Arbitrageurs in Segmented Markets

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minimally complex autonomous software bargaining agents, agent-based economics

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ABSTRACT

This paper describes a system of autonomous software agents able to engage in bargaining in an electronic market. The introduction of arbitrageurs is shown to have a stabilising effect on prices in segmented markets. This type of agent could be used in applications such as market-based control, electronic commerce and economic modelling. The paper describes the need for arbitrage in segmented markets and reports the results of simulating experiments with different market structures.

KEYWORDS: minimally complex autonomous software bargaining agents, agent-based economics

1. INTRODUCTION

The size and complexity of computer systems is vastly increasing and as a natural consequence so are the difficulties in controlling these systems. One problem in controlling these systems is the allocation of scarce resources (e.g. processor time, bandwidth). Ideally the allocation is done in a decentralised way. With a central controller the time to gather the information needed to determine an optimal allocation as well as the time to determine this allocation increases with the size and complexity of the system. Furthermore if this central controller fails so will the system itself. In free-market economics it is claimed that groups of self-optimising individuals engaged in trading are able to optimally allocate scarce resources. The theories used in economics can thus be used as an inspiration for solving problems in distributed resource allocation and control (i.e. market-based control).

The system used to perform experiments in this paper consists of autonomous self-interested agents that engage in bargaining in a continuous double-auction market. These agents can incorporate the economic theories to control complex systems in an automatic and decentralised way. Other application areas for the agents are for example e-commerce where they can bargain for goods on behalf of a human user and economic modelling where the agents can be used to try to explain economic phenomena.

This paper describes the development of minimally complex autonomous software agents able to engage in bargaining which can be used for the application areas identified above. First some of the work by the experimental economist Vernon Smith (Smith, 1962) is explained followed by Dave Cliff's (Cliff, 1997) work on minimally complex bargaining software agents. The market structures used in Cliff's work do not reflect the real world situation because all the individuals involved reside in one logical market place and every individual is aware of the actions of all the other individuals. This is not likely to be the case in the possible application areas of the agents where markets generally consist of several possibly overlapping segments. In real markets arbitrage stabilises trade prices to equilibrium over all the market segments. This is a useful property because the optimal allocation of goods occurs at equilibrium. This paper introduces electronic arbitrageurs and studies their role in stabilising the trade prices in segmented markets.

2. BACKGROUND

The system used to perform the experiments described in this paper is based on the work of Vernon Smith on competitive market behaviour. In his experiments a group of human subjects is divided into two subgroups representing sellers and buyers. The sellers are given one unit of the commodity to sell and a lower limit price to sell at. Each seller is to consider the difference between this reservation price and the actual price at which it trades as pure profit. Furthermore sellers are encouraged to trade at the margin i.e. they would rather trade at their limit price than not trade at all. Similarly each buyer is given an upper limit at which to buy one unit of the commodity as well as the money to buy it at that price. The buyers are also told to maximise their profit, which is equal to the difference between their limit price and the price at which they trade and encouraged to trade at the margin. The limit prices of the subjects are kept private.

An experiment in this setup consists of several trading periods in which the subjects are allowed to shout bids and offers. At any time the subjects are free to accept a bid or an offer after which a binding contract between the two subjects is made. At this point neither subject has an urge to buy or sell and they therefore leave the market. The shouts continue until either no contracts are being made or time runs out after which the market is closed. The sellers are once more endowed with a unit of the commodity and the buyers with the money and the urge to buy a unit of the commodity. The market now reopens for a new trading period.

The distribution of reservation prices sets the supply and demand curves and thus the equilibrium price for the experiments (figure 1b).

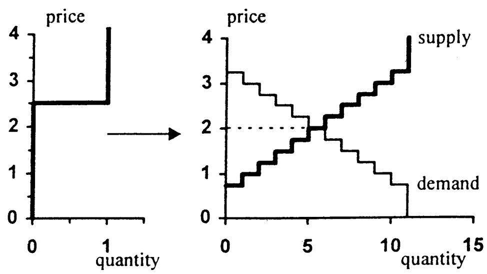

Figure 1a Figure 1b

Figure 1a shows the individual supply curve of a seller with a reservation price of £2.5. Horizontal summation of the individual supply and demand curves is used to construct the total supply and demand curves (figure 1b).

The trade prices typically differ significantly from the theoretical equilibrium price at the start of an experiment but as time progresses trade prices approach equilibrium value. Smith introduced a quantitative measure of this convergence α that is defined as $100*\sigma_0/P_0$, where σ_0 is the standard deviation of trade prices around equilibrium price and P_0 this equilibrium price. At the end of each trading period α is computed. If for instance α equals 5 then trade prices in that period have fluctuated within a 5 percent margin around equilibrium price. In most of Smith's experiments α declines as time progresses.

