These expressions are unique up to the commutativity of \wedge , provided certain simple conditions are satisfied, [5, Theorem 2.1]. We can hence associate to F a max-plus matrix, M , by choosing for the i -th row of the matrix, M_i , any of the $l(i)$ conjunctions in the expression for F_i . Hence $M_i = A_{i\cdot}^k$ where $1 \leq k \leq l(i)$ and k need not be fixed as i changes. It is clear that there are $\prod_{1 \leq i \leq d} l(i)$ such matrices that we could construct. They are called the max-only projections of F and they are uniquely defined.

Theorem 2.3 ([5, Theorem 5.1].) *Let F be a min-max function of any dimension and let $M(1), M(2), \dots, M(N)$ be the associated max-only projections. If F has a fixed point then*

$$\chi(F) = \mu M(1) \wedge \mu M(2) \wedge \dots \wedge \mu M(N).$$

Since example (3) is already in canonical form it is easy to write down its four max-plus projections. Since we know that it has a fixed point we can use Theorem 2.3 to compute the cycle time. We find that

$$\chi(F) = (\beta_1 \vee \frac{\beta_2 + \delta_1}{2} \vee \delta_2) \wedge (\beta_1 \vee \frac{\beta_2 + \delta_3}{2} \vee \delta_4) \wedge (\beta_3 \vee \frac{\beta_4 + \delta_1}{2} \vee \delta_2) \wedge (\beta_3 \vee \frac{\beta_4 + \delta_3}{2} \vee \delta_4)$$

which gives a closed-form solution for the cycle time of our example.

3 Clock schedule verification

The SMO formulation for multi-phase clocking in latched synchronous circuits, [9], has generated several papers on the problem of verifying clock schedules, [8, 10, 11]. These papers

have two things in common: the verification problem is reduced to finding the fixed point of a min-max function and the fixed point is found by generating the series of values

$$x, F(x), F^2(x), \dots$$

and hoping that the process stops at the fixed point. It was pointed out in [11] that (a) the process may not stop, (b) it may take unboundedly long to stop and (c) even if it does stop the fixed point may not be unique.

None of these observations is surprising. They apply to any min-max function. Furthermore, in more complex examples, the iterative process may both fail to stop and yet not go anywhere: it might cycle. In this section we shall give simple geometric explanations of these observations and present some results about the structure of the fixed points of F in dimension 2. For the applications to clock schedule verification, the dimension of the resulting min-max function is proportional to the number of latches in the circuit. Hence the discussion here is largely heuristic but, we hope, none the less useful for that.

We begin by explaining the clock-schedule verification problem and showing that it can be formulated in terms of min-max functions. Let n be the number of latches in a synchronous circuit. Assume that the latches are numbered from 1 to n and that $i \rightarrow j$ is the “fans out to” relation on latches. That is, $i \rightarrow j$, if, and only if, there is a path of combinational logic from the output of i to the input of j . Define the min-max functions $D, d : \mathbf{R}^{n+1} \rightarrow \mathbf{R}^{n+1}$ by the following equations:

$$\begin{aligned} D_i(\vec{x}) &= \max\{x_j + \Lambda_{j,i}\} \wedge (x_{n+1} + B_i) \quad \text{for } 1 \leq i \leq n \\ D_{n+1}(\vec{x}) &= x_{n+1} \\ d_i(\vec{x}) &= \min\{x_j + \lambda_{j,i}\} \wedge (x_{n+1} + B_i) \quad \text{for } 1 \leq i \leq n \\ d_{n+1}(\vec{x}) &= x_{n+1} \end{aligned}$$

where $\Lambda_{i,j}$, $\lambda_{i,j}$ and B_i are constants defined by the clocking schedule to be verified and the minimum and maximum delays through the combinational logic, [11, Figure 2]. x_{n+1} is a dummy variable whose only purpose is to make it clear that D and d are genuine min-max functions. Because the value of x_{n+1} never changes, it is clear that if the cycle time is defined then $\chi(D) = 0$. Similarly for d .

