## The Opposite of Green is Purple?

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## Keyword(s):

opponent colors, after images, color spaces, chromatic axes


#### Abstract

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The conventional understanding of opponent colors has red and green as one axis and yellow and blue on a second axis. This perceptual opponency is a result of the trichromatic nature of human color vision in combination with subsequent processing in the visual system. This red-green and yellow-blue opponency is fundamental to many different color spaces. CIELAB, CIELUV, CIECAM02, IPT, YCC and more all incorporate this concept of chromatic opponency. In most cases the yellow and blue opponent axes are reasonable. However for the red-green axis it is more like a purplegreen axis due to a consistent, significant bending of the red-green axis. Is dark purple the opposite of green? This paper summarizes the result of analyzing a wide range of color spaces based on their actual opponency. The consistent limitation of a shared matrix formulation for opponency is discussed and finally a simple, invertible color space is considered. The angular differences between quadrants and computed antonyms is shown to be significantly more consistent using this hypothetical alternative color space.


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## 1. INTRODUCTION

Color science and color vision reference books often describe color as being three dimensional. For instance in Color Appearance Models by Mark D. Fairchild, ${ }^{1}$ Figure 3-9 has a minimalist representation of the CIELAB color space with the color axes labeled "red", "yellow", "green" and "blue". The opponent nature of color vision is based on phenomena, such as simultaneous contrast and afterimages, which exhibit red-green and yellow-blue opponency. In Color Vision by Leo M. Hurvich ${ }^{2}$ it is noted that "the seen afterimage usually lies about 180 degrees from the hue of the initially inspected stimulus on the opposite side of the hue circle." In Color: Essence and Logic Rolf G. Kuehni ${ }^{3}$ shows an ideal arrangement of a color space in Figure 5-5 with red and green 180 degrees apart and yellow and blue 180 degrees apart although he notes "while we have described a valid blueprint for a color solid, there is actually no system in existence as described." As a quick experiment to consider what the afterimage of purple is refer to figure one below. Stare at the central purple square for about a minute and then look to the side to the white around the colored squares. The red, green, yellow and blue squares to the author have color names corresponding roughly to greenish, reddish, bluish and yellowish. The purple square to the author has a color name that would be roughly a chartreuse or yellow-green. This figure is consistent with the previous Hurvich quote although his quote doesn't necessarily specify the location of purple in the hue circle. Based on the perceptual opponency implicit in the after-image demonstration shown in Figure 1, the opposite of purple might be a hue circle with yellow-green 180 degrees apart.


Figure 1. Test stimulus for after-image demo. Stare at the central purple square for about a minute and then look to the side. What color is the afterimage of purple?

[^0]In comparison, early color theorists and artists often ordered color in range of ways as can be seen in Color Ordered by Kuehni and Schwarz. ${ }^{4}$ In 1611 Fosini had system in which the opposite of green was yellow and the opposite of purple was sky blue. In 1810 Runge has a system in which the opposite of violet was green. At about the same time Goethe proposed his hue circle in Farbenlehre in which the opposite of green is red and the opposite of violet is yellow. Psychological color order systems, starting with Hering in 1905 have the opposite of green being red and the opposite of purple being a yellow-green. This ordering is roughly consistent for the systems of Johansson, Hesselgren, and the Natural Color System. A notable exception in the $20^{\text {th }}$ century is the five primary system of Munsell, in which the opposite of green is red and the opposite of purple is yellow.
Following the psychological ordering systems, early applied color spaces, such as those for television encoding made use of various $\mathrm{YC}_{\mathrm{r}} \mathrm{C}_{\mathrm{b}}$ type colors spaces where the Y is luminance and there are two chrominance or opponent channels. One of the opponent color channels corresponds to a red-green axis while the other corresponds to a yellow-blue axis. Typically this was computed given a specific set of RGB primaries using an equation such as:

$$
\left[\begin{array}{c}
\text { Luminance }  \tag{1}\\
\text { Red - Green } \\
\text { Yellow - Blue }
\end{array}\right]=\left[\begin{array}{ccc}
c_{1} & c_{2} & c_{3} \\
c_{4} & -c_{5} & c_{6} \\
c_{7} & c_{8} & -c_{9}
\end{array}\right]\left[\begin{array}{l}
R \\
G \\
B
\end{array}\right]
$$

