Repairing Bell's Theorem for a Class of Down-Conversion Experiments

Sandu Popescu[1], Lucien Hardy[2], Marek Zukowski[3]
Basic Research Institute in the
Mathematical Sciences
HP Laboratories Bristol
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[1] Isaac Newton Institute, University of Cambridge, Cambridge, CB3 0EH, U.K.
[2] Dipartimento di Fisica, Universita degli Studi di Roma "La Sapienza" I-00185 Roma, Italy
[3] Instytut Fizyki Teoretycznej i Astrofizyki, Uniwersytet Gdanski, PL-80-952 Gdansk, Poland

REPAIRING BELL'S THEOREM FOR A CLASS OF DOWN-CONVERSION EXPERIMENTS

Sandu Popescu[1], Lucien Hardy[2] and Marek Żukowski[3]

[1]*Isaac Newton Institute, University of Cambridge,*
Cambridge, CB3 0EH, U.K.
[2]*Dipartimento di Fisica, Università degli Studi di Roma "La Sapienza"*
I-00185 Roma, Italy
[3]*Instytut Fizyki Teoretycznej i Astrofizyki, Uniwersytet Gdański,*
PL-80-952 Gdańsk, Poland

ABSTRACT

A certain class of parametric down-conversion Bell type experiments has the following features. In the idealized perfect situation it is in only 50% of cases that each observer receives a photon; in the other 50% of cases one observer receives both photons of a pair while the other observer receives none. The standard approach is to discard the events of the second type. Only the remaining ones are used as the data input to some Bell inequalities. This raises justified doubts whether such experiments could be ever genuine tests of local realism. We propose to take into account these "unfavorable" cases and to analyze the entire pattern of polarization and localization correlations. This departure from the standard reasoning enables one to show that indeed the experiments are true tests of local realism.

1. INTRODUCTION

The tremendous progress in experimental quantum optics during recent years, and especially the advent of parametric down-conversion techniques have generated a new wave of experiments purposing to test Bell's inequalities [1]. The experiments like those of C. O. Alley and Y. H. Shih [2], Z. Y. Ou and L. Mandel [3], and T. E. Kiss, Y. H. Shih, A. V. Sergienko and C. O. Alley [4] are some of the most famous examples. They form a distinctive class: polarization correlations are measured, and to produce the required phenomena the use is made of a technique involving essentially two linear optical devices, namely a wave plate and a single beamsplitter (fig.1). Although these experiments have successfully reproduced certain long-distance quantum mechanical correlations, it is generally believed [5] that they could never, *not even their idealized versions*, be considered as true tests of local realism. In the present paper we prove that this general belief is wrong and that the above experiments are in fact much better than they were previously thought

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to be: in principle they *could* be tests of the most general premises of local realism, and the only obstacles they face are purely technical (such as present day low efficiency of the photon detectors, misalignments etc.).

The claim that the above experiments can never be true tests of local realism is based usually on the following argument. In the original setting, proposed by J. Bell and then widely analyzed by many others [6], experiments are performed on an ensemble of pairs of particles prepared in such a way that one particle in each pair is directed towards an observer while the other particle is directed toward another observer situated far from the first one. However, in the parametric down-conversion experiments we are interested in [2-4] the pairs of particles (photons) are prepared in a different way - only in 50% of the cases each observer receives a photon; in the other 50% of cases one observer receives both photons of a pair while the other observer receives none. The usual superficial method to deal with this situation is to discard all the unfavorable the cases in which both photons end at the same observer and to retain only the cases in which each observer receives a photon. Therefore, it can be justifiably claimed, one runs directly into the well-known problem of subensemble post-selection. If one restricts the analysis to a small enough subensemble of the original ensemble of pairs one can never rule out all possible local hidden variable models.

However, a more careful look at the the argument formulated in the last two sentences shows its weakness. First of all, the idea that the "unfavorable" cases should be discarded appeared probably because of the desire to forcefully map the problem under investigation into the one originally formulated by Bell. But instead of doing this one could actually make use of these "unfavorable" cases by taking them also into account and analyzing the entire pattern of correlations. (This is the approach we shall take in the present paper.) Second, while it is widely believed that subensemble selection (when the selected subensemble is small enough) *always* prevents observing violations of local realism, this is actually not true. Indeed there are cases in which post-selection is a serious drawback (e.g., "detector efficiency" problem [7]) but there are also other cases in which subensemble selection raises no problems [8, 9]. Consequently one should not dismiss from the beginning the possibility of observing violations of local realism just because subensemble selection has been performed, but each situation should be investigated carefully.

