Comment: 'M-Sequences for OFDM Peak-to-Average Power Ratio Reduction and Error Correction'

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A recent letter proposed the use of a block coding scheme for OFDM based on m-sequences. We show that the peak-to-average power for this scheme was not calculated correctly and we recommend against its use.

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Let $\mathbf{c} = (c_0, c_1, \ldots, c_{N-1})$ be a block codeword for orthogonal frequency domain multiplexing (OFDM) across N carriers with symbol rate f. Then, up to normalisation, the transmitted signal over one symbol period using this codeword is given by the continuous function $s(t) = (\sqrt{N})^{-1} \sum_{n=0}^{N-1} c_n \exp(j2\pi nft)$ for $0 \leq ft \leq 1$. The envelope power over this period is $|s(t)|^2$ [1]. Since the mean value of $|s(t)|^2$ is 1, the peak-to-average envelope power ratio (PAP) of the codeword $\mathbf{c}$ is $\max_{0 \leq ft \leq 1} |s(t)|^2$.

Li and Ritcey [2] noted that the time domain OFDM samples are given by $C_t = (\sqrt{N})^{-1} \sum_{n=0}^{N-1} c_n \exp(j2\pi nt/N)$ evaluated at the N discrete values $t = 0, 1, \ldots N-1$. However their assumption, that the maximum value of $|C_t|$ over these N discrete values is equal to the maximum value of $|s(t)|$ over the continuous interval $0 \leq ft \leq 1$, does not follow.

Indeed, the proposed scheme of [2] uses a cyclic code of length $N = 2^m - 1$ based on an m-sequence, which itself arises from a primitive polynomial of degree m over GF(2) [3]. For example, for $m = 3$ the primitive polynomial $x^3 + x + 1$ gives rise to a cyclic code containing the codeword 1001110. For this codeword the maximum value of $|C_t|^2$ is 1.14 but the maximum value of $|s(t)|^2$ is 2.90, so that the PAP is 4.6dB and not 0.58dB as given in [2]. Similarly, for $m = 4$ the primitive polynomial $x^4 + x + 1$ gives rise to a cyclic code containing the codeword 100011110101100. For this codeword the maximum value of $|C_t|^2$ is 1.07 but the maximum value of $|s(t)|^2$ is 3.48, so that the PAP is 5.4dB and not 0.28dB as given in [2].

In view of this we recommend against the use of the scheme proposed in [2]. We refer the reader to [4] for a description of an OFDM scheme using $N = 2^m$ carriers attaining a PAP of at most 3dB while allowing simple encoding and decoding at high code rates for binary, quaternary or higher-phase signalling together with good error correction. Full details of this scheme and its variations will be given in a forthcoming paper.

References

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