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Nonlinear Waves, Nonlinear Optics and Your Communications Future

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A historical account of the development of optical communications in connection with nonlinear optics is given contrasting NRZ and soliton systems. A short summary of interesting problems in nonlinear optics dealing with nonlinear dynamics and nonlinear waves is provided with many references.

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If you read this article from your web browser the data has probably traveled part of its journey through an optical fiber. In existing commercial optical fiber communications links, information is encoded with square-wave pulses in the non-return-to-zero (NRZ) format as shown in Fig. 1. These pulses propagate essentially linearly in existing systems. Dispersion, absorption and imperfections in the fiber cables deform them causing errors in the transmitted signal. To keep the signal error free, it is periodically corrected and amplified. Transoceanic links typically have the highest data rates. They now reach rates as high as 2.5Gb/s. In contrast, data navigates your local network and enters your computer at rates many orders of magnitude slower than this. In most cases the cables closest to you are not optical at all.

Several years from now if you read this article from the archives of *Nonlinear Science Today* many more of the components that transport and process your data will be optical. Some of the links may run at data rates as high as 40-100Gb/s. During part of its journey your data may be encoded in a return-to-zero (RZ) format using solitons to represent the bits as shown in Fig. 1. At these high bit rates optical signals will propagate nonlinearly whether encoded in the NRZ format or as solitons in the RZ format. In Fig. 2 the linear and nonlinear evolution of pulses encoded in RZ and NRZ formats is shown. As local-area networks speed up they will eventually need to process optically encoded signals using all-optical devices in which signals are typically processed by employing the dynamics and interactions of nonlinear optical waves.

The story of modern optical communications really begins with the introduction of

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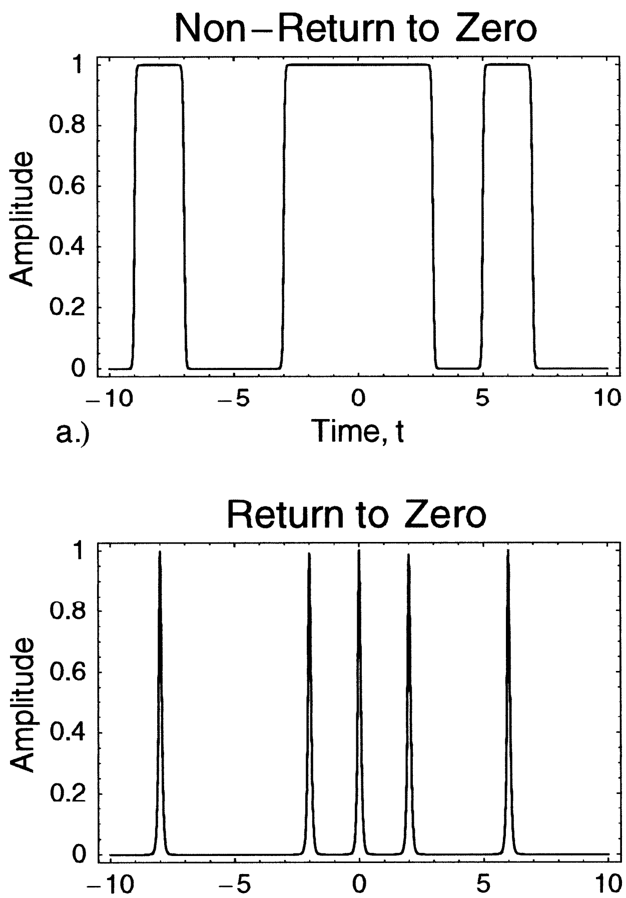


Figure 1: The (a) NRZ and (b) RZ format for encoding the eight bit sequence (10011101).

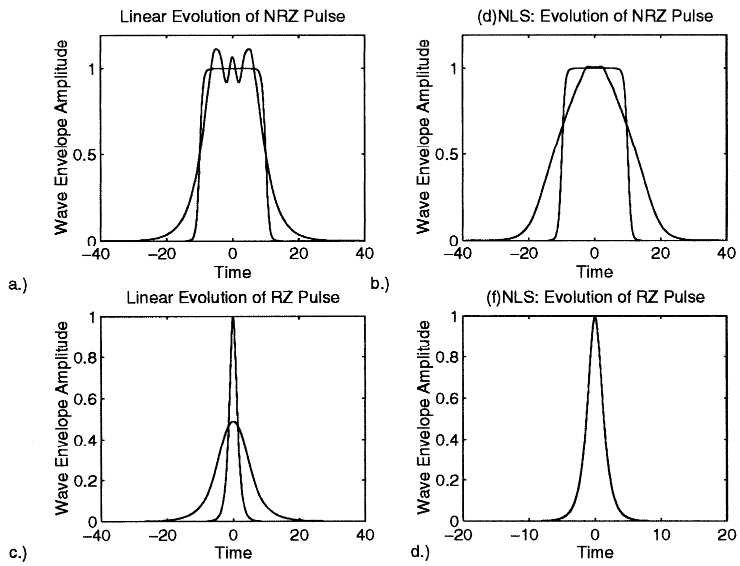


Figure 2: The input and output profiles for propagation of an (a) linear dispersive and (b) nonlinear dispersive NRZ pulse are contrasted against those for propagation of a (c) linear dispersive and (d) nonlinear dispersive RZ soliton pulse.

the laser in the early 60's. By modulating the amplitude of the intense coherent light emitted by a laser, information could be encoded on a light wave. What made this particularly interesting for communications was the development of low-absorption glass fibers. In these fibers, the absorption was low enough that information could be transmitted over commercially significant distances. The erbium-doped fiber amplifier gave an enormous boost to optical communications in the late 80's by providing a good way to combat absorption and increase the error-free transmission distance even further. More recently nonlinear optics and the dynamics of nonlinear waves have been playing an increasingly important role in optical communications.