Consider first the experimental setting used in [2-4] and illustrated in fig. 1. A type I parametric down-conversion source is used to generate pairs of photons in which

both photons have the same energy and linear polarization (say $\hat{x}$) but propagate in two different directions. One of the photons passes through a 90° polarization rotator, the wave-plate WP, emerging polarized along $\hat{y}$. The two photons are then directed by two mirrors, M, onto the two sides of a (polarization independent) beamsplitter BS which, for simplicity, we consider to be symmetric. Each observer is equipped with a polarizing beamsplitter, orientated along an arbitrary axis, randomly chosen just before the photons are supposed to arrive. Each polarizing beamsplitter is followed by two detectors, D_1^+, D_1^- and D_2^+, D_2^- respectively, where the lower index indicates the corresponding observer and the upper index the two exit ports of the polarized beamsplitter ("+" meaning parallel with the polarization axis of the beamsplitter and "-" meaning orthogonal to this axis). All optical paths are assumed to be equal.

The quantum state of the two photons just before entering the detection stations is

$$|\Psi\rangle = \frac{1}{2}\left(|1\hat{x}\rangle_1|1\hat{y}\rangle_2 - |1\hat{y}\rangle_1|1\hat{x}\rangle_2 + i|1\hat{x}, 1\hat{y}\rangle_1|0\rangle_2 + i|0\rangle_1|1\hat{x}, 1\hat{y}\rangle_2\right), \qquad (1)$$

where the subscript 1 or 2 on the ket vectors represent the two regions of space where the photons arrive, i.e. near observer 1 and near observer 2, the notation 0 inside the ket vectors denotes vacuum and $1\hat{x}$ and $1\hat{y}$ represents 1 photon polarized along the $\hat{x}$ or $\hat{y}$ directions respectively. The first two terms in (1) correspond to cases in which each observer will register a single photon while the last two terms correspond to cases in which one of the observers will register two photons while the other observer will register none.

Quite often the discussion starts with just simply chopping off the last two terms in (1). This approach, since it is effectively a post-selection procedure, raises serious, justified doubts [5], whether the experiments indeed are tests of local realism. But in fact there is no reason why one should reject the other cases.

Let us denote by $P(i, \hat{\xi}; j, \hat{\eta})$ the joint probability for the outcome i to be registered by observer 1 when his polarizing beamsplitter BS1 is oriented along the direction $\hat{\xi}$ and the outcome j to be registered by observer 2 when her polarizing beamsplitter is oriented along $\hat{\eta}$. Here $i, j = 1 - 6$ and have the following meaning:

1 = one photon in D^-, no photon in D^+,

2 = one photon in D^+, no photon in D^-,

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3 = no photons,

4 = one photon in D^+ and one photon in D^-,

5 = two photons in D^+, no photon in D^-,

6 = two photons in D^-, no photons in D^+.

We have not included the possibility of more than one pair of photons being emitted. The probability of this happening, under the usual experimental conditions, is very small, and thus will lead to negligible effects.

A Clauser-Horne-Shimony-Holt (CHSH) type inequality which is obeyed by all local hidden variables models but is violated by quantum mechanics for the full state (1) can be obtained in the following way. Let us associate with each outcome registered by observer 1 and 2 a corresponding value a_i^ξ and b_j^η respectively, where $a_1^\xi = b_1^\eta = -1$ while all the other values are equal to 1, and let us denote by $E(a^\xi b^\eta)$ the expectation value of their product

$$E(a^\xi b^\eta) = \sum_{i,j} a_i^\xi b_j^\eta P(i,\hat{\xi};j,\hat{\eta}). \tag{2}$$

Now, in a local hidden variables model

$$P(i,\hat{\xi};j,\hat{\eta}) = \int d\lambda \rho(\lambda) P_1(i,\hat{\xi};\lambda) P_2(j,\hat{\eta};\lambda), \tag{3}$$

where λ is the local hidden variable with ρ its distribution function ($\int d\lambda \rho(\lambda) = 1$), and $P_1(i,\hat{\xi};\lambda)$ and $P_2(j,\hat{\eta};\lambda)$ the local probabilities ($\sum_i P_1(i,\hat{\xi};\lambda) = 1 = \sum_j P_2(j,\hat{\eta};\lambda)$). It is straightforward to see that according to any local hidden variables model the CHSH inequality holds, i.e.

$$|E(a^\xi b^\eta) + E(a^\xi b^{\eta'}) + E(a^{\xi'} b^\eta) - E(a^{\xi'} b^{\eta'})| \leq 2, \tag{4}$$

for any directions $\hat{\xi}$, $\hat{\xi}'$, $\hat{\eta}$ and $\hat{\eta}'$.

