Frequency-offset Invariant Equaliser for Broadband Wireless Networks

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ABSTRACT

An equaliser algorithm has been developed for use with a differential detector operating in a time dispersive channel. Although differential detection allows the stringent requirements on frequency accuracy generally imposed on coherent receivers to be relaxed, its performance is more severely degraded by intersymbol interference. The algorithm, described in this paper, provides reliable performance even after differential detection. Results for the *differential* equaliser operating in a typical indoor wireless channel are presented and are shown to be comparable to those of a coherent receiver, using decision feedback equalisation.

I. INTRODUCTION

The benefits of wireless networking, mobility and portability are widely acknowledged. Supporting multimedia services in such networks requires transmission rates, which are comparable to those provided by fixed networking solutions. In order to obtain the necessary bandwidth to support such rates, wireless networks have to operate at increasingly higher frequencies. This presents two problems. Firstly, since the signal bandwidth is relatively broad, some form of equalisation is generally required to mitigate the channel ISI. Secondly, frequency offsets introduced during the process of converting to, and from, the transmit frequency will increase the error rate of the demodulator, unless frequency tracking is used at the receiver.

The use of differential detection is attractive when the frequency offset is sufficiently large to prohibit reliable training of coherent equalisers. However, a major disadvantage is its increased sensitivity to channel time dispersion, due to the nonlinear detection process [1]. Several methods of equalising a differentially detected signal have been proposed [2,3,4]. In [2], linear equalisation is used *prior* to differential detection in order to maintain linearity. Although in [3], a two-state maximum likelihood sequence estimator is used after the differential detector, it also relies on some pre-detection processing. Here, as in [4], we choose to consider the detection process and the propagation channel as elements of a composite nonlinear channel with all signal processing applied to the output of the nonlinear channel. [1]In [4], a nonlinear processor was used for determining the elements in the equaliser input vector, but the method was of $O(L^2)$ complexity, where L is the number of significant symbol-spaced multipath components in the channel.

The differential equaliser algorithm presented in this paper (section II), which is similar to the decision feedback Bayesian equaliser in [5], is more computationally efficient. For comparison, a decision feedback equaliser (DFE), with a frequency tracking loop incorporated in the feedback loop, is described briefly in section III. The computational complexity of the two equalisers is compared in section IV. In section V, performance results for the differential equaliser and DFE are presented. Section VI summarises the main conclusions.

II. THE DIFFERENTIAL EQUALISER

The output of the propagation channel is given by

$$y_k = \sum_{i=0}^{L-1} x_{k-i} h_i + n_k \ , \qquad (1)$$

where h_i are the complex taps of the combined impulse response from the GMSK modulator and the propagation channel, n_k is a complex valued additive white Gaussian noise sample with zero mean and variance σ_n^2. Note that we have combined the modulation filter impulse response and the channel impulse response into the multipath components h_i and therefore the symbols x_k, $x_k = j\alpha_k x_{k-1}$, correspond to symbol rate samples from an unfiltered MSK signal. For MSK modulation, x_k, $x_k \in \{-1,+1,-j,+j\}$, are the transmitted symbols and α_k, $\alpha_k \in \{-1,+1\}$, are the information bits to be recovered. The differential detector multiplies the current received signal sample, y_k, with the conjugate of the previous

[1] Note: In Fig.1 and in the remainder of the paper we have assumed the use of GMSK modulation as this is of current practical interest.

sample, yielding the composite channel output, $r_k = y_k y_{k-1}^*$. From eqn.1, r_k can be expanded as

$$r_k = \sum_{i=0}^{L-1} x_{k-i} x_{k-i-1}^* |h_i|^2 + \sum_{\substack{n=0 \\ n \neq i}}^{L-1} \sum_{\substack{i=0 \\ i \neq n}}^{L-1} x_{k-i} x_{k-n-1}^* h_i h_n^* + z_k + n_k n_{k-1}^*, \quad (2)$$

where, $z_k = n_k \sum_{n=0}^{L-1} x_{k-n-1}^* h_n^* + n_{k-1}^* \sum_{i=0}^{L-1} x_{k-i}^* h_i^*$.

