# Attosecond-Resolution Measurement of Polarization Mode Dispersion in Short Sections of Optical Fiber

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A new technique is presented for measurement of the differential group delay (DGD) between principal states of polarization of an optical device, with a demonstrated accuracy and resolution of roughly 50 attoseconds ($50 \times 10^{-18}$ s). Accuracy was directly assessed by measurement of a crystal quartz DGD standard. For the first time, this allows complete measurement of the polarization mode dispersion (PMD) of straight, sub-meter lengths of optical fiber. PMD can be reliably measured only when the fiber is straight, since spooling of the fiber changes its PMD by introducing additional mode coupling. Measurement of four fiber samples revealed correlation between DGD and fiber core ovality.

## 1 Introduction

The importance of accurate, repeatable measurement of polarization mode dispersion (PMD) to the design of single-mode fiber data links continues to increase as these links advance in length, complexity and data rate. PMD, which may limit transmission bandwidths in practical systems, is a fundamental characteristic of a fiber or device that describes its propensity to split a narrow-band optical input pulse into two temporally separate output pulses according to state of polarization (SOP). PMD is completely characterized by specifying of a pair of principal states of polarization (PSP) and a differential group delay (DGD) as a function of wavelength.

Current PMD measurement techniques are not suitable for measurement of DGD's of less than 3 fs [1-3]. This limitation may at first seem inconsequential in view of the much longer pulses commonly used in fiber-optic transmission, but there exist important applications which will benefit from the higher temporal resolution and accuracy afforded by the technique to be described. In the manufacture and test of single-mode fiber, sub-femtosecond resolution is essential to measure the DGD expected of a fiber less than one or two meters long. A longer fiber would be very awkward to test without placing it on a spool, but the spooling process changes the fiber's PMD by increasing the level of bending-induced mode coupling [4,5]. Furthermore, when hundreds or even thousands of elements such as isolators, amplifiers and taps are concatenated, limits on the DGD of the entire system may lead to sub-femtosecond resolution requirements for DGD measurements of individual components.

## 2 Theory

The author has previously shown [3] that PMD can be obtained by measuring the Jones matrix $\mathbf{T}(\omega_n)$ of an unknown device at a series of at least two discrete optical radian frequencies $\omega_n$. The DGD $\Delta\tau_n$ over the frequency interval $\omega_{n-1}$ to $\omega_n$ is then given by

$$\Delta\tau_n = \left| \frac{\mathrm{Arg}(\rho_1 / \rho_2)}{\omega_n - \omega_{n-1}} \right| , \qquad (1)$$

where $\rho_1$ and $\rho_2$ are the eigenvalues of the matrix product $\mathbf{T}(\omega_n)\mathbf{T}^{-1}(\omega_{n-1})$ and Arg denotes the argument function, e.g. $\mathrm{Arg}(\alpha\ \mathrm{Exp}(i\theta)) = \theta$ . The eigenvectors of $\mathbf{T}(\omega_n)\mathbf{T}^{-1}(\omega_{n-1})$ locate the output PSPs as a function of frequency or wavelength.

Jones matrices can be measured using the procedure given by R. C. Jones for experimentally determining the matrix of an unknown linear, time-invariant optical device [6]. This procedure involves stimulating the unknown device with three known linear input SOP's and measuring the corresponding output SOP's. From these six SOP's, the device Jones matrix can be calculated to within a multiplicative complex constant.

In addition to its practical convenience, a polarimeter whose optical input is coupled to experiments through a single-mode fiber has inherent advantages which can contribute to highly accurate measurements. An open beam polarimeter requires careful spatial alignment of an optical input beam to correctly illuminate the detector areas of the polarimeter for proper operation. In contrast, when the input is coupled through a single-mode fiber, an imperfect optical launch into the fiber will result in a simple loss of power instead of erroneous polarization measurements. However, a disadvantage of a fiber-coupled polarimeter is that any practical single-mode input fiber will exhibit non-zero PMD, which will act as a bias on top of the characteristics of the device to be measured. Typically, a few meters of low-birefringence fiber including a fiber connector and a number of twists and turns exhibits a DGD of 3 to 6 fs. While this is insignificant compared to the DGD expected of many devices and long fibers, the DGD of short sections of fiber to be tested may be less than this bias DGD. Moreover, the bias DGD cannot be simply subtracted from the final measurement because the DGD's of concatenated devices add together in a manner strongly dependent on the relative orientations of their principal states of polarization [7].