While the laser represents a turning point in all branches of optics, it ushered in the young field of nonlinear optics in the early 60's. Nonlinear laser-matter interactions were first observed when intense light beams were focused into transparent materials. The now classic nonlinear wave interactions like harmonic generation, wave mixing, parametric interactions, and self focusing were soon identified. Today these are the basic building blocks for many nonlinear optical devices and phenomena.

When irradiated by such intense beams of light, the refractive index of many materials becomes intensity dependent. The nonlinear Schrödinger equation models the propagation of wavepackets of light in weakly nonlinear dispersive materials. As the peak intensity of a light beam is increased, localization induced by the cubic nonlinearity begins to compete with diffraction. Chiao, Garmire and Townes[1] recognized in 1964 that the stationary ground state solution of the Nonlinear Schrödinger equation in two spatial and one time dimension modeled the balance between these opposing effects. In doing so they apparently introduced the first optical solitary wave.

Since then much effort has been exerted to identify and analyze self-trapped or stable localized pulses in optical systems. The solitons associated with integrable evolution equations in one spatial and one time dimension were introduced in the early 1970's for the nonlinear Schrödinger equation, the Maxwell-Bloch equations, the three-wave equations, and the coupled nonlinear Schrödinger equations (see for instance [2]). Through the efforts of Zakharov and Shabat[3] in 1972 the nonlinear Schrödinger equation was solved using the newly developed inverse scattering transform. Unfortunately these solutions are unstable to transverse modulations when embedded in two spatial dimensions[4]. What is more, a little too much energy causes solutions such as the Townes soliton to self-focus forming a singularity in the amplitude in finite-time. A general source covering the mathematical foundations of nonlinear optics can be found in the book by Newell and Moloney[5].

By guiding the waves, nonlinear optical fibers avoided the issue of diffraction and suggested important applications in optical communications. In 1973 Hasegawa and Tappert[6] showed that optical fibers could sustain envelope solitons in the longitudinal dimension having a single mode guided in the direction perpendicular to the propagation direction. Still it would not be until 1980 that intense short-pulse laser sources would permit Mollenauer, Stolen and Gordon[7] to observe these guided-wave optical solitons.

In contrast to the Towns soliton, optical solitons in fibers form as the nonlinearity balances the *dispersive* spreading of a guided wavepacket. Wavetrains with different frequencies that make up the wavepacket have different speeds. This causes a linear pulse to spread as it propagates. Because the refractive index in optical fibers is proportional to the local intensity of the light, the wave speeds of the component wavetrains are slightly shifted. Slow waves speed up and fast waves slow. The magnitude of this shift depends on the intensity of the wavetrain. In media with cubic response, the intensity-dependent shift of sech-shaped pulses is just enough to balance dispersive spreading. The nonlinear Schrödinger equation captures this balance. In Fig. 2 we see that the soliton in 2d retains its shape while the NRZ pulse does not. Due to the nonlinear response, the NRZ pulse also generates new frequencies as it propagates while the soliton does not.

The Commercial use of solitons in communications began to become possible just after 1990 when a solution to the timing-jitter problem was provided[8, 9]. While the erbium-doped fiber amplifier was extremely successful at countering absorption, it also introduced errors. Uncertainties in the amplification process introduce random variations in the spectrum of each soliton pulse. In response, the solitons reshape dynamically shedding very low amplitude, nearly linear waves and obtaining an overall shift in their center frequencies (the frequencies associated with their spectral peaks). These frequency shifts are randomly distributed and they produce small changes in the group velocities of each of the solitons.

As the solitons propagate with their shifted group velocities, their relative positions change, and these shifts are randomly distributed. In this way erbium-doped amplifiers introduce errors in the arrival times of the soliton bits. This error is called the timing jitter[10]. The further the information is transported, the more amplification stages are required, so the larger the jitter can become. As the jitter grows, the probability of an error occurring in a bit sequence grows.

Filters are used to periodically strip away unwanted noise and radiation modes. The modes with frequencies under the soliton spectrum remain and continue to be amplified. By shifting the center frequency of each filter slightly in the same direction, these extra modes are extinguished. Solitons adjust to the asymmetry imposed by the shifted filters by shifting their own center frequencies. In doing so they keep up with the shifting filters and pass through the system. Non-soliton components do not keep up and eventually are absorbed. In Fig. 3 the evolution of a soliton in a system with filters is shown. Here the filters do not slide and modes are seen to grow up in the background of the averaged soliton. In Fig. 4 the filters do slide and the spurious modes are extinguished. Accounts of the development of these techniques can be found in articles by Haus[11] and Mollenauer[12] and in the article by Mollenauer, Gordon and Mamyshev[13].