On the other hand, according to quantum mechanics

$$E(a^\xi b^\eta) = \langle \Psi | A^\xi B^\eta | \Psi \rangle, \tag{5}$$

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where $A^{\hat{\xi}}$ and $B^{\hat{\eta}}$ are the corresponding quantum observables, defined as follows: $A^{\hat{\xi}}$ is an operator which has a nondegenerate eigenvalue $A^{\hat{\xi}} = -1$ corresponding to the eigenstate $|1\hat{\xi}_{\perp}\rangle_1$ (which represents 1 photon polarized orthogonal to $\hat{\xi}$ and which corresponds to observer 1 obtaining the outcome $i = 1$) and a multiple degenerate eigenvalue $A^{\hat{\xi}} = 1$ corresponding to the rest of the Hilbert space (the space spanned by $|1\hat{\xi}\rangle_1$, $|0\rangle_1$, $|1\hat{\xi}, 1\hat{\xi}_{\perp}\rangle_1$, $|2\hat{\xi}\rangle_1$ and $|2\hat{\xi}_{\perp}\rangle_1$ and which correspond to observer 1 obtaining outcomes $i = 2 - 6$); $B^{\hat{\eta}}$ is defined in a similar way. In other words, the operators A and B are equal to the usual polarization operators on the subspace of "favorable" cases (yielding +1 if the photon's polarization is parallel with that of the polarization analyzer and -1 if it is perpendicular) and equal to the identity operator on the subspace of "unfavorable" cases. Let us also define $|\Psi_1\rangle$ and $|\Psi_2\rangle$ as the normalized projections of $|\Psi\rangle$ on the subspaces of "unfavorable" and "favorable" cases respectively, i.e.

$$|\Psi_1\rangle = \frac{i}{\sqrt{2}}\left(|1\hat{x}, 1\hat{y}\rangle_1 |0\rangle_2 + |0\rangle_1 |1\hat{x}, 1\hat{y}\rangle_2\right) \tag{6}$$

and

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}\left(|1\hat{x}\rangle_1 |1\hat{y}\rangle_2 + |1\hat{y}\rangle_1 |1\hat{x}\rangle_2\right) \tag{7}.$$

Then, as the local operators A and B do not mix the local vacuum and local two photons states with the local one photon states, it follows that

$$CHSH_Q = \frac{1}{2}\langle\Psi_1|A^{\hat{\xi}}B^{\hat{\eta}} + A^{\hat{\xi}}B^{\hat{\eta}'} + A^{\hat{\xi}'}B^{\hat{\eta}} - A^{\hat{\xi}'}B^{\hat{\eta}'}|\Psi_1\rangle +$$

$$\frac{1}{2}\langle\Psi_2|A^{\hat{\xi}}B^{\hat{\eta}} + A^{\hat{\xi}}B^{\hat{\eta}'} + A^{\hat{\xi}'}B^{\hat{\eta}} - A^{\hat{\xi}'}B^{\hat{\eta}'}|\Psi_2\rangle$$

$$= \frac{1}{2} \times 2 + \frac{1}{2} \times \langle\Psi_2|A^{\hat{\xi}}B^{\hat{\eta}} + A^{\hat{\xi}}B^{\hat{\eta}'} + A^{\hat{\xi}'}B^{\hat{\eta}} - A^{\hat{\xi}'}B^{\hat{\eta}'}|\Psi_2\rangle \tag{8}$$

The expectation value in the last term in (8) is nothing else than the usual quantum CHSH expression computed in the $|\Psi_2\rangle$ state which for suitable chosen directions $\hat{\xi}$, $\hat{\xi}'$, $\hat{\eta}$ and $\hat{\eta}'$ can yield $2\sqrt{2}$. Choosing such directions in (8) it follows that, for the idealized perfect experiment

$$CHSH_Q = 1 + \sqrt{2} > 2 \tag{9}$$

which is in contradiction with the limit imposed by local hidden variables models.

