A Kinematically Stable Hybrid Position/Force Control Scheme

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A correction to the position formulation of the hybrid position/force control scheme is presented. In the original position formulation, the derivation that uses the inverse of the Jacobian matrix to map from Cartesian space to joint space is shown to be algebraically incorrect. This incorrect derivation causes the kinematic instability attributed to the hybrid position/force control scheme. The correct derivation shows that the transformation between joint space and Cartesian space involves both the selection matrix and the Jocobian matrix, making the inverse mapping more complex since the two matrix transformations can no longer be treated independently. A sufficient condition for system stability using only the kinematic information is defined. With the corrected position formulation, the hybrid position/force control scheme demonstrates kinematically stable behavior and is applied to an example that was previously shown to be unstable. This correction generalizes the applicability of the hybrid position/force control scheme by providing system robustness for all robot manipulators.

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Contents

1 Introduction and Background

The hybrid position/force control scheme, also referred to as hybrid control, was first proposed over ten years ago by Craig and Raibert [2]. The advantage of hybrid control was that the position and force information are analyzed independently to take advantage of well-known control techniques for each, and are combined only at the final stage when both have been converted to joint torques. Hence, having one control scheme to handle both position and force information in an integrated manner allowed the end effector to be moved more intelligently in nondeterministic environments.

We will use a simplified representation of the original hybrid control scheme shown in Figure 1 to review the basic concepts proposed by Craig and Raibert. The **x**'s in the top half of Figure 1

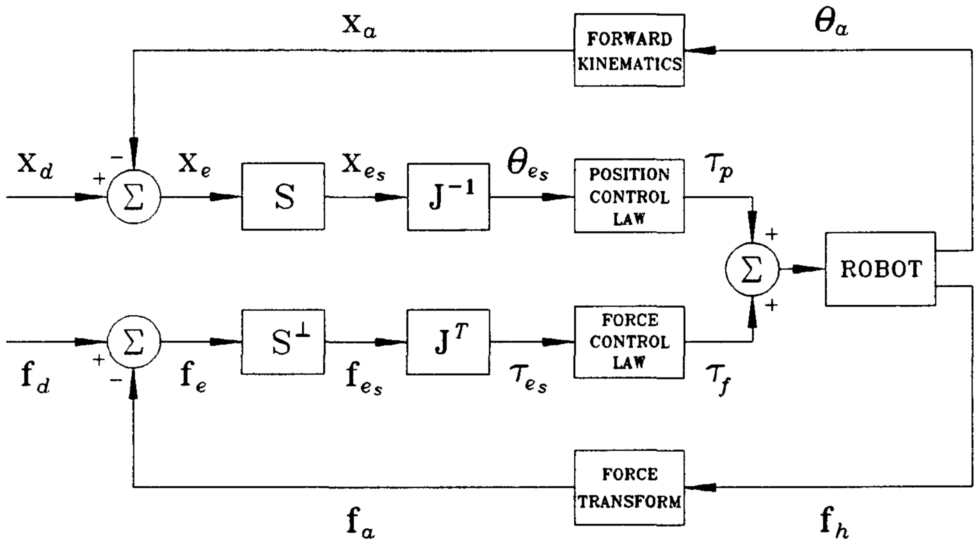

Figure 1: Original Hybrid Control Scheme

are 6×1 vectors representing Cartesian position and orientation,[1] and the **f**'s in the bottom half are 6×1 vectors representing the Cartesian force and moment. In this paper, the term Cartesian position includes both the position and orientation information. Likewise, the term Cartesian force includes both force and moment information. The manipulator joint position and torque values are represented by $n \times 1$ vectors $\boldsymbol{\theta}$ and $\boldsymbol{\tau}$, respectively, where n is the number of manipulator joints. All of the subscripts should be self-explanatory from the context in which they are presented. The boldface upper case letters in the boxes are matrices of the appropriate dimensions to keep the vector sizes consistent and will be explained later. The other boxes are labeled with generic terms (e.g., position control law) due to their variability in any given implementation and have no direct effect on the analysis in this paper.

About five years ago, An and Hollerbach [1] proved that some well-accepted force control methods, including hybrid control, are unstable. Using the simplest assumption of positive definite constant diagonal gain matrices $\mathbf{K}_p$ and $\mathbf{K}_d$ in a PD position control law, they showed by example how the position part of the hybrid control scheme for a two revolute joint manipulator produces an unstable

[1] Since this paper is concerned only with the kinematic behavior of the hybrid control scheme, we have intentionally omitted the Cartesian velocity and acceleration terms, while knowing their inclusion may be necessary for improved system performance.

system. Zhang [10] also showed that the hybrid control scheme may become unstable in certain manipulator configurations using revolute joints. He used a linearized state space approach to show that the term $\mathbf{J}^{-1}\,\mathbf{S}\,\mathbf{J}$ results in an unstable system for certain manipulator configurations. In both papers, only the position part of the hybrid control scheme was examined to reveal a kinematic instability, which was claimed to be an inherent characteristic of the hybrid control formulation. These results were derived from the equations that are represented by the block diagram shown in Figure 1, that were defined in the architectural framework of the hybrid control scheme given by Raibert and Craig [7].

