



## **An OWL Full Interpretation**

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This report is an appendix to report HPL-2008-59. It gives a worked example of the construction used in the proof from that report. For finiteness, a reduced datatype map consisting of only `xsd:boolean` is used. Each of the graphs in the construction is listed explicitly, with some redundancy eliminated. The final Herbrand graph contains about 15,000 triples.

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## Abstract

This report is an appendix to report HPL-2008-59. It gives a worked example of the construction used in the proof from that report. For finiteness, a reduced datatype map consisting of only `xsd:boolean` is used. Each of the graphs in the construction is listed explicitly, with some redundancy eliminated. The final Herbrand graph contains about 15,000 triples.

## 1 Introduction

This is an appendix to [1, 2]. The construction in those reports builds a Herbrand graph in which the property extension of every property is totally explicit. This appendix presents a complete worked example.

The construction is essentially an infinite one. We make it finite in the following ways:

- We have a very small datatype map  $D$ , being `{ xsd:boolean }`
- We pretend  $L_{\text{plain}} = \{ "" \}$
- We use the ter Horst technique of ignoring every `rdf:_i` except the first; these only appear because of their presence in the RDF and RDFS axioms.

The first of these two are non-conformant. Datatype maps are required to include `xsd:string`, `xsd:integer` and `XMLLiteral`.  $L_{\text{plain}}$  is infinite. The differences are of no great consequence here.

Each section presents one of the graphs in the construction of section 9 of the two papers. Each graph is given by listing the new triples in that graph that were not in the previous triples.

### 1.1 Notation

In addition to the abbreviations specified in [2], we have the following: `disjointWith`, `intersectionOf`, `equivalentClass`, `minCardinality`, `maxCardinality`, `cardinality`, `differentFrom`, `AllDifferent`, `distinctMembers`, `FunctionalProperty`, `subclassOf`, `allValuesFrom`, `someValuesFrom`, `ContainerMembershipProperty`, `versionInfo`. and `^^xsd:boolean`.

Some of the triples in the first few sections (5 to 8), are shown with a wavy line under, such as ("0"^^b.type; eg:c). This means that the triple is not included in the graphs  $H_i$ , i.e. sections 9 to 18.

To avoid too much redundancy we also use the following sets of nodes, identified by their first element, with an overline.

| <i>Relating to classes</i>    |   |
|-------------------------------|---|
| $\overline{b_{10}}$           | $b_{10}, b_9,$  |
| $\overline{\text{Nothing}}$   | Nothing, $b_2$ , DataRange, DataProp, DeprClass, DeprProp, Ontology, Restrict, Alt, Bag, Seq, Statement, XMLLiteral, Container,   |
| $\overline{b_3}$              | $b_3, b_6, b_7,$  |
| $\overline{\text{owl:Class}}$ | owl:Class, rdfs:Class,  |
| $\overline{\text{ObjProp}}$   | ObjProp, Prop,  |
| $\overline{\text{Thing}}$     | Thing, Resource,  |
| <i>Relating to properties</i> |   |
| $\overline{b_{11}}$           | $b_{11}, b_9,$  |
| $\overline{\text{priorVers}}$ | priorVers, allVals, backComp, card, hasValue, imports, incompat, maxCard, minCard, onProperty, someVals, versInfo, _1, object, predicate, subject, value, comment, isDefinedBy, label, member, seeAlso, |

These are used in *metatriples* like  $(\overline{\text{owl:Class}}, \text{equivC}, \overline{\text{owl:Class}})$  in the listing. This one stands for four triples in the graph, by taking each member of the set for both subject and object. Some of these may have occurred earlier in the construction.

## 2 The initial graph

This is given in section 5. It has several features intended to illustrate the action of the construction on literals, which while somewhat artificial in this example, are part of the D-semantics [3], and covered by our method.

- We use non-canonical lexical forms. The notion of ‘canonical’ used in  $\mathcal{V}\mathcal{L}_D$  is not, in general, the same as that in XML Schema [4], which provides several different ‘canonical’ forms for the same value depending on which derived datatype is used. So in general, the literal replacement step between  $G_3$  and  $H_0$  is necessary. In our example, it is artificial.
- We use URI and blank nodes which implicitly represent literals; and then use some of these in predicate position.

## 3 The Function $\psi$

The function  $\psi$ , see section 9.2 of [2], is determined by some D-interpretation of  $G_3$ . We take  $\psi$  to be:

$$\psi(x) = \begin{cases} \text{"true"}^{\wedge\wedge\mathbf{b}} & x = \text{"1"}^{\wedge\wedge\mathbf{b}} \\ \text{"false"}^{\wedge\wedge\mathbf{b}} & x = \text{"0"}^{\wedge\wedge\mathbf{b}} \\ \text{"true"}^{\wedge\wedge\mathbf{b}} & x = \text{eg:v} \\ \text{"false"}^{\wedge\wedge\mathbf{b}} & x = b_{15} \\ x & \text{otherwise} \end{cases} \quad (1)$$

Others would be found from different D-interpretations.

## 4 The Interpretation

The interpretation in section 10 of [2], uses the graph presented in this document, along with the empty sting, the boolean datatype, and its values as the domain of discourse. The function  $\chi$  is

then given as:

$$\chi(x) = \begin{cases} \text{the boolean datatype} & x = \text{boolean} \\ "" & x = "" \\ \text{TRUE} & x = \text{"true"}^{\wedge\wedge\text{b}} \\ \text{FALSE} & x = \text{"false"}^{\wedge\wedge\text{b}} \\ x & \text{otherwise} \end{cases} \quad (2)$$

With the last line meaning the occurrences of the appropriate text string in triples (and implicitly in metatriples) in sections 5 to 18, except where deleted by the wavy line. The actual interpretation is built as specified in [2], via the function  $\theta$ , which we modify, following ter Horst [5] to deal with the `rdf:⊥`:

$$\theta(x) = \begin{cases} \chi(\perp) & x \in \{\text{rdf:}\perp : i = 2, 3 \dots\} \\ \chi(x) & x \in \text{nd}(H_9) \\ \chi(\psi(x)) & x \in \text{nd}(G_3) \end{cases} \quad (3)$$

## 5 $G_0$

|  |  |                                     |
|--|--|-------------------------------------|
| <code>("", type, eg:c)</code>                      | <code>(eg:a, eg:p, eg:v)</code>                  | <code>(eg:p, range, boolean)</code> |
| <code>("<u>0</u>"<sup>⋈</sup>b, type, eg:c)</code> | <code>(eg:b, eg:p, <u>b</u><sub>15</sub>)</code> |                                     |
| <code>(eg:a, eg:p, "<u>1</u>"<sup>⋈</sup>b)</code> | <code>(eg:v, eg:v, eg:v)</code>                  |                                     |

## 6 $G_1$

|  |  |
|--|--|
| <code>(boolean, type, Datatype)</code> | <code>("false"<sup>⋈</sup>b, type, boolean)</code> |
| <code>("", type, Literal)</code>       | <code>("true"<sup>⋈</sup>b, type, boolean)</code>  |

