Aharonov-Bohm Type Forces Between Magnetic Fluxons

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Interactions related to A-B phases exist between fluxons with $\Phi = \alpha\Phi_0$, $\alpha \neq$ *integer* when the fluxons are actually immersed in the region with non vanishing mobile charge density. In particular for the interesting case of $\alpha = ½$ we find that this force is *attractive*. We briefly comment on the prospects of observing such forces.

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There is no magnetic field outside an ideal infinitely long and thin solenoid and no electromagnetic force per unit length between such parallel solenoids or fluxons. In the following we note that the presence of mobile charged particles between and around the fluxons induces a new type of force between them. This force is of some theoretical interest and conceivably can be detected in a suitable experimental set-up.

Let us assume that n_F fluxons $\Phi_1, \Phi_2, ..., \Phi_{n_F}$ have been introduced at locations $\vec{R}_1, \vec{R}_2, ..., \vec{R}_{n_F}$ where the ground state wave function of a system of N charges $\Psi^{(0)}(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N)$ is non vanishing. The modification of the Schrödinger equation via $\vec{\partial}_i \rightarrow \vec{\partial}_i + \frac{e}{c}\vec{A}_i$ will then change the wave function

$$\Psi^{(0)}(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N) \rightarrow \Psi^{(0)}(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N; \vec{R}_1, \Phi_1, \vec{R}_2, \Phi_2, ..., \vec{R}_{n_F}, \Phi_{n_F}) \tag{1}$$

and shift the initial ground state energy $E^{(0)}$ to

$$E^{(0)} \rightarrow E^{(0)} + \delta E^{(0)}(\vec{R}_1, \Phi_1, \vec{R}_2, \Phi_2, ..., \vec{R}_{n_F}, \Phi_{n_F}). \tag{2}$$

This energy shift induces an interaction energy between the fluxons:

$$\delta E^{(0)}(\vec{R}_j, \Phi_j) = W(\vec{R}_j, \Phi_j) \tag{3}$$

and the gradients $\nabla_{\vec{R}_i} W$ will then yield forces $\vec{F}_i$ acting on the fluxons Φ_i. The discussion of such forces and in particular the novel possibility of attraction between two semi-fluxons of equal sign ($\Phi_1 = \Phi_2 = \Phi_0/2$) is the main purpose of the present note.

To simplify the following we neglect first mutual interaction of the charged particles. The energy shift is then a sum:

$$\delta E^{(0)}(\vec{R}_j, \Phi_j) = \sum_{i=1}^{N} \delta E_{\gamma_i}(\vec{R}_j, \Phi_j) \tag{4}$$

over the shifts of the individual energies E_{γ_i} of the N charges. The latter will now be assumed to be fermions.

Consider first one fluxon, $\Phi = \alpha\Phi_0$, introduced in the center of a cylindrical region of radius R. The free wave functions will be eigenfunctions of the ($\hat{z}$ component of) angular momentum denoted by l. For $\Phi = 0$, l assumes the conventional positive and negative integer values $l = 0, \pm1, \pm2, \pm3, \cdots$. If the radial degrees of freedom were frozen then we would have simply $E^{(0)}_{l,r} = \frac{\hbar^2|l|^2}{2mr^2}$. The introduction of the fluxon effectively shifts up by α all the l values

$$|l| \to |l| + \alpha \quad for \quad l \geq 0, \qquad |l| \to |l| - \alpha \quad for \quad l < 0 \tag{5}$$

The *sum* of the energy shifts of the *pair* of levels $l = \pm|l|$ is then

$$\delta E^{(0)}_{|l|,r} = \frac{\hbar^2\xi(\alpha)}{2mr^2} \tag{6}$$

Due to the invariance under $\Phi \to \Phi + n\Phi_0$, $\xi(\alpha)$ is periodic: $\xi(\alpha + n) = \xi(\alpha)$. Specifically we find $\xi(\alpha) = \{\alpha\}^2$ if $\{\alpha\} < 1/2$, or $\xi(\alpha) = (1 - \{\alpha\})^2$ if $\{\alpha\} \geq 1/2$, with $\{\alpha\}$ the non integer part of α (i.e. $\{\alpha\} = \alpha mod 1$).

