Proceedings of the Workshop on Nonlinear Wave Modulation in Optical and Other Systems

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HPL-BRIMS-97-11
June, 1997

nonlinear wave modulation, optical systems

This BRIMS report contains a summary of talks and discussions from the joint workshop between ICMS and BRIMS held following the 1997 BAMC. Mathematical techniques for integrable and near-integrable evolution equations have been essential in both basic and applied nonlinear optics. Several talks explore new ways that this mathematical theory could be used to advance state of the art optical transmission and processing devices. The material also covers new developments in applications of the theory of nonlinear waves and patterns to basic nonlinear optics.

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INTRODUCTION:

Mathematical techniques for integrable and near-integrable evolution equations have been essential in both basic and applied nonlinear optics. One goal of the workshop was to explore new ways that this mathematical theory could be used to advance state of the art optical transmission and processing devices. A second goal of equal importance was to identify interesting mathematical problems suggested in the context of these applications.

The workshop also provided an opportunity for researchers to exchange ideas and to describe new developments in applications of the modulation theory of nonlinear waves and of the theory of nonlinear pulse dynamics to optical and other electronic communications or data-handling systems. It included both presentations and discussions.

MINUTES OF THE WORKSHOP:

On April 4th and 5th BRIMS, Hewlett-Packard Laboratories and the International Centre for Mathematical Sciences held a short workshop devoted to the mathematical techniques and ideas used in nonlinear optics and its applications. The meeting immediately followed the 1997 British Applied Mathematics Colloquium held at the University of Edinburgh 1-4 April. During BAMC there was a minisymposium on Nonlinear Optics organized by John Elgin of Imperial College and two invited lectures by Alan Newell of the University of Warwick. In addition, a related workshop was held prior to BAMC at ICMS entitled Asymptotic and Numerical Methods in Wave Propagation that emphasized classical linear problems in electromagnetics. See Appendix A below for more information on these related programs.

The workshop started off on Friday afternoon April 4th following the BAMC minisymposium on Nonlinear Optics that had been held that morning. The first talk of the afternoon was by Alexander Mikhailov. He described the use of inverse scattering theory on the semi-infinite interval to address stimulated Raman scattering of a laser pulse seeded by a Stokes pulse. He showed that the birth of solitons corresponds to sudden large variations of the phase of the output pump pulse. His analysis also showed that initial Stokes phase flips induce Raman spikes in the pump output. He used his analysis to interpret results obtained in short-pulse experiments.

After a coffee break and discussion period, Stefan Wabnitz presented recent experimental and theoretical results on induced modulational instability and parametric frequency generation in optical birefringent fibres in the strong conversion regime. The experiments showed for the first time the break up of intense quasi-cw waves into trains of soliton-like pulses at THz repetition rates in the normal dispersion regime of optical fibres. These results were shown to be in perfect agreement with theory. While predictions for the conditions for maximum frequency conversion were far from the predictions of the linearized analysis a secondary instability analysis was shown to give correct results.

The important topic of dispersion management in optical soliton transmission was covered next by Wladek Forysiak. He examined the role of dispersion management in future single-channel and WDM optical soliton transmission systems. He considered two forms of dispersion management: stepwise exponential tapering, using fibres of anomalous dispersion only, and strong dispersion management, using fibres of anomalous and normal dispersion. In the first case, he considered the specific forms of dispersion profiles that give rise to improved soliton propagation in single-channel systems, and reduced channel cross-talk in WDM systems. In the second case, he described the new, periodically oscillating soliton-like solutions, and examined their properties in the context of optical fibre communications.

The final talk on Friday was by John Arnold. He described a novel technique for encoding information on the periodic nonlinear waves that are naturally supported in optical fibers. This work makes use of the integrable structure of the governing evolution equations and discrete approximations thereof. Healthy discussions accompanied each of these talks and spilled over into the reception that followed the last talk. Most of the group stayed on and went together to dinner, where discussion continued.

On Saturday the workshop continued with a pair of talks on patterns and their control. These complemented the talks by Oppo and Saffman given on Friday at the BAMC minisymposium. The first talk described optical systems which appear to show dynamics typically associated with waves in excitable media. These results were obtained in two and three- level laser systems with multimode transverse dynamics. The second talk from the same laboratory was given by Weiping Lu describing control and tracking of optical patterns into regimes exhibiting spatio-temporal chaos. Algorithms were described to achieve this control and tracking based on the idea of stabilization of unstable periodic wave patterns. The algorithms were demonstrated through numerical simulations.

