An Optical Method for Teleportation

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A teleportation method using standard present day optical technology is presented. In an ideal case this method has 100% rates of success, as opposite to the optical teleportation methods suggested until now which even in an ideal case can yield only a 50% success rate.

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About two years ago, C. Bennett and collaborators invented "teleportation" [1], a method for transmitting quantum states from one place to another by using a "nonlocal" communication channel in parallel with a standard classical one. Ever since its invention there was a strong desire for the experimental verification of teleportation, both in order to test such a fundamental aspect of quantum mechanics (the nonlocality revealed by teleportation is different from the one revealed by the usual Bell type measurements [2]) and also in view of its potential applications in quantum information transmission, quantum cryptography, etc. But the experimental realization of teleportation is not easy. The main difficulty lies in implementing the transmitter's actions. According to the original scheme, the transmitter has to measure a quantum operator A which acts on two different particles and whose (nondegenerate) eigenstates are entangled (non direct-product) states of these two particles. But despite the recent progress in manipulating entangled states - is is now relatively easy to prepare two particles in a given entangled state (the most common case being that of two photons) - no one has measured yet such an operator.

At present there are a few proposal for experimental verification of teleportation. Some of them rely on some very new and rather exotical techniques such as atoms interacting with electro-magnetic cavities [3-5] and optical parametric up-conversion (the opposite of the already common technique of optical parametric down conversion) [6]. These schemes are supposed to implement the original teleportation scheme as described in [1]. There exist also experiment proposals [7,8] based on more traditional quantum optical techniques, but these schemes differ from the original one and, even in the ideal limit, only 50% of the attempted teleportations succeed while in the other cases the state which has to be

teleported is completely and irremediably destroyed.

In the present paper I propose yet another teleportation scheme based on standard quantum optical technology but which achieves, in the ideal limit, 100% success. This scheme is different from the original one in the way in which the state which has to be transmitted is presented to the transmitter. The original teleportation scheme involves three observers, a Preparer, a Transmitter and a Receiver (the last two usually called Alice and Bob), and three particles: a pair of particles in an entangled state (shared by Alice and Bob and constituing the "nonlocal transmission channel"), and a particle originally prepared by the Preparer in a state Ψ and then given to Alice. (The state Ψ is unknown for to Alice). In the present scheme instead of encoding Ψ in a third particle, the Preparer encodes the state Ψ in a different degree of freedom of Alice's member of the pair shared by her and Bob. This avoids altogether the difficult problem originally facing Alice, that is, the measurement of an operator acting on two particles and having entangled eigenstates. Instead Alice has to measure a formally identical operator but acting this time on two different degrees of freedom of the same particle - a much simpler task. Following I shall first describe the experimental proposal and after that I will discuss the significance of my departure from the original teleportation scheme.

The optical setup is illustrated in fig. 1. The first stage is to produce by standard parametric down-conversion two photons both with the same polarization, say horizontal (h), and entangled in directions, such that their initial state is

$$|\Phi_{in}\rangle = \frac{1}{\sqrt{2}}(|a\rangle_1|a'\rangle_2 + |b\rangle_1|b'\rangle_2)|\mathbf{h}\rangle_1|\mathbf{h}\rangle_2, \qquad (1)$$

where $|a\rangle_1$ and $|b\rangle_1$ represent photon 1 being in beams a and b respectively, and similarly

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$|a'\rangle_2$ and $|b'\rangle_2$ represent photon 2 in beams a' and b' respectively. The boldfaced vectors represent the polarization, in this case horizontal, of photons 1 and 2. One of these photons, say photon 1, is sent to Alice and the other one to Bob. These two photons represent the "nonlocal transmission channel", the equivalent of the two entangled spin 1/2 particles in the original teleportation paper.

On the way to Alice, photon 1 is intercepted by the Preparer who changes its polarization from horizontal to some arbitrary (not necessary linear) polarization $|\Psi\rangle_1$. The Preparer affects the polarization in both beams a and b in the same way. The state Ψ is the state Alice has to transmit to Bob.

After having the polarization changed, photon 1 reaches Alice. The state of the two photons, just before reaching Alice is thus

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|a\rangle_1|a'\rangle_2 + |b\rangle_1|b'\rangle_2)|\Psi\rangle_1|\mathbf{h}\rangle_2, \qquad (2)$$

which is the formal analogue of state Ψ_{123} in [1] (eq. 4 in [1]). The polarization degree of freedom in which the state given by the Preparer is encoded is totally uncorrelated to the direction degrees of freedom in which the two photons are entangled.