The total energy shift is found by summing $\delta E^{(0)}_{|l|,r}$ over l and r values. Bohr-Sommerfeld quantization implies discrete $r = r_n$ orbits. Along with the discrete l in each annular $r_{n+1} > r > r_n$ region this yields one state per area a_0^2 with a_0 the typical distance between the charged particles which in the ground state where we fill up the Fermi "circle". The two dimensional ("one layer") density is $n_2 \sim a_0^{-2}$. Thus [1], in the limit of many states:

$$W^{tot}_\alpha = \sum_{|l|,n} \frac{\hbar^2\xi(\alpha)}{2mr_n^2} \frac{1}{2} \sum_{l,n} \frac{\hbar^2\xi(\alpha)}{2mr_n^2} \tag{7}$$

$$\to \frac{\xi(\alpha)}{2}\frac{n_2\hbar^2}{2m}\int_{a_0}^{R}\frac{2\pi r dr}{r^2} = \frac{\pi}{2}\frac{\xi(\alpha)n_2\hbar^2}{2m}\ln(R/a_0) \tag{8}$$

is the total energy required in order to insert one fluxon at the center of a cylinder of radius R [2]. An extra factor 1/2 is due to the fact that there are only half as many ($l = \pm|l|$) pairs as l values. Also the minimal "Bohr radius" a_0 serves as a cutoff.

Let us address next the interaction of two *semi-fluxons* at a relative distance $|\vec{R}_1 - \vec{R}_2| = a$ introduced near the origin at the center of a large domain containing the mobile charged particles, with constant two dimensional density n_2. At a point $\vec{r}$ the vector potentials of the semi fluxons are: $\vec{A}_i(r) = \frac{1}{2}\Phi_0 \frac{\hat{e}_{\theta_i}}{|\vec{r}-\vec{R}_i|}$ $i = 1, 2$ where $\hat{e}_{\theta_i}$ refer to the tangential unit vector with respect to $\vec{R}_i$ as origin. For $r >> a$ $\vec{A}_1 + \vec{A}_2$ add, up to small corrections, to the vector potential of a single quantized fluxon at the origin. Insofar as the topological effect of interest are concerned the latter is just a gauge artifact. Hence we expect that only a region of size of order a around each fluxon and between the fluxons will be affected and consequently that mutual interaction energy behave like $W^{tot}(r)$ of eq. (7) with $r \sim a$ and $\xi(\alpha) = \xi(1/2) = 1/4$:

$$W_{(\alpha_1=1/2,\alpha_2=1/2)}(a) = \kappa \frac{\pi}{16} \frac{n_2 \hbar^2}{m} \ln(a/a_0) \tag{9}$$

The numerical factor of order one κ represents the effects of having a two center system.

This interaction leads then to an attractive force between the two semi-fluxons

$$F_{(1/2,1/2)}(a) \simeq \frac{\kappa\pi}{16} \frac{n_2 \hbar^2}{m} \frac{1}{a} \tag{10}$$

Recalling that W and F represent the effect of "one layer" and n_2 is the two dimensional number density in this layer, we can rewrite the last equation in a more useful form as

$$\frac{F_{(1/2,1/2)}(a)}{unit\ \ fluxon\ \ length} \simeq \frac{\kappa\pi}{16} \frac{n \hbar^2}{m} \frac{1}{a} \tag{11}$$

with n the true three dimensional density of the charges.

For two arbitrary fluxons α_1 and α_2 the force is given by an expression similar with (11) but with κ a function of α_1 and α_2. Obviously $\kappa(\alpha_1, \alpha_2)$ is a periodic function in both α_1 and α_2. The exact form of $\kappa(\alpha_1, \alpha_2)$ is difficult to calculate, and depends on many details, but it is easy to see when the force is attractive and when the force is repulsive. For this one has to compare the energy in the case when the two fluxons overlap with the energy when the two fluxons are far apart: From (8), the energy required to insert two overlapping fluxons is W_{α}^{tot} with $\alpha = \alpha_1 + \alpha_2$. The energy required to insert two fluxons when the fluxons are well separated is simply the sum of the energies required to insert each fluxon, i.e. $W_{\alpha_1}^{tot} + W_{\alpha_2}^{tot}$. Consequently, for $\xi(\alpha_1 + \alpha_2) > \xi(\alpha_1) + \xi(\alpha_2)$ the force is repulsive, while for $\xi(\alpha_1 + \alpha_2) < \xi(\alpha_1) + \xi(\alpha_2)$ the force is attractive. Clearly, for two semi-fluxons the force is attractive.