We will show that the kinematic instability problem attributed to hybrid control is not fundamental, as the above published works conclude, but the result of an incorrect formulation and implementation of the hybrid control scheme. It is not clear whether the originators of the hybrid control scheme were aware of the subtleties in their approach or the ramifications it would have on different manipulator kinematics. We intend to show that the hybrid control scheme as still a very powerful, stable, and robust approach to robot manipulator control. The objective now is to understand and explain why a kinematic instability problem arises in the position part of the hybrid control scheme and to show theoretically why it should never happen. The crux of the problem has to do with the mapping of Cartesian position errors to joint errors using the inverse of the manipulator Jacobian matrix. The derivation of this equation will be analyzed in the next section.

2 The Position Related Equations

For any given task in Cartesian space, the position constraints are separated from the force constraints by the selection matrix S shown in Figure 1. S is a 6×6 diagonal matrix with each diagonal element being either a one for position control or zero for no position control for each degree of freedom in the Cartesian reference frame of interest. The relevant or *selected* Cartesian position errors are determined as

$$\mathbf{x}_{e_s} = \mathbf{S}\,\mathbf{x}_e \tag{1}$$

where the Cartesian error vector $\mathbf{x}_e$ is the difference between the *desired* and *actual* Cartesian locations of the manipulator. The next step is to map the Cartesian error $\mathbf{x}_{e_s}$ to a corresponding joint error θ_{e_s} for controlling the manipulator.

The manipulator Jacobian matrix $\mathbf{J}$ is the first order approximation for transforming differential motions in joint space to differential motions in Cartesian space [6]. The following linearized relationship

$$\mathbf{x}_e = \mathbf{J}\,\theta_e \tag{2}$$

is used to map small joint errors θ_e to their corresponding Cartesian errors $\mathbf{x}_e$. A unique inverse mapping exists in Equation 2 when $\mathbf{J}$ is a square matrix of maximal rank. Under this condition, the joint errors are calculated from the Cartesian errors as

$$\theta_e = \mathbf{J}^{-1}\,\mathbf{x}_e \tag{3}$$

By the Craig and Raibert approach for the hybrid control scheme, the selected joint errors are simply

$$\theta_{e_s} = \mathbf{J}^{-1}\,\mathbf{x}_{e_s} \tag{4}$$

when using the selected Cartesian errors determined in Equation 1. We will refer to Equation 4 as the *original position solution* for θ_{e_s}.

Clearly, when $\mathbf{J}$ is singular or even near a singularity, a more general and numerically stable method is required to determine θ_{e_s} in Equation 4. While this needs to be remembered, it is not the problem we plan to address in this paper. The kinematic instability problem was shown to occur even when the manipulator Jacobian matrix is well conditioned [1] [10], meaning it is far away from a singular region in the numerical sense. Equation 4 is, however, an incorrect solution that causes the hybrid control scheme to be kinematically unstable. We will show that Equation 4 is not the only solution for θ_{e_s}, and that there is a general position solution from which a kinematically stable formulation for the hybrid control scheme may be found.

3 A Correction to the Position Formulation

The kinematic instability of hybrid control has been shown to exist only in the position part of the formulation. In this section, we will show how Equation 4 in Section 2 is actually an incorrect derivation and that there is a general position solution for θ_{e_s}. The derivation of θ_{e_s} in Equation 4 happens to be only one solution out of an infinite number of possible solutions. A fundamental assumption of maximal rank is made implicitly when the inverse of the manipulator Jacobian matrix is used in Equation 3 to map the Cartesian errors to their corresponding joint errors. The only Cartesian error vector of interest in the hybrid control scheme is $\mathbf{x}_{e_s}$ in Equation 1, which is incorrectly used in Equation 3. The problem with the original position control formulation is that Equations 1 and 2 were treated as two independent mappings. When Equation 1 is combined with Equation 2 the relationship between Cartesian space and joint space becomes

$$\mathbf{S}\,\mathbf{x}_e = (\mathbf{S}\,\mathbf{J})\,\theta_e \tag{5}$$

This crucial step was omitted in the original position solution of the hybrid control scheme. The significance of Equation 5 is that $\mathbf{S}$ reduces the Cartesian space on the left side of the expression, while $(\mathbf{S}\,\mathbf{J})$ maps a redundant number of manipulator joints onto this Cartesian subspace on the right. In essence, there are now more joints than necessary to satisfy the Cartesian position constraints of the end effector. By substituting Equation 1 into Equation 5, the correct relationship between the selected Cartesian errors and the joint errors is