It can be shown that the damping and driving provided by the combination of

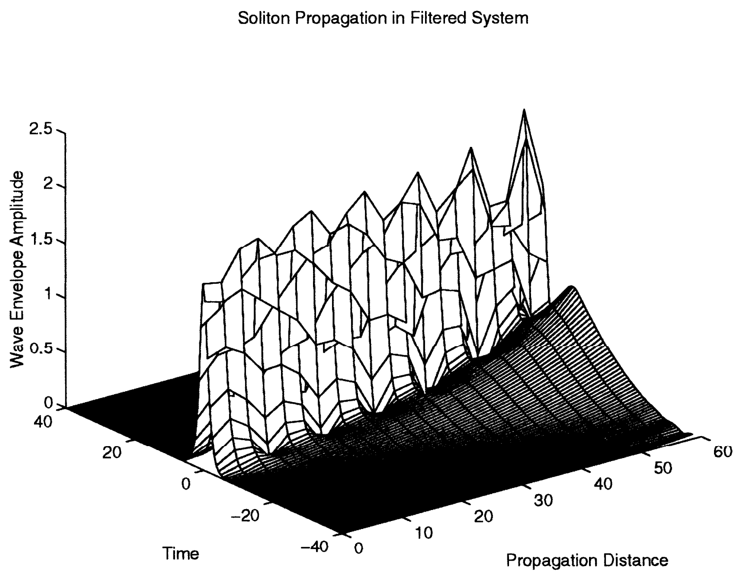


Figure 3: Here a sech-shaped pulse propagates along under the action of the focusing nonlinear Schrödinger equation with absorption. To counter the absorption the pulse undergoes 240 stages of amplification and filtering. The solution is plotted at 40 equally spaced positions along the system. Over one period the wave undergoes large deviations from the soliton solution, but on average the dynamics is that of the perturbed nonlinear Schrödinger equation. In this system the center frequency of the filter remains fixed allowing other modes to grow in the background of the average soliton.

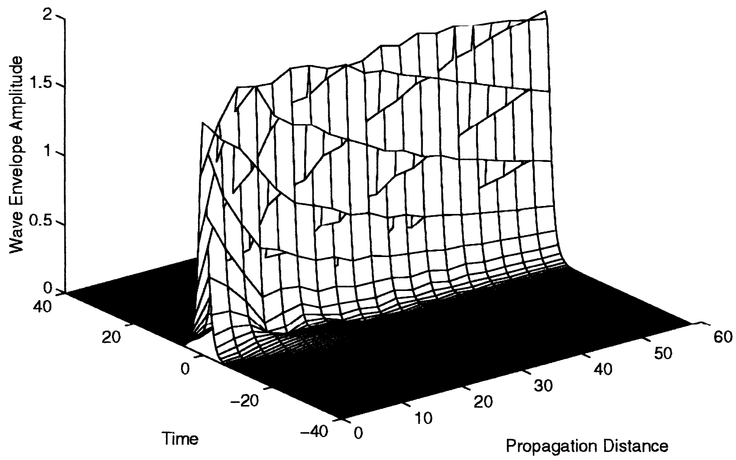


Figure 4: Here the system is exactly the same as in the previous figure, but now the center frequency of the filter undergoes a constant shift extinguishing the modes in the background of the average soliton.

amplification and filtering generates a soliton that is independent of the initial conditions. The soliton parameters evolve towards a fixed point under this perturbation. The analytical theory is detailed in[9, 14, 15]. Much of the detailed mathematical analysis of soliton communications systems can be found in the recent book by Hasegawa and Kodama[16]. A more general book on analytical techniques for optical solitons is also available[17].

With an eye toward satisfying the increasing demand for communications, much of the current research is directed toward 40-100Gb/s systems. To achieve these high bit rates, several signals at different frequencies, encoded either in NRZ or soliton format, are used simultaneously. These wavelength-division-multiplexed (WDM) systems permit the simultaneous use of several independent information carrying channels in a single fiber thereby increasing the data rate.

One important limitation for these systems is cross-talk among channels. Collisions between solitons of the nonlinear Schrödinger equation at different frequencies induce shifts in phase and position. What is more, amplification, filtering and absorption permit four-wave mixing components generated during the collision of pulses to grow and to fill in the gaps in the spectrum between channels. Both of these effects tend to introduce errors. If on average the four-wave mixing components are phase matched they have enhanced growth rates. This poses a significant problem at high bit rates. The quasi-phase matching effect is essentially eliminated by varying the amount of dispersion along each section of fiber so that the four-wave mixing components are detuned even on average[18, 19].

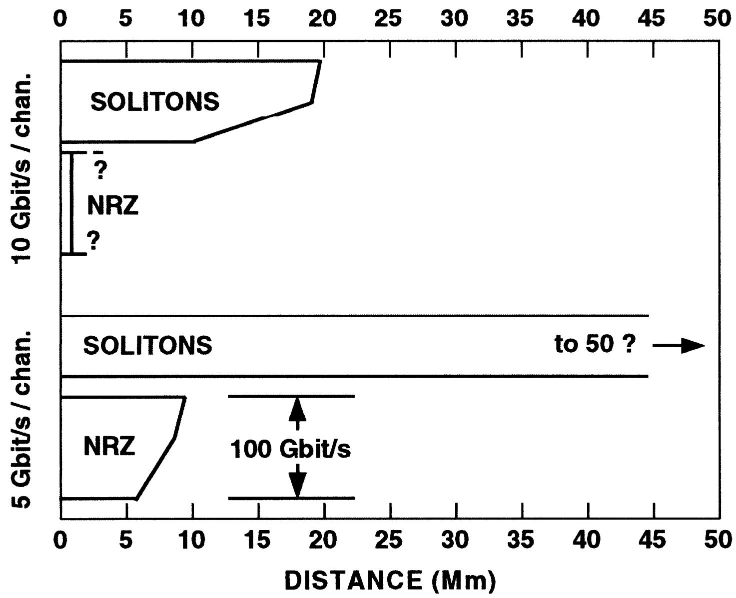
While the analysis of solitons in optical fiber devices has relied heavily on the mathematics of nonlinear waves, the impact of these ideas and techniques has become much more general with recent advances in the analysis of NRZ systems. In the high-bit-rate regime, NRZ signals carry a large number of nonlinear modes whose evolution is governed by the defocusing nonlinear Schrödinger equation. This group of nonlinear modes evolves in a rather complex way compared to its single nonlinear-mode counterpart in soliton systems. This contrast is clear in Fig. 2. Using the semi-classical modulation theory of the defocusing nonlinear Schrödinger equation, Kodama and Wabnitz have introduced the nonlinear theory of NRZ pulse trains[20] based partly on the work in[21].