Giampaolo D'Alessandro continued with a presentation on stability in the interaction between a pump wave and a counter-propagating second harmonic wave phase-matched in a periodically poled $\chi^{(2)}$ material. In contrast to the more usual co-propagating case, backward phase-matched frequency doubling displays richer and more complex behavior, as a result of the presence of built-in feedback. A matched asymptotic expansion was used to understand the boundary layer that is formed in this system when it is posed on the finite line. Numerical results using the Chebychev-based spectral method were also presented.

Following a break for coffee and discussion, Dave Hutchings described his analysis of nonlinear refractive coupling and mixed-mode spatial solitary waves in anisotropic cubic media. Using the slowly-varying envelope approximation, equations of motion were derived for the propagation of orthogonal polarization components in an anisotropic cubically nonlinear medium including birefringence. The Hamiltonian of this interaction was generated and used to study stationary solutions (eigenpolarizations) and their stability. It was found that as the modulus of the ratio of the nonlinearity to birefringence increases, the normally stable TE and TM modes of a waveguide become unstable; there is a bifurcation of the stable eigenpolarizations to states that are elliptically or linearly polarized in a particular orientation. This analysis was also extended to study the propagation of mixed-mode spatial solitary waves and comparison with recent experimental results in AlGaAs planar waveguides was described. Patric Bontemps next presented a variational analysis of pulse propagation and resonant interaction in quadratic media using a Gaussian anzats and described some potential implications for optical devices.

At noon the meeting was adjourned for lunch. We went as a group nearby where we had time to eat and continue our discussions in some detail. Following the lunch break we reconvened at ICMS where Sioned Baker presented her results on soliton jitter in optical communications systems due to random birefringence. She discussed the effect of random variations in the orientation of the birefringence axes along an optical fibre on the velocity of optical solitons, and how this leads to an uncertainty in the pulse to pulse arrival times. An analytic expression for the square deviation in arrival times was derived using soliton perturbation theory. This formula was shown to be consistent with numerical simulations of the full evolution equations. The analytic results assume the application of an ergodic hypothesis, whereby results derived from an ensemble average over a collection of fibres are the same as those found using a single fibre. Details of the mathematical model presented showed that randomness of the fibre axes leads to the stochastic jitter in a soliton pulse train.

Tassos Fragos continued with an approximate analysis of the interaction of radiation modes with a soliton in nonlinear Schrödinger equation. The soliton dressing method was considered for the A sech(x) initial value problem. It was shown that given an initial condition which is a perturbation of A sech(x) it is possible to predict quite accurately the emerging soliton characteristics from the consideration of the integrals of motion of the NLS system. Analytical results were compared with numerical simulations of the NLS initial value problem in order to assess the limits of the approach. Subsequently, using the lagrangian method to describe the propagation of the continuum, an approximate description of the main features of the propagation of the soliton plus radiation initial condition was achieved.

The final talk was presented by Greg Luther who gave a geometrical description of quasi-phase-matching in second harmonic generation and provided a new understanding of dispersion managed solitons. He began by using Poisson reduction to obtain a reduced phase space for three-wave mixing. This reduced phase space generalizes the Poincaré sphere. He went on to discuss both the existing linear theory of QPM and to extend it into the nonlinear regime by composing the dynamics of alternating dynamical systems on the reduced invariant phase phase. He finished by describing numerical results in which the finite gap spectrum of dispersion-managed solitons is monitored during their propagation. These numerics demonstrate the existence of nearly periodic orbits for the composition of the self-focusing and self-defocusing dynamics in ideal dispersion managed systems. Following this final talk, the meeting broke for a period of coffee and discussion. Many of the participants lingered on for an hour or two of detailed discussions before returning home.

SUMMARY:

The participants were quite happy with the workshop. They assured us that it was very profitable and that it would impact their research efforts significantly. We were quite pleased that even with only a day and a half, the meeting represented progress on so many areas of central interest in mathematical nonlinear optics. The participants also expressed an enthusiastic interest in continuing to having meetings of similar type in and around the UK. The interdisciplinary nature of the participants enabled many of the participants to identify new ways to apply their mathematical expertise to problems arising in nonlinear optics. At the same time several participants found that new mathematical issues were suggested during both talks and the ensuing discussions.

