Causality and Nonlocality as Axioms for Quantum Mechanics

Sandu Popescu, Daniel Rohrlich*
Basic Research Institute in the Mathematical Sciences
HP Laboratories Bristol
HPL-BRIMS-97-23
October, 1997

foundations of quantum mechanics, nonlocality, axioms

Quantum mechanics permits nonlocality – both nonlocal correlations and nonlocal equations of motion – while respecting relativistic causality. Is quantum mechanics the unique theory that reconciles nonlocality and causality? We consider two models, going beyond quantum mechanics, of nonlocality: "superquantum" correlations, nonlocal "jamming" of correlations. These models are consistent with some definitions of nonlocality and causality.

*School of Physics and Astronomy, Tel Aviv University, Ramat Aviv, Israel

TAUP 2452-97

CAUSALITY AND NONLOCALITY AS AXIOMS FOR QUANTUM MECHANICS*

Sandu Popescu

University of Cambridge, Isaac Newton Institute, 20 Clarkson Road, Cambridge, U.K. CB3 0EH and BRIMS Hewlett-Packard Labs. Bristol BS12 6QZ, U.K.

Daniel Rohrlich

School of Physics and Astronomy, Tel Aviv University, Ramat Aviv 69978 Israel

(October 7, 1997)

Abstract

Quantum mechanics permits nonlocality—both nonlocal correlations and nonlocal equations of motion—while respecting relativistic causality. Is quantum mechanics the unique theory that reconciles nonlocality and causality? We consider two models, going beyond quantum mechanics, of nonlocality: "superquantum" correlations, and nonlocal "jamming" of correlations. These models are consistent with some definitions of nonlocality and causality.

I. INTRODUCTION

"But how can it be like that?" This question, which every student of quantum mechanics asks, is unanswerable, wrote Feynman; we should not keep asking ourselves "But how can it

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*For the proceedings of the symposium on *Causality and Locality in Modern Physics and Astronomy: Open Questions and Possible Solutions*, York University, Toronto, August 25-29, 1997 be like that?" lest we end up in a blind alley from which no one has yet escaped. According to Feynman, the question reflects an utterly vain desire to see quantum mechanics in terms of something familiar [1]. What exactly is the problem? Is it that an electron can be in two places at once? No, we are used to such behavior in electrons. The problem is deeper. We see the problem if we compare quantum mechanics with the special theory of relativity. Special relativity can be deduced in its entirety from two axioms: the equivalence of inertial reference frames, and the constancy of the speed of light. Both axioms have clear physical meaning. By contrast, the numerous axioms of quantum mechanics have no clear physical meaning.

Shimony [2,3] and Aharonov [4,5] offer hope and a new approach to this problem. Their point of departure is the remarkable coexistence (peaceful or otherwise) of quantum nonlocality and the theory of relativity. Shimony focussed on the subtle nonlocality of quantum correlations. On the one hand, quantum correlations themselves obey relativistic causality, in the sense that we cannot exploit quantum correlations to transmit signals at superluminal speeds [8] (or at any speed). On the one hand, as Bell [6] showed, quantum correlations could not arise in any theory of local hidden variables [7]. That quantum mechanics combines nonlocality and causality is wondrous. Nonlocality and causality seem *prima facie* incompatible. Yet quantum correlations do not permit action at a distance, and Shimony [2] has aptly called the nonlocality manifest in quantum correlations "passion at a distance". Shimony has raised the question whether nonlocality and causality can peacefully coexist in any other theory besides quantum mechanics [2,3].

Quantum mechanics also implies nonlocal equations of motion, as Aharonov [4,5] has pointed out. In one version of the Aharonov-Bohm effect [9], a solenoid carrying an isolated magnetic flux, inserted between two slits, shifts the interference pattern of electrons passing through the slits. The electrons therefore obey a nonlocal equation of motion: they never pass through the flux yet the flux affects their final positions on the screen [10]. Aharonov has shown that the solenoid and the electrons exchange a physical quantity, the *modular momentum*, nonlocally. In general, modular momentum is measurable and obeys a nonlocal equation of motion. But when the flux is constrained to lie between the slits, its modular momentum is completely uncertain, and this uncertainty keeps us from seeing a violation of causality. Nonlocal equations of motion imply action at a distance, but quantum mechanics just barely manages to respect relativistic causality. Could it be, Aharonov [5] has asked, that quantum mechanics is the *unique* theory combining them?