Assuming the symbols α_k and, therefore, x_k are uncorrelated, the variance of z_k, $\sigma_{z_k}^2$, is $\sigma_{z_k}^2 = 2\sigma_n^2 \sum_{i=0}^{L-1} |h_i|^2$. For convenience, we assume that the propagation channel has an impulse response of unit energy $\sum_{i=0}^{L-1} |h_i|^2 = 1$, and therefore $\sigma_{z_k}^2 = 2N_o$, where N_o is the single sided noise power spectral density. Using the complex valued output samples from the composite channel, r_k, the equaliser must recover the information symbol, $\alpha_{k-\gamma}$, where $\alpha_{k-\gamma} = -jx_{k-\gamma} x_{k-\gamma-1}^*$, and γ is an arbitrary delay.

The equalisation method is based on the Decision Feedback Bayesian equaliser proposed in [5]. Its basic structure is shown in figure.1a, which is characterised by the feedforward order (*M*) the feedback order (*N*) and the decision delay (*D*). Synchronisation is achieved by cross-correlating a preamble sequence with the received signal and identifying the start of frame from the peak in the correlation profile. Defining $N_{pk} \in \{0,1,..,L-1\}$, as the position of the highest peak in the composite channel impulse response, then the equaliser is configured as $M = N_{pk} + 1$, $D = N_{pk}$ and $N = L - 1$. This has been shown to provide a sufficient solution for a coherent receiver, i.e. a Bayesian Equaliser with $M = D + 1$ has the same performance as those with $M > D + 1$ [5].

Decisions are formed by the equaliser, using a vector containing the most recent output samples from the composite channel, together with the most recent decisions (figure 1a). In the absence of noise, there will only be a finite number of channel outputs, referred to here as *channel states.* Each channel state is assigned a label corresponding to the decision to be produced by the equaliser given the current state. The correct labels are determined, during an initial training phase. During data detection, the distance between each noisy channel output sample and each set of channel states with the same label is computed. The label associated with the set resulting in the least total distance is accepted as the decision. This idea generalises to allow the use of multiple consecutive channel outputs in forming each decision. Clearly, the complexity grows exponentially with the number of channel output samples used to form each decision. However, by performing the distance computations using only a subset of channel states, identified by the vector of previous decisions, the complexity can be maintained within easily manageable limits, whilst actually enhancing performance. The algorithm details are summarised in figure 1a. The training method adopted for the differential equaliser is supervised clustering [7], which is naturally suited to deal with the nonlinearity of the differential detector.

III. DECISION FEEDBACK EQUALISATION WITH CARRIER RECOVERY

The use of decision feedback equalisation is widely documented and has recently been investigated in the context of indoor radio LANs [9,10]. It is well known that a frequency offset can severely impair the DFE performance. We therefore consider the use of a frequency tracking loop, integrated within the DFE adaptation loop, as proposed in [11]. Our interest in examining frequency tracking is to allow a fair performance and complexity comparison with the proposed differential equaliser. The DFE and frequency tracking loop structure is shown in figure 1b, together with the modified LMS algorithm [11] used for DFE training and frequency tracking. In the figure, FFF denotes the feedforward filter and FBF denotes the feedback filter. Vectors $\mathbf{W}(n)$ and $\mathbf{U}(n)$, denote filter coefficients and filter input data respectively; the subscripts *f* and *b* denote variables associated with the FFF and FBF respectively. The frequency tracking loop parameters α and β are used to set the loop bandwidth.

IV. EQUALISER COMPARISON

This section compares the two equalisation methods described above. An overview of some important implementation issues is given for both methods followed by a breakdown of the required computations.

