Action and Passion at a Distance

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We consider two models, going beyond quantum mechanics, of nonlocality – "superquantum" correlations, and nonlocal "jamming" of correlations – and derive new results for the jamming model. In one space dimension, jamming allows reversal of the sequence of cause and effect: in higher dimensions, however, effect never precedes cause.

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ACTION AND PASSION AT A DISTANCE

An Essay in Honor of Professor Abner Shimony*

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Abstract

Quantum mechanics permits nonlocality—both nonlocal correlations and nonlocal equations of motion—while respecting relativistic causality. Is quantum mechanics the unique theory that reconciles nonlocality and causality? We consider two models, going beyond quantum mechanics, of nonlocality—"superquantum" correlations, and nonlocal "jamming" of correlations—and derive new results for the jamming model. In one space dimension, jamming allows reversal of the sequence of cause and effect; in higher dimensions, however, effect never precedes cause.

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I. INTRODUCTION

Why is quantum mechanics what it is? Many a student has asked this question. Some physicists have continued to ask it. Few have done so with the passion of Abner Shimony. "Why is quantum mechanics what it is?" we, too, ask ourselves, and of course we haven't got an answer. But we are working on an answer, and we are honored to dedicate this work to you, Abner, on your birthday.

What is the problem? Quantum mechanics has an axiomatic structure, exposed by von Neumann, Dirac and others. The axioms of quantum mechanics tell us that every state of a system corresponds to a vector in a complex Hilbert space, every physical observable corresponds to a linear hermitian operator acting on that Hilbert space, etc. We see the problem in comparison with the special theory of relativity. Special relativity can be deduced in its entirety from two axioms: the equivalence of inertial reference frames, and the constancy of the speed of light. Both axioms have clear physical meaning. By contrast, the numerous axioms of quantum mechanics have no clear physical meaning. Despite many attempts, starting with von Neumann, to derive the Hilbert space structure of quantum mechanics from a "quantum logic", the new axioms are hardly more natural than the old.

Abner Shimony offers hope, and a different approach. His point of departure is a remarkable property of quantum mechanics: nonlocality. Quantum correlations display a subtle nonlocality. On the one hand, as Bell[1] showed, quantum correlations could not arise in any theory in which all variables obey relativistic causality[2]. On the other hand, quantum correlations themselves obey relativistic causality—we cannot exploit quantum correlations to transmit signals at superluminal speeds[3] (or at any speed). That quantum mechanics combines nonlocality and causality is wondrous. Nonlocality and causality seem *prima facie* incompatible. Einstein's causality contradicts Newton's action at a distance. Yet quantum correlations do not permit action at a distance, and Shimony[4] has aptly called the nonlocality manifest in quantum correlations "passion at a distance". Shimony has raised the question whether nonlocality and causality can peacefully coexist in any other theory besides quantum mechanics[4, 5].

Quantum mechanics also implies nonlocal equations of motion, as Yakir Aharonov[6, 7] has pointed out. In one version of the Aharonov-Bohm effect[8], a solenoid carrying an isolated magnetic flux, inserted between two slits, shifts the interference pattern of electrons passing through the slits. The electrons therefore obey a nonlocal equation of motion: they never pass through the flux yet the flux affects their positions when they reach the screen[9]. Aharonov has shown that the solenoid and the electrons exchange a physical quantity, the *modular momentum*, nonlocally. In general, modular momentum is measurable and obeys a nonlocal equation of motion. But when the flux is constrained to lie between the slits, its modular momentum is completely uncertain, and this uncertainty is just sufficient to keep us from seeing a violation of causality. Nonlocal equations of motion imply action at a distance, but quantum mechanics manages to respect relativistic causality. Still, nonlocal equations of motion seem so contrary to relativistic causality that Aharonov[7] has asked whether quantum mechanics is the *unique* theory combining them.