## 7 $G_2$

|   |   |  |
|---|---|--|
| <code>(nil, type, List)</code>            | <code>(XMLLiteral, type, Datatype)</code>   | <code>(domain, domain, Prop)</code>            |
| <code>(⊥, type, Prop)</code>              | <code>(⊥, domain, Resource)</code>          | <code>(isDefinedBy, domain, Resource)</code>   |
| <code>(first, type, Prop)</code>          | <code>(first, domain, List)</code>          | <code>(label, domain, Resource)</code>         |
| <code>(object, type, Prop)</code>         | <code>(object, domain, Statement)</code>    | <code>(member, domain, Resource)</code>        |
| <code>(predicate, type, Prop)</code>      | <code>(predicate, domain, Statement)</code> | <code>(range, domain, Prop)</code>             |
| <code>(rest, type, Prop)</code>           | <code>(rest, domain, List)</code>           | <code>(seeAlso, domain, Resource)</code>       |
| <code>(subject, type, Prop)</code>        | <code>(subject, domain, Statement)</code>   | <code>(subClass, domain, rdfs:Class)</code>    |
| <code>(type, type, Prop)</code>           | <code>(type, domain, Resource)</code>       | <code>(subPropOf, domain, Prop)</code>         |
| <code>(value, type, Prop)</code>          | <code>(value, domain, Resource)</code>      | <code>(⊥, range, Resource)</code>              |
| <code>(⊥, type, CMemProp)</code>          | <code>(comment, domain, Resource)</code>    | <code>(first, range, Resource)</code>          |
| <code>(object, range, Resource)</code>    | <code>(isDefinedBy, range, Resource)</code> | <code>(Bag, subClass, Container)</code>        |
| <code>(predicate, range, Resource)</code> | <code>(label, range, Literal)</code>        | <code>(Seq, subClass, Container)</code>        |
| <code>(rest, range, List)</code>          | <code>(member, range, Resource)</code>      | <code>(XMLLiteral, subClass, Literal)</code>   |
| <code>(subject, range, Resource)</code>   | <code>(range, range, rdfs:Class)</code>     | <code>(CMemProp, subClass, Prop)</code>        |
| <code>(type, range, rdfs:Class)</code>    | <code>(seeAlso, range, Resource)</code>     | <code>(Datatype, subClass, rdfs:Class)</code>  |
| <code>(value, range, Resource)</code>     | <code>(subClass, range, rdfs:Class)</code>  | <code>(isDefinedBy, subPropOf, seeAlso)</code> |
| <code>(comment, range, Literal)</code>    | <code>(subPropOf, range, Prop)</code>       |  |
| <code>(domain, range, rdfs:Class)</code>  | <code>(Alt, subClass, Container)</code>     |  |

## 8 $G_3$

|   |  |  |
|---|--|--|
| (eg:p, type, Prop)                        | (subPropOf, type, Prop)                  | (Container, type, rdfs:Class)            |
| (eg:v, type, Prop)                        | (eg:c, type, rdfs:Class)                 | (CMemProp, type, rdfs:Class)             |
| (comment, type, Prop)                     | (Alt, type, rdfs:Class)                  | (Datatype, type, rdfs:Class)             |
| (domain, type, Prop)                      | (Bag, type, rdfs:Class)                  | (Literal, type, rdfs:Class)              |
| (isDefinedBy, type, Prop)                 | (List, type, rdfs:Class)                 | (Resource, type, rdfs:Class)             |
| (label, type, Prop)                       | (Prop, type, rdfs:Class)                 | (boolean, type, rdfs:Class)              |
| (member, type, Prop)                      | (Seq, type, rdfs:Class)                  | ( <u>"1"^^b, type, Literal</u> )         |
| (range, type, Prop)                       | (Statement, type, rdfs:Class)            | ("false"^^b, type, Literal)              |
| (seeAlso, type, Prop)                     | (XMLLiteral, type, rdfs:Class)           | ("true"^^b, type, Literal)               |
| (subClass, type, Prop)                    | (rdfs:Class, type, rdfs:Class)           | ( <u>b<sub>15</sub>, type, Literal</u> ) |
| <br>                                      |  |  |
| (eg:v, type, Literal)                     | (eg:p, type, Resource)                   | (first, type, Resource)                  |
| ("", type, Resource)                      | (eg:v, type, Resource)                   | (nil, type, Resource)                    |
| ( <u>"0"^^b, type, Resource</u> )         | (Alt, type, Resource)                    | (object, type, Resource)                 |
| ( <u>"1"^^b, type, Resource</u> )         | (Bag, type, Resource)                    | (predicate, type, Resource)              |
| ("false"^^b, type, Resource)              | (List, type, Resource)                   | (rest, type, Resource)                   |
| ("true"^^b, type, Resource)               | (Prop, type, Resource)                   | (subject, type, Resource)                |
| ( <u>b<sub>15</sub>, type, Resource</u> ) | (Seq, type, Resource)                    | (type, type, Resource)                   |
| (eg:a, type, Resource)                    | (Statement, type, Resource)              | (value, type, Resource)                  |
| (eg:b, type, Resource)                    | (XMLLiteral, type, Resource)             | (rdfs:Class, type, Resource)             |
| (eg:c, type, Resource)                    | (-1, type, Resource)                     | (Container, type, Resource)              |
| <br>                                      |  |  |
| (CMemProp, type, Resource)                | (seeAlso, type, Resource)                | (List, subClass, List)                   |
| (Datatype, type, Resource)                | (subClass, type, Resource)               | (Prop, subClass, Prop)                   |
| (Literal, type, Resource)                 | (subPropOf, type, Resource)              | (Seq, subClass, Seq)                     |
| (Resource, type, Resource)                | (boolean, type, Resource)                | (Statement, subClass, Statement)         |
| (comment, type, Resource)                 | ( <u>"1"^^b, type, boolean</u> )         | (XMLLiteral, subClass, XMLLiteral)       |
| (domain, type, Resource)                  | ( <u>b<sub>15</sub>, type, boolean</u> ) | (rdfs:Class, subClass, rdfs:Class)       |
| (isDefinedBy, type, Resource)             | (eg:v, type, boolean)                    | (Container, subClass, Container)         |
| (label, type, Resource)                   | (eg:c, subClass, eg:c)                   | (CMemProp, subClass, CMemProp)           |
| (member, type, Resource)                  | (Alt, subClass, Alt)                     | (Datatype, subClass, Datatype)           |
| (range, type, Resource)                   | (Bag, subClass, Bag)                     | (Literal, subClass, Literal)             |
| <br>                                      |  |  |
| (Resource, subClass, Resource)            | (object, subPropOf, object)              | (isDefinedBy, subPropOf, isDefinedBy)    |
| (boolean, subClass, Literal)              | (predicate, subPropOf, predicate)        | (label, subPropOf, label)                |
| (boolean, subClass, boolean)              | (rest, subPropOf, rest)                  | (member, subPropOf, member)              |
| (eg:p, subPropOf, eg:p)                   | (subject, subPropOf, subject)            | (range, subPropOf, range)                |
| (eg:v, subPropOf, eg:v)                   | (type, subPropOf, type)                  | (seeAlso, subPropOf, seeAlso)            |
| (-1, subPropOf, -1)                       | (value, subPropOf, value)                | (subClass, subPropOf, subClass)          |
| (-1, subPropOf, member)                   | (comment, subPropOf, comment)            | (subPropOf, subPropOf, subPropOf)        |
| (first, subPropOf, first)                 | (domain, subPropOf, domain)              |  |