A. *The differential equaliser*

The differential equaliser relies on the training algorithm to produce an accurate and *complete* set of noise free channel states. For a channel with *L* significant multipath components, completing the set requires that all combinations of *L*-tuple symbols are transmitted through the channel. It is also important that several replications of the same *L*-tuple exist in the transmitted sequence in order to average the noise perturbation produced by the channel. If we consider a length 32 augmented m-sequence, then all 5-tuples exist, but in a noisy channel an inaccurate estimate would exist as only one sample per channel state exists. In this example, a more suitable choice would be a 3-tuple. For the HIPERLAN training sequence we note that it consists of all 5-tuples except the all-zero sequence; but for 6-tuple there are three missing sequences, and so on. As it stands, the HIPERLAN training sequence will only permit a complete set of noise free channel states for an ISI span of four symbols. Therefore, for greater ISI time spans, there will be a significant degradation in performance.

The training algorithm to determine the channel states is based on supervised clustering (e.g. [7]). This is highly stable, relative to traditional gradient search methods and is of relatively low

complexity; training is based on averaging the noisy channel states. The complexity of the equaliser is essentially a function of the number of precursor paths in the channel impulse response. For all values of delay spread, the feedback order of the differential equaliser is fixed at $N = 4$ and $N_{pk} = 1$, giving $M = 2$. N_{pk} was chosen to be 1, since the Gaussian transmit filter, with $BT = 0.3$, introduces only one significant precursor ISI component, while the propagation channel is exponential and, therefore, results in predominantly postcursor ISI. Based on these parameters, the differential equaliser can tolerate ISI distortion spanning up to 5 symbol periods ($L = 5$). In this case, the number of squared distance calculations per information symbol is $N_s / N_f = 2^{N_{pk}+1} = 4$ [5].

Other receiver functions, such as correlation and decimation are equivalent to those used with the DFE, and the process of differential detection is of similar complexity to coherent phase tracking. It should be mentioned, that the differential equaliser requires AGC for maintaining the linearity of envelope, so maintaining the distance between the ISI states in the channel. However, the differential equaliser is much more robust than the DFE to nonlinear distortion occurring in the AGC stages.

B. *The decision feedback equaliser*

An important consideration in the use of the LMS algorithm is the choice of the step-size value. Whilst instability due to poorly conditioned input data has not generally been a difficulty in the simulations presented here, the variation in the input signal power has been problematic. Since the variation in the input signal power affects the speed of frequency tracking, as well as DFE adaptation speed, the choice of value for this parameter becomes very critical. To try and minimise the effect of signal power variation, when comparing the performance of various DFE configurations in different channel conditions, the step-size has been normalised by the power of the input samples in the FFF and multiplied by the FFF length. This ensures a step-size, which is relatively constant for different simulations. The step-size for the FBF is simply scaled by the FBF length, since the FBF input samples are of constant magnitude. A functional listing for the DFE is shown in table 1. The operation count is in terms of real valued operations.

Table 1: The operations/iteration for a DFE using a 2nd order frequency tracking loop

Function	Operations	
	Mults	**Adds**
FFF & FBF outputs	$4L$	$8L$[Y]
Phase rotate FFF output	4	2
Error computation and step-size multiplication	2	1
Phase error estimate	2	0
Phase error update	2	2
Cosine & sine look-up	0	0
Phase rotate error	4	2
FFF & FBF coeff. Updates	$4L$	$8L$[Y]
TOTAL	$\mathbf{8L+14}$	$\mathbf{16L+5}$

Notes: [Y] It is assumed that the FBF input data is quantised to one of two signs in both I and Q demodulator branches as in QPSK modulation, which allows multiplication operations in the FBF to be replaced by additions.

Two configurations, (5,5) and (8,8), are used for the DFE, where the bracketed values refer to the number of FFF and FBF coefficients respectively. For the (5,5) DFE 54 multiplication and 85 addition operations per iteration are required; for the (8,8) DFE 78 multiplications and 133 additions are required. During decision directed mode, this is significantly greater than the number of computations performed by the differential equaliser.

