More than Zero Intelligence Needed for Continuous Double-Auction Trading

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ABSTRACT

Gode & Sunder's (1993) results from using "zero-intelligence" (ZI) traders, that act randomly within a continuous double-auction (CDA) market, appear to imply that human-like convergence to the theoretical equilibrium price in such markets is determined more by market structure than by the intelligence of the traders in that market. This paper presents a mathematical analysis that predicts serious failures in ZI-trader CDA markets. The analytical predictions are confirmed by computer simulations. Thus, more than zero intelligence is required of trading agents to yield human-like CDA market behavior.

1 INTRODUCTION

Smith (1962) demonstrated that the transaction prices of remarkably small groups of human traders, operating in experimental CDA markets, rapidly approach the theoretical equilibrium price. But human beings are notoriously smart creatures: the question of just how much intelligence is required of an agent to achieve human-level trading performance is an intriguing one. This question was addressed by Gode & Sunder (1993), whose results appear to indicate that no intelligence at all is required of the traders, so long as they are prevented from trading at a loss.

Gode & Sunder (1993) reported results from market experiments where "zero-intelligence constrained" (ZI-C) trader-programs, that submit random bids and offers, are used to replace human traders in CDA markets. They found that the imposition of the budget constraint (that prevents ZI traders from entering into loss-making deals), is sufficient to raise the allocative efficiency of the auctions to values near 100 percent. They conclude that the traders' motivation, intelligence, or learning have little effect on the allocative efficiency, which derives instead largely from the structure of the CDA markets. Thus, they claim, "Adam Smith's invisible hand may be more powerful than some may have thought; it can generate aggregate rationality not only from individual rationality but also from individual irrationality." (Gode & Sunder, 1993, p.119).

This important work has often been cited approvingly in the experimental economics literature. See, e.g., Friedman and Rust (1992, p.*xxiii*), Friedman (1992, p.19), Rust, Miller, and Palmer (1992, pp.160–161, 175), Bollerslev and Domowitz (1992, pp.230–231), Cason and Friedman (1992, pp.253, 258), Kagel and Vogt (1992, pp.292, 294), Davis and Holt (1993, p.132), Roth (1995, pp.52–55, 80–81), Holt (1995, p.370), Kagel (1995, pp.570, 580), and Camerer (1995, p.674); and has even been discussed in a recent book on the philosophy of mind (Clark, 1997, pp.183–184).

This paper presents an analysis of the probability functions underlying CDA markets populated by Gode & Sunder's ZI-C traders. This analysis, in markets similar to those used by Smith (1962), leads to predictions of market conditions in which ZI-C traders fail to trade at equilibrium prices. These analytic results are supported by empiricial results from simulation experiments in which the ZI-C traders are demonstrated to fail as predicted. Thus, it is claimed here that the ZI-C traders lack sufficient rationality to exhibit human-like equilibration in CDA markets.

Section 2 presents a brief overview of Gode & Sunder's work, prior to the critique in Section 3. Herein, Gode & Sunder are referred to as G&S.

2 ZERO-INTELLIGENCE TRADERS

It is beyond the scope of this paper to provide a full description of G&S's ZI-C work here: their 1993 paper is the definitive account; for a comprehensive summary, see Cliff (1997).

G&S used an electronic CDA market, where traders are connected on a computer network. G&S's experiments with human traders were performed in a manner similar to that established by Smith (1962): the subjects are divided into a group of sellers and a group of buyers. Sellers are given a number of units of an arbitrary commodity, and each unit has a limit price (below which it cannot be sold), which is private (i.e., known only to the seller of that unit). Buyers are given the rights and means to buy a number of units, and for each unit they are given a private limit price above which they must not pay. The array of sellers' limit prices determines the market supply curve, and the array of buyer's limit prices determines the market demand curve. In the experiments with human traders, traders 'quote' bid and offer prices by typing them into their computer terminals: the quotes are then distributed to the other traders, and at any time a buyer can accept a seller's offer or a seller can accept a buyer's bid. This continuous trading process is broken into discrete periods or 'days': at the start of each day, new allocations of selling or buying rights are distributed to the traders. In experimental CDA markets such as these, as with real human CDA markets, transaction prices rapidly approach the theoretical equilibrium value given by the intersection of the supply curve and the demand curve.