We now explore how to use a fiber-coupled polarimeter to measure the PMD of a device or fiber under test (FUT) independently of the PMD of the coupling fiber. Referring to Fig. 1, polarization transformations presented by the FUT and the coupling fiber are represented by the Jones matrices $\mathbf{F}$ and $\mathbf{R}$, respectively. Although direct measurement of $\mathbf{F}$ is impossible, it can easily be calculated from the measurable matrices $\mathbf{R}$ and $\mathbf{M}=\mathbf{RF}$. Using subscripts 1 and 2 to denote measurement wavelengths, it is easily shown that $\mathbf{F}_1\mathbf{F}_2^{-1}$, whose eigenvalues yield the FUT's DGD through Eq. (1), is equivalent to $\mathbf{R}_1^{-1}\mathbf{M}_1\mathbf{M}_2^{-1}\mathbf{R}_2$.

## 3 Experimental results and discussion

The experimental apparatus is shown in Fig. 1. This technique requires that the birefringent transformation presented by SMF1 must remain constant, so the fiber end was first rigidly fixed in position and SMF1 was immobilized by taping it to the optical bench. Two Fabry-Perot diode lasers were used, each with a spectrum in which 80% of the power was concentrated within a span of 3 nm. The laser center wavelengths were 1304 nm and 1540 nm. The two lasers were coupled into SMF2 via a wavelength-dependent fused coupler, and the other end of SMF2 was attached to a movable arm. This arm carried a lens and provided a shelf on which a small polarizer could be placed. The polarizer was mounted in a triangular housing so that it could be set upon the shelf in one of three angular orientations. The entire arm, including the polarizer, lens, and SMF2, could be adjusted to maximize the power received by the polarimeter. The FUT was butt-coupled to SMF1. Realignment time was minimized by locating each end of the FUT in a V-groove, so that coupling of a new fiber could be quickly optimized by slight adjustment of the two translation stages.

Six SOP's were measured with the fiber or device under test in place, one SOP for each wavelength at each polarizer orientation. These measurements are sufficient to calculate a Jones matrix product $\mathbf{M}_1=\mathbf{R}_1\mathbf{F}_1$ at 1304 nm and a matrix $\mathbf{M}_2=\mathbf{R}_2\mathbf{F}_2$ at 1540 nm. After one or more fibers were measured and removed, the movable arm was repositioned to launch light directly into SMF1. Again, six SOP's were measured, allowing calculation of $\mathbf{R}_1$ and $\mathbf{R}_2$.

Estimation of the DGD measurement accuracy requires an accurately known DGD standard. An antireflection-coated crystal quartz slab can serve as such a standard because the optical properties of quartz are accurately known. The following dispersion formula, kindly supplied by V. Vats of the Karl Lambrecht Corporation, allows calculation of the refractive indices of quartz as a function of the wavelength $\lambda$ given in micrometers:

$$n^2(\lambda) = A + B\lambda^2 + C\lambda^4 + \frac{D}{\lambda^2} + \frac{E\lambda^2(\lambda^2 - F)}{(\lambda^2 - F)^2 + G\lambda^2} \tag{2}$$

The dispersion constants A through G are given in Table 1. Group indices $n_g$ were calculated from the refractive indices $n$ using the relation $n_g = n - \lambda(dn/d\lambda)$, using either the ordinary or extraordinary dispersion constants as appropriate. The optical thickness of the slab was first determined by measuring the retardance of the slab at 20-nm intervals over the range 1460 nm to 1560 nm. Choosing a slab thickness $z = 0.366379$ mm, the retardances calculated using Eq. (2) differed from the six measured retardances by less than 0.15 degree at each of these six wavelengths. When Jones matrices are measured at wavelengths $\lambda_1$ and $\lambda_2$, the Jones matrix eigenanalysis relation (1) yields a DGD $\Delta\tau$ given by

$$\Delta\tau = \frac{z}{c(\lambda_2 - \lambda_1)} \int_{\lambda_1}^{\lambda_2} \Delta n_g(\lambda)\, d\lambda \tag{3}$$

where $\Delta n_g$ is the wavelength-dependent difference between the extraordinary and ordinary group indices and $c$ is the vacuum speed of light. Numerical integration yields $\Delta\tau = 11.423$ fs for the quartz slab and wavelengths used in this experiment.