$$\mathbf{x}_{e_s} = (\mathbf{S}\,\mathbf{J})\,\theta_e \tag{6}$$

It should be noted that $(\mathbf{S}\,\mathbf{J})$ is a singular matrix and does not have a unique inverse. The general position solution for the selected joint errors in Equation 6 is[2]

$$\theta_{e_s} = (\mathbf{S}\,\mathbf{J})^{+}\,\mathbf{x}_{e_s} + [\mathbf{I} - (\mathbf{S}\,\mathbf{J})^{+}(\mathbf{S}\,\mathbf{J})]\,\mathbf{z} \tag{7}$$

where $(\mathbf{S}\,\mathbf{J})^{+}$ is the pseudoinverse [8] of $(\mathbf{S}\,\mathbf{J})$. The vector $\mathbf{z}$ in Equation 7 is an arbitrary $n \times 1$ vector in the manipulator joint space. It should be obvious that the original position solution for θ_{e_s} computed in Equation 4 will not always produce the same results as those computed in Equation 7. The original position solution for θ_{e_s} in Equation 4 is only one solution out of the infinite set of possible solutions that could be generated using Equation 7. We will refer to Equation 7 as the *general position solution* for θ_{e_s}.

[2]The reasons for the change in notation of the joint error θ_e to θ_{e_s} are both to signify that these are the *selected* joint errors computed from the selected Cartesian error $\mathbf{X}_{e_s}$ and to be consistent with the notation used in the original position solution formulation.

4 A Comparison to the Original Position Solution

To fully appreciate the relationship between the original position solution for θ_{e_s} given in Equation 4 and the general position solution for θ_{e_s} in Equation 7, we use the properties of projection matrices for linear systems [3]. All selected joint errors may be projected into the sum of two orthogonal vectors using the $(\mathbf{S}\,\mathbf{J})$ transformation matrix given in Equation 6 as

$$\theta_{e_s} = (\mathbf{S}\,\mathbf{J})^{+}\,(\mathbf{S}\,\mathbf{J})\,\theta_{e_s} + [\,\mathbf{I} - (\mathbf{S}\,\mathbf{J})^{+}(\mathbf{S}\,\mathbf{J})\,]\,\theta_{e_s} \tag{8}$$

where $(\mathbf{S}\,\mathbf{J})^{+}(\mathbf{S}\,\mathbf{J})$ and $[\,\mathbf{I} - (\mathbf{S}\,\mathbf{J})^{+}(\mathbf{S}\,\mathbf{J})\,]$ are the joint space projection matrices for the system. Substitute the original position solution for θ_{e_s} from Equation 4 into Equation 8 to get

$$\mathbf{J}^{-1}\,\mathbf{x}_{e_s} = (\mathbf{S}\,\mathbf{J})^{+}\,(\mathbf{S}\,\mathbf{J})\,\mathbf{J}^{-1}\,\mathbf{x}_{e_s} + [\,\mathbf{I} - (\mathbf{S}\,\mathbf{J})^{+}(\mathbf{S}\,\mathbf{J})\,]\,\mathbf{J}^{-1}\,\mathbf{x}_{e_s} \tag{9}$$

The projected vector $(\mathbf{S}\,\mathbf{J})^{+}\,(\mathbf{S}\,\mathbf{J})\,\mathbf{J}^{-1}\,\mathbf{x}_{e_s}$ in Equation 9 may be simplified by realizing $\mathbf{J}\,\mathbf{J}^{-1} = \mathbf{I}$ and $\mathbf{S}\,\mathbf{x}_{e_s} = \mathbf{x}_{e_s}$ to obtain

$$\mathbf{J}^{-1}\,\mathbf{x}_{e_s} = (\mathbf{S}\,\mathbf{J})^{+}\,\mathbf{x}_{e_s} + [\,\mathbf{I} - (\mathbf{S}\,\mathbf{J})^{+}(\mathbf{S}\,\mathbf{J})\,]\,\mathbf{J}^{-1}\,\mathbf{x}_{e_s} \tag{10}$$