In G&S's work with zero-intelligence (ZI) traders, the humans were replaced with software 'agents' (simple programs). G&S tested the software agents in markets with supply and demand curves similar or identical to those used with their human subjects. Two types of ZI CDA markets were investigated. In the first type, the humans were replaced with *unconstrained* (ZI-U) traders. The ZI-U traders, whether they are buyers or sellers, simply quote random prices in the range $\{1, 2, \ldots, 200\}$, regardless of whether the price quoted would lead to a loss-making transaction. In the second type of ZI CDA market, the humans were replaced with *constrained* (ZI-C) traders. Each ZI-C trader generates random bid or offer prices, but using a distribution constrained by the limit price for the current unit: each buyer is constrained to bid a price chosen randomly between the market minimum (1 currency unit) and that buyer's current limit price; each seller is constrained to offer at a price chosen randomly between that seller's limit price and the market maximum (200 currency units).

G&S showed results from five types of market. For each type of market, they show time-series of transaction prices from one experiment with ZI-U traders, from one experiment with ZI-C traders, and from one with human traders. Each experiment is divided into a small number of trading 'days'. The surprising and significant observation that G&S make is that the results from ZI-C traders appear to be much more similar to those of human traders than of ZI-U traders. In particular, G&S monitored allocative efficiency (profit extracted from the market as a proportion of maximum possible profit in that market) and found that the allocative efficiency of humans and ZI-C traders were not significantly different, while the ZI-U traders showed poor allocative efficiency. Thus, they conclude that no intelligence other than the budget constraint is required of trading agents to exhibit human-like behavior in CDA markets. G&S also speculate that no intelligence is necessary for the transaction prices of the traders to converge to the equilibrium value. It is this claim that is criticized in the next section.

3 CRITIQUE

G&S's central argument, that the structure of a double auction market is largely responsible for achieving high levels of allocative efficiency, regardless of the intelligence, motivation, or learning of the agents in the market, is not in doubt. However, serious concerns about the equilibrating tendencies of the ZI-C traders are discussed below. G&S state (1993, p.131): *"...the convergence of transaction price in ZI-C markets is a consequence of the market discipline; trader's attempts to maximize their profits, or even their ability to remember or learn about events of the market, are not necessary for such convergence."* This statement is demonstrated below to be incorrect.

In Section 3.1 the probability distributions underlying the ZI-C markets are discussed qualitatively. Then, in Section 3.2, analytic results are presented that demonstrate that the expected value of ZI-C transaction prices is equal to the equilibrium price only in certain special cases, differing significantly from equilibrium in other situations. To reinforce this result, empirical results from simulation studies are presented in Section 3.3, and discussed further in Section 3.4.

3.1 Qualitative Discussion

Fig. 1 shows the supply and demand curves for four types of market, labelled A, B, C, and D. In market A, the supply curve SS starts at some minimum price $S_{\min}$ at the minimum quantity supplied and slopes upwards to a price $S_{\max}$ at the maximum quantity supplied in the market, beyond which the supply curve is undefined (represented by the vertical segment of the curve). Similarly, the demand curve DD starts at some high price $D_{\max}$ for the minimum quantity demanded and slopes downwards to some minimum price $D_{\min}$ at the maximum quantity demanded, beyond which there is no demand (represented by the vertical segment of the curve). In market A, the supply and demand curves have gradients that are approximately equal in magnitude but opposite in sign: such markets are referred to here as *symmetric* because the supply and demand curves are mirror-symmetric, by reflection in the line of constant price at the equilibrium value P_0, over the range of quantities from zero to Q_0.