Recently autonomous software agents have been developed (Cliff, 1997) which reproduce the results of Smith's work with humans. In the electronic markets that form the basis of this work the human subjects are represented by autonomous software agents which are able to either buy or sell. At the start of an experiment each agent sets a random price to buy or sell at with the reservation price it is given as a constraint. The market is opened and agents are allowed to shout to buy or sell. As in Smith's experiments the agents can respond to a shout at any time in which case a binding contract between the two participants is made.

Agents adapt their buying or selling price based on what they hear other agents shout and the contracts they see taking place. The way in which they adapt has been designed to try to capture the essential elements of human behaviour and can best be explained by an example. A seller agent has a sell price of £5 i.e. it tries to sell a unit of the commodity for £5 or more. If this agent hears a deal taking place at more than £5 it will adjust its sell price towards this price because the buying community seems to accept higher prices. The adjusting function is based on what the agents hear in the market i.e. a shout to offer or bid at a certain price and whether a deal takes place or not (Montfort, 1997).

Smith's experiments were replicated using the autonomous agents as subjects. The results of these agents were compared with the results Vernon Smith achieved with the human subjects. The computational agents exhibit the same characteristics and performance i.e. converge to trade at equilibrium price (for example figure 2).

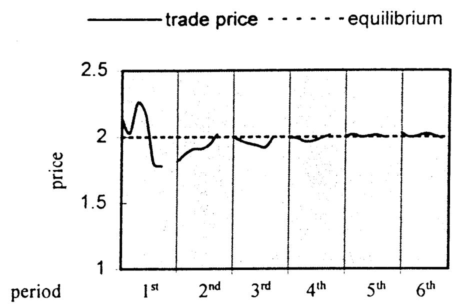

Figure 2

3. SEGMENTED MARKETS

If the autonomous agents were to be used in for example e-commerce applications some more realistic market structures need to be experimented with. In particular markets in the real world tend to be segmented i.e. total supply and demand for a certain good is divided into possibly overlapping parts. This paper examines the performance of agents capable of arbitrage in segmented markets. Arbitrage is defined as the buying and selling of goods to take advantage of varying prices in different markets. Profit is made by buying goods in low price markets and selling them in high price markets. Arbitrage has the effect of stabilising the price for a good in the total market.

The equilibrium price in a market segment is determined by the supply and demand curves in that segment. The equilibrium price of the total market is defined by the combined supply and demand curves of all the market segments. When equilibrium prices between segments differ the shift in supply and demand curves in a particular segment needed to change the segment equilibrium price to total equilibrium price can easily be computed.

If for example the supply curve is linear the needed shift in supply is equal to:

$(P_S-P_0)/\text{slope(supply)}$

where P_0 is the total equilibrium price and P_S is the price at which the sellers are willing to sell exactly the amount the buyers are willing to buy at equilibrium price (figure 3). The same can be said about the needed shift in demand:

$(P_D-P_0)/\text{slope(demand)}$

The needed shift in supply and demand in order to bring a market segment equilibrium price to total equilibrium price can be built out of any combination of the two. The size of these shifts can be seen as the need for arbitrage.

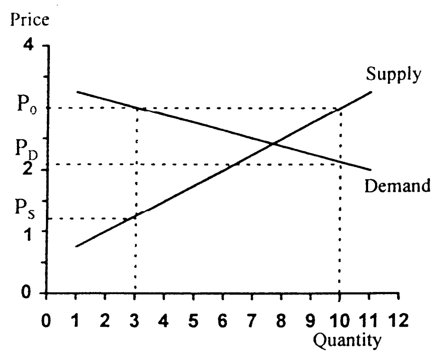

Figure 3

In the following experiments the agents reside in a market segment and can thus only hear the offers and bids tendered and the contracts that result from these shouts in that segment. In the experiments the market consists of two segments with a varying overlap (figure 4).

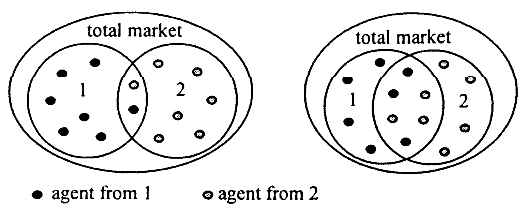

figure 4

The agents in the overlapping part can deal with all the agents in the market. This market structure provides a basis for arbitrage when prices between the two segments differ. When human traders are confronted with a situation such as the one above and are able to see the price difference in the two segments they will start buying on the low price market and selling on the high price market in order to profit from their position. The experiments are to show whether the electronic arbitrageurs will exhibit the same behaviour and if they do how this will influence trade prices in the total market.