They introduce the theory by showing that the average dynamics is governed by the defocusing nonlinear Schrödinger equation in the weak dispersion limit. Using a WKB analysis, they show that the evolution of an NRZ pulse before the first caustic is related to the classic dam breaking problem from fluid dynamics, which is solved using the method of characteristics. They then show that these pulses can be controlled by using phase shifts, which are the equivalent of velocity gradients in the dam problem[22]. The detailed analysis uses higher genus solutions along with the semi-classical modulation theory. Recently the theory was extended to the coupled nonlinear Schrödinger equation to provide predictions for the limits on channel spacing in wavelength-division-multiplexed NRZ systems[23].

The mathematical techniques and ideas upon which the NRZ theory rests were developed over many years. Even those with casual acquaintance with the theory of nonlinear waves may recall the Whitham theory for modulated waves. During the 1970's this theory was beginning to be applied to integrable systems and in the 80's it encompassed near-integrable systems and the semi-classical limit of integrable systems (see [24] and [25] for references to the extensive mathematical literature on these topics).

With the increase in demand for communications services, optical systems have begun to be implemented in local networks. This means that the mathematics of nonlinear waves and nonlinear dynamics will continue to play an important role in the development of optical communications technologies for some time. New techniques will be needed as well as new ways of using old ones. Both soliton and NRZ systems seem to have a role to play in the market for communications. This is shown in Fig. 5, where the two formats are contrasted. This data shows that the RZ or soliton format tends to work better for high bit rates over long distances. At shorter distances and lower bit rates the two technologies tend to become comparable and other considerations become more important.

Interesting and challenging problems continue to come up in the analysis of both



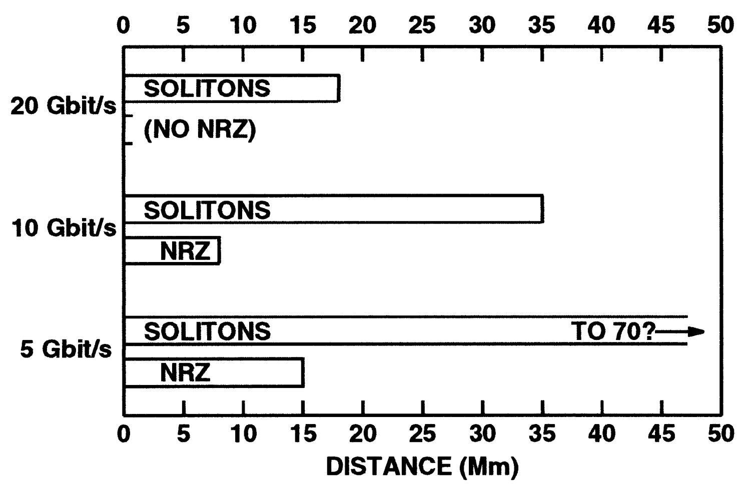


Figure 5: The (top) graph shows a comparison between the error-free propagation distance of sliding-frequency-soliton systems and NRZ systems for a single channel. The (bottom) graph shows a comparison between the error-free propagation distance of the two types of encoding for wavelength-division-multiplexed systems. The trans-pacific distance is about 10Mm. *These graphs were provided courtesy of L. F. Mollenauer, G. P. Gordon and P. V. Mamyshev and originally appear in "Solitons in High-Rate, Long-Distance Transmission" which appears in Optical Fiber Telecommunications, Vol. IIIA, Academic Press, 1997.*

soliton and NRZ systems. Much of the emphasis of present work is on new techniques for manipulating and controlling nonlinear waves with existing materials constraints. This work is really the beginning of a nonlinear systems theory for optics. Some of the recent advances are represented in [26]-[37].

Though the original motivation to create solitons in optical fibers was for communications, the nonlinear phenomena that occur in optical fibers have been employed to produce new laser sources, measurements, sensors, and switches. Many of these applications turn out to be fascinating dynamical systems. The impact of nonlinear science in optical communications is a dramatic demonstration of the importance of the field in the lives of even the most unsuspecting web browsers. More generally, nonlinear optics exhibits a wonderful world of nonlinear phenomena that touches on many if not all of the basic questions dealt with in nonlinear science today. In the Appendix we point out a few recent contributions to the nonlinear dynamics of optical systems. Certainly this is not a comprehensive list of topics or important contributors, but in most cases original references and key contributions can be found by checking the references provided. We hope you find it useful.