The parallel questions raised by Shimony and Aharonov lead us to consider models for theories, going beyond quantum mechanics, that reconcile nonlocality and causality. Is quantum mechanics the only such theory? If so, nonlocality and relativistic causality together imply quantum theory, just as the special theory of relativity can be deduced in its entirety from two axioms[7]. In this paper, we will discuss model theories[10, 11, 12] manifesting nonlocality while respecting causality. The first model manifests nonlocality in the sense of Shimony: nonlocal correlations. The second model manifests nonlocality in the sense of Aharonov: nonlocal dynamics. We find that quantum mechanics is *not* the only theory that reconciles nonlocality and relativistic causality. These models raise new theoretical and experimental possibilities. They imply that quantum mechanics is only one of a class of theories combining nonlocality and causality; in some sense, it is not even the most nonlocal of such theories. Our models raise a question: What is the minimal set of physical principles—"*nonlocality* plus *no signalling* plus *something else simple and fundamental*" as Shimony put it[13]—from which we may derive quantum mechanics?

II. NONLOCALITY I: NONLOCAL CORRELATIONS

The Clauser, Horne, Shimony, and Holt[14] form of Bell's inequality holds in any classical theory (that is, any theory of local hidden variables). It states that a certain combination of correlations lies between -2 and 2:

$$-2 \leq E(A,B) + E(A,B') + E(A',B) - E(A',B') \leq 2 \quad . \tag{1}$$

Besides 2, two other numbers, $2\sqrt{2}$ and 4, are important bounds on the CHSH sum of correlations. If the four correlations in Eq. (1) were independent, the absolute value of the sum could be as much as 4. For quantum correlations, however, the CHSH sum of correlations is bounded [15] in absolute value by $2\sqrt{2}$. Where does this bound come from? Rather than asking why quantum correlations violate the CHSH inequality, we might ask why they do not violate it *more*. Suppose that quantum nonlocality implies that quantum correlations violate the CHSH inequality at least sometimes. We might then guess that relativistic causality is the reason that quantum correlations do not violate it maximally. Could relativistic causality restrict the violation to $2\sqrt{2}$ instead of 4? If so, then nonlocality and causality would together determine the quantum violation of the CHSH inequality, and we would be closer to a proof that they determine all of quantum mechanics. If not, then quantum mechanics cannot be the unique theory combining nonlocality and causality. To answer the question, we ask what restrictions relativistic causality imposes on joint probabilities. Relativistic causality forbids sending messages faster than light. Thus, if one observer measures the observable A, the probabilities for the outcomes $A = 1$ and $A = -1$ must be independent of whether the other observer chooses to measure B or B'. However, it can be shown[10, 16] that this constraint does not limit the CHSH sum of quantum correlations to $2\sqrt{2}$. For example, imagine a "superquantum" correlation function E for spin measurements along given axes. Assume E depends only on the relative angle θ between axes. For any pair of axes, the outcomes $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ are equally likely, and similarly for $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$. These four probabilities sum to 1, so the probabilities for $|\uparrow\downarrow\rangle$

and $|\downarrow\downarrow\rangle$ sum to 1/2. In any direction, the probability of $|\uparrow\rangle$ or $|\downarrow\rangle$ is 1/2 irrespective of a measurement on the other particle. Measurements on one particle yield no information about measurements on the other, so relativistic causality holds. The correlation function then satisfies $E(\pi-\theta) = -E(\theta)$. Now let $E(\theta)$ have the form

(i) $E(\theta) = 1$ for $0 \leq \theta \leq \pi/4$;

(ii) $E(\theta)$ decreases monotonically and smoothly from 1 to -1 as θ increases from $\pi/4$ to $3\pi/4$;

(iii) $E(\theta) = -1$ for $3\pi/4 \leq \theta \leq \pi$.