4. ARTIFICIAL ARBITRAGEURS

The goal is to find a minimally complex mechanism that provides an agent with the ability to act as an arbitrageur. The mechanism used for the agents in this paper uses the reservation price the agents are given as an upper limit for buying a unit of the commodity as well as a lower limit for selling one unit of the commodity. The agents use their reservation price to set a price to sell and a price to buy the commodity at. In the experiments each agent receives a unit of the commodity to sell as well as money to buy a unit of the commodity at its reservation price. The agents can thus act as sellers as well as buyers and will decide their action based on the profit they can make. The price adjustment mechanism of the agents now adjusts two prices after the agent receives market information (i.e. the agent's sell and buy price). After an agent has participated in a deal it will not leave the market because for example after buying a unit of the commodity the buying agent can decide to sell it again. An agent is not allowed to shout to buy or to respond to a seller's offer if it does not have the money to fulfil the contract. Similarly an agent is not allowed to shout to sell or to respond to a buyer's bid if it does not have a unit of the commodity left to sell.

Human arbitrageurs do not value a good in its own right they only use it as a means to make a profit. The agents used in these experiments all place a certain value on a unit of the commodity (i.e. their reservation price) and do not explicitly buy the commodity on one market and sell it on the other. The overlapping agents are thus not real arbitrageurs. However the expectation is that the result of their actions will be the same. This expectation stems from the initial price difference between the markets as used in the experiments. The overlapping agents offer to sell or bid to buy for a certain price in the total market. The 'higher priced' market is more likely to accept the offer to sell and the 'lower price' market is more likely to accept the bid to buy. Therefore the flow of goods from one market to the other will be established by the overlapping agents and they will emerge as arbitrageurs whenever the environment provides the opportunity i.e. whenever a price difference between the markets exists.

5. INFORMATION DISTRIBUTION

The experiments show that the way market information is distributed amongst the agents influences the learning capabilities of the agents. Smith's α is used as a quantitative measure of learning of the agents. Agents are considered to perform well (i.e. learn well) when α quickly drops to a stable level (around 1 percent) after a few trading periods.

The experiments are performed with four different ways of distributing the information over the agents that are not able to see the whole market.

1. They only receive information from within their segment
2. As 1 and they receive information from deals from all the overlapping agents with other overlapping agents
3. As 1 and they receive information from all deals from the overlapping agents that originally belong to their segment
4. As in 1 and they receive information from all deals from all the overlapping agents

Information here is defined as the price that was shouted, what was shouted (an offer or a bid) and whether a deal has taken place or not. Limiting the information received by the overlapping agents is not considered here.

A series of experiments was performed to find the best way of distributing the information amongst the agents. A single experiment consists of twelve trading sessions where the number of overlapping agents increases from zero to total overlap (both segments have eleven agents). Each trading session in itself consists of 20 trading periods with α recorded at the end of each period. The only difference between the single experiments is the difference in equilibrium prices in the two segments.

The first way of distributing the information performs badly when the number of overlapping agents is large (except for total overlap). This can easily be explained by the fact that the non-overlapping agents receive less and less market information as the overlap increases and thus are less able to conform to the equilibrium price. In (Montfort, 1997) it is empirically proven that the third way of distributing the data gives the best results. This is also a reasonable solution because here the non-overlapping agents are able to learn from all the information produced by the ones in their segment.

6. RESULTS

In order to analyse the performance of the arbitrageurs a new quantitative measure is needed to express the work done by the arbitrageurs to stabilise the market prices. Therefore the above experiments are repeated but now the shift established by the overlapping agents as a percentage of the theoretically needed shift is also recorded. The α is recorded with P_0 equal to the equilibrium price in the total market. If α is small, every agent in the market is dealing close to the total equilibrium price.

The results shown are the mean results of one hundred runs of the same experiment in order to eliminate irregularities due to the randomness in the experiments. The figure below depicts α and the actual shift of goods as a percentage of the theoretically needed shift. The equilibrium prices of the segments are £4 and £2 respectively and thus there is a shift of goods from the second segment to the first. The number of overlapping agents from each segments equals 5 and there are 20 trading periods (figure 5).

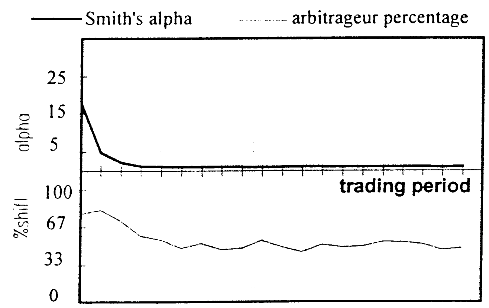

Figure 5

It is clear from figure 5 that in spite of the initial price difference between the two segments the trade price in the total market stabilises to equilibrium (i.e. α approximately 1 percent). This is due to the shift of goods from the low price market to the high price market accomplished by the arbitrageurs. The degree of the shift diminishes as the price difference between the segments decreases as can be seen from figure 5.