## 9 $H_0$

|                          |                                   |                                   |
|--------------------------|-----------------------------------|-----------------------------------|
| ("false"^^b, type, eg:c) | ("true"^^b, "true"^^b, "true"^^b) | (eg:b, eg:p, "false"^^b)          |
| ("true"^^b, type, Prop)  | (eg:a, eg:p, "true"^^b)           | ("true"^^b, subPropOf, "true"^^b) |

## 10 $H_1$

|  |                                   |                                       |
|--|-----------------------------------|---------------------------------------|
| ( <u>rdfs:Class</u> , type, $\overline{b_3}$ ) | (priorVers, type, FunProp)        | ( $b_9$ , type, priorVers)            |
| ( $b_2$ , type, $b_8$ )                        | (priorVers, type, InvFunProp)     | (allVals, type, Prop)                 |
| ( $b_2$ , type, $b_9$ )                        | (backComp, type, OntProp)         | (card, type, Prop)                    |
| ( $b_3$ , type, $b_9$ )                        | (imports, type, OntProp)          | (complmntOf, type, Prop)              |
| ( $b_1$ , type, AllDiff)                       | (incompat, type, OntProp)         | (different, type, Prop)               |
| (versInfo, type, AnnProp)                      | (priorVers, type, OntProp)        | (disjoint, type, Prop)                |
| (comment, type, AnnProp)                       | (equivProp, type, SymProp)        | (dstnctMems, type, Prop)              |
| (isDefinedBy, type, AnnProp)                   | (inverseOf, type, SymProp)        | (hasValue, type, Prop)                |
| (label, type, AnnProp)                         | (subClass, type, TransProp)       | (intersect, type, Prop)               |
| (seeAlso, type, AnnProp)                       | (subPropOf, type, TransProp)      | (inverseOf, type, Prop)               |
| (maxCard, type, Prop)                          | (DeprProp, type, rdfs:Class)      | ( $b_2, b_{11}, b_3$ )                |
| (minCard, type, Prop)                          | (Nothing, type, rdfs:Class)       | ( $b_3, b_{12}, b_2$ )                |
| (onProperty, type, Prop)                       | (Ontology, type, rdfs:Class)      | (Nothing, complmntOf, <u>Thing</u> )  |
| (oneOf, type, Prop)                            | (Literal, type, Datatype)         | ( <u>Thing</u> , complmntOf, Nothing) |
| (someVals, type, Prop)                         | ( $b_2, b_8, b_4$ )               | (Thing, different, Nothing)           |
| (unionOf, type, Prop)                          | ( $b_3, b_8, b_2$ )               | (Nothing, disjoint, Nothing)          |
| ( $b_2$ , type, rdfs:Class)                    | ( $b_8, b_8, b_2$ )               | (Nothing, disjoint, Resource)         |
| (owl:Class, type, rdfs:Class)                  | ( $b_8, b_8, b_3$ )               | (Resource, disjoint, Nothing)         |
| (DataRange, type, rdfs:Class)                  | ( $b_2, b_9, b_3$ )               | ( $b_1$ , dstnctMems, $b_2$ )         |
| (DeprClass, type, rdfs:Class)                  | ( $b_4, b_{10}, b_5$ )            | ( $b_1$ , dstnctMems, $b_3$ )         |
| ( <u>Thing</u> , equivC, <u>Thing</u> )        | (equivProp, inverseOf, equivProp) | ( $b_7$ , oneOf, $b_3$ )              |
| ( $b_8$ , equivProp, $b_8$ )                   | (imports, inverseOf, priorVers)   | (Thing, sameAs, Thing)                |
| (incompat, equivProp, backComp)                | (incompat, inverseOf, backComp)   | ( $b_2$ , unionOf, nil)               |
| (priorVers, equivProp, backComp)               | (inverseOf, inverseOf, inverseOf) | (owl:Class, unionOf, $b_2$ )          |
| (priorVers, equivProp, priorVers)              | (priorVers, inverseOf, backComp)  | (rdfs:Class, unionOf, $b_3$ )         |
| ( $b_3$ , intersect, $b_5$ )                   | (priorVers, inverseOf, priorVers) | ( $b_2$ , first, rdfs:Class)          |
| (owl:Class, intersect, $b_2$ )                 | ( $b_2$ , oneOf, nil)             | ( $b_3$ , first, rdfs:Class)          |
| (rdfs:Class, intersect, $b_3$ )                | ( $b_3$ , oneOf, $b_2$ )          | ( $b_4$ , first, Nothing)             |
| ( $b_9$ , inverseOf, $b_{12}$ )                | ( $b_6$ , oneOf, $b_2$ )          | ( $b_5$ , first, $b_3$ )              |
| ( $b_{12}$ , inverseOf, $b_9$ )                | ( $b_6$ , oneOf, $b_3$ )          | ( $b_2$ , rest, nil)                  |
| ( $b_3$ , rest, nil)                           | ( $b_{10}$ , range, $b_8$ )       | (rdfs:Class, subClass, owl:Class)     |
| ( $b_4$ , rest, nil)                           | (AnnProp, subClass, Prop)         | (Resource, subClass, Thing)           |
| ( $b_5$ , rest, nil)                           | (DataProp, subClass, Prop)        | ( $b_8$ , subPropOf, $b_8$ )          |
| ( $b_9$ , domain, $b_{10}$ )                   | (OntProp, subClass, Prop)         | (inverseOf, subPropOf, inverseOf)     |
| ( $b_{10}$ , domain, $b_8$ )                   | (Restrict, subClass, rdfs:Class)  | (priorVers, subPropOf, $b_9$ )        |
| ( $b_{12}$ , domain, $b_{10}$ )                | (Prop, subClass, ObjProp)         | (priorVers, subPropOf, backComp)      |
| ( $b_9$ , range, $b_{10}$ )                    | (Prop, subClass, Thing)           |                                       |

## 11 $H_2$

|   |                               |   |
|---|-------------------------------|---|
| ( $b_4$ , type, $b_8$ )                         | (owl:Class, type, owl:Class)  | (List, type, owl:Class)                       |
| ( $b_5$ , type, $b_8$ )                         | (FunProp, type, owl:Class)    | (CMemProp, type, owl:Class)                   |
| ( $b_2$ , type, $b_{10}$ )                      | (InvFunProp, type, owl:Class) | (Datatype, type, owl:Class)                   |
| ( $b_3$ , type, $b_{10}$ )                      | (Nothing, type, owl:Class)    | (Literal, type, owl:Class)                    |
| ( <u><math>b_3</math>, type, owl:Class</u> )    | (ObjProp, type, owl:Class)    | (boolean, type, owl:Class)                    |
| ( $b_8$ , type, owl:Class)                      | (OntProp, type, owl:Class)    | ("true"^^b, type, ObjProp)                    |
| ( <u><math>b_{10}</math>, type, owl:Class</u> ) | (SymProp, type, owl:Class)    | ( $b_8$ , type, ObjProp)                      |
| (eg: c, type, owl:Class)                        | (Thing, type, owl:Class)      | ( $b_{10}$ , type, ObjProp)                   |
| (AllDiff, type, owl:Class)                      | (TransProp, type, owl:Class)  | ( <u><math>b_{11}</math>, type, ObjProp</u> ) |
| (AnnProp, type, owl:Class)                      | (priorVers, type, owl:Class)  | ( $b_{12}$ , type, ObjProp)                   |

