An Algorithm for Equalising a Differentially Detected Signal

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ABSTRACT

An equalizer algorithm is presented which is suitable for use with a differential detector operating in a time dispersive channel. Although differential detection allows the stringent requirements on frequency stability generally imposed on coherent receivers to be relaxed, its performance is more severely degraded by intersymbol interference. The algorithm described in this paper, which has been derived from previous Bayesian coherent methods, is able to provide reliable performance even after differential detection. Results for the *differential* equalizer operating in a typical indoor wireless channel are presented and are shown to compare favourably with those of a coherent receiver, using decision feedback equalisation, in the presence of a frequency offset.

Index Terms – Differential detection, equalization, indoor channel, GMSK.

I. Introduction

The use of differential detection is attractive in a receiver when the frequency offset is sufficiently large to prohibit reliable training of coherent equalizers. However, a major disadvantage is its increased sensitivity to channel time dispersion, due to the nonlinear detection process [1]. Several methods of equalizing a differentially detected signal have been proposed [2,3,4]. In [2], linear equalization was still used *prior* to differential detection in order to maintain linearity. Although in [3], a two-state maximum likelihood sequence estimator was used after the differential detector, it still also relied on some pre-detection processing. Here, as in [4], we choose to consider the detection process and the propagation channel as elements of a composite nonlinear channel (Fig.1) with all signal processing applied to the output of the nonlinear channel. [1]In [4], a nonlinear processor was used for determining the elements in the equalizer input vector, but the method was of $O(L^2)$ complexity, where L is the number of significant symbol-spaced multipath components in the channel. The differential equaliser algorithm presented in this paper (section III), based on the decision feedback Bayesian equaliser in [5], is more computationally efficient. In section IV, performance results for the differential equaliser are presented and compared with those of a decision feedback equaliser. Section V summarises the main conclusions. Section II first introduces the problem and the modeling assumptions.

II. Background

The output of the propagation channel is given by

$$y_k = \sum_{i=0}^{L-1} x_{k-i} h_i + n_k \ , \qquad (1)$$

where h_i are the complex taps of the combined impulse response from the GMSK modulator and the propagation channel, n_k is a complex valued additive white Gaussian noise sample with zero mean and variance σ_n^2. Note that we have combined the modulation filter impulse response and the channel impulse response into the multipath components h_i and therefore the symbols x_k, $x_k = j\alpha_k x_{k-1}$, correspond to symbol rate samples from an unfiltered MSK signal. For MSK modulation, x_k, $x_k \in \{-1,+1,-j,+j\}$, are the transmitted symbols and α_k, $\alpha_k \in \{-1,+1\}$, are the information bits to be recovered. The differential detector multiplies

[1] Note, that in Fig.1 and in the remainder of the paper we have assumed the use of GMSK modulation as this is of current practical interest.

the current received signal sample, y_k, with the conjugate of the previous sample, yielding the composite channel output, $r_k = y_k y_{k-1}^*$. From eqn.1, r_k can be expanded as

$$r_k = \sum_{i=0}^{L-1} x_{k-i} x_{k-i-1}^* |h_i|^2 + \sum_{\substack{n=0 \\ n \neq i}}^{L-1} \sum_{\substack{i=0 \\ i \neq n}}^{L-1} x_{k-i} x_{k-n-1}^* h_i h_n^* + z_k + n_k n_{k-1}^*, \quad (2)$$

where, $z_k = n_k \sum_{n=0}^{L-1} x_{k-n-1}^* h_n^* + n_{k-1}^* \sum_{i=0}^{L-1} x_{k-i}^* h_i^*$. Assuming the symbols α_k and therefore x_k are uncorrelated, the variance of z_k, $\sigma_{z_k}^2$, is $\sigma_{z_k}^2 = 2\sigma_n^2 \sum_{i=0}^{L-1} |h_i|^2$. For convenience, we assume that the propagation channel has an impulse response of unit energy $\sum_{i=0}^{L-1} |h_i|^2 = 1$, and therefore $\sigma_{z_k}^2 = 2N_o$, where N_o is the single sided noise power spectral density. Using the complex valued output samples from the composite channel, r_k, the equalizer must recover the information symbol, $\alpha_{k-\gamma}$, where $\alpha_{k-\gamma} = -j x_{k-\gamma} x_{k-\gamma-1}^*$, and γ is an arbitrary delay.