It is important to note that the first projection term in Equation 10 is the minimum norm solution part of the general form for θ_{e_s} in Equation 7. Equation 10 explicitly shows that the traditional approach of using the inverse of the manipulator Jacobian matrix to solve for θ_{e_s} in Equation 4 will inadvertently add an orthogonal vector $[\,\mathbf{I} - (\mathbf{S}\,\mathbf{J})^{+}(\mathbf{S}\,\mathbf{J})\,]\,\mathbf{J}^{-1}\,\mathbf{x}_{e_s}$ to the minimum norm solution. For the general position solution in Equation 7 to behave the same as the original position solution in Equation 4, compare the orthogonal projection terms in Equations 10 and 7 to immediately see that one obvious choice for the arbitrary vector $\mathbf{z}$ would be

$$\mathbf{z} = \mathbf{J}^{-1}\,\mathbf{x}_{e_s} \tag{11}$$

Our claim is the projection of this unbeknown choice for $\mathbf{z}$ in Equation 11 onto the null space of $(\mathbf{S}\,\mathbf{J})$ causes the kinematic instability in the original hybrid control scheme. We will justify this assertion in the next section by giving a sufficient condition for system stability using just the kinematic information.

5 A Stable Hybrid Control Scheme

It may be argued that the orthogonal vector $[\,\mathbf{I} - (\mathbf{S}\,\mathbf{J})^{+}(\mathbf{S}\,\mathbf{J})\,]\,\mathbf{z}$ in Equation 7 adds flexibility to the solution for θ_{e_s} and may be used to optimize θ_{e_s} based on some desired criterion (e.g., to minimize joint energy or to keep the joints in the middle of their operating range). It is important to keep in mind that the general position solution given in Equation 7 was derived without taking into account the rest of the hybrid control scheme. The orthogonal vector contribution is from the joint space that is not available to the minimum norm solution, which is the space potentially used by the force control part of the formulation. To avoid any conflicts with the force part of the hybrid control scheme, we will not use the orthogonal vector defined in the general position solution for θ_{e_s} by assigning the arbitrary vector to be $\mathbf{z} = \mathbf{0}$ and use only the minimum norm solution as follows

$$\theta_{e_s} = (\mathbf{S}\,\mathbf{J})^{+}\,\mathbf{x}_{e_s} \tag{12}$$

We will show that the minimum norm solution for θ_{e_s} guarantees that the linear transformation from Cartesian space to the manipulator joint space will never generate a vector that would be considered

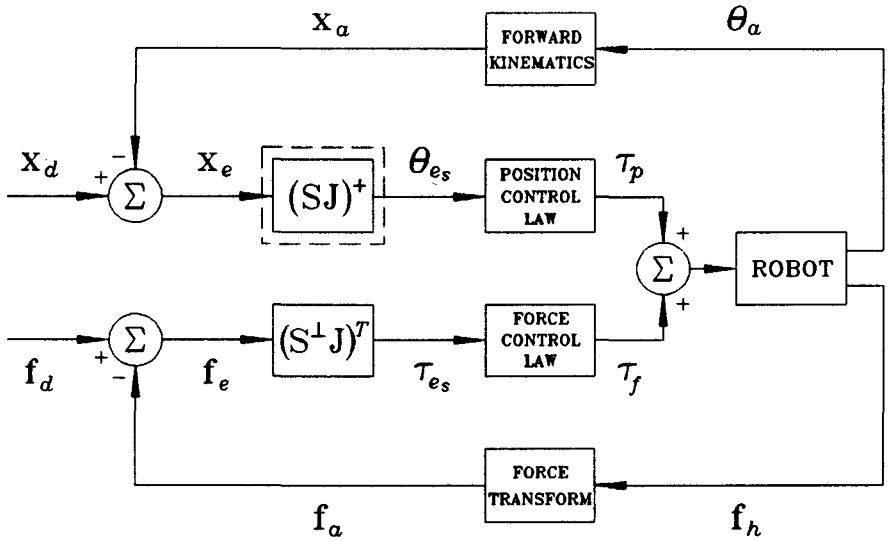

Figure 2: A Stable Hybrid Control Scheme

in the opposite direction to the joint error vector $\boldsymbol{\theta}_e$ when $\mathbf{S} = \mathbf{I}$ and never cause an increase in the joint error vector norm, resulting in always maintaining a kinematically stable system. Figure 2 shows a kinematically stable version for the hybrid control scheme using just the minimum norm solution for $\boldsymbol{\theta}_{e_s}$ given in Equation 12. The dotted outline shown in Figure 2 is the only difference from the original block diagram shown in Figure 1. Although the difference may seem small, it has a tremendous impact on the stability and robustness of hybrid position/force control. Some explanation is needed to fully appreciate and agree with the results of the block diagram shown in Figure 2. One observation is the transformation from $\mathbf{x}_e$ to $\mathbf{x}_{e_s}$ using the selection matrix in Equation 1 is missing. It can be shown that both $(\mathbf{S}\,\mathbf{J})^+\,\mathbf{S}$ and $(\mathbf{S}\,\mathbf{J})^+$ satisfy the four Moore-Penrose properties of a pseudoinverse [4] and since the pseudoinverse of a matrix is unique, means that $(\mathbf{S}\,\mathbf{J})^+\,\mathbf{S} = (\mathbf{S}\,\mathbf{J})^+$. The change in the force part is only for maintaining symmetry in the diagram. The two blocks shown in Figure 1 representing the matrix transformations for mapping Cartesian forces to joint torques have been combined into the one block shown in Figure 2 using standard matrix operations [5] [8].