Consider four measurements along axes defined by unit vectors $\hat{a}'$, $\hat{b}$, $\hat{a}$, and $\hat{b}'$ separated by successive angles of $\pi/4$ and lying in a plane. If we now apply the CHSH inequality Eq. (1) to these directions, we find that the sum of correlations

$$E(\hat{a},\hat{b}) + E(\hat{a}',\hat{b}) + E(\hat{a},\hat{b}') - E(\hat{a}',\hat{b}') = 3E(\pi/4) - E(3\pi/4) = 4 \qquad (2)$$

violates the CHSH inequality with the maximal value 4. Thus, a correlation function could satisfy relativistic causality and still violate the CHSH inequality with the maximal value 4.

III. NONLOCALITY II: NONLOCAL EQUATIONS OF MOTION

Although quantum mechanics is not the unique theory combining causality and nonlocal correlations, could it be the unique theory combining causality and nonlocal equations of motion? Perhaps the nonlocality in quantum dynamics has deeper physical signficance. Here we consider a model that in a sense combines the two forms of nonlocality: nonlocal equations of motion where one of the physical variables is a nonlocal correlation. *Jamming*, discussed by Grunhaus, Popescu and Rohrlich[11] is such a model. The jamming paradigm involves three experimenters. Two experimenters, call them Alice and Bob, make measurements on systems that have locally interacted in the past. Alice's measurements are spacelike separated from Bob's. A third experimenter, Jim (the jammer), presses a button on a black box. This event is spacelike separated from Alice's measurements and from Bob's. The black box acts at a distance on the correlations between the two sets of systems. For the sake of definiteness, let us assume that the systems are pairs of spin-1/2 particles entangled in a singlet state, and that the measurements of Alice and Bob yield violations of the CHSH inequality, in the absence of jamming; but when there is jamming, their measurements yield classical correlations (no violations of the CHSH inequality).

Indeed, Shimony[4] considered such a paradigm in the context of the experiment of Aspect, Dalibard, and Roger[17]. To probe the implications of certain hidden-variable theories[18], he wrote, "Suppose that in the interval after the commutators of that experiment have been actuated, but before the polarization analysis of the photons has been completed, a strong burst of laser light is propagated transverse to but intersecting the paths of the propagating photons.... Because of the nonlinearity of the fundamental material medium which has been postulated [in these models], this burst would be expected to generate excitations, which could conceivably interfere with the nonlocal propagation that is responsible for polarization correlations." Thus, Shimony asked whether certain hidden-variable theories would predict classical correlations after such a burst. (Quantum mechanics, of course, does not.)

Here, our concern is not with hidden-variable theories or with a mechanism for jamming; rather, we ask whether such a nonlocal equation of motion (or one, say, allowing the third experimenter nonlocally to create, rather than jam, nonlocal correlations) could respect causality. The jamming model[11] addresses this question. In general, jamming would allow Jim to send superluminal signals. But remarkably, some forms of jamming would not; Jim could tamper with nonlocal correlations without violating causality. Jamming preserves causality if it satisfies two constraints, the *unary* condition and the *binary* condition. The unary condition states that Jim cannot use jamming to send a superluminal signal that Alice (or Bob), by examining her (or his) results alone, could read. To satisfy this condition, let us assume that Alice and Bob each measure zero average spin along any axis, with or without jamming. In order to preserve causality, jamming must affect correlations only, not average measured values for one spin component. The binary condition states that Jim cannot use jamming to send a signal that Alice and Bob *together* could read by comparing their results, if they could do so in less time than would be required for a light signal to reach the place where they meet and compare results. This condition restricts spacetime configurations for jamming. Let a, b and j denote the three events generated by Alice, Bob, and Jim, respectively: a denotes Alice's measurements, b denotes Bob's, and j denotes Jim's pressing of the button. To satisfy the binary condition, the overlap of the forward light cones of a and b must lie entirely *within* the forward light cone of j. The reason is that Alice and Bob can compare their results only in the overlap of their forward light cones. If this overlap is entirely contained in the forward light cone of j, then a light signal from j can reach any point in spacetime where Alice and Bob can compare their results. This restriction on jamming configurations also rules out another violation of the unary condition. If Jim could obtain the results of Alice's measurements prior to deciding whether to press the button, he could send a superluminal signal to Bob by *selectively* jamming[11].