III. The Differential Equaliser

The equalization method is based on the Decision Feedback Bayesian equaliser proposed in [5]. Its basic structure is shown in Fig.2, which is characterised by the feedforward order (*M*) the feedback order (*N*) and the decision delay (*D*). Synchronisation is performed by cross-correlating a preamble sequence with the received signal and identifying the start of frame from the peak in the correlation profile. Defining $N_{pk} \in \{0, 1, .., L-1\}$, as the position of the highest peak in the composite channel impulse response, then the equaliser is configured as $M = N_{pk} + 1$, $D = N_{pk}$ and $N = L - 1$. This has been shown [5] to provide a sufficient solution for a coherent receiver, i.e. a Bayesian Equalizer with $M = D + 1$ has the same performance as those with $M > D + 1$.

Decisions are formed by the equalizer using a vector of the most recent output samples from the composite channel and the most recent decisions. In the absence of noise, there will only be a finite number of channel outputs, referred to here as *channel states*. Each channel state is assigned a label corresponding to the decision to be produced by the equaliser given the current state. The correct labels are determined, during an initial training phase. During data detection, the distance between each noisy channel output sample and each set of channel states with the same label is computed. The label associated with the set resulting in the least total distance is accepted as the decision. This idea generalises to allow the use of multiple consecutive channel outputs in forming each decision. Clearly, the complexity grows exponentially with the number of channel output samples used to form each decision. However, by performing the distance computations using only a subset of channel states, identified by the vector of previous decisions, the complexity can be maintained within easily manageable limits, whilst actually enhancing performance. We now elaborate on the algorithm details.

The input to the equalizer at time k, from the differential detector, is the vector $\mathbf{r}_k = \left[r_k \cdots r_{k-N_{pk}}\right]^T$. We denote the corresponding noise free channel state vector as $\hat{\mathbf{r}}_k = \left[\hat{r}_k \cdots \hat{r}_{k-N_{pk}}\right]^T$, which is obtained from eqn.2 with $n_k = 0$, and the set of all possible channel states is denoted by R. For $M = N_{pk} + 1$, the differentially detected symbols that currently lie in the time span of the feedforward section at time *k* are represented by the vector

$$\mathrm{\alpha}_k = \left[\alpha_k \cdots \alpha_{k-N_{pk}-L+1}\right]^T ,$$

where $\alpha_k = -jx_k x^*_{k-1}$. There are $N_s = 2^{L+n_{pk}}$ combinations for vector $\mathrm{\alpha}_k$, which yields N_s noise free channel states $\hat{\mathbf{r}}_k$. The feedback vector, $\mathrm{\alpha}_{fb,k-N_{pk}}$, consists of binary elements ±1 and is formed from the last elements of $\mathrm{\alpha}_k$, such that

$$\hat{\alpha}_{fb,k-N_{pk}} = \left[\hat{\alpha}_{k-1-N_{pk}} \cdots \hat{\alpha}_{k-L+1-N_{pk}}\right]^T ,$$

where ^ denotes a decision. The feedback vector has $N_f = 2^{L-1}$ combinations. We define a set of N_f subsets of R, R_i, $0 \le i \le N_f - 1$, conditioned on, $R_i = \left\{ \hat{\mathbf{r}}_k \mid \hat{\alpha}_{fb,k-N_{pk}} \overset{\Delta}{\to} i \right\}$; i.e. we group the channel states into disjoint subsets, $R = \bigcup_{0 \le i \le N_f - 1} R_i$, sharing the same feedback vector component. Note that the mapping Δ, is a one-to-one mapping from the feedback vectors of binary elements into the range of integers $(0..N_f - 1)$[2]. This can then be implemented using a look-up-table. R_i is further sub-divided into two disjoint subsets, R_i^+ and R_i^-, where

$$R_i^+ = \left\{ \mathbf{r}_k \middle| \left[\alpha_{k-n_{pk}} = +1, \hat{\alpha}_{f,k-n_{pk}}\right] \right\} \quad R_i^- = \left\{ \mathbf{r}_k \middle| \left[\alpha_{k-n_{pk}} = -1, \hat{\alpha}_{f,k-n_{pk}}\right] \right\}.$$