For further information you might also wish to consult the proceedings of "Nonlinear Dynamics in Optical Systems," which is held every two years by the Optical Society of America. This meeting is a good source of information on current research in the field. The American Optical Society also runs "Nonlinear Guided Waves and Their Applications" (see Vol. 15, 1996 OSA Technical Digest Series (Optical Society of America, Washington DC, 1996)) which deals directly with applications of nonlinear waves in optical systems (see aslo `http://w3.osa.org/MTG_CONF/INDCAL/FTPINDEX.htm`). Recent meetings held at the University of Notre Dame[38] (`http://www.science.nd.edu/math/symposium.html` and `http://www.nd.edu/~malber/optics.html`), the Center for Nonlinear Studies at Los Alamos National Laboratories and BRIMS, Hewlett-Packard Labs[39] (`http://www-uk.hpl.hp.com:80/brims/events96.html#optics`) brought together researchers from mathematics, physics and engineering to discuss the analysis of nonlinear optical systems and phenomena. Applications of nonlinear dynamics in optics also appear through the SIAM activity group in dynamical systems. At "The Fourth SIAM Conference on Applications of Dynamical Systems" held in May of 1997, mathematical problems that arise in nonlinear optics and its applications were also well represented (`http://www.siam.org/meetings/ds97/ds97home.htm`).

References

[1] R.Y. Chiao, E. Garmire, and C.H. Townes, "Self-trapping of optical beams," *Phys. Rev. Lett.* **13**, 479-482 (1964).

[2] Ablowitz, M.J. and H. Segur, Solitons and the Inverse Scattering Transform, (SIAM, Philadelphia, 1981).

[3] Zakharov, V.E. and A.B. Shabat, "Exact theory of two-dimensional self-focusing and one-dimensional self-modulation waves in nonlinear media," *Sov.Phys.JETP* **34**, 62-69 (1972).

[4] E.A. Kuznetsov, A.M. Rubenchik, and V.E. Zakharov, "Soliton stability in plasmas and hydrodynamics," *Phys. Rep.* **142**, 103-165 (1986); V.E. Zakharov, E.A. Kuznetsov, and A.M. Rubenchik, "Soliton stability," in Solitons, eds. S.E. Trullinger, V.E. Zakharov, and V.L. Pokrovsky, Chap. 10, (Elsevier Science Publishers B. V., 1986), pp. 503-554.

[5] A.C. Newell and J.V. Moloney, Nonlinear Optics, (Addison-Wesley, Redwood City, 1992).

[6] A. Hasegawa and F. Tappert, "Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers, I; Anomalous dispersion," *Appl. Phys. Lett.* **23**, 142 - 144 (1973).

[7] L.F. Mollenauer, R.H. Stolen and J.P. Gordon, "Experimental observation of picosecond pulse narrowing and solitons in optical fibers," *Phys. Rev. Lett.* **45**, 1095-1098 (1980).

[8] A. Mecozzi, J.D. Moores, H.A. Haus and Y. Lai, "Soliton transmission control," *Opt. Lett.* **16**, 1841 (1991).

[9] Y. Kodama and A. Hasegawa, "Generation of asymptotically stable optical solitons and suppression of the Gordon-Haus effect," *Opt. Lett.* **17**, 31 (1992).

[10] J.P. Gordon and H.A. Haus, "Random walk of coherently amplified solitons in optical fiber transmission," *Optics Lett.* **11** 665-667 (1986).

[11] H.A. Haus, "Molding light into solitons," *IEEE Spectrum* pp. 48-53 March 1993.

[12] L.F. Mollenauer, "Soliton transmission speeds greatly multiplied by sliding-frequency guiding filters," *Optics & Photonics News* pp. 15-19 April 1994.

[13] L. F. Mollenauer, G. P. Gordon and P. V. Mamyshev, "Solitons in High-Rate, Long-Distance Transmission," in Optical Fiber Telecommunications, Vol. IIIA, (Academic Press, New York, 1997).

[14] J.N. Elgin, "Stochastic perturbations of optical systems," *Opt. Lett.* **18**, 10 (1993).

[15] J.N. Elgin, "Stochastic perturbations of optical systems," *Phys. Lett. A* **181**, 54 (1993).

[16] A. Hasegawa and Y. Kodama, Solitons in Optical Communications, Oxford series in optical and imaging sciences, Vol. 7, (Clarendon Press, Oxford, 1995).

[17] F. Abdullaev, S. Darmanyan and P. Khabibullaev, Optical Solitons, Springer Series in Nonlinear Dynamics, (Springer-Verlag, Berlin, 1993).

[18] P.V. Mamyshev and L.F. Mollenauer, "Pseudo-phase-matched four-wave mixing in soliton wavelength-division multiplexing transmission," *Opt. Lett.* **21**, 396 (1996).

[19] M.J. Ablowitz, G. Biondini and J.R. Sauer, "Four-wave mixing in wavelength-division-multiplexed soliton systems: damping and amplification," *Opt. Lett.* **21**, 1646 (1996).

[20] Y. Kodama and S. Wabnitz, "Analytical theory of guiding-center nonreturn-to-zero and return-to-zero signal transmission in normally dispersive nonlinear optical fibers," *Opt. Lett.* **20**, 2291-2293 (1995).

[21] A.M. Bloch and Y. Kodama, "Dispersive regularization of the Whitham equation for the Toda Lattice," *SIAM J. Applied Math.* **52**, 909-928 (1992).

[22] Y. Kodama and S. Wabnitz "Compensation of NRZ signal distortion by initial frequency shifting," *Electronics Lett.* **31**, 1761-1762 (1995).

[23] Y. Kodama, A. Maruta and S. Wabnitz, "Minimum channel spacing in wavelength-division-multiplexed nonreturn-to-zero optical fiber transmission," *Opt. Lett.* to appear.

[24] N.M Ercolani, M.G. Forest, D.W. McLaughlin, and A. Sinha, "Strongly nonlinear modal equations for nearly integrable PDEs," *J. Nonlinear Sci.* **3**, 393-426 (1993).