IV. AN EFFECT CAN PRECEDE ITS CAUSE!

If jamming satisfies the unary and binary conditions, it preserves causality. These conditions restrict but do not preclude jamming. There are configurations with spacelike separated a, b and j that satisfy the unary and binary conditions. We conclude that quantum mechanics is not the only theory combining nonlocal equations of motion with causality. In this section we consider another remarkable aspect of jamming, which concerns the time sequence of the events a, b and j defined above. The unary and binary conditions are manifestly Lorentz invariant, but the time sequence of the events a, b and j is not. A time sequence a, j, b in one Lorentz frame may transform into b, j, a in another Lorentz frame. Furthermore, the jamming model presents us with reversals of the sequence of *cause* and *effect*: while j may precede both a and b in one Lorentz frame, in another frame both a and b may precede j.

To see how jamming can reverse the sequence of cause and effect, we specialize to the case of one space dimension. Since a and b are spacelike separated, there is a Lorentz frame in which they are simultaneous. Choosing this frame and the pair (x, t) as coordinates for space and time, respectively, we assign a to the point (-1,0) and b to the point (1,0). What are possible points at which j can cause jamming? The answer is given by the binary condition. It is particularly easy to apply the binary condition in 1+1 dimensions, since in 1+1 dimensions the overlap of two light cones is itself a light cone. The overlap of the two forward light cones of a and b is the forward light cone issuing from (0,1), so the jammer, Jim, may act as late as $\Delta t = 1$ *after Alice and Bob have completed their measurements* and still jam their results. More generally, the binary condition allows us to place j anywhere in the backward light cone of (0,1) that is also in the forward light cone of (0,-1), but not on the boundaries of this region, since we assume that a, b and j are mutually spacelike separated. (In particular, j cannot be at (0,1) itself.)

Such reversals may boggle the mind, but they do not lead to any inconsistency as long as they do not generate self-contradictory causal loops[19, 20]. Consistency and causality are intimately related. We have used the term *relativistic causality* for the constraint that others call *no signalling.* What is causal about this constraint? Suppose that an event (a "cause") could influence another event (an "effect") at a spacelike separation. In one Lorentz frame the cause precedes the effect, but in some other Lorentz frame the effect precedes the cause; and if an effect can precede its cause, the effect could react back on the cause, at a still earlier time, in such a way as to prevent it. A self-contradictory causal loop could arise. A man could kill his parents before they met. Relativistic causality prevents such causal contradictions[19]. Jamming *allows* an event to precede its cause, but does not allow self-contradictory causal loops. It is not hard to show[11] that if jamming satisfies the unary and binary conditions, it does not lead to self-contradictory causal loops, regardless of the number of jammers. Thus, the reversal of the sequence of cause and effect in jamming is consistent. It is, however, sufficiently remarkable to warrant further comment below, and we also show that the sequence of cause and effect in jamming depends on the space dimension in a surprising way.

The unary and binary conditions restrict the possible jamming configurations; however, they do not require that jamming be allowed for all configurations satisfying the two conditions. Nevertheless, we have made the natural assumption that jamming is allowed for all such configurations. This assumption is manifestly Lorentz invariant. It allows a and b to both precede j. In a sense, it means that Jim acts along the backward light cone of j; whenever a and b are outside the backward light cone of j and fulfill the unary and binary conditions, jamming occurs.