If we assume a sufficiently high signal-to-noise ratio, such that the term $n_k n_{k-1}^*$ may be reasonably neglected [6], then the noise variance is simply $\sigma_{z_k}^2$ for all states. Consequently, the Bayesian decision function is given by [5]

$$f_B\left(\mathbf{r}_k \middle| \hat{\alpha}_{f,k-n_{pk}}\right) = \sum_{l=1}^{n_i^+} \exp\left(\frac{-\left\|\mathbf{r}_k - \mathbf{r}_l^+\right\|^2}{\sigma_{z_k}^2} \right) - \sum_{l=1}^{n_i^-} \exp\left(\frac{-\left\|\mathbf{r}_k - \mathbf{r}_l^-\right\|^2}{\sigma_{z_k}^2} \right), \tag{3}$$

where the number of states in the selected subsets are given by N_i^- and N_i^+, and $\mathbf{r}_l^- \in R_i^-$, $\mathbf{r}_l^+ \in R_i^+$. Assuming that all the channel states are distinct, and that the noise variance is small in comparison to the shortest distance between any two states, then searching for the maximum value in the first and second summations of eqn.3 yields a reasonable approximation of the full summation. It will be shown in section IV that the equaliser can operate effectively, even when the above conditions are not well satisfied. Therefore, the decision function can be simplified to

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[2] For example, if $N_f = 4$, then $\Delta(-1,-1) \to 0, \Delta(-1,+1) \to 1, \Delta(+1,-1) \to 2, \Delta(+1,+1) \to 3$

$$f'_B\left(\mathbf{r}_k \middle| \hat{\alpha}_{fb,k-n_{pk}}\right) = \max_l\left(\exp\left(\frac{-\left\|\mathbf{r}_k - \mathbf{r}_l^+\right\|^2}{\sigma_{z_k}^2}\right)\right) - \max_l\left(\exp\left(\frac{-\left\|\mathbf{r}_k - \mathbf{r}_l^-\right\|^2}{\sigma_{z_k}^2}\right)\right). \tag{4}$$

Furthermore, since $\exp(-x),\ x \geq 0$, is a strictly monotonic decreasing function and noting that the noise variance is common to all terms, eqn.4 becomes

$$f'_B\left(\mathbf{r}_k \middle| \hat{\boldsymbol{\alpha}}_{fb,k-n_{pk}}\right) = \min_l\left\|\mathbf{r}_k - \mathbf{r}_l^+\right\|^2 - \min_l\left\|\mathbf{r}_k - \mathbf{r}_l^-\right\|^2 . \tag{5}$$

The decision on the transmitted symbol is therefore given by

$$\alpha_{k-N_{pk}} = \text{sgn}\left(f'_B\left(\mathbf{r}_k \middle| \hat{\alpha}_{fb,k-n_{pk}}\right)\right), \tag{6}$$

where $\text{sgn}(\bullet)$ is the *signum* function. It has been implicitly assumed in the above that channel states are known. In practice, these must be determined, together with their associated labeling, during an initial training period. Since the states must be determined from the noisy outputs of the channel, they are therefore only approximations of the exact channel states. The training method adopted here is supervised clustering, which is naturally suited to deal with the nonlinearity of the differential detector. Details regarding the algorithm can be found in [7].

IV. Simulation Results

The performance of the proposed differential equalizer and a coherent decision feedback equaliser (DFE), using 5 feedforward and 5 feedback coefficients, are compared in this section using a simulation of a typical indoor radio channel. The LMS algorithm has been used to train the DFE using the 450-bit training sequence and continues to adapt the DFE throughout the data frame.