[25] S. Jin, C.D. Levermore and D.W. Mclaughlin, The behavior of solutions of the NLS equation in the semiclassical limit, N.M. Ercolani et al., eds., *NATO ASI Series B* (Plenum Press, New York) **320**, 235-256 (1994).

[26] J.N. Elgin, "Perturbations of optical solitons," *Phys. Rev. A* **47**, 4331-4341 (1993).

[27] R.D. Li, P. Kumar and W.L. Kath, "Dispersion compensation with phase-sensitive optical amplifiers," *J. Lightwave Tech.* **12**, 541 (1994).

[28] N. Akhmediev and M. Karlsson, "Cherenkov radiation emitted by solitons in optical fibers," *Phys. Rev. A* bf 51, 2602-2607 (1995).

[29] S. Chakravarty, M.J. Ablowitz and R.B. Jenkins and J.R. Sauer, "Multisoliton interactions and wavelength-division multiplexing," *Opt. Lett.* **20**, 136 (1995).

[30] R.B. Jenkins, J.R. Sauer and M.J. Ablowitz, "Data-dependent timing jitter in wavelength-division-multiplexing soliton systems," *Opt. Lett.* **20**, 1964 (1995).

[31] E.A. Kuznetsov, A.V. Mikhailov and I.A. Shimokhin, "Nonlinear interaction of solitons and radiation," *Physica D* **87**, 201-215 (1995).

[32] J.C. Bronski and J. Nathan Kutz, "Modulational stability of plane waves in non-return-to-zero communications systems with dispersion management," *Opt. Lett.* **21** (1996).

[33] J.C. Bronski and J. Nathan Kutz, "Asymptotic behavior of the nonlinear Schrödinger equation with rapidly-varying, mean-zero dispersion," preprint submitted to Elsevier Science, 29 March 1996.

[34] S. Burtsev, D.J. Kaup and B.A. Malomed, "Optimum reshaping of an optical soliton by a nonlinear amplifier," *J. Opt. Soc. B* **13**, 888 (1996).

[35] R.-J. Essiambre and G.P. Agrawal, "Timing jitter analysis for optical communications systems using ultrashort solitons and dispersion-decreasing fibers," *Opt. Lett.* , (1996).

[36] I.R. Gabitov and S.K. Turitsyn, "Averaged pulse dynamics in a cascaded transmission system with passive dispersion compensation," *Opt. Lett.* **21**, 327-329 (1996).

[37] N.J. Smith, F.M. Knox, N.J. Doran, K.J. Blow and I. Bennion, "Enhanced power solitons in optical fibers with periodic dispersion management," *Elecronics Lett.* **32**, 54-55 (1996).

[38] Proceedings of the First Notre Dame Workshop on Mathematical Methods in Nonlinear Optics, The University of Notre Dame, Notre Dame, Indiana, April 18th-21st, 1996. Part of the Symposium on Current and Future Directions in Applied Mathematics, M.S. Alber and G.G. Luther, August, 1996, BRIMS Report HPL-BRIMS-96-19.

[39] Proceedings of the BRIMS Workshop on Mathematical Methods in Nonlinear Optics, BRIMS, Hewlett-Packard Laboratories, Bristol, September 2-3, 1996, G.G. Luther and Y.S. Kivshar, September, 1996, BRIMS Report HPL-BRIMS-96-21.

Appendix

•*Nonlinear Optical Switching*: In $\chi^{(2)}$ materials, which respond quadratically in the presence of light, three-wave mixing and second harmonic generation take place. These materials hold great promise for switching in nonlinear guided wave optics. Inspired by recent experimental successes, this field has seen a resurgence of theoretical activity. New predictions about the existence and stability of solitary waves including self-trapped parametricly interacting pulses in higher dimensional systems have been made. Technologies for generating short wavelength light sources and for all optical switching rely heavily on the advancement of our understanding of the interaction of light with $\chi^{(2)}$ materials. One important theoretical result is the extension of the Vakhitov-Kolokolov stability criterion for Hamiltonian wave systems.

M.G. Vakhitov and A.A. Kolokolov, Radiphys. Quantum Electron. **16**, 783 (1973).

D.J. Mitchell and A.W. Snyder, J. Opt. Soc. Am. B **10**, 1572 (1993).

G.I. Stegeman and A. Miller, "Physics of all-optical switching devices," in Photonics Switching Vol. 1, J. Midwinter, ed. (Academic Press., Orlando, Fla., 1993) pp. 81-146.

A.V. Buryak, Y.S. Kivshar, and V.S. Steblina, "Self-trapping of light beams and parametric solitons in diffractive quadratic media," Phys. Rev. A **52**, 1670-1674 (1995).

Y.S. Kivshar, Phys. Rev. E **51**, 1613 (1995).

W.E. Torruellas, Z. Wang, D.J. Hagan, E.W. Van Stryland, G.I. Stegeman, L. Torner, and C.R. Menyuk, Phys. Rev. Lett. **74**, 5036 (1995).

D.E. Pelinovsky, A.V. Buryak, and Y.S. Kivshar, "Instability of solitons governed by quadratic nonlinearities," Phys. Rev. Lett. **75**, 591 (1995).

•*Self Focusing*: With the development of laser sources producing intense pulses of light with sub-picosecond time scales, the theory of the self-focusing singularity was re-examined. Over nearly thirty years the evolution of the self focusing singularity of the nonlinear Schrödinger equation with two spatial dimensions had been carefully studied. In the late eighties, the log log scaling for the evolution of the amplitude near the singularity was proved for critical self focusing. With this background and the advent of intense short pulse lasers the question it was unclear what a small amount of normal dispersion would do to a focusing pulse. After a series of numerical studies showing that pulses seemed not to self-focus but to break up, several theoretical results followed yielding modulation equations describing the focusing process. It was clear that continuum generation was part of the process. Surprisingly, this work has also lead to the theoretical generalization of a heuristic numerically determined equation for the focusing of Gaussian pulses obtained in the mid seventies.