In Market B, the supply curve is flat over the range of quantities supplied, so $S_{\min} = S_{\max} = P_0$. In Markets C and D, both the supply curve and the demand curve are flat: thus, in both C and D, $S_{\min} = S_{\max}$ and $D_{\min} = D_{\max}$. However, in C, demand exceeds supply, and so the equilibrium price $P_0 = D_{\max}$ because the excess demand encourages price competition among buyers that will lead to bid-price increases until the maximum buyer limit price is reached. Similarly, in D, supply exceeds demand and so the excess supply encourages offer-price cuts, driving the price down to equilibrium at $P_0 = S_{\min}$.

In the five experiments presented by G&S, the market supply and demand were all similar to A, although not so perfectly symmetric over the range of quantities 0 to Q_0. Yet markets such as as B, C, and D have also been studied in the litera-

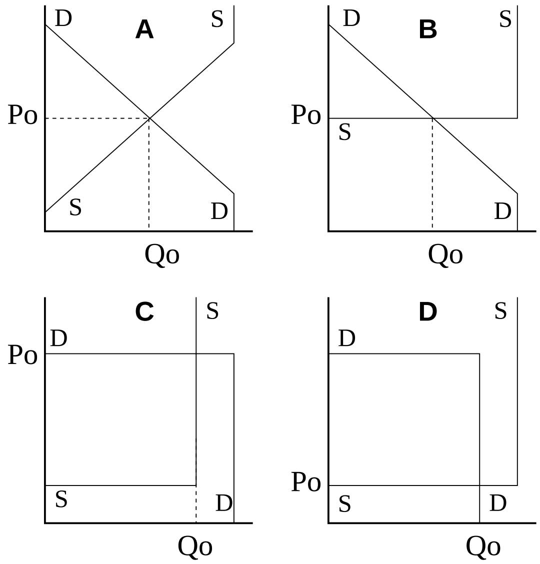

Figure 1: Four types of market. In each graph, the horizontal axis is quantity and the vertical is price. The supply curve is labelled SS and the demand curve is labelled DD; the intersection of these curves gives the equilibrium price P_0 and quantity Q_0. At top left is a market labelled A, where the supply and demand curves are symmetric about the line of constant price P_0. The top right market, labelled B, has a flat supply curve. At bottom left there is a 'box' market with excess demand, labelled C. At bottom right there is an excess-supply 'box' market labelled D.

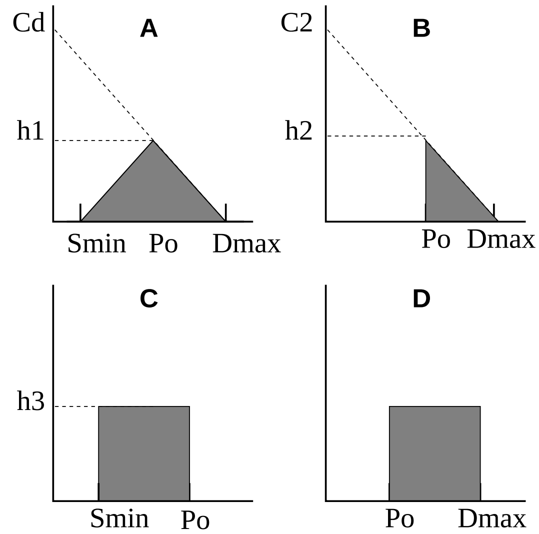

Figure 2: ZI-C transaction-price probability density functions (PDFs) for the four markets introduced in Fig. 1. In each graph, the horizontal axis is price and the vertical axis is the probability of a transaction occuring at a given price in a ZI-C CDA market. The values c_d, c_2, and $h_{i=1,2,3}$ are used in the analysis in Section 3.2.

ture. For example, market B is similar to Smith's (1962) "Chart 4", and markets C and D are similar to Smith's (1962) "Chart 6". Markets C and D are also known as "box-design" schedules (Davis & Holt, 1993, p.141).