The DGD of the quartz slab was measured using the apparatus shown in Fig. 1, with the quartz slab substituted for the FUT. The slab was measured at five different angular orientations at relative angles 0, 35, 70, 105, and 140 degrees. DGD is unaffected by rotation of the slab, so the five results give some indication of the repeatability of the overall measurement. Using the eigenvalues of $\mathbf{R}_1^{-1}\mathbf{M}_1\mathbf{M}_2^{-1}\mathbf{R}_2$ in Eq. (1) yielded the five results 11.468, 11.379, 11.377, 11.402, and 11.444 fs, with a mean of 11.414 fs. Over these five results the largest deviation from the calculated DGD is 0.046 fs, roughly indicating the accuracy of this technique.

To demonstrate the utility of this method toward the measurement of short sections of single-mode fiber, the DGD of four different fibers was measured as a function of the location along the fiber. Ten 30.2-cm sections of fiber were sequentially cleaved from each fiber sample, and each section was measured while held straight in the apparatus of Fig. 1. Results are plotted in Fig. 2, where each line segment indicates the DGD and position of the corresponding fiber section. Fibers D, C and B were samples of experimental fibers selected for large core ovality. The core ovalities of these fibers were approximately 0.157, 0.093 and 0.012, respectively. (Approximating the core cross section as an ellipse with minor and major axes a and b, we define ovality as $2(b-a)/(b+a)$.) It can be seen that core ovality correlates with DGD, as might be expected given the geometric contribution to fiber birefringence. Fiber A was a sample of Corning SMF-28, and exhibited a consistently low DGD near 0.51 fs for each 30.2-cm section. This technique allows measurement of DGD values less than 0.1 fs, whereas low values of ovality are difficult to measure. Moreover, the true DGD is measured, (as well as the principal states of polarization,) including the effects of both geometric and stress-induced birefringence. The position-dependent DGD exhibited by fiber D may reflect a position-dependent ovality, or it may be caused by an uneven residual stress near the fiber core. PMD of km-length fibers cannot be simply extrapolated from sub-meter measurements: Power transfer between principal states must also be considered [4].

Three sources of gross error can be avoided by careful experimental technique. First, the fixed fiber leading from the fiber or device under test to the polarimeter must remain in a fixed position throughout all measurements, so that the reference frame matrices $\mathbf{R}_k$ do not change. A second type of gross error occurs when the polarization transmitted through the FUT is strongly dependent on the position of either XYZ translation stage, i.e. on the details of the optical coupling. This behavior indicates that cladding modes or higher-order core modes are not sufficiently attenuated in the short FUT, making accurate measurements impossible. In the reported measurements, as the optical coupling was adjusted to reduce the transmitted power by 3 dB from its peak value, the measured SOP typically moved less than one degree on the Poincare sphere. In other, unreported measurements where the effects of undesired modes were observed, these effects were eliminated by forming the FUT into a loop to lower the effective cutoff wavelength and improve the mode-stripping action of the plastic jacket. Of course, bending the fiber in this way will induce a small amount of DGD through the photo-elastic effect, but a 1-meter section bent into a single loop would suffer a stress-induced DGD of only 0.07 fs, according to [8]. Finally, a third source of gross error arises from the possibility of alias differential phase measurements caused by the finite wavelength difference between measured Jones matrices along with the $2\pi$ ambiguity of the multiple-valued argument function in Eq. (1). Just as a time-sampled sequence can lead to aliasing in the frequency domain, a frequency-sampled Jones matrix sequence can lead to aliasing in the time (DGD) domain. The practical limitation set by aliasing is to require the argument of the ratio of the eigenvalues of $\mathbf{R}_1^{-1}\mathbf{M}_1\mathbf{M}_2^{-1}\mathbf{R}_2$ to be less than $\pi$ in absolute value. This limitation, together with the optical frequency difference of the two wavelengths used in this experiment, required the DGD of the FUT-SMF1 concatenation to be less than 14.2 fs.

## 4 Summary

In conclusion, this technique for the first time affords a DGD resolution and accuracy roughly of the order of 50 attoseconds, allowing accurate measurement of short (unspooled) sections of single-mode fiber. Accuracy was directly assessed by measurement of a crystal quartz DGD standard. This technique allows measurement of any linear, time-invariant device, even in the presence of relatively large DGD in connecting fibers. Measurements of four fibers showed a correlation between DGD and fiber core ovality, and revealed position-dependent PMD in one sample. These measurement capabilities have obvious application to testing and manufacture of single-mode fiber.