|  |   |  |
|--|---|--|
| (eg:p, type, ObjProp)                                | ( <u>priorVers</u> , type, <u>ObjProp</u> )         | ("", type, Thing)                              |
| (complmntOf, type, ObjProp)                          | (sameAs, type, <u>ObjProp</u> )                     | ("false"^^b, type, Thing)                      |
| (different, type, ObjProp)                           | (unionOf, type, ObjProp)                            | ("true"^^b, type, Thing)                       |
| (disjoint, type, ObjProp)                            | (first, type, ObjProp)                              | (b <sub>1</sub> , type, <u>Thing</u> )         |
| (dstnctMems, type, ObjProp)                          | (rest, type, ObjProp)                               | ( <u>b<sub>3</sub></u> , type, <u>Thing</u> )  |
| (equivC, type, <u>ObjProp</u> )                      | (type, type, ObjProp)                               | (b <sub>4</sub> , type, <u>Thing</u> )         |
| (equivProp, type, <u>ObjProp</u> )                   | (domain, type, ObjProp)                             | (b <sub>5</sub> , type, <u>Thing</u> )         |
| (intersect, type, ObjProp)                           | (range, type, ObjProp)                              | (b <sub>8</sub> , type, <u>Thing</u> )         |
| (inverseOf, type, ObjProp)                           | (subClass, type, ObjProp)                           | (b <sub>10</sub> , type, <u>Thing</u> )        |
| (oneOf, type, ObjProp)                               | (subPropOf, type, ObjProp)                          | ( <u>b<sub>11</sub></u> , type, <u>Thing</u> ) |
| <br>   |   |  |
| (b <sub>12</sub> , type, <u>Thing</u> )              | ( <u>Nothing</u> , type, <u>Thing</u> )             | (equivC, type, <u>Thing</u> )                  |
| (eg:a, type, Thing)                                  | ( <u>ObjProp</u> , type, <u>Thing</u> )             | (equivProp, type, <u>Thing</u> )               |
| (eg:b, type, Thing)                                  | (OntProp, type, <u>Thing</u> )                      | (intersect, type, <u>Thing</u> )               |
| (eg:c, type, Thing)                                  | (SymProp, type, <u>Thing</u> )                      | (inverseOf, type, <u>Thing</u> )               |
| (eg:p, type, Thing)                                  | ( <u>Thing</u> , type, <u>Thing</u> )               | (oneOf, type, <u>Thing</u> )                   |
| (AllDiff, type, <u>Thing</u> )                       | (TransProp, type, <u>Thing</u> )                    | ( <u>priorVers</u> , type, <u>Thing</u> )      |
| (AnnProp, type, <u>Thing</u> )                       | (complmntOf, type, <u>Thing</u> )                   | (sameAs, type, <u>Thing</u> )                  |
| (owl:Class, type, <u>Thing</u> )                     | (different, type, <u>Thing</u> )                    | (unionOf, type, <u>Thing</u> )                 |
| (FunProp, type, <u>Thing</u> )                       | (disjoint, type, <u>Thing</u> )                     | (List, type, Thing)                            |
| (InvFunProp, type, <u>Thing</u> )                    | (dstnctMems, type, <u>Thing</u> )                   | (first, type, Thing)                           |
| <br>   |   |  |
| (nil, type, Thing)                                   | (boolean, type, Thing)                              | (eg:p, domain, Resource)                       |
| (rest, type, Thing)                                  | (b <sub>2</sub> , type, List)                       | (complmntOf, domain, Resource)                 |
| (type, type, Thing)                                  | (b <sub>3</sub> , type, List)                       | (different, domain, Resource)                  |
| (CMemProp, type, Thing)                              | (b <sub>4</sub> , type, List)                       | (disjoint, domain, Resource)                   |
| (Datatype, type, Thing)                              | (b <sub>5</sub> , type, List)                       | (dstnctMems, domain, Resource)                 |
| (Literal, type, Thing)                               | ("true"^^b, domain, Resource)                       | (equivC, domain, Resource)                     |
| (domain, type, Thing)                                | (b <sub>8</sub> , domain, Resource)                 | (equivProp, domain, Resource)                  |
| (range, type, Thing)                                 | (b <sub>10</sub> , domain, Resource)                | (intersect, domain, Resource)                  |
| (subClass, type, Thing)                              | ( <u>b<sub>11</sub></u> , domain, Resource)         | (inverseOf, domain, Resource)                  |
| (subPropOf, type, Thing)                             | (b <sub>12</sub> , domain, Resource)                | (oneOf, domain, Resource)                      |
| <br>   |   |  |
| ( <u>priorVers</u> , domain, <u>b<sub>3</sub></u> )  | ( <u>priorVers</u> , domain, <u>ObjProp</u> )       | ( <u>priorVers</u> , domain, Resource)         |
| ( <u>priorVers</u> , domain, <u>b<sub>8</sub></u> )  | ( <u>priorVers</u> , domain, OntProp)               | ( <u>priorVers</u> , domain, boolean)          |
| ( <u>priorVers</u> , domain, <u>b<sub>10</sub></u> ) | ( <u>priorVers</u> , domain, SymProp)               | (sameAs, domain, Resource)                     |
| ( <u>priorVers</u> , domain, eg:c)                   | ( <u>priorVers</u> , domain, Thing)                 | (unionOf, domain, Resource)                    |
| ( <u>priorVers</u> , domain, AllDiff)                | ( <u>priorVers</u> , domain, TransProp)             | (first, domain, Resource)                      |
| ( <u>priorVers</u> , domain, AnnProp)                | ( <u>priorVers</u> , domain, <u>priorVers</u> )     | (rest, domain, Resource)                       |
| ( <u>priorVers</u> , domain, owl:Class)              | ( <u>priorVers</u> , domain, List)                  | (domain, domain, Resource)                     |
| ( <u>priorVers</u> , domain, FunProp)                | ( <u>priorVers</u> , domain, CMemProp)              | (range, domain, Resource)                      |
| ( <u>priorVers</u> , domain, InvFunProp)             | ( <u>priorVers</u> , domain, Datatype)              | (subClass, domain, Resource)                   |
| ( <u>priorVers</u> , domain, <u>Nothing</u> )        | ( <u>priorVers</u> , domain, Literal)               | (subPropOf, domain, Resource)                  |
| <br>   |   |  |
| ("true"^^b, range, Resource)                         | (equivC, range, Resource)                           | ( <u>priorVers</u> , range, AnnProp)           |
| (b <sub>8</sub> , range, Resource)                   | (equivProp, range, Resource)                        | ( <u>priorVers</u> , range, owl:Class)         |
| (b <sub>10</sub> , range, Resource)                  | (intersect, range, Resource)                        | ( <u>priorVers</u> , range, FunProp)           |
| ( <u>b<sub>11</sub></u> , range, Resource)           | (inverseOf, range, Resource)                        | ( <u>priorVers</u> , range, InvFunProp)        |
| (b <sub>12</sub> , range, Resource)                  | (oneOf, range, Resource)                            | ( <u>priorVers</u> , range, <u>Nothing</u> )   |
| (eg:p, range, Resource)                              | ( <u>priorVers</u> , range, <u>b<sub>3</sub></u> )  | ( <u>priorVers</u> , range, <u>ObjProp</u> )   |
| (complmntOf, range, Resource)                        | ( <u>priorVers</u> , range, b <sub>8</sub> )        | ( <u>priorVers</u> , range, OntProp)           |
| (different, range, Resource)                         | ( <u>priorVers</u> , range, <u>b<sub>10</sub></u> ) | ( <u>priorVers</u> , range, SymProp)           |
| (disjoint, range, Resource)                          | ( <u>priorVers</u> , range, eg:c)                   | ( <u>priorVers</u> , range, Thing)             |
| (dstnctMems, range, Resource)                        | ( <u>priorVers</u> , range, AllDiff)                | ( <u>priorVers</u> , range, TransProp)         |