The propagation channel is modeled by a tapped delay line, which has an exponential power delay profile with Rayleigh fading on the individual taps. The root mean square (RMS) delay spread, σ , of the power delay profile is used here as a measure of the channel time dispersion. In the simulation, 10^4 different channels for each value of σ were considered, which ensured that the average power delay profile did indeed converge to an exponentially decaying profile.

In the simulation model, the HIPERLAN physical layer was adopted [8]. The length of the packet is equal to 946 bits including the 450 training bits. This corresponds to the shortest packet in the HIPERLAN standard (note no coding has been used). The modulation scheme is GMSK with BT=0.3 and synchronisation is performed using the 450 bit preamble. The oversampling rate used in the simulation is 8 and the IF filter is a fourth order Butterworth with a normalised bandwidth of unity. The performance of the equalisers is compared both with and without a frequency offset. The frequency offset used was 10kHz, which for a 5GHz transmitter frequency corresponds to 1ppm accuracy. This is much greater accuracy than the 10ppm stability specified in the HIPERLAN standard [8]. A 10kHz frequency offset was chosen to demonstrate the relatively severe degradation suffered by a coherent DFE. At greater offsets than 10kHz, the performance of the DFE is largely dominated by the frequency offset.

For all values of delay spread, the feedback order of the equalizer is fixed at $N = 4$ and $N_{pk} = 1$, giving $M = 2$. N_{pk} was chosen to be 1, since the Gaussian transmit filter, with $BT = 0.3$, introduces only one significant precursor ISI component, while the propagation channel is exponential and should, therefore, result in predominantly postcursor ISI. Based on these parameters, the equaliser can tolerate an ISI span of up to 5 symbol periods ($L = 5$) and the number of squared distance calculations per information symbol is $N_s/N_f = 2^{N_{pk}+1} = 4$ [5]. It is observed that the performance of the differential equaliser was relatively poor when there were a large number of precursor ISI components.

The BER curves of the unequalised and equalized systems are shown in Fig.3 and Fig.4 for $E_b/N_o = 30dB$ and $E_b/N_o = 50dB$ respectively. Clearly, the unequalized system is highly susceptible, even to relatively low values of delay spread, and can only operate with bit error rates $< 10^{-3}$ for $\sigma \le 0.2$. From Fig.3, the equalized system using differential detection can achieve error rates $< 10^{-3}$ up to $\sigma = 0.5$. From Fig. 4, this error rate can be extended to $\sigma = 0.75$. This improvement is due to the reduced noise variance, which allows more accurate determination of the channel states and which improves the accuracy of the approximation used to develop eqn. 4. It is clear that the coherent receiver using a (5,5) DFE and the LMS training algorithm (step-size = 0.045) provides significantly superior performance only in the absence of a frequency offset and for high values of delay spread. Note that the performance of the DFE is dependent on the step-size chosen for the DFE. As the normalised RMS delay spread is reduced, it was found that the mean signal power for certain channels was relatively low, requiring a larger step-size to increase convergence speed. The DFE performance curves flatten for $\sigma \le 1.0$ due to the selection of a fixed step-size and the reduced time diversity in the channel as σ is decreased. Improvements in the DFE performance are possible by optimizing the step-size on a packet-by-packet basis, but such an idealized approach is not possible in practice. Consequently, the results shown are for a fixed step-size only.

When the frequency offset is included, the DFE performance degrades significantly as can be seen in Figs 3 and 4. Increasing the frequency offset beyond 20kHz degraded the DFE BER such that its performance was poorer than that of the differential equaliser. In contrast, the differential equaliser performance is unaffected by the frequency offset. Although a frequency tracking loop may be incorporated in the DFE, this adds additional complexity and is unable to fully recover the lost performance.