For each of the four styles of market shown in Fig. 1, analytic expressions will be derived below for the expected values of transaction prices of ZI-C traders. It is shown that the expected value of transaction prices of ZI-C traders in symmetric markets such as A can be identical to the equilibrium price P_0, and thus they appear to converge on the theoretical equilibrium.

If convergence is a property of the CDA market institution, and ZI-C traders converge in near-symmetric markets similar to A, then it seems reasonable to assume that ZI-C traders would also exhibit convergence in markets B, C, and D. As will be shown, this assumption does not hold, because the convergence of ZI-C traders in CDA markets such as A is largely a matter of coincidence.

To explain why this is so, it is necessary to consider the probability density function (PDF) for transaction prices in each ZI-C markets. Transactions occur between ZI-C traders when a (randomly-generated) bid-price and (randomly-generated) offer-price 'cross'. Thus the PDF for transaction prices is given by the intersection of the PDFs for the ZI-C sellers' offer prices and ZI-C buyers' bid-prices. Valid ZI-C offer prices are generated at random from a distribution bounded from below by the supply curve and bounded from above by the system maximum price (200 in G&S's experiments). Valid ZI-C bid prices are bounded from below by the system minimum (one in G&S's experiments) and from above by the demand curve. Thus, the PDF for ZI-C transaction-prices (where both the bid-price and the offer-price are valid) is determined by the supply and demand curves, in a manner illustrated in Fig. 2. For full discussion of how these PDFs are derived, and further explanatory figures, see (Cliff, 1997).

As is clear from Fig. 2, only market A has a transaction-price PDF that is symmetric about the equilibrium price P_0. In markets B, C, and D, the transaction-price PDF has P_0 as a *bound*. When coupled to an intuitive notion of the average or expected value of a random variable as the "center of gravity" of the PDF (formally, the expected value of a random variable is the first moment about the origin), it becomes clear that only in markets similar to A will average transaction prices be close to P_0. This is established formally below.

3.2 Analytic Arguments

Let P denote the ZI-C transaction prices in a CDA market: P is a random variable; let $f(p)$ denote its PDF. If $f(p)$ is known, then the mean or expected value $E(P)$ of the ZI-C transaction prices can be calculated from the standard formula for the first moment:

$$E(P) = \int_{-\infty}^{\infty} p \cdot f(p)\, \mathrm{d}p \qquad (1)$$

Consider the case where the supply and demand curves are symmetric (i.e., have opposite sign and equal magnitude), as illustrated in Fig. 1A. The corresponding PDF is shown in Fig. 2A. As was mentioned above, G&S's ZI-C markets were all roughly symmetric.

The transaction-price PDF can be written as:

$$f_1(p) = \begin{cases} m_1 p + c_s & S_{\min} \le p \le P_0 \\ -m_1 p + c_d & P_0 \le p \le D_{\max} \\ 0 & \text{Otherwise} \end{cases} \qquad (2)$$

For $m_1 = 1/(D_{\max} - P_0)^2$. Substituting Equation 2 into Equation 1 and solving the integral gives:

$$\begin{aligned} E(P) &= \int_{S_{\min}}^{D_{\max}} p f_1(p)\, \mathrm{d}p \\ &= P_0 \end{aligned} \qquad (3)$$

The proof of Equation 3 requires only straightforward algebra, as do the proofs of Equations 5 and 7 (below). For completeness, the proofs are given in Cliff (1997). Thus, from Equation 3, when the supply and demand curves are linear and have opposite sign and equal magnitude, the mean ZI-C transaction price is equal to the equilibrium price.