It follows from [11, Figure 2] that, if $D(\vec{x}) = \vec{x}$ then $x_i - x_{n+1}$ is the latest signal departure time from latch i . Similarly, if $d(\vec{x}) = \vec{x}$ then $x_i - x_{n+1}$ is the earliest signal departure time from latch i . If the fixed points can be found and the departure times (strictly speaking, the arrival times) satisfy the setup and hold constraints for each latch, then the clock schedule is verified.

D is a max-only function. This forms the basis of the analysis presented in [11]. Many of the results which were proved by *ad hoc* methods in that paper are instances of general theorems in max-plus algebra. Compare, for instance, [11, Theorem 3.3] with [1, Theorem 3.17] and [11, Lemma 2.7] with [1, Theorem 3.20]. Nevertheless, the algebraic methods of [1] are not ideally suited to discussing the iterative behaviour of D and d . In the rest of this section we will give some intuition for the observations made in [11] by discussing the iterative behaviour of functions in dimension 2.

Let F be a min-max function of dimension 2 and let H be its auxiliary function. It is easier—and equivalent—to iterate H rather than F . (It is worth noting that any piecewise-nice function is the auxiliary function of some min-max function of dimension 2, [5, Theorem 3.1], which

gives an idea of the extent of the set of min-max functions.) Consider the graph of H plotted on the x, y plane in the usual way. H has a fixed point where its graph crosses the main diagonal, $y = x$. It is clear that we can draw a piecewise nice function which does not cross the main diagonal. Such a function does not have a fixed point and, by the first part of Theorem 2.1, the values $x, F(x), F^2(x), \dots$, never converge to a fixed point.

Now suppose that the graph of H has a segment of slope $+1$. This segment is parallel to the main diagonal. If the segment is close to the main diagonal then it is easy to see that, in graphical terms, the iterative process describes a “staircase” between the segment and the main diagonal which must eventually converge to a fixed point, if such a point exists at all. The closer the segment gets to the main diagonal, the longer the staircase becomes. It follows that the time taken in the iterative process may become unboundedly long.

In the limit, the segment of slope $+1$ will coincide with the main diagonal. There are then several fixed points. Indeed, it must be the case that there is either a unique fixed point or there are infinitely many fixed points. This was pointed out in [11, Theorem 3.11] by more complex reasoning. It was also pointed out that the fixed points could become unboundedly large. This, however, is a specific property of the clock schedule verification problem. If the segment of slope $+1$ is bounded in size then the fixed points form a bounded subset.

Proposition 3.1 (*Adapted from [6, Proposition 6.2].*) *Let F be a min-max function of dimension 2 and let*

$$P_k = \{u \in \mathbf{R} \mid F \text{ is periodic at } (u, 0) \text{ with period } \leq k\}.$$

Then P_k is a closed, connected subset of \mathbf{R} .

In higher dimensions the structure of P_k appears quite complex. It is shown in [11, Theorem 2.9] that D and d have a least fixed point if they have a fixed point at all. This is too much to hope for in a general min-max function. It is worth noting that non-uniqueness of fixed points always occurs in the limit as iterative convergence becomes unboundedly long.

Finally, consider what happens if a segment of slope -1 intersects the main diagonal. There is then an unique fixed point but any iteration towards it is likely to get trapped in a cycle of period 2. However, it is easy to see by symmetry that the following result holds.

Proposition 3.2 (*[6, Proposition 6.1]*) *Let F be a min-max function of dimension 2. If F has a periodic point of period 2 then it has a unique fixed point which lies at the centroid of any 2-period.*

Hence if \vec{v} is a periodic point of period 2 then the fixed point can be immediately calculated as $(\vec{v} + F(\vec{v}))/2$. This result is conjectured to hold in higher dimensions. For max-only functions rather more is known, [2].

This completes our discussion of the pitfalls of iterative convergence towards fixed points. We have explained in simple terms the observations made in [11].

4 Conclusion

The theory of min-max functions is still in its infancy. We hope the account given here will suggest new practical applications and encourage others to delve into it and to develop it further.

Copies of some of the papers listed below can be obtained by anonymous ftp from `hplose.hpl.hp.com` (IP address 15.254.100.100) in the directory “pub/jhcg”. Use “ftp” as user name and password.

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