|                                       |   |  |
|---------------------------------------|---|--|
| (priorVers, range, priorVers)         | (type, range, Resource)                         | (b <sub>9</sub> , subClass, b <sub>9</sub> )   |
| (priorVers, range, List)              | (domain, range, Resource)                       | (b <sub>10</sub> , subClass, b <sub>10</sub> ) |
| (priorVers, range, CMemProp)          | (range, range, Resource)                        | (AllDiff, subClass, AllDiff)                   |
| (priorVers, range, Datatype)          | (subClass, range, Resource)                     | (AnnProp, subClass, AnnProp)                   |
| (priorVers, range, Literal)           | (subPropOf, range, Resource)                    | (owl:Class, subClass, owl:Class)               |
| ( <u>priorVers</u> , range, Resource) | (b <sub>2</sub> , subClass, b <sub>2</sub> )    | (DataRange, subClass, DataRange)               |
| (priorVers, range, boolean)           | (b <sub>3</sub> , subClass, b <sub>3</sub> )    | (DataProp, subClass, DataProp)                 |
| (sameAs, range, Resource)             | (b <sub>6</sub> , subClass, b <sub>6</sub> )    | (DeprClass, subClass, DeprClass)               |
| (unionOf, range, Resource)            | (b <sub>7</sub> , subClass, b <sub>7</sub> )    | (DeprProp, subClass, DeprProp)                 |
| (rest, range, Resource)               | (b <sub>8</sub> , subClass, b <sub>8</sub> )    | (FunProp, subClass, FunProp)                   |
| (InvFunProp, subClass, InvFunProp)    | (b <sub>9</sub> , subPropOf, b <sub>9</sub> )   | (dstnctMems, subPropOf, dstnctMems)            |
| (Nothing, subClass, Nothing)          | (b <sub>10</sub> , subPropOf, b <sub>10</sub> ) | (equivC, subPropOf, equivC)                    |
| (ObjProp, subClass, ObjProp)          | (b <sub>11</sub> , subPropOf, b <sub>11</sub> ) | (equivProp, subPropOf, equivProp)              |
| (Ontology, subClass, Ontology)        | (b <sub>12</sub> , subPropOf, b <sub>12</sub> ) | (hasValue, subPropOf, hasValue)                |
| (OntProp, subClass, OntProp)          | (allVals, subPropOf, allVals)                   | (imports, subPropOf, imports)                  |
| (Restrict, subClass, Restrict)        | (backComp, subPropOf, backComp)                 | (incompat, subPropOf, incompat)                |
| (SymProp, subClass, SymProp)          | (card, subPropOf, card)                         | (intersect, subPropOf, intersect)              |
| (Thing, subClass, Thing)              | (complmntOf, subPropOf, complmntOf)             | (maxCard, subPropOf, maxCard)                  |
| (TransProp, subClass, TransProp)      | (different, subPropOf, different)               | (minCard, subPropOf, minCard)                  |
| (priorVers, subClass, priorVers)      | (disjoint, subPropOf, disjoint)                 | (onProperty, subPropOf, onProperty)            |
| (oneOf, subPropOf, oneOf)             | (sameAs, subPropOf, sameAs)                     | (unionOf, subPropOf, unionOf)                  |
| (priorVers, subPropOf, priorVers)     | (someVals, subPropOf, someVals)                 | (versInfo, subPropOf, versInfo)                |

## 12 $H_3$

The nodes of the graph  $H_2$  are: { "", "false"^^b, "true"^^b, b<sub>1</sub>, b<sub>2</sub>, b<sub>3</sub>, b<sub>4</sub>, b<sub>5</sub>, b<sub>6</sub>, b<sub>7</sub>, b<sub>8</sub>, b<sub>9</sub>, b<sub>10</sub>, b<sub>11</sub>, b<sub>12</sub>, eg:a, eg:b, eg:c, eg:p, AllDiff, AnnProp, owl:Class, DataRange, DataProp, DeprClass, DeprProp, FunProp, InvFunProp, Nothing, ObjProp, Ontology, OntProp, Restrict, SymProp, Thing, TransProp, allVals, backComp, card, complmntOf, different, disjoint, dstnctMems, equivC, equivProp, hasValue, imports, incompat, intersect, inverseOf, maxCard, minCard, onProperty, oneOf, priorVers, sameAs, someVals, unionOf, versInfo, Alt, Bag, List, Prop, Seq, Statement, XMLLiteral, \_l, first, nil, object, predicate, rest, subject, type, value, rdfs:Class, Container, CMemProp, Datatype, Literal, Resource, comment, domain, isDefinedBy, label, member, range, seeAlso, subClass, subPropOf, boolean, }.

For any single node  $n$  in this set, we add  $(n, \text{sameAs}, n)$ .

For any pair of distinct nodes  $n, n'$  in this set, we add  $(n, \text{different}, n')$ .

## 13 $H_4$

|  |   |  |
|--|---|--|
| ("true"^^b, type, FunProp)               | (b <sub>10</sub> , type, InvFunProp)        | (disjoint, type, SymProp)                  |
| (b <sub>10</sub> , type, FunProp)        | ( <u>b<sub>11</sub></u> , type, InvFunProp) | (equivC, type, SymProp)                    |
| ( <u>b<sub>11</sub></u> , type, FunProp) | (b <sub>12</sub> , type, InvFunProp)        | ( <u>priorVers</u> , type, SymProp)        |
| (b <sub>12</sub> , type, FunProp)        | (eg:p, type, InvFunProp)                    | (sameAs, type, SymProp)                    |
| (eg:p, type, FunProp)                    | (dstnctMems, type, InvFunProp)              | ("true"^^b, type, TransProp)               |
| ( <u>priorVers</u> , type, FunProp)      | ( <u>priorVers</u> , type, InvFunProp)      | (b <sub>10</sub> , type, TransProp)        |
| (sameAs, type, FunProp)                  | (sameAs, type, InvFunProp)                  | ( <u>b<sub>11</sub></u> , type, TransProp) |
| (first, type, FunProp)                   | ("true"^^b, type, SymProp)                  | (b <sub>12</sub> , type, TransProp)        |
| (rest, type, FunProp)                    | (complmntOf, type, SymProp)                 | (eg:p, type, TransProp)                    |
| ("true"^^b, type, InvFunProp)            | (different, type, SymProp)                  | (dstnctMems, type, TransProp)              |
| (equivC, type, TransProp)                | (sameAs, type, TransProp)                   | (eg:c, subClass, Literal)                  |
| (equivProp, type, TransProp)             | (rest, type, TransProp)                     |  |
| (priorVers, type, TransProp)             | (eg:c, type, Datatype)                      |  |