If photon storage and photon-photon interaction would be an easy thing to do, the Preparer would not have had to intercept the photon Alice will use for transmission but could have given her another photon, say photon 0 with polarization Ψ. Alice could have then stored this photon and after receiving photon1 she could have interchanged their polarization

$$|\Psi\rangle_0|\mathbf{h}\rangle_1 \rightarrow |\mathbf{h}\rangle_0|\Psi\rangle_1 \qquad (3)$$

and then proceed with the method I will describe bellow. In such a case teleportation

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would have been realized exactly like originally proposed. Unfortunately photon storage and photon-photon interactions are very difficult to realize in practice, and the Preparer has to help Alice by encoding the unknown state Ψ directly in the polarization of photon 1.

Now, following the teleportation procedure, Alice has to measure the operator A whose (nondegenerate) eigenstates are

$$\frac{1}{\sqrt{2}}(|a\rangle_1|\mathbf{v}\rangle_1 + |b\rangle_1|\mathbf{h}\rangle_1)$$

$$\frac{1}{\sqrt{2}}(|a\rangle_1|\mathbf{v}\rangle_1 - |b\rangle_1|\mathbf{h}\rangle_1)$$

$$\frac{1}{\sqrt{2}}(|a\rangle_1|\mathbf{h}\rangle_1 + |b\rangle_1|\mathbf{v}\rangle_1)$$

$$\frac{1}{\sqrt{2}}(|a\rangle_1|\mathbf{h}\rangle_1 - |b\rangle_1|\mathbf{v}\rangle_1). \tag{4}$$

The measurement of the operator A effectively implies an interaction between the two degrees of freedom (direction and polarization) of photon 1. Indeed, suppose that we start with the state $|a\rangle_1|\mathbf{v}\rangle_1$, measure A and obtain, say, $A = a_1$, the eigenvalue corresponding to the first eigenstate (4). Then after the measurement photon 1 is in state $\frac{1}{\sqrt{2}}(|a\rangle_1|\mathbf{v}\rangle_1 + |b\rangle_1|\mathbf{h}\rangle_1)$, meaning that the direction and the polarization got entangled. In the original teleportation paper the operator A acts on two different particles and effectively implies an interaction between them. But controlled interaction between two quantum particles is much more difficult to realize - that's why the experimental implementation of

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the original teleportation method is so difficult. On the other hand, to realize an interaction between the direction and the polarization of the same photon is trivially simple: any polarizing beam splitter does that.

To measure A Alice first splits each of her incoming beams, a and b, into two beams, by use of polarizing beam-splitters and then, once the information about polarization is encoded in the position of the photon, she rotates the polarization in all the beams to the same direction. These unitary transformations, as illustrated in fig.1, are

$$|a\rangle_1|\mathbf{v}\rangle_1 \rightarrow |1\rangle_1|\mathbf{v}\rangle_1 \rightarrow |1\rangle_1|\mathbf{h}\rangle_1$$

$$|a\rangle_1|\mathbf{h}\rangle_1 \rightarrow |2\rangle_1|\mathbf{h}\rangle_1 \rightarrow |2\rangle_1|\mathbf{h}\rangle_1$$

$$|b\rangle_1|\mathbf{v}\rangle_1 \rightarrow |3\rangle_1|\mathbf{v}\rangle_1 \rightarrow |3\rangle_1|\mathbf{h}\rangle_1$$

$$|b\rangle_1|\mathbf{h}\rangle_1 \rightarrow |4\rangle_1|\mathbf{h}\rangle_1 \rightarrow |4\rangle_1|\mathbf{h}\rangle_1. \tag{5}$$

Measuring the operator A reduces now to measuring an operator A' acting solely on the position of photon 1 and having the eigenstates

$$\frac{1}{\sqrt{2}}(|1\rangle_1 + |4\rangle_1)$$

$$\frac{1}{\sqrt{2}}(|1\rangle_1 - |4\rangle_1)$$

$$\frac{1}{\sqrt{2}}(|2\rangle_1 + |3\rangle_1)$$

$$\frac{1}{\sqrt{2}}(|2\rangle_1 - |3\rangle_1). \tag{6}$$

This can easily be done by letting beams 1 and 4 impinge onto a symmetric beam-splitter S_1, followed by detectors D_1 and D_4, and beams 2 and 3 impinge onto beam-splitter S_2