These forces act only on fluxons which are actually immersed in the charged particle background but are absent for fluxons which are outside this region. Thus if in the example of the concentric cylindrical geometry discussed above, we move the fluxon inside a hole, the energy of the system is unchanged and no force is expected. Since there are no charged particles in the hole we can continue using the same vector potential $\vec{A}(r) = \frac{\alpha\Phi_0}{r}\hat{e}_\theta$ even when the fluxon is not in the center. The key point is that for every path enclosing the fluxon (or fluxons - if there are several fluxons inside the hole) that a charge particle confined to the anular domain can *actually* perform - the AB phase will be the same.

The interaction (9) is quasi - confining ($W(r) \to \infty$ with $r \to \infty$) just like the two dimensional coulomb interaction. For such cases the system may find it energetically favorable once $r \geq \lambda_s = (screening\ \ length)$, to screen the charges (or fluxons in the present case). Indeed such a screening is generated by the circulation of all the charged particles of charge e (for fluxon of $\Phi = \Phi_0/2$ say). The corresponding current density at a distance r is

$$\vec{J}(r) = \frac{\hbar\alpha n}{mr}\hat{e}_\theta \tag{12}$$

The screening of the B field is found from Maxwell's equation:

$$\frac{dB_z^{induced}}{dr} = \frac{1}{c}J_\theta = \frac{\alpha e\hbar n}{mcr} \tag{13}$$

The α in eq. (13) depends on r due to the partial screening of the initial fluxon $\alpha = \alpha(r=0)$

by currents circulating between the origin and r:

$$\alpha(r) = (\alpha\Phi_0 - \int_0^r 2\pi r B_z^{induced}(r)dr)/\Phi_0 \tag{14}$$

The coupled equations (13) (14) yield r profiles for $\alpha(r)$ and $B_z^{induced}(r)$ which are exponentially falling off like $\exp(-r/\lambda_s)$, thus defining λ_s. Approximating $\alpha(r) = \alpha\theta(\lambda_s - r)$ we readily find λ_s from

$$2\pi \int_0^{\lambda_s+\epsilon} rB_z^{induced}(r) = \pi \int_0^{\lambda_s+\epsilon} r^2\frac{dB_z}{dr} = \tag{15}$$

$$= \frac{\pi}{mc}\frac{e\alpha\hbar\lambda_s^2}{2} = \alpha\Phi_0 = \frac{2\pi\alpha\hbar c}{e} \tag{16}$$

where we used integration by parts, eq. (13), and demanded that the net induced flux exactly cancel $\alpha\Phi_0$.

We have so far discussed the force between non integer fluxons in abstract - referring to ideal infinitely thin fluxons and ideal mobile non-interacting charged particles in the space between the fluxons. We will next briefly address the feasibility of actually detecting such forces.

In principle the conduction electrons in metals could serve as the charged medium particles. Their number density n is large enough, $n \sim 3 \cdot 10^{22}$ so as to make the putative $1/r$ force (eq. 11) appreciable ($\sim 10^{-3}$ $dyne/cm$ at $r = 10\mu$). Clearly for the force to operate over a range r the wave function of the electrons should coherently spread over this distance. Such extended coherent states in which the circulating pattern of currents associated with the modified ground states do not dissipate, have indeed been manifested in the related experiments of Webb [3] utilizing mesoscopic rings at low temperatures. A magnetic fluxon was introduced there at the hole of the ring. However in order to generate the forces of interest here the fluxons should be, according to the previous discussion, actually immersed inside the metal or be pressed right next to the surface. The fluxons in this case would be carried inside microscopic solenoid and it is not clear how a practical arrangement can be made where the forces of interest can actually compete with the metals rigidity.