V. SIMULATION RESULTS

The performance of the proposed differential equaliser and a coherent DFE are compared in this section using a simulation of a typical indoor radio channel. The propagation channel is modeled by a tapped delay line filter, which has an exponential power delay profile with independent Rayleigh fading on the individual taps. The root mean square (RMS) delay spread, σ, of the power delay profile is used here as a measure of the channel time dispersion. In the simulation, 10^4 channel realisations, for each value of σ, were used.

In the simulation model, the HIPERLAN physical layer was adopted [8]. The length of each transmitted packet is 946 bits including the 450 training bits, which corresponds to the shortest packet in the HIPERLAN standard (the results shown are without coding). The modulation scheme is GMSK with BT=0.3 and synchronisation is performed using the 450-bit preamble. The oversampling rate used in the simulation is 8 and the IF filter is a fourth order Butterworth with a normalised bandwidth of unity. The performance of the equalisers are compared with both 10kHz and 100kHz frequency offsets corresponding to frequency accuracies of 1ppm and 10ppm, respectively, for a 5GHz carrier frequency (10ppm accuracy is specified in [8]).

The BER curves for the differential equaliser and DFE equaliser are shown in figures 2a and 2b respectively. From figure 2a, the differential equalised system can achieve error rates $< 10^{-3}$ for delay spreads up to $\sigma = 0.5$ for $E_b / N_o = 30dB$ while for $E_b / N_o = 50dB$ this same error rate can be achieved even at $\sigma = 0.75$. This improvement is due to the reduced noise variance, which allows more accurate determination of the channel states. In addition, the Bayesian decision function was approximated by the minimum Euclidean distance[2]. This approximation relies on each noisy ISI state being both distinct, and well separated, which is more closely satisfied at $E_b / N_o = 50dB$.

[2] This is the familiar equivalence between the maximum-likelihood and minimum distance detector [6].

For the DFE, the frequency tracking loop parameters were set to $\alpha = 0.05, \beta = 0.01$, which from simulation were found to restore the DFE performance close to that obtained when operating without any offset. The results shown for the DFE demonstrate superior performance, but require considerably more computational effort. Note that, without a frequency tracking loop and with a fixed step-size of 0.03, it can be seen that the DFE performance is poorer than that of the differential equaliser even with a 10kHz offset. This indicates the sensitivity of the DFEs performance on the value chosen for the step-size.

VI. CONCLUSIONS

An equaliser algorithm, suitable for equalising the output of a differential detector for a high rate mobile receiver has been presented. The motivation for this work has been to develop an algorithm to enable the use of differential reception to increase robustness to large frequency offsets, even in the presence of intersymbol interference. The algorithm and processing structure described in the paper are easily scaleable, allowing the equaliser to be configured for widely varying levels of channel time dispersion. Simulated performance results for the differential equaliser, using a typical indoor channel model, have shown the feasibility of achieving bit error rates comparable to a coherent DFE in the presence of a frequency offset.