|   |   |   |
|---|---|---|
| $(\overline{\text{ObjProp}}, \text{disjoint}, \text{AllDiff})$              | $(\text{OntProp}, \text{disjoint}, \overline{\text{Nothing}})$          | $(\text{SymProp}, \text{disjoint}, \text{eg:c})$                        |
| $(\overline{\text{ObjProp}}, \text{disjoint}, \overline{\text{Nothing}})$   | $(\text{OntProp}, \text{disjoint}, \text{priorVers})$                   | $(\text{SymProp}, \text{disjoint}, \text{AllDiff})$                     |
| $(\overline{\text{ObjProp}}, \text{disjoint}, \text{List})$                 | $(\text{OntProp}, \text{disjoint}, \text{List})$                        | $(\text{SymProp}, \text{disjoint}, \overline{\text{Nothing}})$          |
| $(\overline{\text{ObjProp}}, \text{disjoint}, \text{Datatype})$             | $(\text{OntProp}, \text{disjoint}, \text{CMemProp})$                    | $(\text{SymProp}, \text{disjoint}, \text{priorVers})$                   |
| $(\text{OntProp}, \text{disjoint}, \overline{b_3})$                         | $(\text{OntProp}, \text{disjoint}, \text{Datatype})$                    | $(\text{SymProp}, \text{disjoint}, \text{List})$                        |
| $(\text{OntProp}, \text{disjoint}, b_8)$                                    | $(\text{OntProp}, \text{disjoint}, \text{Literal})$                     | $(\text{SymProp}, \text{disjoint}, \text{Datatype})$                    |
| $(\text{OntProp}, \text{disjoint}, \overline{b_{10}})$                      | $(\text{OntProp}, \text{disjoint}, \text{boolean})$                     | $(\overline{\text{Thing}}, \text{disjoint}, \overline{\text{Nothing}})$ |
| $(\text{OntProp}, \text{disjoint}, \text{eg:c})$                            | $(\text{SymProp}, \text{disjoint}, \overline{b_3})$                     | $(\text{TransProp}, \text{disjoint}, \overline{b_3})$                   |
| $(\text{OntProp}, \text{disjoint}, \text{AllDiff})$                         | $(\text{SymProp}, \text{disjoint}, b_8)$                                | $(\text{TransProp}, \text{disjoint}, b_8)$                              |
| $(\text{OntProp}, \text{disjoint}, \text{AnnProp})$                         | $(\text{SymProp}, \text{disjoint}, \overline{b_{10}})$                  | $(\text{TransProp}, \text{disjoint}, \overline{b_{10}})$                |
| $(\text{TransProp}, \text{disjoint}, \text{eg:c})$                          | $(\text{priorVers}, \text{disjoint}, \text{AnnProp})$                   | $(\text{List}, \text{disjoint}, \text{eg:c})$                           |
| $(\text{TransProp}, \text{disjoint}, \text{AllDiff})$                       | $(\text{priorVers}, \text{disjoint}, \overline{\text{Nothing}})$        | $(\text{List}, \text{disjoint}, \text{AllDiff})$                        |
| $(\text{TransProp}, \text{disjoint}, \overline{\text{Nothing}})$            | $(\text{priorVers}, \text{disjoint}, \text{OntProp})$                   | $(\text{List}, \text{disjoint}, \text{AnnProp})$                        |
| $(\text{TransProp}, \text{disjoint}, \text{List})$                          | $(\text{priorVers}, \text{disjoint}, \text{SymProp})$                   | $(\text{List}, \text{disjoint}, \text{FunProp})$                        |
| $(\text{TransProp}, \text{disjoint}, \text{Datatype})$                      | $(\text{priorVers}, \text{disjoint}, \text{List})$                      | $(\text{List}, \text{disjoint}, \text{InvFunProp})$                     |
| $(\text{priorVers}, \text{disjoint}, \overline{b_3})$                       | $(\text{priorVers}, \text{disjoint}, \text{CMemProp})$                  | $(\text{List}, \text{disjoint}, \overline{\text{Nothing}})$             |
| $(\text{priorVers}, \text{disjoint}, b_8)$                                  | $(\text{priorVers}, \text{disjoint}, \text{Datatype})$                  | $(\text{List}, \text{disjoint}, \overline{\text{ObjProp}})$             |
| $(\text{priorVers}, \text{disjoint}, \overline{b_{10}})$                    | $(\text{priorVers}, \text{disjoint}, \text{Literal})$                   | $(\text{List}, \text{disjoint}, \text{OntProp})$                        |
| $(\text{priorVers}, \text{disjoint}, \text{eg:c})$                          | $(\text{priorVers}, \text{disjoint}, \text{boolean})$                   | $(\text{List}, \text{disjoint}, \text{SymProp})$                        |
| $(\text{priorVers}, \text{disjoint}, \text{AllDiff})$                       | $(\text{List}, \text{disjoint}, \overline{b_3})$                        | $(\text{List}, \text{disjoint}, \text{TransProp})$                      |
| $(\text{List}, \text{disjoint}, \text{priorVers})$                          | $(\text{CMemProp}, \text{disjoint}, \text{AnnProp})$                    | $(\text{Datatype}, \text{disjoint}, b_8)$                               |
| $(\text{List}, \text{disjoint}, \text{CMemProp})$                           | $(\text{CMemProp}, \text{disjoint}, \overline{\text{owl:Class}})$       | $(\text{Datatype}, \text{disjoint}, \overline{b_{10}})$                 |
| $(\text{List}, \text{disjoint}, \text{Datatype})$                           | $(\text{CMemProp}, \text{disjoint}, \overline{\text{Nothing}})$         | $(\text{Datatype}, \text{disjoint}, \text{eg:c})$                       |
| $(\text{List}, \text{disjoint}, \text{Literal})$                            | $(\text{CMemProp}, \text{disjoint}, \text{OntProp})$                    | $(\text{Datatype}, \text{disjoint}, \text{AllDiff})$                    |
| $(\text{List}, \text{disjoint}, \text{boolean})$                            | $(\text{CMemProp}, \text{disjoint}, \text{priorVers})$                  | $(\text{Datatype}, \text{disjoint}, \text{AnnProp})$                    |
| $(\text{CMemProp}, \text{disjoint}, \overline{b_3})$                        | $(\text{CMemProp}, \text{disjoint}, \text{List})$                       | $(\text{Datatype}, \text{disjoint}, \text{FunProp})$                    |
| $(\text{CMemProp}, \text{disjoint}, b_8)$                                   | $(\text{CMemProp}, \text{disjoint}, \text{Datatype})$                   | $(\text{Datatype}, \text{disjoint}, \text{InvFunProp})$                 |
| $(\text{CMemProp}, \text{disjoint}, \overline{b_{10}})$                     | $(\text{CMemProp}, \text{disjoint}, \text{Literal})$                    | $(\text{Datatype}, \text{disjoint}, \overline{\text{Nothing}})$         |
| $(\text{CMemProp}, \text{disjoint}, \text{eg:c})$                           | $(\text{CMemProp}, \text{disjoint}, \text{boolean})$                    | $(\text{Datatype}, \text{disjoint}, \overline{\text{ObjProp}})$         |
| $(\text{CMemProp}, \text{disjoint}, \text{AllDiff})$                        | $(\text{Datatype}, \text{disjoint}, \overline{b_3})$                    | $(\text{Datatype}, \text{disjoint}, \text{OntProp})$                    |
| $(\text{Datatype}, \text{disjoint}, \text{SymProp})$                        | $(\text{Literal}, \text{disjoint}, \text{AllDiff})$                     | $(\text{boolean}, \text{disjoint}, b_8)$                                |
| $(\text{Datatype}, \text{disjoint}, \text{TransProp})$                      | $(\text{Literal}, \text{disjoint}, \text{AnnProp})$                     | $(\text{boolean}, \text{disjoint}, \overline{b_{10}})$                  |
| $(\text{Datatype}, \text{disjoint}, \text{priorVers})$                      | $(\text{Literal}, \text{disjoint}, \overline{\text{owl:Class}})$        | $(\text{boolean}, \text{disjoint}, \text{AllDiff})$                     |
| $(\text{Datatype}, \text{disjoint}, \text{List})$                           | $(\text{Literal}, \text{disjoint}, \overline{\text{Nothing}})$          | $(\text{boolean}, \text{disjoint}, \text{AnnProp})$                     |
| $(\text{Datatype}, \text{disjoint}, \text{CMemProp})$                       | $(\text{Literal}, \text{disjoint}, \text{OntProp})$                     | $(\text{boolean}, \text{disjoint}, \overline{\text{owl:Class}})$        |
| $(\text{Datatype}, \text{disjoint}, \text{Literal})$                        | $(\text{Literal}, \text{disjoint}, \text{priorVers})$                   | $(\text{boolean}, \text{disjoint}, \overline{\text{Nothing}})$          |
| $(\text{Datatype}, \text{disjoint}, \text{boolean})$                        | $(\text{Literal}, \text{disjoint}, \text{List})$                        | $(\text{boolean}, \text{disjoint}, \text{OntProp})$                     |
| $(\text{Literal}, \text{disjoint}, \overline{b_3})$                         | $(\text{Literal}, \text{disjoint}, \text{CMemProp})$                    | $(\text{boolean}, \text{disjoint}, \text{priorVers})$                   |
| $(\text{Literal}, \text{disjoint}, b_8)$                                    | $(\text{Literal}, \text{disjoint}, \text{Datatype})$                    | $(\text{boolean}, \text{disjoint}, \text{List})$                        |
| $(\text{Literal}, \text{disjoint}, \overline{b_{10}})$                      | $(\text{boolean}, \text{disjoint}, \overline{b_3})$                     | $(\text{boolean}, \text{disjoint}, \text{CMemProp})$                    |
| $(\text{boolean}, \text{disjoint}, \text{Datatype})$                        | $(\overline{\text{Nothing}}, \text{equivC}, \overline{\text{Nothing}})$ | $(\text{boolean}, \text{equivC}, \text{boolean})$                       |
| $(\overline{b_3}, \text{equivC}, \overline{b_3})$                           | $(\overline{\text{ObjProp}}, \text{equivC}, \overline{\text{ObjProp}})$ | $(\overline{b_3}, \text{subClass}, b_3)$                                |
| $(b_8, \text{equivC}, b_8)$   | $(\text{OntProp}, \text{equivC}, \text{OntProp})$                       | $(\overline{b_3}, \text{subClass}, \overline{\text{owl:Class}})$        |
| $(\overline{b_{10}}, \text{equivC}, \overline{b_{10}})$                     | $(\text{SymProp}, \text{equivC}, \text{SymProp})$                       | $(\overline{b_3}, \text{subClass}, \overline{\text{Thing}})$            |
| $(\text{eg:c}, \text{equivC}, \text{eg:c})$                                 | $(\text{TransProp}, \text{equivC}, \text{TransProp})$                   | $(b_8, \text{subClass}, \overline{\text{Thing}})$                       |
| $(\text{AllDiff}, \text{equivC}, \text{AllDiff})$                           | $(\text{priorVers}, \text{equivC}, \text{priorVers})$                   | $(b_8, \text{subClass}, \text{List})$                                   |
| $(\text{AnnProp}, \text{equivC}, \text{AnnProp})$                           | $(\text{List}, \text{equivC}, \text{List})$                             | $(b_9, \text{subClass}, b_{10})$  |
| $(\overline{\text{owl:Class}}, \text{equivC}, \overline{\text{owl:Class}})$ | $(\text{CMemProp}, \text{equivC}, \text{CMemProp})$                     | $(b_{10}, \text{subClass}, b_9)$  |
| $(\text{FunProp}, \text{equivC}, \text{FunProp})$                           | $(\text{Datatype}, \text{equivC}, \text{Datatype})$                     | $(\overline{b_{10}}, \text{subClass}, \overline{\text{owl:Class}})$     |
| $(\text{InvFunProp}, \text{equivC}, \text{InvFunProp})$                     | $(\text{Literal}, \text{equivC}, \text{Literal})$                       | $(\overline{b_{10}}, \text{subClass}, \overline{\text{Thing}})$         |