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followed by detectors D_2 and D_3. Indeed, the symmetric beam-splitters S_1 and S_2 transform the input states $|1\rangle_1$ and $|4\rangle_1$ and respectively $|2\rangle_1$ and $|3\rangle_1$ into the corresponding linear transformations

$$\frac{1}{\sqrt{2}}(|1\rangle_1 + |4\rangle_1) \rightarrow |1'\rangle_1$$

$$\frac{1}{\sqrt{2}}(|1\rangle_1 - |4\rangle_1) \rightarrow |4'\rangle_1$$

$$\frac{1}{\sqrt{2}}(|2\rangle_1 + |3\rangle_1) \rightarrow |2'\rangle_1$$

$$\frac{1}{\sqrt{2}}(|2\rangle_1 - |3\rangle_1) \rightarrow |3'\rangle_1 \tag{7}$$

which are directly detected by $D_1 - D_4$.

To summarize, if the polarization state prepared by the Preparer was

$$|\Psi\rangle_1 = \alpha|\mathbf{h}\rangle_1 + \beta|\mathbf{v}\rangle_1, \tag{8}$$

then after all the actions made by Alice, the state of the two photons just before the detection of photon 1 by one of the detectors $D_1 - D_4$ is

$$\frac{1}{2}\{|1'\rangle_1(\beta|a'\rangle_2 + \alpha|b'\rangle_2) + |2'\rangle_1(\alpha|a'\rangle_2 + \beta|b'\rangle_2) + |3'\rangle_1(\alpha|a'\rangle_2 - \beta|b'\rangle_2) +$$

$$|4'\rangle_1(\beta|a'\rangle_2 - \alpha|b'\rangle_2)\}|\mathbf{h}\rangle_1|\mathbf{h}\rangle_2, \tag{9}$$

the analog of Eq.(5) in the original paper [1]. Therefore, depending on which detector registers photon 1, photon 2 is left in one of the four states

$$(\beta|a'\rangle_2 + \alpha|b'\rangle_2)|\mathbf{h}\rangle_2,$$

$$(\alpha|a'\rangle_2 + \beta|b'\rangle_2)|\mathbf{h}\rangle_2,$$

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$$\big(\alpha|a'\rangle_2 - \beta|b'\rangle_2\big)|\mathbf{h}\rangle_2,$$

$$\big(\beta|a'\rangle_2 - \alpha|b'\rangle_2\big)|\mathbf{h}\rangle_2 \tag{10}$$

which are isomorphic to Ψ up to some standard unitary transformations, independent of Ψ.

Having finished with Alice's actions, let us now analyze Bob's. Bob's task is to reconstruct the polarization state Ψ into his photon. To this end he first translates the information which is contained in the states (10) in the direction degree of freedom into polarization. This is done by rotating the polarization in beam a' from horizontal to vertical, and then merging the two beams a' and b' into a single one (denoted by o) by use of a polarized beam splitter. That is, Bob's actions are

$$|a'\rangle_2|\mathbf{h}\rangle_2 \rightarrow |a'\rangle_2|\mathbf{v}\rangle_2 \rightarrow |o\rangle_2|\mathbf{v}\rangle_2$$

$$|b'\rangle_2|\mathbf{h}\rangle_2 \rightarrow |b'\rangle_2|\mathbf{h}\rangle_2 \rightarrow |o\rangle_2|\mathbf{h}\rangle_2. \tag{11}$$

As a result the four possible states (10) are transformed into

$$\big(\beta|a\rangle_2 + \alpha|b\rangle_2\big)|\mathbf{h}\rangle_2 \rightarrow |o\rangle_2\big(\beta|\mathbf{v}\rangle_2 + \alpha|\mathbf{h}\rangle_2\big)$$

$$\big(\alpha|a\rangle_2 + \beta|b\rangle_2\big)|\mathbf{h}\rangle_2 \rightarrow |o\rangle_2\big(\alpha|\mathbf{v}\rangle_2 + \beta|\mathbf{h}\rangle_2\big)$$

$$\big(\alpha|a\rangle_2 - \beta|b\rangle_2\big)|\mathbf{h}\rangle_2 \rightarrow |o\rangle_2\big(\alpha|\mathbf{v}\rangle_2 - \beta|\mathbf{h}\rangle_2\big)$$

$$\big(\beta|a\rangle_2 - \alpha|b\rangle_2\big)|\mathbf{h}\rangle_2 \rightarrow |o\rangle_2\big(\beta|\mathbf{v}\rangle_2 - \alpha|\mathbf{h}\rangle_2\big). \tag{12}$$

Photon 2 is then kept in flight, by sending it on a long optical path (illustrated by the dotted line in fig. 1) until the signal comes from Alice which of the detectors $D_1 - D_4$