Stability of a system is determined by the interaction among the kinematics, dynamics, and the control law. The notion of a system being kinematically unstable in hybrid control is not very meaningful. The real issue is how the selection matrix influences system stability. Naturally, the system is assumed stable when in pure position control (i.e., $\mathbf{S} = \mathbf{I}$) so that the $\boldsymbol{\theta}_e$'s corresponding to the $\mathbf{x}_e$'s under these *normal conditions* do not produce any system instabilities. The purpose of the selection matrix is not to stabilize an inherently unstable system or to cause an inherently stable system to become unstable. The issue then becomes one of "comparing" the selected joint position errors $\boldsymbol{\theta}_{e_s}$ for arbitrary $\mathbf{S}$ to the stable joint errors $\boldsymbol{\theta}_e$ that would have been calculated under normal conditions with $\mathbf{S} = \mathbf{I}$. We do this by defining the following sufficient condition for system stability using inner products of the joint error vectors as

Sufficient Condition for System Stability:

$$0 \leq {\theta_e}^T \theta_{e_s} \leq {\theta_e}^T \theta_e\,, \quad \forall\, \theta_e \tag{13}$$

It is important to note that both θ_e and θ_{e_s} are computed using the same Cartesian position error $\mathbf{x}_e$, making any solution for θ_{e_s} always related to θ_e. The lower bound in Equation 13 does not allow the projection of θ_{e_s} onto θ_e be in the opposite direction of θ_e, thereby eliminating any potentially unstable conditions of introducing positive feedback into the system. The upper bound in Equation 13 restricts the projection of θ_{e_s} onto θ_e to be no larger than θ_e, so that the end effector will not overshoot its destinations and cause uncontrollable system oscillations. The system may still be stable when the value of ${\theta_e}^T \theta_{e_s}$ is outside the bounds given in Equation 13; determining how far outside the bounds before the system becomes unstable is not straightforward. We will show in the next section a situation where the lower bound in Equation 13 may become negative and still maintain a stable system, reinforcing the fact that Equation 13 is only a sufficient and not a necessary condition for stability.

To determine the relationship between θ_e and θ_{e_s} for the position control formulation shown in Figure 2, we combine Equation 12 with Equation 6 to get

$$\theta_{e_s} = (\mathbf{S}\,\mathbf{J})^{+}\,(\mathbf{S}\,\mathbf{J})\,\theta_e \tag{14}$$

Since $(\mathbf{S}\,\mathbf{J})^{+}(\mathbf{S}\,\mathbf{J})$ is a projection matrix [3], it satisfies the definition of a positive semidefinite matrix [8], meaning the following quadratic expression

$${\theta_e}^T\,(\mathbf{S}\,\mathbf{J})^{+}(\mathbf{S}\,\mathbf{J})\,\theta_e \geq 0 \tag{15}$$

is true for all vectors θ_e. When θ_{e_s} in Equation 14 is substituted into Equation 13, the inner product is exactly Equation 15 and hence, the minimum norm solution for θ_{e_s} will always satisfy the lower bound in Equation 13. Another property of a projection matrix is that the norm of a projected vector is bounded by the norm of the original vector [3], meaning the norm of θ_{e_s} given in Equation 14 is bounded by

$$\|\,(\mathbf{S}\,\mathbf{J})^{+}(\mathbf{S}\,\mathbf{J})\,\theta_e\,\| \leq \|\,\theta_e\,\| \tag{16}$$

and so, the upper bound in Equation 13 is also satisfied for all θ_e when using the minimum norm solution for θ_{e_s}. This proves that the minimum norm solution for θ_{e_s} in Equation 12 will always satisfy the sufficient condition for system stability as defined in Equation 13.

6 A Test Case: The An and Hollerbach Example

In the original hybrid control formulation, the relationship between θ_e and θ_{e_s} is easily determined by combining Equations 4, 1, and 2 to get

$$\theta_{e_s} = (\mathbf{J}^{-1}\,\mathbf{S}\,\mathbf{J})\,\theta_e \tag{17}$$

An and Hollerbach [1] showed by example that the position part of the system could become unstable and that the cause was somehow related to the interaction between the kinematic $\mathbf{J}^{-1}\,\mathbf{S}\,\mathbf{J}$ transformation matrix and the system inertia matrix. They used a linear state space model of the position part of the system to test for the instability by doing a root locus plot of varying manipulator configurations. We argue that it is the $\mathbf{J}^{-1}\,\mathbf{S}\,\mathbf{J}$ term that causes the system poles to migrate into the unstable right half plane for various end effector motions.