V. AN EFFECT CAN PRECEDE ITS CAUSE??

That Jim may act after Alice and Bob have completed their measurements (in the given Lorentz frame) is what may boggle the mind. How can Jim change his own past? We may also put the question in a different way. Once Alice and Bob have completed their measurements, there can after all be no doubt about whether or not their correlations have been jammed; Alice and Bob cannot compare their results and find out until after Jim has already acted, but whether or not jamming has taken place is already an immutable fact. This fact apparently contradicts the assumption that Jim is a free agent, i.e. that he can freely choose whether or not to jam. If Alice and Bob have completed their measurements, Jim is *not* a free agent: he must push the button, or not push it, in accordance with the results of Alice and Bob's measurements.

We may be uncomfortable even if Jim acts before Alice and Bob have both completed their measurements, because the time sequence of the events a, b and j is not Lorentz invariant; a, j, b in one Lorentz frame may transform to b, j, a in another. The reversal in the time sequences does not lead to a contradiction because the effect cannot be isolated to a single spacetime event: there is no observable effect at either a or b, only correlations between a and b are changed. All the same, if we assume that Jim acts on either Alice or Bob—whoever measures later—we conclude he could not have acted on either of them, because both come earlier in some Lorentz frame.

What, then, do we make of cause and effect in the jamming model? We offer two points of view on this question. One point of view is that we don't have to worry; jamming does not lead to any causal paradoxes, and that is all that matters. Of course, experience teaches that causes precede their effects. Yet experience also teaches that causes and effects are locally related. In jamming, causes and effects are nonlocally related. So we cannot assume that causes must precede their effects; it is contrary to the spirit of special relativity to impose such a demand. Indeed, it is contrary to the spirit of general relativity to assign absolute meaning to any sequence of three mutually spacelike separated events, even when such a sequence has a Lorentz-invariant meaning in special relativity [20]. We only demand that no sequence of causes and effects close upon itself, for a closed causal loop—a time-travel paradox—would be self-contradictory. If an effect can precede its cause *and both are spacetime events*, then a closed causal loop can arise. But in jamming, the cause is a spacetime event and the effect involves two spacelike separated events; no closed causal loop can arise[11].

This point of view interprets cause and effect in jamming as Lorentz invariant; observers in all Lorentz frames agree that jamming is the effect and Jim's action is the cause. A second point of view asks whether the jamming model could have any other interpretation. In a world with jamming, might observers in different Lorentz frames give different accounts of jamming? Could a sequence a, j, b have a covariant interpretation, with two observers coming to different conclusions about which measurements were affected by Jim? (No experiment could ever prove one of them wrong and the other right[21].) Likewise, perhaps observers in a Lorentz frame where both a and b precede j would interpret jamming as a form of *telesthesia*: Jim knows whether the correlations measured by Alice and Bob are nonlocal before he could have received both sets of results. We must assume, however, that observers in such a world would notice that jamming always turns out to benefit Jim; they would not interpret jamming as mere telesthesia, so the jamming model could not have this covariant interpretation.

Finally, we note that a question of interpreting cause and effect arises in quantum mechanics, as well. Consider the measurements of Alice and Bob in the absence of jamming. Their measured results do not indicate any relation of cause and effect between Alice and Bob; Alice can do nothing to affect Bob's results, and vice versa. According to the conventional interpretation of quantum mechanics, however, the first measurement on a pair of particles entangled in a singlet state causes collapse of the state. The question whether Alice or Bob caused the collapse of the singlet state has no Lorentz-invariant answer[11, 22].