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has registered photon 1. This signal is processed by some fast electronic device which can then activate some appropriate active optical elements so that when photon 2 goes through them its polarization is rotated to $|\Psi\rangle_2$. This could be obtained for example by two active optical elements (such as two Kerr or two Pockel cells) placed one after the other (C1 and C2 on fig. 1) where the first one just reverses the signs of the horizontal polarization

$$|v\rangle_2 \rightarrow |v\rangle_2$$

$$|h\rangle_2 \rightarrow -|h\rangle_2 \tag{13}$$

while the other rotates horizontal into vertical polarization and vice-versa,

$$|v\rangle_2 \rightarrow |h\rangle_2$$

$$|h\rangle_2 \rightarrow |v\rangle_2. \tag{14}$$

By activating none, one, or both of these cells, according to the result obtained by Alice, Bob accomplishes his task.

The success of teleportation can be verified by letting Bob's photon impinge on a polarizer parallel with the one used by the Preparer - Bob's photon must pass it with certainty. In fact even if one does not succeed to implement the rapid communication between Alice and Bob, and the on-flight rotation of Bob's photon, one could still verify the success of the non-local part of the transmission. In such a case Bob's station could end with the photon passing through the polarizing beam-splitter (SP3) and from here the photon enters the verification station. The verification station consists simply of a polarizer which is set at random in one of the four positions which could transmit with

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certainty one of the states (12). (The four states (12) are not orthogonal on each other, that's why one cannot devise a measurement which can tell them apart. The most one can do is to set a polarizer such as one of these states passes with certainty, while the others have some smaller probability to pass.) To verify the success of teleportation one should analyze the subensemble of runs in which it happened that Alice got one of the four results corresponding to the particular setting of the verification polarizer in that run. In this subensemble Bob's photon must pass the verification polarizer with certainty. (In fact if we just wish to check the success of the non-local part of the teleportation and do not request the actual reconstruction of the polarization state Ψ, there is no need for Bob's station at all. One could let Bob's input beams a' and b' to enter directly in the verification station, which consists of an arrangement of beam-splitters, phase shifters and detectors which could check, at random, for one of the four states (10). Then, as in the previous case, one should verify that in the subensemble of cases in which it so happened that Alice got the result corresponding to the particular state which was verified, Bob's photon has passed the verification with certainty.

At first sight the teleportation method presented in this paper seems very different from the original scheme. In the original method the Preparer need not know any details about the particular set-up which will be used for teleportation. He just has to prepare his own particle in the state he wants to have transmitted and then give it to Alice. On the other hand, in the above presented method the Preparer has to encode the state which has to be transmitted in the very particle which will be used for transmission. But actually the difference between the two methods is limited only to a local pre processing of data at

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the level of the transmitter (Alice), and has nothing to do with the process of transmission itself, i.e. with the teleportation proper. As in the original method Alice is faced with the same problem - she has only a single replica of the state Ψ and thus she cannot find out what this state is - and she solves it in an identical way - she doesn't even try to find out Ψ but she sends it directly to Bob by using a "nonlocal communication channel" in parallel with a conventional one. So the above presented method deserves the name of teleportation.

In conclusion, I have presented a method for teleportation which can be implemented with present day optical technology. The basic idea is to have the Preparer helping Alice by encoding the state which has to be transmitted directly in Alice's member of the entangled photons which will be used for transmission. Actually this idea is more important than the particular experimental set-up I have described in the present paper. Using the same idea many other set-ups can be designed. An optimization of the above method has already been proposed by H. Bernstein [9]. As yet another possibility, instead of having the two photons entangled in directions and the state Ψ encoded in the polarization degree of freedom, the photons could be entangled in polarization and then Ψ encoded in the direction of one of them (by having the Preparer use a beam-splitter and phase shifters).

Finally, the method presented in this paper modified such that Alice plays also the role of the Preparer and Bob performs randomly chosen measurements on his photon outside the light-cone of Alice's measurements can be used as a generalized Bell's inequalities type measurement. This is a generalized Bell type measurement because Alice performs a generalized measurement (a POVM [10]) on her photon, instead of simply measuring

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operators which view her photon as a 2-dimensional system (i.e. taking into account only the fact that her photon lives in beams a or b) as in a usual Bell type measurement carried on a pair of photons emerging from a parametric down-conversion source. It is very possible that such a generalized Bell type measurement could solve the detector efficiency problem which does not allow in a usual Bell type measurement to reach a conclusion in favor of quantum mechanics or local hidden variables models with present day detectors - as Alice can choose among more measurements than in a usual Bell measurement, it might be that contradictions between quantum mechanics and local hidden variables models could be already be observed even with low detector efficiencies.