By using the same example, we will show that the $\mathbf{J}^{-1}\,\mathbf{S}\,\mathbf{J}$ term is clearly responsible for causing an unstable system when applying the sufficient condition test in Equation 13. In Case 2 of their paper, $\mathbf{S} = \mathrm{diag}[0,1]$ and the Jacobian matrix for the two revolute joint manipulator was given as

$$\mathbf{J} = \begin{bmatrix} -l_1\,s_1 - l_2\,s_{12} & -l_2\,s_{12} \\ l_1\,c_1 + l_2\,c_{12} & l_2\,c_{12} \end{bmatrix} \tag{18}$$

where $s_i = \sin(\theta_i)$, $c_i = \cos(\theta_i)$, $s_{12} = \sin(\theta_1 + \theta_2)$, and $c_{12} = \cos(\theta_1 + \theta_2)$. The link lengths were $l_1 = 0.462\,m$ and $l_2 = 0.4445\,m$. The inner product of θ_e with θ_{e_s} using Equation 17 is $\theta_e{}^T\,(\mathbf{J}^{-1}\,\mathbf{S}\,\mathbf{J})\,\theta_e$. To test this inner product against the bounds set forth by the sufficient condition in Equation 13, we will consider the situation where $\theta_1 = 0^o$ and θ_2 varies from -180^o to 180^o with $\theta_e = [\,0, \pm 1\,]^T$. A plot of the results is shown in Figure 3. For comparison, a plot of the inner

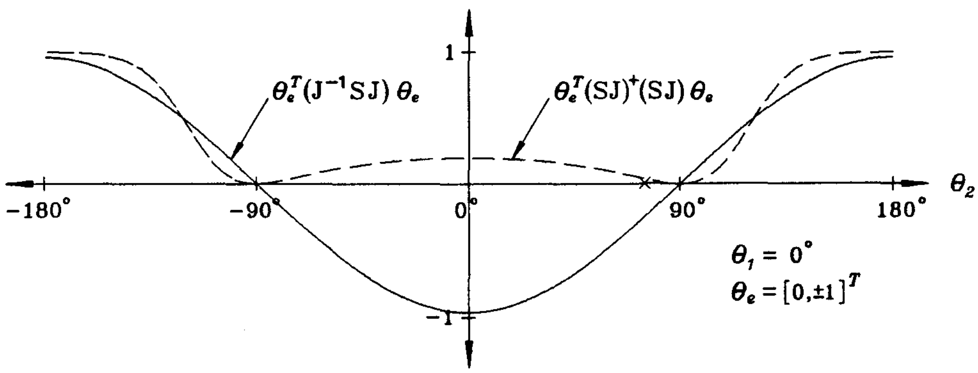

Figure 3: The Sufficient Stability Test for both Position Formulations

product using the minimum norm solution for θ_{e_s} is also shown in Figure 3. It should be clear from Figure 3 that the original position solution for θ_{e_s} violates the sufficiency condition for system stability when $\|\theta_2\| \leq 90^o$. The point marked with an $\times$ in Figure 3 was identified by An and Hollerbach as the value of θ_2 where the system transitions from the stable region to the unstable region. Even though the inner product is negative for the values of θ_2 from $\times$ to 90^o, the system is still stable. The sufficient condition in Equation 13 does not imply the system will be unstable for values outside the stated bounds; it only indicates that an unstable situation could occur.

We have included for reference the equations from Section 2.1 of the An and Hollerbach paper regarding the stability analysis of hybrid control. It was stated that the closed-loop system described as

$$\delta\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\,\mathbf{K}_p\,\mathbf{J}^{-1}\,\mathbf{S}\,\mathbf{J} & -\mathbf{M}^{-1}\,\mathbf{K}_v\,\mathbf{J}^{-1}\,\mathbf{S}\,\mathbf{J} \end{bmatrix} \delta\mathbf{x} \tag{19}$$

must have negative real parts for the eigenvalues of the matrix to guarantee local stability at the equilibrium points. The inertia matrix was

$$\mathbf{M} = \begin{bmatrix} \mathrm{m}_{11} & \mathrm{m}_{12} \\ \mathrm{m}_{21} & \mathrm{m}_{22} \end{bmatrix} \tag{20}$$

with

$$
\begin{aligned}
m_{11} &= I_1 + I_2 + m_2 l_1 l_2 c_2 + \tfrac{1}{4}(m_1 l_1^2 + m_2 l_2^2) + m_2 l_1^2 \\
m_{12} = m_{21} &= I_2 + \tfrac{1}{4} m_2 l_2^2 + \tfrac{1}{2} m_2 l_1 l_2 c_2 \\
m_{22} &= I_2 + \tfrac{1}{4} m_2 l_2^2
\end{aligned}
\tag{21}
$$