This leads us than to consider type II superconductors where half fluxons (that is, integer fluxons in the charge of the Cooper pair) naturally arise. In ideal samples of sufficient purity and resultant minimal pinning two relatively near by fluxons may move toward each other under the effect of the force generated due to the presence of a fraction f of unpaired electrons.

It should be emphasized that despite superficial similarities, the effect considered here is novel and quite distinct from conventional forces between the fluxons in superconductors. Thus in type II superconductors an effective logarithmic interaction - cutoff at a distance of the order of the penetration length λ_p- exists between the fluxons. This is due to the $\vec{B}_1 \cdot \vec{B}_2$ interaction of the overlapping magnetic field extending out of the fluxons cores. Alternatively, it can be viewed as the interaction between the supercurrents maintaining the two fluxons [4]. This force is repulsive (attractive) between equal sign fluxons - a feature underlying the formation of the Abrikosov lattice - since the $\vec{B}_i$'s ($\vec{J}_i$) are parallel (anti-parallel).

Our force is completely different in every possible respect. First it is *attractive* also between two equal sign semi-fluxons. Second it is generated by the response and currents due to the *ordinary unpaired* electrons. The fraction of f of these varies like:

$$f = \frac{n_e}{n_e + n_s} \sim \left(\frac{T}{T_c}\right)^{3/2} \tag{17}$$

Thus for small f the present force will be weaker but potentially could have a longer range. Indeed from eq. (16) we find that the screening length due to the electrons is [5]:

$$\lambda_s = \frac{2}{\alpha_{em}} a_{Bohr} \Big(\frac{a_{Bohr}^{-3}}{n}\Big)^{1/2} = 150 \Big(\frac{10^{25}}{n}\Big)^{1/2} A^0 \tag{18}$$

We note that the phases of the unpaired electrons wave function and ensuing currents are not manifest in the Ginsburg Landau approximation. The latter - basic framework for much of the superconductivity research - is concerned only with the overall unpaired fraction (reflected in the magnitude of the order parameter) and hence cannot capture the delicate effect proposed here. Hopefully it is not altogether impossible that the effect can be observed in some special experimental arrangement [6].

We will conclude now with few remarks:

(i) We considered so far a system of free charged fermions. Let us consider the induced interactions for charged bosons. At $T = 0$ all bosons would be in the same nodeless ground state $\psi_0(x, y)$. In the cylindrically symmetric geometry considered above ψ_0 has $l = 0$ and is roughly constant radially. The introduction of the semi-fluxon will shift it to $l = 1/2$. The expectation of $\frac{l^2}{2mr^2}$ yield then a $\sim \frac{\hbar^2}{8m} \ln R$ estimate for the energy shift of a single state and $\frac{n\hbar^2}{8m} \ln R$ for a density n of particles.

(ii) The semi-fluxons in a $T = 0$ charged bosonic background exhibit an amusing confining - screening interplay, somewhat reminiscent of this effect in QCD. There is a confining linear $\bar{Q}Q$ potential $V = \sigma R$ at "large" distances and the same is expected for QQ "baryons" in $SU(2)_c$. Creation of $\bar{q}q$ pairs tends to screen the confining potential - and only exponentially falling Yukawa like potentials exist between physical, color neutral, hadrons.

The QCD quarks with non zero triality (screening the confining interaction between $\bar{Q}Q$) are analogous to the electrons which transform non trivially under the "Z_2" of the fluxon in our example, and generate currents screening the interaction between the semi-fluxons. The mechanisms for screening and confinement tend to be mutually exclusive: both in QCD and in our example the screening of charges reduce the long range forces and resulting putative pairing of $Q\bar{Q}$, (two semi-fluxons here). Also $\bar{q}q$'s which are already paired by confinement to triality (and color) singlets will not screen the $Q\bar{Q}$ force. The introduction of the two semi-fluxons will not induce here large scale pairing of the electrons. Yet the mechanism of semi-fluxons confinement may quench the screening currents. This could be the case for the Bose-Einstein condensate example. A null line will form between the fluxons or between the fluxons and the boundary of the medium [7]. This line impedes the circulation of screening currents around Φ_1 or Φ_2 separately.