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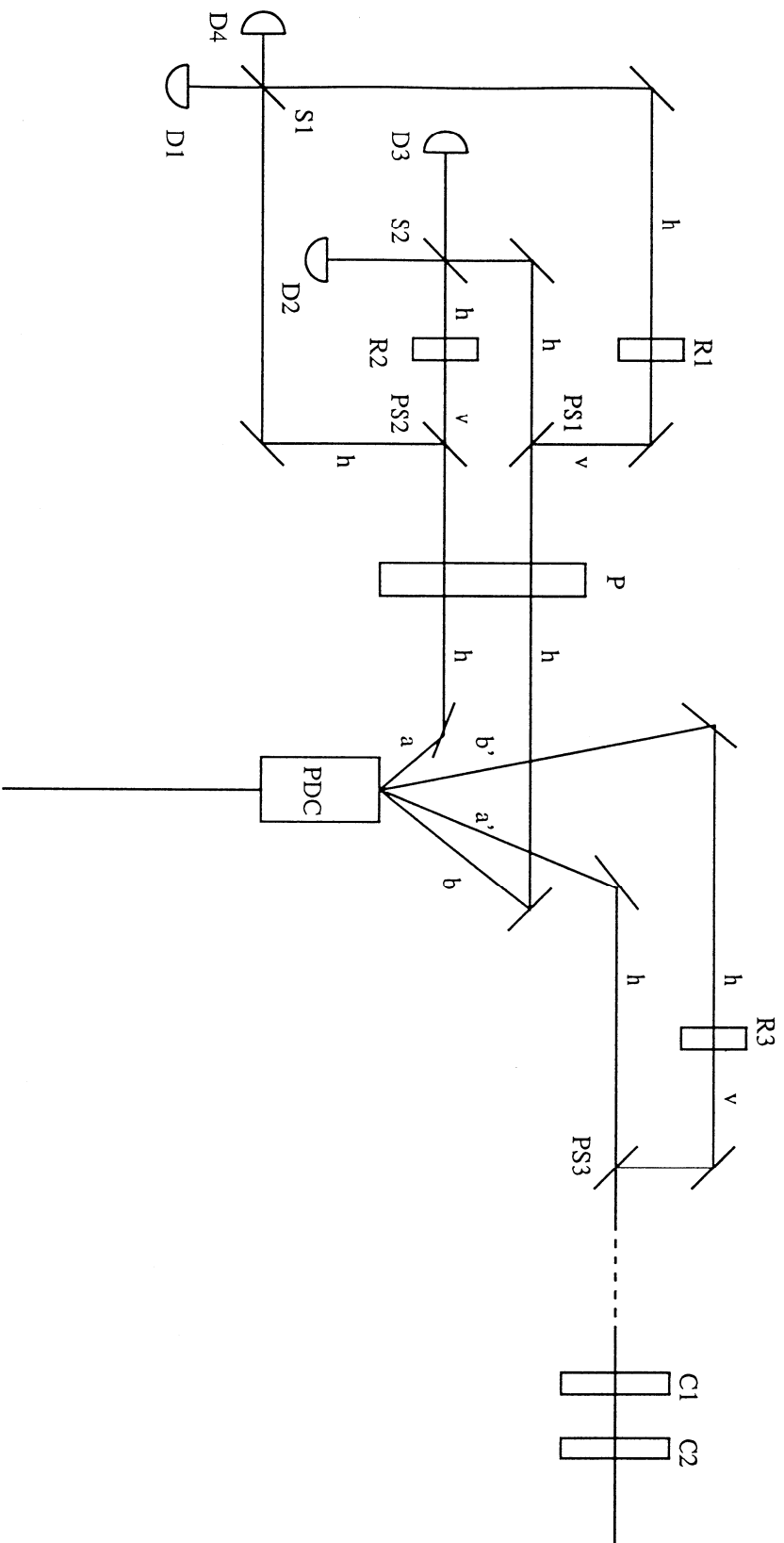
Fig. 1. PDC represents a parametric-down-conversion-cristal, S1,S2 regular beamsplitters, PS1-PS3 polarizing beam splitters, R 90 polarization rotators, P the polarization rotator used by the Preparer, D1-D4 detectors and C1,C2 active optical elements. The dotted line represents a long optical path, and the letters h,v and denote polarization directions. The signal from the detectors is proccessed by electronics (not illustrated in the picture) which then controls the active cells C1 and C2.