Where the luminance or Y is computed based a specific positive weighting of red, green and blue, the red-green axis is computed primarily based on a scaled difference between red and green and the yellow-blue axis is calculated based roughly on the sum of the red and green values minus the blue. The specific values for the 3 by 3 matrix depend on the specific set of input RGB primaries and on the design requirements for the YCC space. ${ }^{5}$ Similar differencing schemes are also used in other color spaces, such as IPT, ${ }^{6}$ CIELAB, ${ }^{7}$ CIECAM02 ${ }^{8}$ and many other color spaces. One limitation implicit in the formulation of the yellow-blue axis is that the red-green axis becomes distorted such that reds move away from the 0 degree line towards yellow and greens move away from the 180 degree line and also toward the yellow. This is a result of computing the yellow-blue axis based on the sum of the red and green. Various scaling factors can be used but the results are still generally distorted.
This distortion was evident during the computation of color antonyms during the formulation of the Color Thesaurus. ${ }^{9}$ Initially it was assumed that a reasonable antonym would be the color name that was as close as possible to the inverse of the luminance and the hue. However with extensive testing the opposite of green was consistently dark purple. Certainly it is a fair question whether perceptual inverses correspond to linguistic inverses but it was not even possible to make calculations in which this was the case. Likewise as even the simple example in Figure 1 shows, there may be some debate about whether the afterimage of green is red or a magenta-red. But the fact that all of the color spaces tested had dark purple as the opposite of green was frustrating.

## 2. ANALYSIS

Based on the data collected from an unconstrained online color naming experiment ${ }^{10}$ it is possible to compute monolexical naming centroids for red, green, yellow and blue. The four points were calculated assuming sRGB as a nominal web display although the results for the hue angles of these centroids has been shown to be consistent with centroids obtained under laboratory conditions. The results for six of the color spaces or diagrams tested are shown in Figure 2. The results for CIELAB are in the upper left and the wishbone appearance of the four color names is evident. Note the lack of blue hue constancy is such that green and blue are almost 180 degrees apart. The results for IPT are shown in the upper right and again the red and green are bent although in this case the red and blue are almost 180 degrees apart. The results for CIELUV and CIECAM02 are shown on the left and right of the middle row. Although 26 years separate these two color spaces, the Joshua tree appearance of these results is remarkably similar. Finally the results for a YCC encoding and a Macleod-Boyton type diagram ${ }^{11}$ are shown on the left and right of the bottom row. Additional color spaces and encodings were also tested but the less than 180 degrees separation between red and green was present.


Figure 2. Red, yellow, green and blue mono-lexically named centroids from the unconstrained online color naming experiment plotted in a six color spaces. The results for CIELAB are shown in the upper left and IPT in the upper right. The results for CIELUV are shown in the middle left and CIECAM02 in the middle right. Finally the results for a YCC encoding are shown in the lower left and a Macleod-Boynton diagram in the lower right.

## 3. A HYPOTHETICAL ALTERNATIVE

Given the caveats and considerations at the end of the introduction, it is not necessarily to be expected that any given color space will have ideal red-green and yellow-blue opponency. In fact it is not clear that this is in fact an optimal or useful color space. However, the inability to compute color antonyms that were also perceptual antonyms in any color space was frustrating. Whether or not red is the opposite of green and chartreuse is the opposite of purple, there is no existing color space in which to make these calculations. This is a direct result of the matrix formulation of opponency shown in equation one. A three by three matrix or a linear differencing of color channels is a ubiquitous feature of the color spaces shown in Figure 2 and many other color spaces not shown in Figure 2. This compact and intuitive formulation is widespread and reasonably useful approximation. Are there alternatives?
After some trial and error, the following highly simplified, which is to say not fully optimized, formulation of opponency was developed:

$$
\begin{align*}
& C_{i}=R-G, \text { and }  \tag{2}\\
& C_{i i}=\min (R, G)-B . \tag{3}
\end{align*}
$$