(iii) Finally it is amusing to compare the topological force (11) with the Casimir force [8] between two parallel conducting wires $\sim \frac{\hbar c}{a^3}$. The ratio is: $\rho \equiv F_{top}/F_{cas} \sim n\hbar a^2/mc^2 \sim n\lambda_{com}a^2$. For electron systems $n \sim a_0^{-3}$ with a_0 of the order of the Bohr radius. Using $\lambda_{com}/a_0 \sim \alpha_{em}$ we have then $\rho \sim \alpha_{em}a^2/a_0^2$. Since generally $a >> a_0$ this ratio is very large. The origin of this large ratio is easy to assess. Only vacuum fluctuations (photons) on scales $\lambda \sim a$ contribute to the Casimir force whereas *all* electron modes down to wavelength $\lambda \sim a_0$ contribute equally to the interaction energy and force proposed here.

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References

[1] The problem of finding $W_\alpha^{tot}(R)$ for the case when we use the eigenfunctions $J_l(k_{n,l}^{(0)}r)$ with energies $(k_{n,l}^{(0)})^2$ is somewhat involved. The $k_{n,l}$ are determined implicitly from the n'th solution of $J_l(kR) = 0$ which is rather complicated for the large l and kr required here.

[2] The logarithmic dependence of $W_\alpha^{tot}(R)$ on R is expected from general scaling arguments. Assume that the fluxon is inserted at the center of a cylindrical hole of radius R_{in}, inside a concentric annulus of external radius R_{out}. The minimal substitution in the regular, symmetric, gauge $\vec{A} = \alpha \frac{\phi_0}{r} \hat{e}_\theta$, $\vec{\partial} \to \vec{\partial} + \frac{e}{c}\vec{A}$, is such that the total energy

$$\sum_\gamma \int |(\vec{\partial} + \frac{e}{c}\vec{A})\psi_\gamma|^2 dxdy$$

remains invariant if we scale $x \to \lambda x$ and $y \to \lambda y$, provided we have a homogeneous uniform (two dimensional) density

$$n_2(x,y) = \sum_\gamma |\psi_\gamma(x,y)|^2 \simeq \quad const.$$

where we assumed that sufficiently many states γ are summed over so that the last eq. is justified. This implies that $W \simeq \log(R_{out}/R_{in})$ The argument holds also for general domains of overall size R and any shape: only the coefficient in $W(R) \simeq c\ln(R)$ would depend on the dimensionless ratios characterizing the shape.

[3] R. Webb, in *Quantum Coherence and Reality,* international conference on fundamental aspects of quantum theory, Univ. of South Carolina, 1992. Ed. J. S. Anandan and J. L. Safko.

[4] In the London approximation $\vec{J}_i \sim \vec{V}_i \sim \vec{A}_i$ with $i = 1, 2$ referring to the first or second fluxon and the interaction is $\frac{e^2}{c^2} \int \vec{A}_1 \cdot \vec{A}_2$.

[5] The circulation of the ordinary electron currents outside λ_p is energetically unfavorable from the point of view of the dominant Cooper pairs as it would generate $\vec{B}$ field outside the original fluxon. However if the coherence length of the electrons wave function is small it will predominantly limit the range of the force.

[6] We need to have sufficiently low temperatures to maximize the coherence length of electrons and yet maintain appreciable fraction f of unpaired electrons. The distance

r between the two fluxons has to be substantially larger then λ_p the range of the much stronger ordinary repulsive forces and pinning effects should not mask the very weak force.

[7] Y. Aharonov, S. Coleman, A. Goldhaber, S. Nussinov, S. Popescu, B. Reznik D. Rohrlich, L. Vaidman, Phys. Rev. Lett. **73**, 918 (1994).

[8] I. H. Duru, Found. of Phys. **23**, 809, (1993) , suggested that a Casimir type $1/a^3$ forces exist in vacuum between fluxons due to the effect of charge Higgs field. The Higgs field, or other virtually created pairs of massive particles, cannot mediate long range forces which are in fact due to two photon exchange. Duru's suggestion amounts therefore to a renormalization due to fluxon effect of the ordinary Casimir force. For example see: G. Feinberg J. Sucher and C. K. Au., Phys. Rep. 180, **83** (1989).