The parallel questions raised by Shimony and Aharonov lead us to consider models for theories, going beyond quantum mechanics, that reconcile nonlocality and causality. Is quantum mechanics the only such theory? If so, nonlocality and relativistic causality together imply quantum theory, just as the special theory of relativity can be deduced in its entirety from two axioms [5]. In this paper, we will discuss model theories [11–13] manifesting nonlocality while respecting causality. The first model manifests nonlocality in the sense of Shimony: nonlocal correlations. The second model manifests nonlocality in the sense of Aharonov: nonlocal dynamics. These models raise new theoretical and experimental possibilities. Apparently, quantum mechanics is *not* the only theory that reconciles nonlocality and relativistic causality. Yet, it is possible that stronger axioms of locality and causality could rule out both models. Thus we have no final answer to the question, Is quantum mechanics the only theory that reconciles nonlocality and causality? It is not surprising that we cannot offer a final answer to the question; it is perhaps surprising that we can offer any answer. But most of all, we offer the *attempt* to formulate and answer the question.

II. NONLOCALITY I: NONLOCAL CORRELATIONS

The Clauser, Horne, Shimony, and Holt [14] form of Bell's inequality holds in any classical theory (that is, any theory of local hidden variables). It states that a certain combination of correlations lies between -2 and 2:

$$-2 \leq E(A,B) + E(A,B') + E(A',B) - E(A',B') \leq 2 \quad . \tag{1}$$

Besides 2, two other numbers, $2\sqrt{2}$ and 4, are important bounds on the CHSH sum of correlations. If the four correlations in Eq. (1) were independent, the absolute value of the sum could be as much as 4. For quantum correlations, however, the CHSH sum of correlations is bounded [15] in absolute value by $2\sqrt{2}$. Where does this bound come from? Rather than asking why quantum correlations violate the CHSH inequality, we might ask why they do not violate it *more*. Suppose that quantum nonlocality implies that quantum correlations violate the CHSH inequality at least sometimes. We might then guess that relativistic causality is the reason that quantum correlations do not violate it maximally. Could relativistic causality restrict the violation to $2\sqrt{2}$ instead of 4? If so, then nonlocality and causality would together determine the quantum violation of the CHSH inequality, and we would be closer to a proof that they determine all of quantum mechanics. If not, then quantum mechanics cannot be the unique theory combining nonlocality and causality. To answer the question, we ask what restrictions relativistic causality imposes on joint probabilities. Relativistic causality forbids sending messages faster than light. Thus, if one observer measures the observable A, the probabilities for the outcomes $A = 1$ and $A = -1$ must be independent of whether the other observer chooses to measure B or B'. However, it can be shown [11,16] that this constraint does not limit the CHSH sum of quantum correlations to $2\sqrt{2}$. For example, imagine a "superquantum" correlation function E for spin measurements along given axes. Assume E depends only on the relative angle θ between axes. For any pair of axes, the outcomes $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ are equally likely, and similarly for $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$. These four probabilities sum to 1, so the probabilities for $|\uparrow\downarrow\rangle$ and $|\downarrow\downarrow\rangle$ sum to 1/2. In any direction, the probability of $|\uparrow\rangle$ or $|\downarrow\rangle$ is 1/2 irrespective of a measurement on the other particle. Measurements on one particle yield no information about measurements on the other, so relativistic causality holds. The correlation function then satisfies $E(\pi - \theta) = -E(\theta)$. Now let $E(\theta)$ have the form

(i) $E(\theta) = 1$ for $0 \leq \theta \leq \pi/4$;

(ii) $E(\theta)$ decreases monotonically and smoothly from 1 to -1 as θ increases from $\pi/4$ to

$3\pi/4$;

(iii) $E(\theta) = -1$ for $3\pi/4 \le \theta \le \pi$.

Consider four measurements along axes defined by unit vectors $\hat{a}'$, $\hat{b}$, $\hat{a}$, and $\hat{b}'$ separated by successive angles of $\pi/4$ and lying in a plane. If we now apply the CHSH inequality Eq. (1) to these directions, we find that the sum of correlations

$$E(\hat{a},\hat{b}) + E(\hat{a}',\hat{b}) + E(\hat{a},\hat{b}') - E(\hat{a}',\hat{b}') = 3E(\pi/4) - E(3\pi/4) = 4 \qquad (2)$$

violates the CHSH inequality with the maximal value 4. Thus, a correlation function could satisfy relativistic causality and still violate the CHSH inequality with the maximal value 4.