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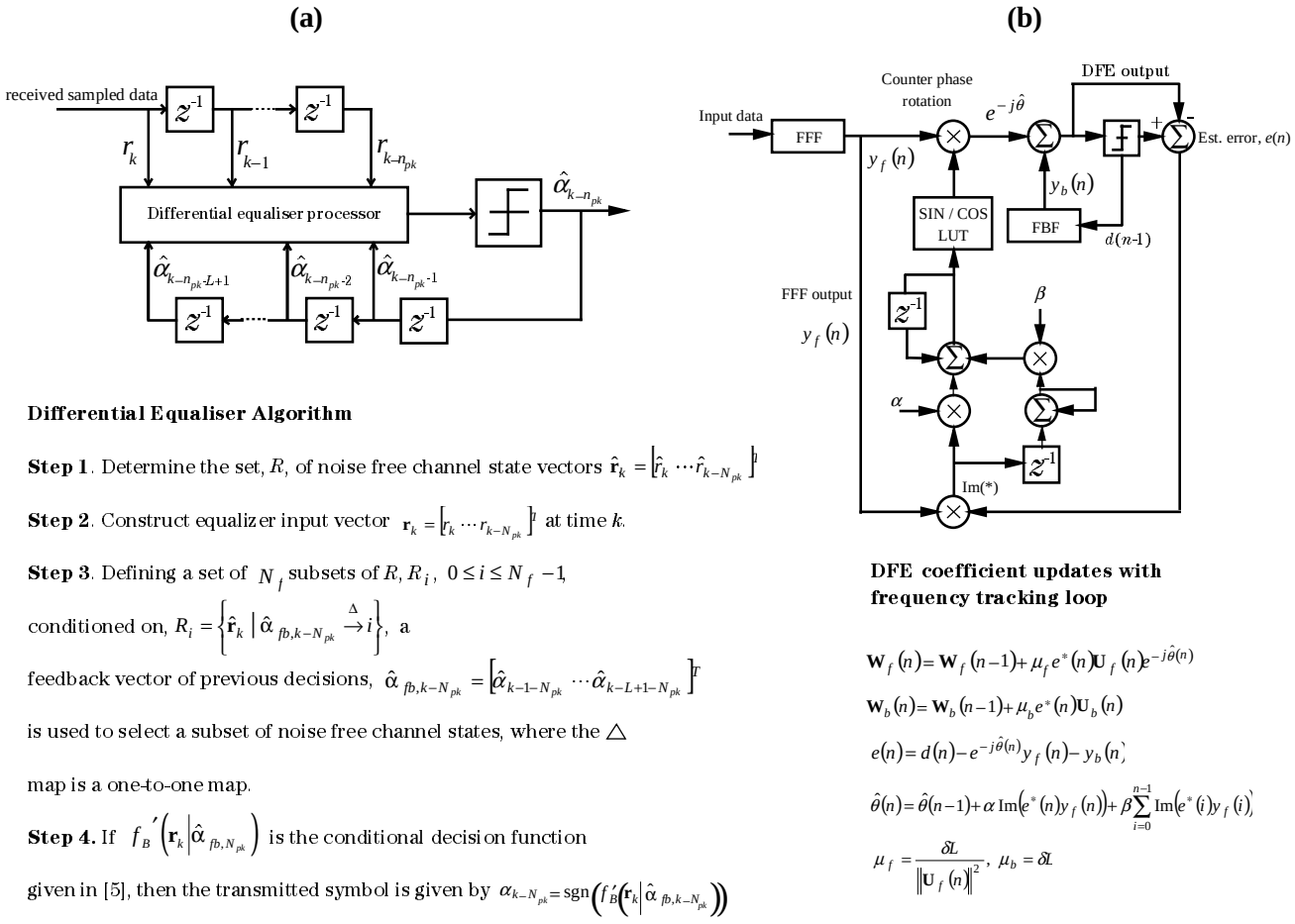
Figures



Differential Equaliser Algorithm

Step 1. Determine the set, R, of noise free channel state vectors $\hat{\mathbf{r}}_k = \left[\hat{r}_k \cdots \hat{r}_{k-N_{pk}}\right]^T$

Step 2. Construct equalizer input vector $\mathbf{r}_k = \left[r_k \cdots r_{k-N_{pk}}\right]^T$ at time k.

Step 3. Defining a set of N_f subsets of R, R_i, $0 \le i \le N_f - 1$, conditioned on, $R_i = \left\{\hat{\mathbf{r}}_k \mid \hat{\alpha}_{fb,k-N_{pk}} \xrightarrow{\Delta} i\right\}$, a feedback vector of previous decisions, $\hat{\alpha}_{fb,k-N_{pk}} = \left[\hat{\alpha}_{k-1-N_{pk}} \cdots \hat{\alpha}_{k-L+1-N_{pk}}\right]^T$ is used to select a subset of noise free channel states, where the $\triangle$ map is a one-to-one map.