|   |   |   |
|---|---|---|
| $(\overline{b_{10}}, \text{subClass}, \text{List})$                     | $(\text{owl:Class}, \text{subClass}, \text{rdfs:Class})$          | $(\overline{\text{Nothing}}, \text{subClass}, \text{AllDiff})$            |
| $(\text{eg:c}, \text{subClass}, \overline{\text{Thing}})$               | $(\text{FunProp}, \text{subClass}, \overline{\text{ObjProp}})$    | $(\overline{\text{Nothing}}, \text{subClass}, \text{AnnProp})$            |
| $(\text{AllDiff}, \text{subClass}, \overline{\text{Thing}})$            | $(\text{FunProp}, \text{subClass}, \overline{\text{Thing}})$      | $(\overline{\text{Nothing}}, \text{subClass}, \text{owl:Class})$          |
| $(\text{AnnProp}, \text{subClass}, \text{FunProp})$                     | $(\text{InvFunProp}, \text{subClass}, \overline{\text{ObjProp}})$ | $(\overline{\text{Nothing}}, \text{subClass}, \text{FunProp})$            |
| $(\text{AnnProp}, \text{subClass}, \text{InvFunProp})$                  | $(\text{InvFunProp}, \text{subClass}, \overline{\text{Thing}})$   | $(\overline{\text{Nothing}}, \text{subClass}, \text{InvFunProp})$         |
| $(\text{AnnProp}, \text{subClass}, \text{ObjProp})$                     | $(\text{InvFunProp}, \text{subClass}, \text{TransProp})$          | $(\overline{\text{Nothing}}, \text{subClass}, \overline{\text{Nothing}})$ |
| $(\text{AnnProp}, \text{subClass}, \text{SymProp})$                     | $(\overline{\text{Nothing}}, \text{subClass}, \overline{b_3})$    | $(\overline{\text{Nothing}}, \text{subClass}, \overline{\text{ObjProp}})$ |
| $(\text{AnnProp}, \text{subClass}, \overline{\text{Thing}})$            | $(\overline{\text{Nothing}}, \text{subClass}, \overline{b_8})$    | $(\overline{\text{Nothing}}, \text{subClass}, \text{OntProp})$            |
| $(\text{AnnProp}, \text{subClass}, \text{TransProp})$                   | $(\overline{\text{Nothing}}, \text{subClass}, \overline{b_{10}})$ | $(\overline{\text{Nothing}}, \text{subClass}, \text{SymProp})$            |
| $(\text{owl:Class}, \text{subClass}, \overline{\text{Thing}})$          | $(\overline{\text{Nothing}}, \text{subClass}, \text{eg:c})$       | $(\overline{\text{Nothing}}, \text{subClass}, \overline{\text{Thing}})$   |
| $(\overline{\text{Nothing}}, \text{subClass}, \text{TransProp})$        | $(\text{OntProp}, \text{subClass}, \text{InvFunProp})$            | $(\text{priorVers}, \text{subClass}, \overline{\text{owl:Class}})$        |
| $(\overline{\text{Nothing}}, \text{subClass}, \text{priorVers})$        | $(\text{OntProp}, \text{subClass}, \text{ObjProp})$               | $(\text{priorVers}, \text{subClass}, \text{FunProp})$                     |
| $(\overline{\text{Nothing}}, \text{subClass}, \text{List})$             | $(\text{OntProp}, \text{subClass}, \text{SymProp})$               | $(\text{priorVers}, \text{subClass}, \text{InvFunProp})$                  |
| $(\overline{\text{Nothing}}, \text{subClass}, \text{CMemProp})$         | $(\text{OntProp}, \text{subClass}, \overline{\text{Thing}})$      | $(\text{priorVers}, \text{subClass}, \overline{\text{ObjProp}})$          |
| $(\overline{\text{Nothing}}, \text{subClass}, \text{Datatype})$         | $(\text{OntProp}, \text{subClass}, \text{TransProp})$             | $(\text{priorVers}, \text{subClass}, \overline{\text{Thing}})$            |
| $(\overline{\text{Nothing}}, \text{subClass}, \text{Literal})$          | $(\text{SymProp}, \text{subClass}, \overline{\text{ObjProp}})$    | $(\text{priorVers}, \text{subClass}, \text{TransProp})$                   |
| $(\overline{\text{Nothing}}, \text{subClass}, \text{boolean})$          | $(\text{SymProp}, \text{subClass}, \overline{\text{Thing}})$      | $(\text{List}, \text{subClass}, \overline{\text{Thing}})$                 |
| $(\overline{\text{ObjProp}}, \text{subClass}, \overline{\text{Thing}})$ | $(\text{Thing}, \text{subClass}, \text{Resource})$                | $(\text{CMemProp}, \text{subClass}, \text{FunProp})$                      |
| $(\overline{\text{ObjProp}}, \text{subClass}, \text{Prop})$             | $(\text{TransProp}, \text{subClass}, \overline{\text{ObjProp}})$  | $(\text{CMemProp}, \text{subClass}, \text{InvFunProp})$                   |
| $(\text{OntProp}, \text{subClass}, \text{FunProp})$                     | $(\text{TransProp}, \text{subClass}, \overline{\text{Thing}})$    | $(\text{CMemProp}, \text{subClass}, \text{ObjProp})$                      |
| $(\text{CMemProp}, \text{subClass}, \text{SymProp})$                    | $(\text{Datatype}, \text{subClass}, \text{owl:Class})$            | $(\text{boolean}, \text{subClass}, \overline{\text{Thing}})$              |
| $(\text{CMemProp}, \text{subClass}, \overline{\text{Thing}})$           | $(\text{Datatype}, \text{subClass}, \overline{\text{Thing}})$     |   |
| $(\text{CMemProp}, \text{subClass}, \text{TransProp})$                  | $(\text{Literal}, \text{subClass}, \overline{\text{Thing}})$      |   |