III. NONLOCALITY II: NONLOCAL EQUATIONS OF MOTION

Although quantum mechanics is not the unique theory combining causality and nonlocal correlations, could it be the unique theory combining causality and nonlocal equations of motion? Perhaps the nonlocality in quantum dynamics has deeper physical signficance. Here we consider a model that in a sense combines the two forms of nonlocality: nonlocal equations of motion where one of the physical variables is a nonlocal correlation. *Jamming*, discussed by Grunhaus, Popescu and Rohrlich [12] is such a model. The jamming paradigm involves three experimenters. Two experimenters, call them Alice and Bob, make measurements on systems that have locally interacted in the past. Alice's measurements are spacelike separated from Bob's. A third experimenter, Jim (the jammer), presses a button on a black box. This event is spacelike separated from Alice's measurements and from Bob's. The black box acts at a distance on the correlations between the two sets of systems. For the sake of definiteness, let us assume that the systems are pairs of spin-1/2 particles entangled in a singlet state, and that the measurements of Alice and Bob yield violations of the CHSH inequality, in the absence of jamming; but when there is jamming, their measurements yield classical correlations (no violations of the CHSH inequality). Indeed, Shimony [2] considered such a paradigm in the context of the experiment of Aspect, Dalibard, and Roger [17]; his concern was to probe hidden-variable theories due to Vigier and others [18].

Here, we ask whether such a nonlocal equation of motion (or one, say, allowing the third experimenter nonlocally to create, rather than jam, nonlocal correlations) could respect causality. The jamming model [12] addresses this question. In general, jamming would allow Jim to send superluminal signals. But remarkably, some forms of jamming would not; Jim could tamper with nonlocal correlations without violating causality. Jamming preserves causality if it satisfies two constraints, the *unary* condition and the *binary* condition. The unary condition states that Jim cannot use jamming to send a superluminal signal that Alice (or Bob), by examining her (or his) results alone, could read. To satisfy this condition, let us assume that Alice and Bob each measure zero average spin along any axis, with or without jamming. In order to preserve causality, jamming must affect correlations only, not average measured values for one spin component. The binary condition states that Jim cannot use jamming to send a signal that Alice and Bob *together* could read by comparing their results, if they could do so in less time than would be required for a light signal to reach the place where they meet and compare results. This condition restricts spacetime configurations for jamming. Let a, b and j denote the three events generated by Alice, Bob, and Jim, respectively: a denotes Alice's measurements, b denotes Bob's, and j denotes Jim's pressing of the button. To satisfy the binary condition, the overlap of the forward light cones of a and b must lie entirely *within* the forward light cone of j. The reason is that Alice and Bob can compare their results only in the overlap of their forward light cones. If this overlap is entirely contained in the forward light cone of j, then a light signal from j can reach any point in spacetime where Alice and Bob can compare their results. This restriction on jamming configurations also rules out another violation of the unary condition. If Jim could obtain the results of Alice's measurements prior to deciding whether to press the button, he could send a superluminal signal to Bob by *selectively* jamming [12].

An odd feature in this model is that, in principle, it allows the effect (the correlations measured at a and b) to precede the effect (the action of Jim at j). Such reversals may boggle the mind, but they do not lead to any inconsistency as long as they do not generate self-contradictory causal loops [19,20]. It is not hard to show [12] that if jamming satisfies the unary and binary conditions, it does not lead to self-contradictory causal loops, regardless of the number of jammers. While we argue that jamming is consistent even if it allows reversals of the sequence of cause and effect, we also point out that such reversals arise only in one space dimension. In higher dimensions, jamming is not possible if both a and b precede j; the binary condition itself eliminates such configurations [21].

IV. CONCLUSIONS

Two related questions of Shimony [2,3] and Aharonov [5] inspire our work. Nonlocality and relativistic causality seem *almost* irreconcilable. The emphasis is on *almost*, because quantum mechanics does reconcile them, and does so in two different ways. But is quantum mechanics the unique theory that does so? Our preliminary answer is that it is not: model theories going beyond quantum mechanics, but respecting causality, allow nonlocality. However, we qualify our answer. First, nonlocality is not completely defined. Nonlocality in quantum mechanics includes both nonlocal correlations and nonlocal equations of motion, and we do not know exactly what kind of nonlocality we are seeking. Second, even causality is not completely defined; there are formulations of causality that we often take to be equivalent, that are not strictly equivalent. Is quantum mechanics the unique theory that reconciles nonlocality and causality? We cannot offer a final answer to the question, but we offer the *attempt* to formulate and answer the question. Whether by strengthening the axioms of nonlocality and causality, or by adding new axioms with clear physical meaning, we hope to rediscover quantum mechanics as the unique theory consistent with these axioms.

ACKNOWLEDGMENTS

D. R. acknowledges support from the State of Israel, Ministry of Immigrant Absorption, Center for Absorption in Science (Giladi Program).

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