Where RGB are the red, green and blue channels for the source color encoding and the min() operator is the minimum of the two input values. In this way equation two is still consistent with equation one but now the computation of the yellow-blue channel is the minimum of red and green minus blue. A luminance or Y channel can be computed as follows:

$$
\begin{equation*}
Y=(((R * 77)+(G * 150) *(B * 29)) / 255) \tag{4}
\end{equation*}
$$

In this way the only true difference for this hypothetical alternative is the calculation of $\mathrm{C}_{\mathrm{ii}}$ in equation 3 . This color space will be referred to as the $\mathrm{YC}_{\mathrm{i}} \mathrm{C}_{\mathrm{i}}$ space for the remainder of this paper. The use of a minimum operator is effectively a logical or comparison. To compute the $\mathrm{YC}_{\mathrm{i}} \mathrm{C}_{\mathrm{ii}}$ inverse the steps are:
If $(\mathrm{Ci}<0)$

$$
\begin{gather*}
R=Y+((150 * C i) / 255)+((29 * C i i) / 255),  \tag{5}\\
G=R-C i, \text { and }  \tag{6}\\
B=R-C i i, \tag{7}
\end{gather*}
$$

else

$$
\begin{gather*}
G=Y-((77 * C i) / 255)+((29 * C i i) / 255),  \tag{8}\\
R=G+C i, \text { and }  \tag{9}\\
B=G-C i i . \tag{10}
\end{gather*}
$$

The minimalist formulae shown in equations two through ten could be further revised and improved by including more specific weighting schemes for a given set of RGB primaries or fitting criterion for the primary hues angles. However for the sake of this paper the equations listed above were used throughout as is. They define a non-matrix based model for opponency that is simple and invertible and can be used as an initial hypothetical alternative to the color spaces shown in Figure 2.

## 4. RESULTS

The results of plotting the red, yellow, green and blue centroids shown in Figure 2 in the $\mathrm{YC}_{\mathrm{i}} \mathrm{C}_{\mathrm{i}}$ color space are shown in Figure 3. In this plot the $x$-axis is the $C_{i}$ value computed using equation two and the $y$-axis is the $C_{i i}$ value computed using equation three. The luminance goes into the graph and the achromatic origin is at the center. The results for $\mathrm{YC}_{\mathrm{i}} \mathrm{C}_{\mathrm{ii}}$ are quite encouraging. The red-green values are almost exactly 180 degrees apart and the yellow-blue axis is close to the $90-270$ degree values. Compared to the psi-shaped results in Figure 2 this slightly slanted X is a considerable improvement. Red is clearly the opposite of green and blue is the opposite of yellow. Likewise at halfway between red and blue, the opposite of purple is halfway between yellow and green or chartreuse.

YCiCii


Figure 3. Red, yellow, green and blue mono-lexically named centroids from the unconstrained online color naming experiment plotted in $\mathrm{YC}_{\mathrm{i}} \mathrm{C}_{\mathrm{ii}}$.