APPENDIX A: RELATED BAMC EVENTS

Thursday April 3

Invited Lecture

Prof. Alan C. Newell of the University of Warwick will Lecture on "Semiconductor Lasers and Kolmogorov Spectra" at **12 noon** in the George Square Theatre of the University of Edinburgh.

Friday April 4

BAMC Minisympósium on Nonlinear Optics

Organizer: John N. Elgin

Description: Two topics will be covered during the minisymposium, chosen to demonstrate the wide range of current interest in problems which arise in Nonlinear Optics. The first is concerned with pattern formations in optical systems. Dr G-L Oppo will discuss and review recent theoretical developments, and Dr M Saffman will give a complementary talk on some recent theoretical experimental work on pattern formations in optical systems. The second topic concerns mathematical problems which arise in studies on telecommunication systems based on optical soliton and 'NRZ' techniques (NRZ: non return to zero). Both schemes are described to leading order by the nonlinear Schrödinger equation, but the relative sign between the dispersive and the nonlinear terms is reversed for the NRZ case - this means that though still integrable, there are no soliton solutions. Prof. Elgin will talk on solitons, and Prof. Wabnitz on NRZ systems.

Venue: Lecture Theatre C of David Hume Tower (adjoining the William Robertson Building) in the George Square area of Edinburgh University.

Talks:

Dr Gian-Luca Oppo, Department of Physics and Applied Physics, University of Strathclyde "Control of Pattern Formation in Nonlinear Optics"

Dr Mark Saffman, Risö National Laboratory, Roskilde, Denmark "Spatial dynamics of light beams in media with cubic nonlinearity: instability, solitons and patterns"

Prof. John N. Elgin, Department of Mathematics, Imperial College, London "Solitons in Optical Fibres"

Prof. Stefan Wabnitz, Laboratoire de Physique, Universite' de Bourgogne, Dijon, France "Nonlinear Theory of NRZ Signal Propagation in Optical Fibers"

APPENDIX B: WORKSHOP PROGRAM

Friday April 4

SESSION I: J.N. Elgin, Chairman

12:45 BAMC closes

2:30 Opening remarks by Luther and Parker

2:45 **A.V. Mikhailov**
"Generation of Solitons in Stimulated Raman Scattering"

3:30 Coffee and Discussion

4:00 **S. Wabnitz**, G. Millot, E. Seve and S. Trillo
"Nonlinear Modulation Dynamics in Birefringent Fibres"

4:45 **W. Forysiak**, N.J. Doran and N.J. Smith
"Dispersion Management in Optical Soliton Transmission"

5:30 **J.M. Arnold**
"The Soliton Communication Channel"

6:15 Reception

8:00 Dinner

Saturday April 5

SESSION II: A.V. Mikhailov, Chairman

9:00 **Dejin Yu**, Weiping Lu and R.G. Harrison
"Wave Patterns in Excitable Optical Systems"

9:30 **Weiping Lu**, Dejin Yu and R.G. Harrison
"Control and Tracking of Wave Patterns in Optical Turbulence"

10:00 **G. D'Alessandro**, P.St.J. Russell and A.A. Wheeler
"Nonlinear Dynamics of a Backward Quasi-Phase-Matched Second Harmonic Generator"

10:30 Coffee and Discussions

11:00 **D. Hutchings**
"Nonlinear Refractive Coupling and Mixed-Mode Spatial Solitary Waves in Anisotropic Cubic Media"

11:30 **A.D. Boardman** and P.Bontemps
"Self-Focusing in Quadratic Media"

12:00 Lunch Break

2:00 **S.M. Baker** and J.N. Elgin
"Soliton Jitter in Optical Fibres with Random Birefringence"

2:30 **T. Fragos** and J.M. Arnold
"Approximate Description of the Soliton-Radiation Interaction"

3:00 **G.G. Luther**, M.S. Alber, J.E. Marsden and J.M. Robbins
"Understanding Quasi-Phase-Matched SHG and Dispersion Managed Solitons"

3:30 Coffee and Discussions

4:00 Farewell

APPENDIX C: ABSTRACTS

The Soliton Communication Channel

J.M. Arnold
Department of Electronics and Electrical Engineering
University of Glasgow