Step 4. If $f_B'\left(\mathbf{r}_k \middle| \hat{\alpha}_{fb,N_{pk}}\right)$ is the conditional decision function given in [5], then the transmitted symbol is given by $\alpha_{k-N_{pk}} = \mathrm{sgn}\left(f_B'\left(\mathbf{r}_k \middle| \hat{\alpha}_{fb,k-N_{pk}}\right)\right)$

DFE coefficient updates with frequency tracking loop

$$\mathbf{W}_f(n) = \mathbf{W}_f(n-1) + \mu_f e^*(n)\mathbf{U}_f(n)e^{-j\hat{\theta}(n)}$$

$$\mathbf{W}_b(n) = \mathbf{W}_b(n-1) + \mu_b e^*(n)\mathbf{U}_b(n)$$

$$e(n) = d(n) - e^{-j\hat{\theta}(n)}y_f(n) - y_b(n)$$

$$\hat{\theta}(n) = \hat{\theta}(n-1) + \alpha\,\mathrm{Im}\left(e^*(n)y_f(n)\right) + \beta\sum_{i=0}^{n-1}\mathrm{Im}\left(e^*(i)y_f(i)\right)$$

$$\mu_f = \frac{\delta L}{\left\|\mathbf{U}_f(n)\right\|^2},\quad \mu_b = \delta L$$

Figure 1: Differential equaliser algorithm (a) and coherent DFE algorithm (b)

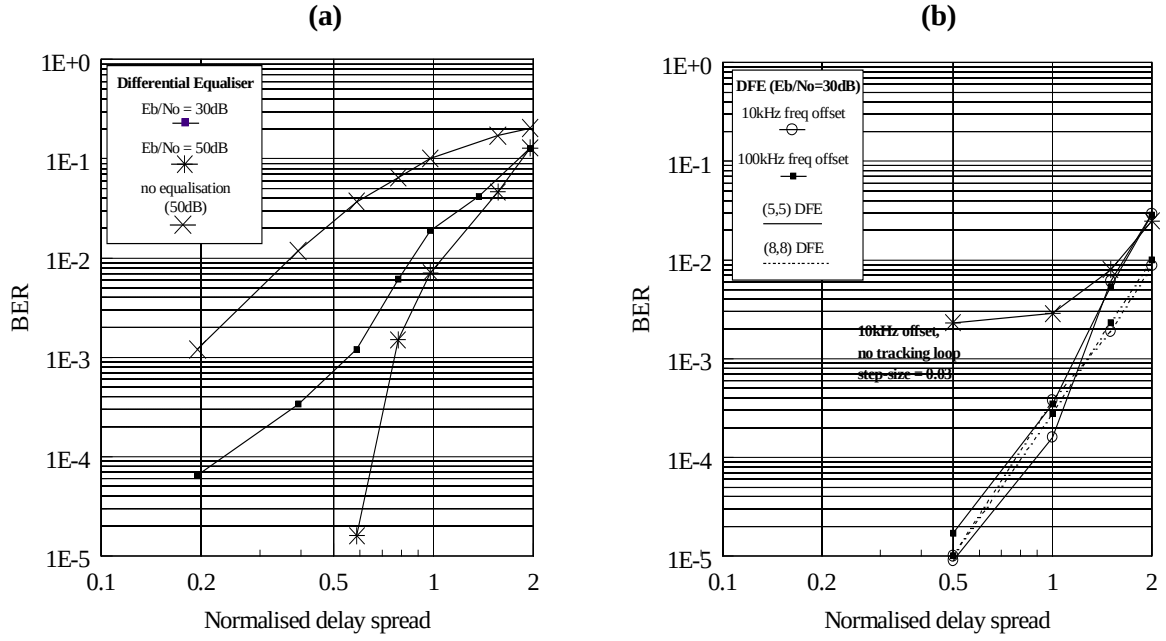


Figure 2: BER performance curves for the differential equaliser (a) and DFE (b)