Now consider a ZI-C market where the supply curve is flat, as in Fig. 1B, with the corresponding transaction-price PDF shown in Fig. 2B. The PDF $f_2(p)$ for such a market has the form:

$$f_2(p) = \begin{cases} m_2 p + c_2 & P_0 \le p \le D_{\max} \\ 0 & \text{Otherwise} \end{cases} \qquad (4)$$

For $m_2 = -h_2/j$ where $j = D_{\max} - P_0$, and $c_2 = 2D_{max}/j^2$. Note also that because $f_2(p)$ is a PDF and a right-triangle, $h_2 j/2 = 1$, so $h_2 = 2/j$ and hence $m_2 = -2/j^2$. Substituting Equation 4 into Equation 1 and solving gives:

$$\begin{aligned} E(P) &= \int_{P_0}^{D_{\max}} p f_2(p)\, \mathrm{d}p \\ &= P_0 + \frac{1}{3}(D_{\max} - P_0) \end{aligned} \qquad (5)$$

So Equation 5 indicates that, when all the sellers have the same limit price, the expected transaction price of ZI-C traders will differ from the equilibrium price P_0 by an amount equal to one third of the difference between P_0 and the maximum buyer price, $D_{\max}$. So long as $P_0 \neq D_{\max}$, the expected value of the ZI-C transaction prices will differ from the equilibrium price P_0.

Finally, consider the case of excess-demand "box" market schedules such as those shown in Fig. 1C: these have a rectangular PDF, as illustrated in Fig. 2C, and represented formally by $f_3(p)$ in Equation 6.

$$f_3(p) = \begin{cases} h_3 & S_{\min} \le p \le P_0 \\ 0 & \text{Otherwise} \end{cases} \qquad (6)$$

Substituting Equation 6 into Equation 1 gives:

$$\begin{aligned} E(P) &= \int_{S_{\min}}^{P_0} p f_3(p)\, \mathrm{d}p \\ &= \frac{1}{2}(P_0 + S_{\min}) \end{aligned} \qquad (7)$$

Hence Equation 7 demonstrates that, in situations where both supply and demand are flat, and there is excess demand, then so long as $S_{\min} \neq P_0$ the expected value $E(P)$ of transaction prices will differ from P_0.

By the same reasoning, in excess-supply 'box' markets such as that shown in Fig. 1D, $P_0 = S_{\min}$; the expected value $E(P)$ is given by Equation 8: $E(P)$ differs from P_0 so long as $D_{\max} \neq P_0$.

$$E(P) = \frac{1}{2}(P_0 + D_{\max}) \qquad (8)$$

These four examples show that, for ZI-C traders, while $E(P) = P_0$ in special circumstances, in general $E(P) \neq P_0$. Similar arguments could be made for ZI-C systems with discrete rather than continuous prices. The following section presents empirical evidence that supports the analytic argument developed here.

3.3 Simulation Studies

To test these analytic predictions, a computer simulation was written to study the behavior of ZI-C traders under different supply and demand schedules. The simulator was written in the C programming language: full details, including the C source-code, are given by Cliff (1997).

Results from four markets are shown here, corresponding to the four types of supply-demand schedules examined analytically in the previous section and illustrated in Fig. 1. In each market, 50 experiments were run, each experiment lasting for ten trading sessions or "days". Each day continued until either eleven transactions had occurred, or no buyers or sellers were able to trade because they were all unable to improve on the current best offer or bid. The parameters for each of the four markets are listed in Table 1. In each market, each trader has one unit to buy or sell, and the theoretical equilibrium values for all four markets are $P_0 = 200$ and $Q_0 = 6$.

Table 2 shows summary results from the four markets: the mean and standard deviation of the ZI-C transaction prices on the first and last trading days, and the correlation coefficient for the mean transaction price over the ten days. For graphs showing the mean and standard deviation of the transaction prices on each day in the four markets, see Cliff (1997). As is clear from the results in table 2, there is no significant change in the mean ZI-C trading price over the course of a ten-day experiment in any of the four markets.