An infinitely long train of soliton pulses is a channel for the transmission of information. When the pulse train is exactly stationary periodic, the information capacity is trivial, and no greater than that of a single 1-soliton. However, when the solitons are allowed to be perturbed from their stationary periodic (equilibrium) configuration, then information is carried in the perturbations which can be detected after transmission over the spatial variable x. A nonstationary nonperiodic pulse train exhibits internal dynamical motion with propagation, and this internal dynamics renders the detection process more complicated, since the internal dynamics must be 'deconvolved' before the original modulation can be identified. The nonlinear deconvolution process is carried out in the presence of noise, which leads to errors in detection. Certain parameters of a soliton train, forming half of its degrees of freedom, are invariant with propagation, and can be detected without deconvolution; the other half of the degrees of freedom are phase variables. Ignoring the phase variables, and only detecting the invariants, is a form of incoherent detection. Coherent detection requires references at the detector for the phase variables. The Inverse Scattering Transform (IST) is a natural framework for the description of this type of analysis in evolution systems which are integrable. It is generally possible to approximate the multi-soliton dynamics of of an integrable wave system such as the Nonlinear Schrödinger equation or KdV equation by an integrable discrete lattice system of Toda type, which is also complex in the case of the Nonlinear Schrödinger equation. The approximating Toda system possesses invariants which can be mapped as invertible linear superpositions of the invariants of the wave system, up to the accuracy of the approximation. The study of incoherent detection can therefore be transferred to the Toda system, which greatly simplifies the mathematical representation of the soliton communication channel. A preliminary version of this theory has already been successfully applied to the description of pulse-position modulation of solitons (J.M. Arnold, Optics Letters, 21, 30-32, 1996); current work is directed at a more general formulation of the theory applicable to a wider class of modulation formats, both analogue and digital.

Self-Focusing.in Quadratic Media

A.D. Boardman and **P.Bontemps**
Department of Physics
University of Salford

Using the paraxial approximation, we show that the evolution of the Gaussian beam widths of the fundamental and second-harmonic is described by a system of two second-order differential equations. We then extend this theory to a system of coupled vector equations in nonlinear quadratic media, bounded by magneto-optic materials. For both cases, the agreement between the theory and the numerics is acceptable. In conclusion, we show that the results can be used in the development of new optical devices.

Nonlinear Dynamics of a Backward Quasi-Phase-Matched Second Harmonic Generator

G. D'Alessandro♣, **P.St.J. Russell**♠ and **A.A. Wheeler**♣
♣ Department of Mathematics, University of Southampton
♠ School of Physics, University of Bath

We study the stability of the interaction between a pump wave and a counter-propagating second harmonic wave phase-matched in a periodically poled $\chi^{(2)}$ material using both analytical and numerical methods. In contrast to the more usual co-propagating case, backward phase-matched frequency doubling displays richer and more complex behavior, owing to the presence of built-in feedback.

Soliton Jitter in Optical Fibres with Random Birefringence

S.M. Baker and **J.N. Elgin**
Department of Mathematics
Imperial College

We will discuss how a random variation in the orientation of the birefringence axes along an optical fibre will affect the velocity of optical solitons, and how this might lead to an uncertainty in their arrival times. An analytic expression for the square deviation in arrival times is derived using an appropriate perturbation

theory, and this is shown to be consistent with a numerical simulation of the full evolution equations. The analytic results assume the application of an ergodic hypothesis, whereby results derived from an ensemble average over a collection of fibres will be the same as those found using a single fibre. Details of this mathematical model presented show how the randomness of the fibre axes leads to a detrimental stochastic jitter in the soliton pulse train.

Dispersion Management in Optical Soliton Transmission

W. Forysiak, **N.J. Doran** and **N.J. Smith**
Department of Electrical Engineering and Applied Physics
Aston University

We examine the role of dispersion management in future single-channel and WDM optical soliton transmission systems. We consider two forms of dispersion management: stepwise exponential tapering, using fibres of anomalous dispersion only, and strong dispersion management, using fibres of anomalous and normal dispersion. In the first case, we consider the specific forms of dispersion profiles that give rise to improved soliton propagation in single-channel systems, and reduced channel cross-talk in WDM systems. In the second case, we describe the new, periodically oscillating soliton-like solutions, and examine their properties in the context of optical fibre communications.

Approximate Description of the Soliton-Radiation Interaction

T. Fragos and **J.M. Arnold**
Department of Electrical and Electronic Engineering
University of Glasgow

The soliton dressing method is considered for the A sech(x) initial value problem of the NLS equation. It is found that given an initial condition which is a perturbation of A sech(x) it is possible to predict quite accurately the emerging soliton characteristics from the consideration of the integrals of motion of the NLS system. The results are compared with numerical simulations of the NLS initial value problem in order to assess the limits of this approach. Subsequently, using the lagrangian method to describe the propagation of the continuum, an approximate description of the main features of the propagation of the soliton plus radiation initial condition is achieved.