In calculating probabilities above we have assumed that the total number of events is equal to the total number of pairs detected. This is equivalent to assuming an event

ready configuration in which the source clicks (or gives off some other appropriate signal) when the photons are emitted. However, the experiments [2-4] were not event ready since there is no way to know that a pair of photons has been emitted (event ready configurations have only been suggested [10]).

Thus, it could be possible, for example, that whether or not two photons are detected at one end (whether we are in a i=4,5,6 or a i=3 case) depends on the setting of the polarizing beamsplitter. Polarizer settings, by biasing the ensemble considered, seemingly introduce the possibility of a loophole.

To solve this problem, first of all, we must decide what we are going to regard as an event. The experiment runs for a certain time, T. This time can be divided up into short intervals of duration τ. At the begining of each time interval the polarizing beamsplitters are set in a new, randomly chosen, direction. The time interval τ must be chosen to be smaller than L/c where L is the distance between the detector stations so that there is no possibility of relativistic causal signals being transmitted during this time interval. Also, to avoid extra complications, it should be chosen such that there is a very small probability of more than one pair being emitted during τ. And it should be bigger than the time resolution of the detectors so that two photons from the same pair are almost certainly detected during the same time interval. A practical choice would probably be $\tau = 10ns$.

The total number of events is N where $T = N\tau$ and T is chosen such that N is integer. The n-th event happens during the interval $(n-1)\tau$ to $n\tau$. During this interval we record the type of event that has occurred at each end of the apparatus ($i, j = 1$ to 6). In this way we can form probabilities in the usual way. Typical counting rates are about 10^4 per second which is much smaller than $\frac{1}{\tau}$. This means almost all events will be of the type where no photons are detected at either end ($i = j = 3$). For CHSH inequalities formulated in the usual way this would be a big problem since these no-photon events would drown out the interesting events and hence the inequalities would not be violated. However, as we will see, the way in which the correlation function has been defined in equation (2) solves this problem.

To include explicitly the vacuum term one can describe the state by

$$|\Psi'\rangle = \alpha|0\rangle_1 |0\rangle_2 + \beta|\Psi\rangle$$

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where $|\Psi\rangle$ is the state (1). The vacuum term is now explicitly included. Either a total of two photons will be detected or no photons. Let N_0 be the number of events for which no photons are detected. Then

$$P(3, \hat{\xi}, 3, \hat{\eta}) = \frac{N_0}{N}$$

Since $a_3^{\hat{\xi}} = b_3^{\hat{\eta}} = 1$ we have from equation (2) that

$$E(a^{\hat{\xi}} b^{\hat{\eta}}) = \frac{N_0}{N} + \frac{N - N_0}{N} \langle \Psi | A^{\hat{\xi}} B^{\hat{\eta}} | \Psi \rangle, \tag{9}$$

Hence, now taking into account the vacuum cases, we have that

$$CHSH_Q = \frac{2N_0}{N} + \frac{N - N_0}{N}(1 + \sqrt{2}) > 2 \tag{10}$$

where the inequality follows since $N_0 < N$. Hence, the CHSH inequalities are still violated.

The magnitude of the violation is not as great but this need not bother us. Any experiment which violates the inequalities when the vacuum events are included will also violate the experiment when they are not and vice versa. Simply, an experiment where every event was a vacuum event would give $CHSH_Q = 2$.

Let us comment on the general case of non-event ready experiments. In their review [6] Clauser and Shimony make the point that the Clauser-Horne inequalities have a 0 as their upper bound and hence they are insensitive to the overall normalization of probabilities making them suitable to non-event ready experiments, and since the CHSH inequalities have non-zero bounds they do rely on knowing how to normalize probabilities and thus are not suitable for non-event ready experiments. However, by employing the two tricks of (i) considering short intervals as events and (ii) putting $a_3^{\hat{\xi}} = b_3^{\hat{\eta}} = 1$ so that the CHSH inequality is saturated by an ensemble of vacuum events, we make it possible to employ the CHSH inequalities in non-event ready type experiments.

To summarize, if one wants to discuss the experiments [2-4] as tests of local realism, one should not discard any "unfavorable" cases but rather one has to analyze them. And there is plenty of information we can obtain apart from what are the polarization correlations. What we have just shown is that the entire pattern of polarization and localization correlations in the experiments [2-4] cannot be explained by any local hidden variable model.

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pump

PDC

x

x

M

M

WP

y

BS

POL

POL

D_1^+

D_1^-

D_2^+

D_2^-