The next figure depicts the same experiment this time for the complete spectrum of overlap (there are 11 agents in each segment figure 6).

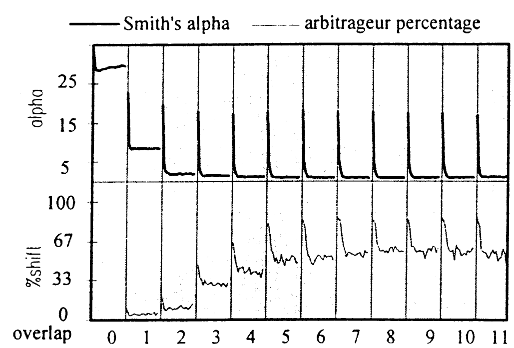

Figure 6

From figure 6 one can see that when then number of agents in the overlap is small the shift that can be accomplished is too small to bring the trade price to equilibrium (i.e. overlap [0...4]). A single arbitrageur can only accomplish a limited shift because each time it wants to shout to buy or sell it has to compete with other agents willing to shout. Agent are not allowed to shout simultaneously. When the arbitrageurs are able to accomplish 75 percent or more of the theoretically needed shift , α drops to a stable level (around 1 percent) and the incentive for the agents to arbitrage diminishes as they can no longer profit from a price difference. This is the explanation for the drop in percentage shift after α becomes stable.

The figure below (figure 7) shows the effect of running the same experiment with both segments having the same equilibrium price (i.e. £2). The percentage shift is not depicted here because when the two equilibrium prices are the same the theoretically needed shift equals zero.

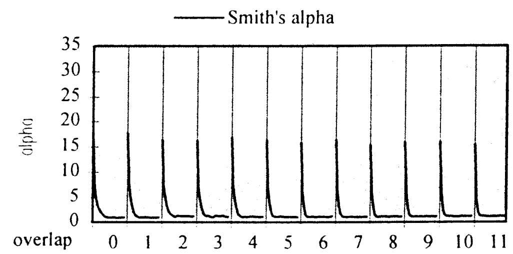

Figure 7

In all cases in the figure above the trade prices stabilise to equilibrium price after the first few trading periods. The arbitrageurs therefore do not badly influence stability. More results are given in (Montfort,1997). For each experiment (i.e. different equilibrium prices) the minimum number of overlapping agents needed to stabilise the trade prices can be given. This minimum is defined as the number of overlapping agents at which the mean as well as the standard deviation of α are small (1 and 0.5 percent). The maximal amount cf shift one overlapping agent can accomplish is indeed limited and thus when equilibrium the price difference between the two segments increases so does the number of overlapping agents needed to stabilise the market.

If the overlapping agents are able to do 75 percent or more of the theoretically needed shift in the first few days α drops down to around 1 percent. Now the price difference between the two segments is too small to encourage arbitrage. However, as agents in the overlap cease to act as arbitrageurs the price difference will begin to increase again, thus also increasing the incentive for arbitrage. The result is a dynamic equilibrium. This is the explanation for the oscillation of the actual shift in figure 8. Figure 8 depicts the prices of deals taking place in the market as well as the actual flow of goods from one segment to the other. The number of overlapping agents from each segment in this case is six. The equilibrium prices of the two segments are once again £4 and £2 thus the total equilibrium price is £3.

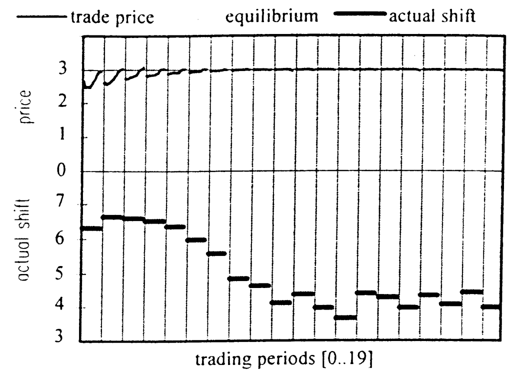

Figure 8

7. CONCLUSIONS AND FURTHER WORK

It has been shown that when the environment provides an opportunity for arbitrage, the autonomous software agents in the overlap do indeed act as arbitrageurs. The number of overlapping agents needed to stabilise the trade prices in the total market is proportional to the initial equilibrium price difference of the market segments. The stabilising effect of arbitrage on trade prices in the total market has also been demonstrated. The results from the simulation correspond with what can be seen happening in the real world.

It is clear that these experiments have been carried out on systems where only one type of good is traded. In order to generalise this work the agents have to be able to trade several types of good at the same time. Now substitution and agent's preferences play a role in the price setting. Also there is a need for more sophisticated market structures (i.e. market segmentation) in order to replicate structures in the real world and study their influence on trade prices.

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