V. Conclusions

An equaliser algorithm, suitable for equalising the output of a differential detector for a high rate mobile receiver has been presented. The motivation for this work has been to develop an algorithm to enable the use of differential reception to increase robustness to large frequency offsets, even in the presence of intersymbol interference. The algorithm and processing structure described in the paper are easily scaleable, allowing the equaliser to be configured for widely varying levels of channel time dispersion. Simulated performance results for the differential equaliser, using a typical indoor channel model, have shown the feasibility of achieving bit error rates comparable to a coherent DFE in the presence of a frequency offset.

VI. References

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FIGURE CAPTIONS

Fig.1: Schematic of nonlinear composite channel

Fig.2: Schematic of differential equalizer with decision feedback

Fig.3: Bit error rate performance for Eb/No=30dB

Fig.4: Bit error rate performance for Eb/No=50dB

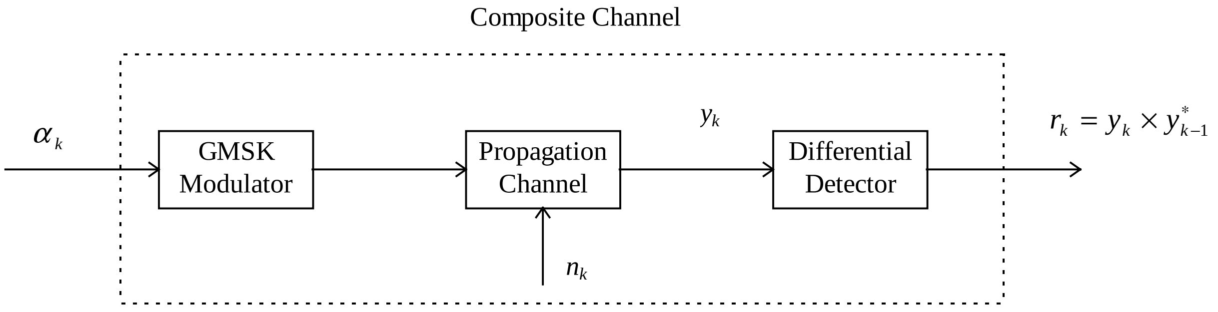
Composite Channel
α_k
GMSK Modulator
Propagation Channel
y_k
Differential Detector
$r_k = y_k \times y_{k-1}^*$
n_k

Fig.1

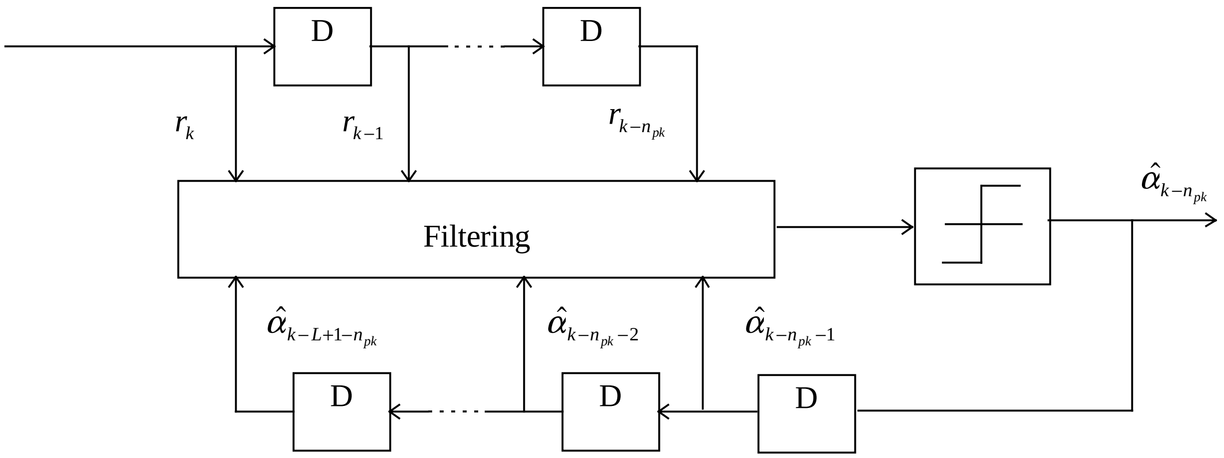
D
D
r_k
r_{k-1}
$r_{k-n_{pk}}$
Filtering
$\hat{\alpha}_{k-n_{pk}}$
$\hat{\alpha}_{k-L+1-n_{pk}}$
$\hat{\alpha}_{k-n_{pk}-2}$
$\hat{\alpha}_{k-n_{pk}-1}$
D
D
D