The values shown in Table 2 are in good agreement with the values predicted from the equations

M	N_b	N_s	S_{min}	S_{max}	D_{min}	D_{max}
A	11	11	75	325	75	325
B	11	11	200	200	75	325
C	11	6	50	50	200	200
D	6	11	200	200	320	320

Table 1: Parameters for the four markets. The column labelled M refers to the market type: one of the four supply-demand schedules shown in Fig. 1. N_b and N_s are the number of buyers and sellers. S_{min} and S_{max} are the minimum and maximum prices on the supply curve, and D_{min} and D_{max} are the minimum and maximum prices on the demand curve.

M	$\mu_p(1)$	$\sigma_p(1)$	$\mu_p(10)$	$\sigma_p(10)$	r
A	200.8	11.8	200.6	14.7	+0.248
B	232.5	8.8	234.6	9.9	−0.132
C	137.9	13.8	136.5	14.7	−0.268
D	248.9	13.5	248.3	13.1	+0.208

Table 2: Summary results from the four markets. In each market, $n = 50$ experiments were conducted, each lasting ten 'days'. The column labelled $\mu_p(1)$ shows the average trading price on day 1, and the column labelled $\sigma_p(1)$ shows the standard deviation. The columns labelled $\mu_p(10)$ and $\sigma_p(10)$ are the same values, for the tenth trading day. The column labelled r shows the correlation coefficient for $\mu_p(d)$ for $d \in \{1, \ldots, 10\}$. None of the r values indicate a significant correlation.

M	P_0	$E(P)$	Obs	$\lvert\text{Obs} - E(P)\rvert/\sigma_p(1)$
A	200.0	200.0	200.7	0.06
B	200.0	233.3	233.5	0.03
C	200.0	125.0	137.2	0.88
D	200.0	260.0	248.6	0.84

Table 3: Summary of differences between theoretical equilibrium price P_0, average ZI-C transaction price predicted ($E(P)$) from the analysis, and average ZI-C transaction price observed ("Obs") in the simulation experiments (calculated by taking the mean of the vales $\mu_p(1)$ and $\mu_p(10)$ from Table 2). The right-most column shows the absolute difference between the observed and predicted values, expressed as a proportion of the $\sigma_p(1)$ value from Table 2.

for $E(P)$ in each market, To demonstrate this, Table 3 shows, for each market, the equilibrium price P_0, the value predicted from the relevant $E(P)$ equation, and the value observed in Table 2. The difference between the predicted and observed values is expressed as a proportion of the standard deviation from the first day ($\sigma_p(1)$ in Table 2). As can be seen, the difference is exceptionally low in markets A and B, and within one standard deviation in markets C and D.[1]

Results from these four sets of simulation experiments clearly lend strong empirical support to the analytic arguments of the previous section. In each case, the average transaction prices of ZI-C traders are close to the value predicted from the relevant $E(P)$ equation, and in the simulations shown here the average transaction prices are only close to the theoretical equilibrium price P_0 in situations where P_0 and $E(P)$ are similar in value.

3.4 Discussion

The mathematics of Section 3.2 could be criticized for ignoring the fact that the market supply and demand curves shift after each transaction: in principle, the analysis applied only to the first transaction in each trading day. Nevertheless, there is such a good agreement between the theoretical predictions of the ZI-C traders' failure and the results from the simulations that, in practice, this criticism can be ignored.

A more subtle point is that G&S's main claim concerned the convergence of transaction prices to equilibrium *within* a trading day: whether this happens cannot be determined from the results presented thus far.

To determine whether the ZI-C traders implemented here exhibit the same convergence to equilibrium as G&S's, the method developed by G&S was used, calculating the root mean square deviation of transaction price from the equilibrium price (a value Smith (1962) referred to as σ_0) as a function of transaction sequence number. Because each day's trading with ZI-C agents is independent and identically distributed (IID), the day number is not relevant, so σ_0 can be calculated for the first transaction in each day of an experiment, then the second transaction of each day, and so on.