## 5 Acknowledgment

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## 6 References

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| | ORDINARY | EXTRAORDINARY |
|---|---|---|
| A | 2.6712295 | 2.6964507 |
| B | −0.011090009 | −0.011596199 |
| C | −0.0001024662 | −0.0001206215 |
| D | 0.011838573 | 0.012383549 |
| E | −0.31467511 | −0.31280678 |
| F | 0.018592511 | 0.019120036 |
| G | 0.015379669 | 0.015545792 |

Table I. Dispersion constants of crystal quartz for use in Eq. (2).

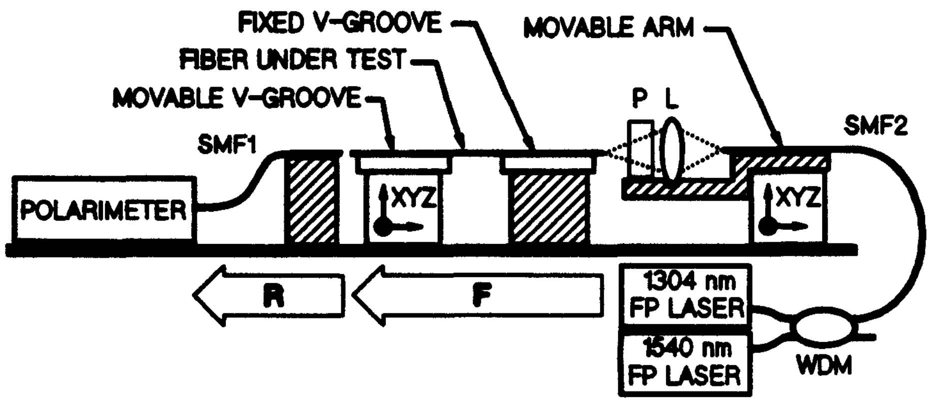


Fig. 1. Apparatus used for measurement of PMD using Jones matrix eigenanalysis. Jones matrices **R** and **F** represent the polarization transformations caused by transmission through SMF1 and the fiber under test, respectively. After measurement of $\mathbf{R}_1\mathbf{F}_1$ and $\mathbf{R}_2\mathbf{F}_2$ with the fiber under test in place, the test fiber is removed, the movable arm is repositioned to launch light directly into SMF1, and $\mathbf{R}_1$ and $\mathbf{R}_2$ are measured. SMF: single-mode fiber; L: lens; P: linear polarizer; WDM: wavelength division multiplexer; XYZ: translation stage.

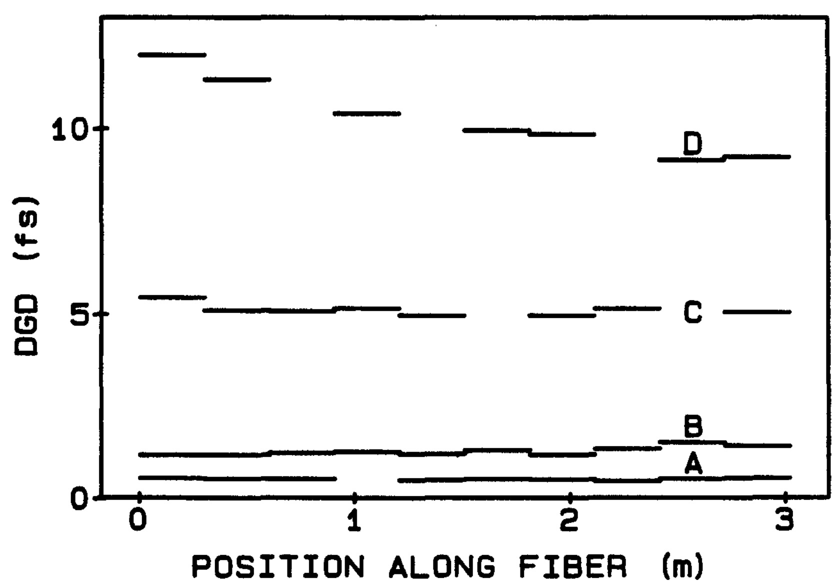


Fig. 2. Differential group delay DGD versus position along the fiber for four fibers of various core ovalities. Line segments indicate relative positions of the fiber sections; broken fibers resulted in some missing segments. A: Corning SMF-28, unknown ovality. Experimental fibers; B: 0.012 ovality; C: 0.093 ovality; D: 0.157 ovality.