J.J. Rasmussen and K. Rypdal, "Blow-up in nonlinear Schroedinger equations-I," *Physica Scripta* **33**, 481-497 (1986).

Wave Collapses, ed. V.E. Zakharov, *Physica D* **52** (1991).

V.M. Malkin, "On the analytical theory for stationary self focusing of radiation," *Physica D* **64**, 251-266 (1993).

G.G. Luther, A.C. Newell, and J.V. Moloney, "The effects of normal dispersion on collapse events," *Physica D* (1994).

G. Fibich, V.M. Malkin and G.C. Papanicolau, "Beam self-focusing in the presence of small normal time dispersion," *Phys. Rev. A* **52**, 4218-4228 (1995).

E.A. Kuznetsov, J. Juul Rasmussen, K. Rypdal, S.K. Turitsyn, "Sharper criteria for the wave collapse," *Physica D* **87**, 273-284 (1995).

L. Bergé, J. Juul Rasmussen, E.A. Kuznetsov, E.G. Shapiro and S.K. Turitsyn, "Self-focusing of chirped optical pulses in media with normal dispersion," *J. Opt. Soc. Am. B* **13**, 1879 (1996).

G. Fibich, "An adiabatic law for self-focusing of optical beams," *Opt. Lett.* **21**, 1735-1737 (1996).

• *Coupled Nonlinear Schrödinger Equation*: The coupled nonlinear Schrödinger equations describe the propagation of two nonlinear dispersive wave-packets coupled through the cubic nonlinearity. They model the propagation of polarized light in optical fibers. Both the near-integrable and far from integrable equations are important in this area. The near-integrable system is attained experimentally either by manipulating the coupling coefficients directly or by randomization.

D. David, D.D. Holm and M.V. Tratnik, "Hamiltonian chaos in nonlinear optical polarization dynamics," *Phys. Rep.* **187**, 281 (1990).

B. Malomed, "Polarization dynamics and interactions of solitons in a birefringent optical fiber," *Phys. Rev. A* **43**, 410-423 (1991).

D.J. Muraki and W.L. Kath, "Hamiltonian dynamics of solitons in optical fibers," *Physica* **D48**, 53-64 (1991).

S. Trillo and S. Wabnitz, "Weak-pulse-activated coherent soliton switching in nonlinear couplers," *Opt. Lett.* **16**, 1-3 (1991).

T. Ueda and W.L. Kath, "Dynamics of optical pulses in randomly birefringent fibers," *Physica* **D55**, 166-181 (1992).

C. De Angelis, S. Wabnitz, and M. Haelterman, "Bandwidth limits due to polarisation multiplexed soliton interactions," *Electron. Lett.* **29**, 1568-1570 (1993).

D.J. Kaup and B.A. Malomed, "Soliton trapping and daughter waves in the Manakov model," *Phys. Rev. A* **48**, 599-604 (1993).

N. Akhmediev and J.M. Soto-Crespo, "Dynamics of soliton like pulse propagation in birefringent optical fibers," *Phys. Rev. E* **49**, 5742-5754 (1994).

C. Sophocleous and D.F. Parker, "Pulse collisions and polarization conversion for optical fibers," *Opt. Comm.* **112**, 214 (1994).

A.V. Buryak, Y.S. Kivshar and D.F. Parker, "Coupling between dark and bright solitons," *Phys. Lett. A* **215**, 57 (1996).

J.U. Kang, G.I. Stegeman, J.S. Aitcheson and N. Akhmediev, "Observation of Manakov spatial solitons in AlGaAs planar waveguides," *Phys. Rev. Lett.* **76**, 3699 (1996).

P.K.A. Wai and C.R. Menyuk, *IEEE J. Lightwave Technol.* **14**, 148 (1996).

•*Arrays of Guided Waves*: When arrays of waveguides couple they produce a host of interesting and complex dynamics. The dynamics of arrays of semiconductor waveguides has been the subject of much study. More recently, the fiber array has been studied. The basic model for the fiber array is a lattice of nonlinear Schrödinger equations with nearest neighbor linear coupling. Localization reminiscent of self focusing organizes the dynamics on long scales seeding the creation of solitary waves. This system also supports complex spatio-temporal dynamics which is close to that of the discrete nonlinear Schrödinger equation. It is one of the important examples of dynamical systems which evolve in continuous time with one discrete dimension and one continuous dimension.

L. Rahman and H.G. Winful, "Nonlinear dynamics of semiconductor arrays: a mean field model," *IEEE J. Quantum. Electron.* **30**, 1405 (1994).

A.B. Aceves, C. De Angelis, G.G. Luther, A.M. Rubenchik, and S.K. Turitsyn, "All-optical-switching, pulse amplification and steering in nonlinear fiber arrays," *Physica D* **87**, 262-272 (1995).

A.V. Buryak and N.N. Akhmediev, "Stationary pulse propagation in N-core nonlinear fiber arrays," *IEEE Journal of Quantum Electronics* **31**, 682-688 (1995).

H. Winful and R.K. DeFreez, "Dynamics of coherent semiconductor laser arrays," in Diod Laser Arrays, ed. D. Botz and D.R. Scifres, Cambridge Studies in Modern Optics, Vol. 14, (Cambridge University Press, 1995).