Full details, including graphs of σ_0 vs. transaction sequence number, are given by Cliff (1997). In the symmetric market A and the flat-supply market B, there is a clear reduction in σ_0 as the day progresses, indicating that the transaction prices are indeed appearing to converge on equilibrium within each day, as observed and explained by G&S.

However, convergence to equilibrium does not occur during trading days in the 'box' markets C and D. On reflection, it is clearly naive to expect ZI-C traders to convergence to equilibrium in such markets, despite the fact that human traders can do so: in markets C and D, all buyers have the same limit price, and all sellers have the same limit price. Therefore each individual ZI-C *transaction* is IID, and so there can be no correlation between transaction sequence number and transaction price in 'box' markets populated by ZI-C traders. Thus, in these markets at least, there is not even a within-day convergence toward the equilibrium price.

3.5 Summary

The ZI-C traders are nothing more than stochastic systems generating random bids and offers. Qualitative consideration of the PDFs underlying G&S's

[1] Equation 5, combined with the parameter values for market B from Table 1 predicts a value of $E(P) \simeq 240$. But only 11 traders each with the right to buy or sell one unit of commodity introduces nonlinearities in the demand curve. Cliff (1997) demonstrates that the true value for the discrete nonlinear curve in market B with the given parameters is $E(P) = 233\frac{1}{3}$.

ZI-C CDA markets led to the analysis demonstrating that, in general, the expected value of ZI-C transaction prices will differ from the equilibrium price. The empirical results presented in Section 3.3 supported these theoretical predictions: in all the simulation studies, the theoretical equilibrium price $P_0 = 200$, yet the mean daily trading price of ZI-C traders was only close to P_0 in market A (when the supply and demand curves were symmetric): in the other cases, the mean ZI-C transaction prices deviated from the P_0 value by amounts predictable from the equations for $E(P)$.

Thus, it has been established here that the mean transaction price observed in ZI-C markets can be predicted from the expected value $E(P)$ of the probability density function (PDF) given by the intersection of the sellers' offer-price PDF and the buyers' bid-price PDF. Only in conditions where $E(P)$ is close to the theoretical equilibrium price P_0, will mean transaction prices *appear* to be close to P_0. In general, $E(P)$ and P_0 will differ, and mean transaction prices will then be at values close to $E(P)$ rather than P_0. In brief, any similarity between ZI-C traders' transaction prices and the theoretical equilibrium price is more likely to be *coincidental* than *causal*.

Moreover, as was discussed in Section 3.4, although G&S's observation of within-day convergence of transaction prices toward the equilibrium value was replicated here in markets A and B, such convergence was not observed (and indeed is theoretically impossible) in the 'box' markets C and D.

From this it is clear that more than zero intelligence is necessary to account for convergence to equilibrium in CDA markets such as B, C, and D.

4 CONCLUSION

G&S's work was an important contribution to the field of experimental economics, providing an absolute lower limit on the mechanistic complexity of CDA trading agents, and demonstrating that allocative efficiency is a poor indicator of the intelligence of agents in CDA markets. However, the critique in Section 3 indicates that some of the tendencies of ZI-C traders towards theoretical equilibrium values are predictable from *a priori* analysis of the probability functions of the system. There is a sense in which the ZI-C simulation experiments (both G&S's, and the ones presented here) are superfluous: the mathematical analysis predicts both the failures and the (apparent) successes of markets populated by ZI-C traders. The failings of the ZI-C traders indicates a need for bargaining mechanisms more complex than constrained stochastic generation of bid and offer prices. Cliff (1997) describes simple adaptive trading strategies that give human-like equilibration in the markets B, C, and D, demonstrating that surprisingly little extra intelligence is required to remedy the problems with ZI-C traders identified here.

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