VI. JAMMING IN MORE THAN ONE SPACE DIMENSION

After arguing that jamming is consistent even if it allows reversals of the sequence of cause and effect, we open this section with a surprise: such reversals arise only in one space dimension! In higher dimensions, the binary condition itself eliminates such configurations; jamming is not possible if both a and b precede j. To prove this result, we first consider the case of 2+1 dimensions. We choose coordinates (x, y, t) and, as before, place a and b on the x-axis, at (-1,0,0) and (1,0,0), respectively. Let A, B and J denote the forward light cones of a, b and j, respectively. The surfaces of A and B intersect in a hyperbola in the yt-plane. To satisfy the binary condition, the intersection of A and B must lie entirely within J. Suppose that this condition is fulfilled, and now we move j so that the intersection of A and B ceases to lie within J. The intersection of A and B ceases to lie within J when its surface touches the surface of J. Either a point on the hyperbola, or a point on the surface of either A of B alone, may touch the surface of J. However, the surfaces of A and J can touch only along a null line (and likewise for B and J); that is, only if j is not spacelike separated from either a or b, contrary to our assumption. Therefore the only new constraint on j is that the hyperbola formed by the intersection of the surfaces of A and B not touch the surface of J. If we place j on the t-axis, at (0,0,t), the latest time t for which this condition is fulfilled is when the asymptotes of the hyperbola lie along the surface of J. They lie along the surface of J when j is the point (0,0,0). If j is the point (0,0,0), moving j in either the x- or y-direction will cause the hyperbola to intersect the surface of J. We conclude that there is no point j, consistent with the binary condition, with t-coordinate greater than 0. Thus, j cannot succeed both a and b in any Lorentz frame (although it could succeed one of them).

For $n > 2$ space dimensions, the proof is similar. The only constraint on j arises from the intersection of the surfaces of A and B. At a given time t, the surfaces of A and B are $(n-1)$-spheres of radius t centered, respectively, at $x = -1$ and $x = 1$ on the x-axis; these $(n-1)$-spheres intersect in an $(n-2)$-sphere of radius $(t^2-1)^{1/2}$ centered at the origin. This $(n-2)$ sphere lies entirely within an $(n-1)$-sphere of radius t centered at the origin, and approaches it asymptotically for $t \to \infty$. The $(n-1)$-spheres centered at the origin are sections of the forward light cone of the origin. Thus, j cannot occur later than a and b.

We find this result both amusing and odd. We argued above that allowing j to succeed both a and b does not entail any inconsistency and that it is contrary to the spirit of the general theory of relativity to exclude such configurations for jamming. Nonetheless, we find that they are automatically excluded for $n \geq 2$.

VII. CONCLUSIONS

Two related questions of Shimony[4, 5] and Aharonov[7] inspire this essay. Nonlocality and relativistic causality seem *almost* irreconcilable. The emphasis is on *almost*, because quantum mechanics does reconcile them, and does so in two different ways. But is quantum mechanics the unique theory that does so? Our answer is that it is not: model theories going beyond quantum mechanics, but respecting causality, allow nonlocality both ways. We qualify our answer by noting that nonlocality is not completely defined. Relativistic causality is well defined, but nonlocality in quantum mechanics includes both nonlocal correlations and nonlocal equations of motion, and we do not know exactly what kind of nonlocality we are seeking. Alternatively, we may ask what additional physical principles can we impose that will single out quantum mechanics as the unique theory. Our "superquantum" and "jamming" models open new experimental and theoretical possibilities. The superquantum model predicts violations of the CHSH inequality exceeding quantum violations, consistent with causality. The jamming model predicts new effects on quantum correlations from some mechanism such as the burst of laser light suggested by Shimony[4]. Most interesting are the theoretical possibilities. They offer hope that we may rediscover quantum mechanics as the unique theory satisfying a small number of fundamental principles: causality plus nonlocality "plus something else simple and fundamental"[13].

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[19] See *e.g.* D. Bohm, *The Special Theory of Relativity*, W. A. Benjamin Inc., New York (1965) 156-158.

[20] We thank Y. Aharonov for a discussion on this point.

[21] They need not be incompatible. An event in one Lorentz frame often is another event in another frame. For example, absorption of a virtual photon in one Lorentz frame corresponds to emission of a virtual photon in another. In jamming, Jim might not only *send instructions* but also *receive information*, in both cases unconsciously. (Jim is conscious only of whether or not he jams.) Suppose that the time reverse of "sending instructions" corresponds to "receiving information". Then each observer interprets the sequence of events correctly for his Lorentz frame.

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