Fig.2

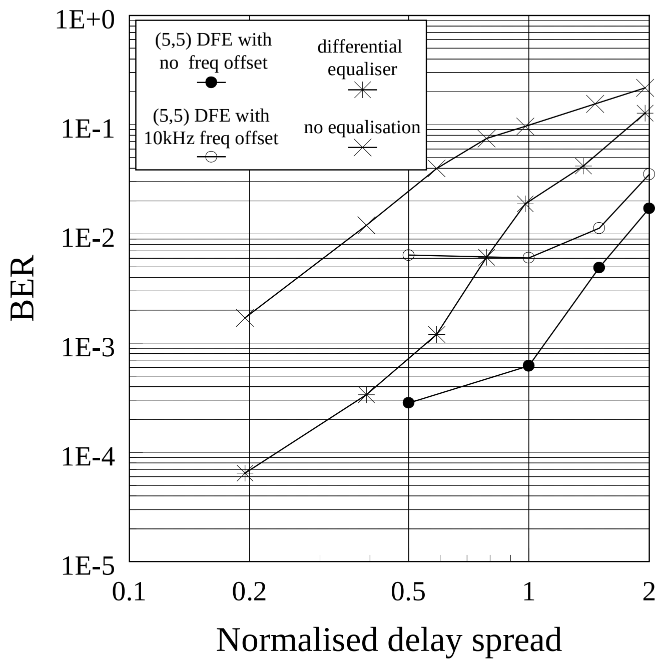
1E+0
1E-1
1E-2
1E-3
1E-4
1E-5
BER
0.1
0.2
0.5
1
2
Normalised delay spread
(5,5) DFE with no freq offset
(5,5) DFE with 10kHz freq offset
differential equaliser
no equalisation

Fig.3

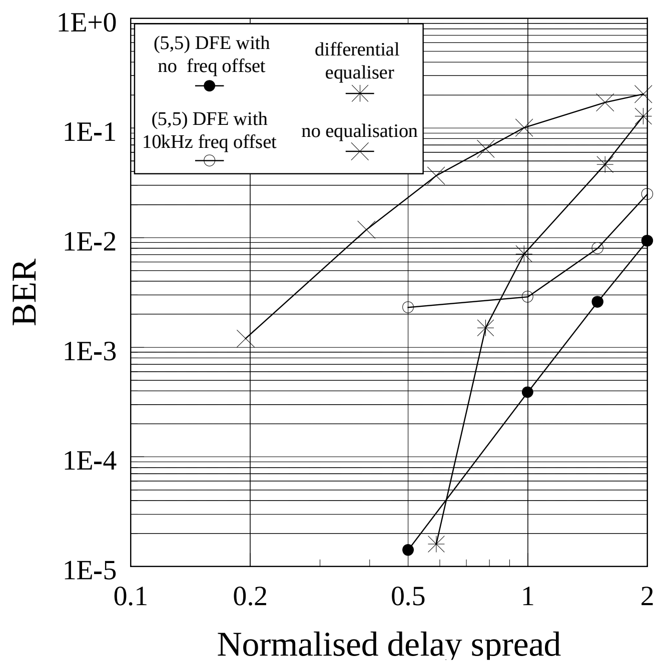
1E+0
1E-1
1E-2
1E-3
1E-4
1E-5
BER
0.1
0.2
0.5
1
2
Normalised delay spread
(5,5) DFE with no freq offset
(5,5) DFE with 10kHz freq offset
differential equaliser
no equalisation

Fig.4