Nonlinear Refractive Coupling and Mixed-Mode Spatial Solitary Waves in Anisotropic Cubic Media

D. Hutchings
Department of Electronics and Electrical Engineering
University of Glasgow

Third-order optical nonlinearities (e.g. nonlinear refraction and two-photon absorption) in materials with cubic symmetry (such as semiconductors with zinc-blend structure) have a different polarization dependence to those in an isotropic medium, such as silica. For a single wavelength, nonlinear refraction requires three independent, non-zero coefficients for a complete description. Using the slowly-varying envelope approximation, equations of motion are derived for the propagation of orthogonal polarization components in an anisotropic cubic nonlinearity including a possible birefringence. The Hamiltonian of this interaction is generated and used to study the stationary solutions (eigenpolarizations) and their stability. It is found that as the modulus of the ratio of the nonlinearity to birefringence increases, the normally stable TE and TM modes of a waveguide become unstable; there is a bifurcation of the stable eigenpolarizations to states that are elliptically or linearly polarized of a particular orientation. This analysis is also extended to study the propagation of mixed-mode spatial solitary waves with a comparison with recent experimental results in AlGaAs planar waveguides.

Control and Tracking of Wave Patterns in Optical Turbulence

Weiping Lu, **Dejin Yu** and **R. G. Harrison**
Department of Physics
Heriot-Watt University

Algorithms are described for control of optical turbulence in partial differential systems based on the idea of stabilization of unstable periodic wave patterns embedded in turbulent states. The algorithms are successfully demonstrated through our numerical simulations in controlling and tracking traveling wave patterns in a transversely extended laser. Issues regarding practical implementation of the tracking procedure are discussed.

Understanding Quasi-Phase-Matched SHG and Dispersion Managed Solitons

G.G. Luther, M.S. Alber, J.E. Marsden and **J.M. Robbins**
BRIMS, Hewlett-Packard Labs
Control and Dynamical Systems, Caltech
Department of Mathematics, University of Notre Dame

Making the most out of nonlinear optical materials and manipulating nonlinear waves are major themes in nonlinear optics. Quasi-phase-matched second harmonic generation and dispersion-management of solitons are two generic examples. Quasi-phase-matching is an example of a simple control that can be used to overcome materials constraints, and dispersion management is an example of a control for nonlinear waves. Despite the apparent differences, the analytical approach for these systems is essentially the same. We review some existing theory from this point of view and provide a useful geometric description that helps in understanding these examples.

Generation of Solitons in Stimulated Raman Scattering

A.V. Mikhailov
Applied Mathematics Department
University of Leeds

Stimulated Raman scattering of a laser pump pulse seeded by a Stokes pulse generically leaves a two-level medium initially at rest in an excited state constituted of *static solitons* and radiation. The soliton birth manifests as sudden very large variations of the phase of the output pump pulse. This is proved by building the IST solution of SRS on the semi-line, which shows moreover that initial Stokes phase flips induce Raman spikes in the pump output also for short pulse experiments.

Nonlinear Modulation Dynamics in Birefringent Fibres

S. Wabnitz, G. Millot, E. Seve
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S. Trillo
Fondazione Ugo Bordoni, Italy

We present experimental results of induced modulational instability and parametric frequency generation in optical birefringent fibres in the strong conversion regime. The observations show for the first time the break up of quasi cw intense waves into trains of soliton-like pulses at THz repetition rates in the normal dispersion regime of optical fibres. We also discovered, in perfect agreement with the theory, that the conditions for maximum frequency conversion are far from the predictions of the linearized analysis. These results may have important applications to parametric devices in nonlinear optics and in other fields.

Wave Patterns in Excitable Optical systems

Dejin Yu, **Weiping Lu** and **R. G. Harrison**
Department of Physics
Heriot-Watt University

We report on formation of nonlinear wave patterns originating from excitability in transversely extended nonlinear optical systems. In particular, we show that the two- and three-level laser systems are excitable, satisfying the same criteria as those for reaction-diffusion systems. These excitable optical systems are found to support stable traveling solitary waves in one-dimensional space and to underlie creation of rotating spiral wave patterns in two-dimensional space. The physical mechanisms underlying excitability are presented.

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