The angular differences between $\mathrm{YC}_{\mathrm{i}} \mathrm{C}_{\mathrm{ii}}$ and three of the other color spaces shown in Figure 2 are shown plotted in Figure 4. These bar chart shows the angular difference between a given quadrant and the next one for $\mathrm{YC}_{\mathrm{i}} \mathrm{C}_{\mathrm{ii}}$, CIELAB, CIECAM02 and YCC. For an ideal red-green and yellow-blue color space the difference between a given quadrant and the next would be 90 degrees. From this plot it is clear that quadrants one and two are significantly under-predicated by YCC, CIELAB and CIECAM02. $\mathrm{YC}_{\mathrm{i}} \mathrm{C}_{\mathrm{ii}}$ in contrast is closer to the 90 degree line. For quadrant three CIELAB and CIECAM02 over-predict the hue angle difference while YCC under-predicts the hue angle difference. Once again the results for $\mathrm{YC}_{\mathrm{i}} \mathrm{C}_{\mathrm{ii}}$ are the closest to the 90 degree difference. Finally, CIELAB and CIECAM02 over-predict quadrant four while YCC and $\mathrm{YC}_{\mathrm{i}} \mathrm{C}_{\mathrm{ii}}$ are reasonably close to the 90 degree difference line. Overall the $\mathrm{YC}_{\mathrm{i}} \mathrm{C}_{\mathrm{ii}}$ space is the space that is most consistently close to a 90 degree difference between quadrants. Although not shown, this is also the case with other color spaces, such as IPT and CIELUV.

With respect to the calculation of color antonyms, the YCiCii color space can be used to compute antonyms that are more consistent with perceptual opponency. This is more of a qualitative test of the hypothetical model. However the results shown in table one are encouraging with respect to this criterion. Both CIELAB and IPT have 'dark purple' as the antonym or color name that is 180 degrees in hue and of inverted lightness for green. In comparison the YCiCii color space results in 'barn red' as the antonym for green. Similarly the YCiCii color space results in the color name 'green
yellow' as the antonym for purple. Interestingly the results for non-basic color names, such as "mud" being an antonym to "cloudy blue" and "asparagus" being and antonym of "soft rose" and "aubergine" being the antonym of "citron" are also intriguing.


Figure 4. Angular differences for four of the color spaces for the four quadrants.

Table 1. Color antonyms computed using three different color spaces.

| Color name | CIELAB antonym | IPT antonym | YCiCii antonym |
| :---: | :---: | :---: | :---: |
| red | dark aqua | cloudy blue | bright lime green |
| green | dark purple | dark purple | barn red |
| yellow | deep blue ocean | midnight purple | real blue |
| blue | brownish green | orangish brown | dark yellow |
| orange | battleship blue | battleship blue | greenish blue |
| pink | forest green | dark grass green | frog green |
| purple | dark grass green | garden green | green yellow |
| brown | country blue | celadon | lichen green |

## 5. DISCUSSION AND CONCLUSIONS

The results shown in Figures three and four and table one are encouraging with respect to conventional theories about color opponency. The simple hypothetical model using the minimum of red or green minus blue to compute the yellowblue axis is simple, invertible and out-performs the other color spaces that were tested. The use of a logical operator in the place of matrix results in a red-green axis that is considerably closer to 180 degrees apart. Specifically while there maybe some debate about specifically which shade of red corresponds to the afterimage of green, a description of 'dark purple' is quite far from the red-green opponent model. Stated differently all existing color spaces were in fact closer to the model of Runge with purple as the opposite of green. It was not possible to compute in closed form red as the opposite of green.

However, there are a number of items to note. As was previously noted this is a completely minimalist approach to achieving more orthogonal axes and no additional optimization was performed. Likewise alternative formulations not based on a logical operation were not considered. There is no reason to expect this model to somehow relate to vision directly but from a computational standpoint, in cases where it is desirable to be able to compute values that are closer to being red-green and yellow-blue opponent, such as calculation of color antonyms, this formulation may be helpful. There are other cases, such as the computation of hue quadrature in color appearance models, where it may be a useful alternative to tabular interpolation of fixed values.
Using the minimum of red or green minus blue to compute a yellow-green axis results in 'barn red' as the opposite of green and 'green yellow' as the opposite of purple.

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[^0]:    ${ }^{1}$ Copyright 2009 Society of Photo-Optical Instrumentation Engineers.
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