The inertia values used were $I_1 = 8.095\,kg \cdot m^2$ and $I_2 = 0.253\,kg \cdot m^2$. The mass values were $m_1 = 120.1\,kg$ and $m_2 = 2.104\,kg$. To maintain a stable system under normal conditions the gain matrices were chosen as $\mathbf{K}_p = \text{diag}[2500, 400]$ and $\mathbf{K}_v = \text{diag}[300, 30]$. A root locus plot of Equation 19 was shown with $\theta_1 = 0^o$ and θ_2 varying from 90^o to 70^o with the transition of the system poles into the unstable right half plane occurring at approximately $\theta_2 = 79^o$. We used all of the information presented in this section to program in Mathematica^{TM} [9] the above equations and definitions to verify their instability result. Our implementation produced a similar root locus plot as in Figure 4 of their paper. We expanded the limits of θ_2 to vary from -180^o to 180^o for a more complete plot and show the results in Figure 4 with all of the singularity points removed. The root

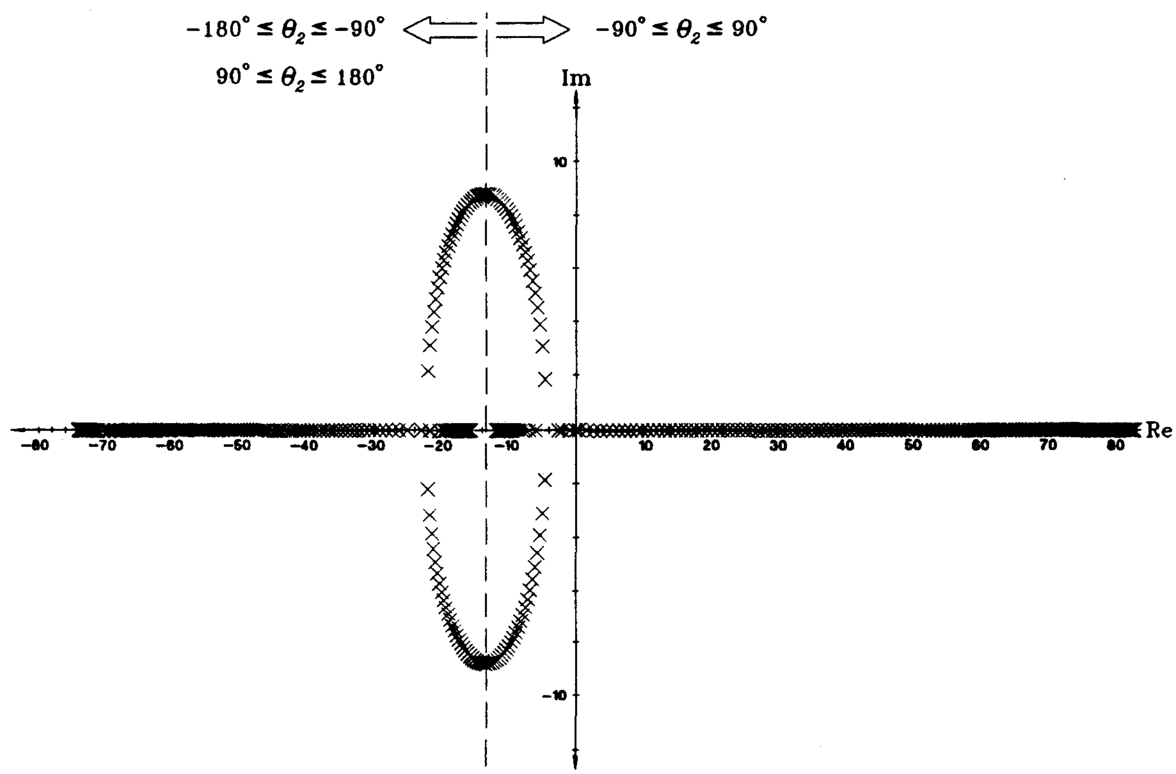

Figure 4: Unstable Root Locus Plot using $\mathbf{J}^{-1}\,\mathbf{S}\,\mathbf{J}$

locus plot shown in Figure 4 confirms our assertion in Figure 3 that the original position formulation for θ_{e_s} can cause the system to become unstable when the sufficient condition in Equation 13 is negative.