M.I. Weinstein and B. Yeary, "Excitation and dynamics of pulses in coupled fiber arrays," *Phys. Lett. A* **222**, 157 (1996).

•*Pattern Formation and Chaos*: It has long been known that chaotic dynamics occurs even in very basic models of lasers. These effects have been observed in a variety of lasers and other nonlinear optical devices. In recent years the emphasis in this area has been on developing homoclinic control techniques to stabilize strongly-driven lasers as well as arrays of lasers. Developments in nonlinear dynamics, in particular ideas about deterministic chaos and pattern formation have had deep implications on the understanding of complex behavior in such nonlinear optical systems. A recent review of these efforts can be found in the article by Dan Gauthier in *Nonlinear Science Today*. It describes the study of bifurcations and chaos in laser systems, the formation of patterns in optical systems, and the control of chaos. More information on pattern forming systems in optics is available in the literature noted below.

P. Coullet, L. Gil and F. Rocca, *Opt. Comm.* **73**, 403 (1989).

W.J. Firth, *J. Mod. Opt.* **37**, 151 (1990).

A.C. Newell, "Patterns in nonlinear optics," Nato Advanced Research Workshop, Spatio-Temporal Patterns in Nonequilibrium Complex Systems, Sante Fe, New Mexico, April 12-17, 1993.

D.J. Gauthier, "The dynamics of optical systems: a renaissance of the 1990's," *Nonlinear Science Today* **4**, 2-11 (1994).

F.T. Arecchi, "Optical morphogenesis: pattern formation and competition in nonlinear optics," *Physica D* **86**, 297 (1995).

J. Lega, J.V. Moloney and A.C. Newell, "Universal description of laser dynamics near threshold *Physica D* **83**, 478 (1995).

R. Martin, A.J. Scroggie, G.-L. Oppo and W.J. Firth, "Stabilization, selection and tracking of unstable patterns by Fourier space techniques," *Phys. Rev. Lett.* **66**, 4007 (1996).

•*Geometric Phases and Soliton Interactions*: Geometric phases or holonomies are a general phenomenon in Hamiltonian systems, both finite- and infinite-dimensional. They have been studied extensively in the last decade in a variety of contexts in optical systems. It has recently been shown that the phase shifts due to soliton collisions have a similar origin. In soliton systems where several signals with different carrier frequencies propagate simultaneously, soliton interactions become important.

J.C. Garrison and R.Y. Chiao, *Phys. Rev. Lett.* **60**, 165 (1988).

J.C. Garrison and E.M. Wright, *Phys. Lett. A* **128**, 177 (1988).

A. Shapere and F. Wilczek, eds., Geometric Phases in Physics, (World Scientific, Singapore, 1989).

M.S. Alber and J.E. Marsden, On geometric phases for soliton equations, *Commun. Math. Phys.* **149**, 217-240 (1992).

D. David and D.D. Holm, "Multiple Lie-Poisson structures, reductions and geometric phases for the Maxwell-Bloch traveling wave equations," *J. Nonlinear Sci.* **2**, 241 (1992).

•*The Maxwell Bloch Equations*: The Maxwell-Bloch equations model the interaction of a light wave with a two-level atom. The Maxwell Bloch equations were shown to be integrable under certain conditions in the early 70's. A renewed interest in the structure of these equations is reflected in the references below.

A.P. Fordy and D.D. Holm, "A tri-Hamiltonian formulation for the self-induced transparency equation," *Phys. Lett. A* **160**, 143 (1991).

D.D. Holm, G. Kovacic and B. Sundaram, "Chaotic laser-matter interaction," *Phys. Lett. A* **154**, 346 (1991).

D.D. Holm, G. Kovacic and T.A. Wettergren, "Homoclinic orbits in the Maxwell-Bloch equations with a probe," *Phys. Rev. E 54*, 243 (1996).

J. Xin and J.V. Moloney, "Global weak solutions and attractors of the three-dimensional Maxwell-Bloch two level laser systems," *Comm. Math. Phys.* **179**, 511 (1996).

•*Identification of Bound States*: Often sophisticated techniques from dynamical systems theory are required to identify bound-state solutions of nonlinear optical systems. Some recent work on the identification of multi-pulse solutions is detailed in the references provided below.

C.K.R.T Jones, "Instability of standing waves for non-linear Schrödinger-type equations," *Ergod. Th. & Dynam. Sys.* **8**, 119-138 (1988).

A.V. Buryak and N.N. Akhmediev, "Stability criterion for stationary bound states of solitons with radiationless oscillating tails," *Phys. Rev. E* **51**, 3572 (1995).

B. Sandstede, C.K.R.T. Jones and J.C. Alexander, "Existence and stability of N-pulses on optical fibers with phase-sensitive amplifiers," Preprint, 1996.

R. Camassa, G. Kovacic and S-K Tin, "A Melnikov method for Homoclinic orbits with many pulses," to be published in *Arch. Rat. Mech. Anal.*

•*Gap Solitons*: Wave propagation in periodic media is of course an important topic in optics. Recently the development of optical fiber Bragg gratings has created a resurgence of interest in the nonlinear propagation of waves in periodic media. The general area of propagation in periodic media holds great promise in nonlinear optics.

C.M. de Sterke, D.G. Slinas and J.E. Sipe, "Coupled-mode theory for light propagation through deep nonlinear gratings," *Phys. Rev. E* **54**, 1969 (1996).

B.J. Eggleton, R.E. Slusher and J.E. Sipe, "Bragg grating solitons," *Phys. Rev. Lett.* **76**, 1627 (1996).