For comparative purposes, we repeated Case 2 replacing the $\mathbf{J}^{-1}\,\mathbf{S}\,\mathbf{J}$ expression in Equation 19 with the solution of $(\mathbf{S}\,\mathbf{J})^{+}(\mathbf{S}\,\mathbf{J})$ from Equation 14. The resulting root locus plot is shown in Figure 5

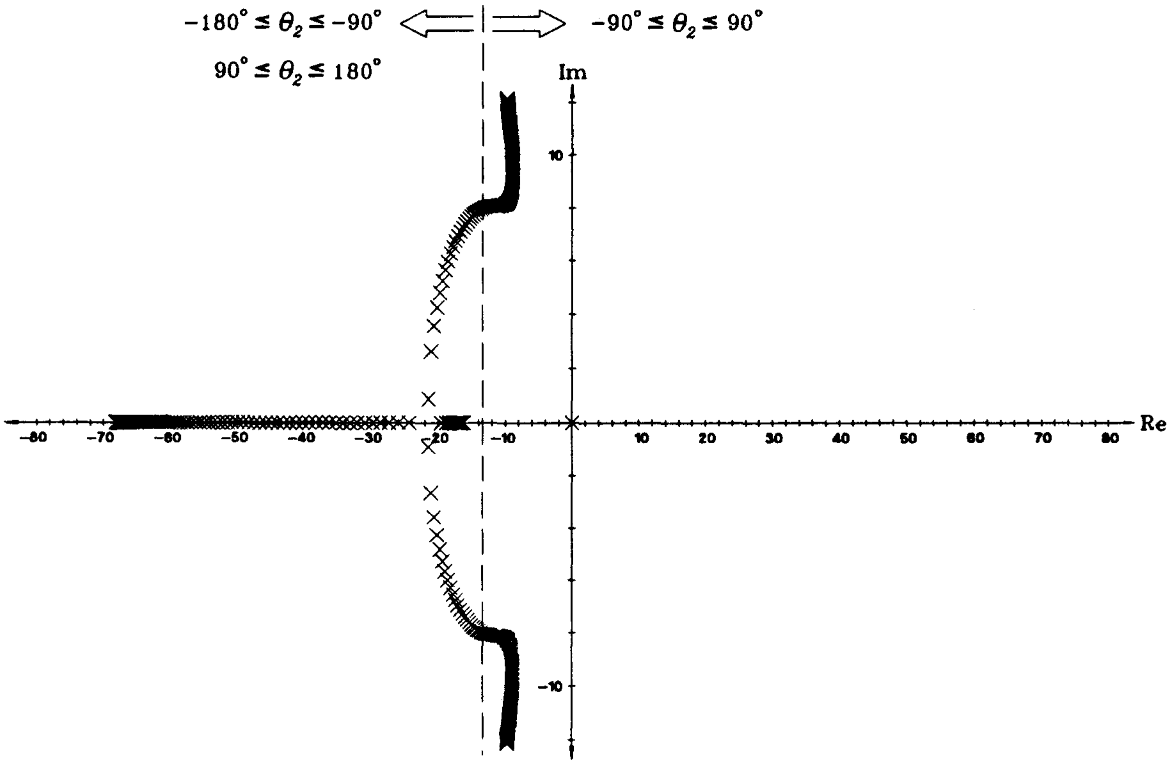

Figure 5: Stable Root Locus Plot using $(\mathbf{S}\,\mathbf{J})^{+}(\mathbf{S}\,\mathbf{J})$

with $\theta_1 = 0^o$ and θ_2 again varying from -180^o to 180^o. Clearly, there are no roots in the unstable right half plane for any value of θ_2 when using the minimum norm solution for θ_{e_s}.

7 Summary and Conclusion

We have presented a correction to the hybrid position/force control scheme for robot manipulators that will enable researchers in the field to exploit fully the power and capability of this control technique. Our efforts and results focused on the basic kinematic equations of the hybrid control scheme to show an error in the original position formulation. The inverse of the manipulator Jacobian matrix was shown to be an incorrect and inappropriate mapping from Cartesian space to joint space. We showed how the original position solution for the joints was only one solution out of an infinite set of possible solutions and named this formulation for the joints the general position solution. From the general position solution, only the minimum norm solution was chosen for the control scheme, since it was fundamental to all possible joint position solutions and avoided any possible conflicts with the force control part of the formulation. We also defined a sufficient condition for system stability that uses only the kinematic information. With the minimum norm position solution, the hybrid position/force control scheme demonstrates kinematically stable behavior and was applied to an example that was previously shown in the literature to be unstable. This correction generalizes the applicability of the hybrid position/force control scheme by providing system stability and robustness for all robot manipulators.

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