## 15 $H_6$

|   |   |   |
|---|---|---|
| $(b_1, \text{dstnctMems}, b_4)$             | $(\overline{\text{Nothing}}, \text{intersect}, b_4)$      | $(\overline{b_3}, \text{unionOf}, b_5)$                   |
| $(b_1, \text{dstnctMems}, b_5)$             | $(\overline{\text{Thing}}, \text{intersect}, \text{nil})$ | $(\text{owl:Class}, \text{unionOf}, b_3)$                 |
| $(b_1, \text{dstnctMems}, \text{nil})$      | $(b_3, \text{oneOf}, b_3)$                                | $(\overline{\text{Nothing}}, \text{unionOf}, b_4)$        |
| $(\overline{b_3}, \text{intersect}, b_5)$   | $(b_7, \text{oneOf}, b_2)$                                | $(\overline{\text{Nothing}}, \text{unionOf}, \text{nil})$ |
| $(\text{owl:Class}, \text{intersect}, b_3)$ | $(\overline{\text{Nothing}}, \text{oneOf}, \text{nil})$   |   |

## 16 $H_7$

|  |  |  |
|--|--|--|
| $(\text{"true"}^{\wedge\wedge b}, \text{inverseOf}, \text{"true"}^{\wedge\wedge b})$ | $(\text{complmntOf}, \text{inverseOf}, \text{complmntOf})$ | $(\text{equivC}, \text{inverseOf}, \text{equivC})$       |
| $(b_{11}, \text{inverseOf}, b_{12})$   | $(\text{different}, \text{inverseOf}, \text{different})$   | $(\text{priorVers}, \text{inverseOf}, \text{priorVers})$ |
| $(b_{12}, \text{inverseOf}, b_{11})$   | $(\text{disjoint}, \text{inverseOf}, \text{disjoint})$     | $(\text{sameAs}, \text{inverseOf}, \text{sameAs})$       |

## 17 $H_8$

|  |   |   |
|--|---|---|
| $(\text{"true"}^{\wedge\wedge b}, \text{domain}, \text{FunProp})$            | $(b_{10}, \text{domain}, \text{Thing})$                           | $(b_{12}, \text{domain}, \text{List})$                            |
| $(\text{"true"}^{\wedge\wedge b}, \text{domain}, \text{InvFunProp})$         | $(b_{10}, \text{domain}, \text{List})$                            | $(\text{eg:p}, \text{domain}, \text{Thing})$                      |
| $(\text{"true"}^{\wedge\wedge b}, \text{domain}, \overline{\text{ObjProp}})$ | $(\overline{b_{11}}, \text{domain}, b_8)$                         | $(\text{complmntOf}, \text{domain}, \overline{\text{owl:Class}})$ |
| $(\text{"true"}^{\wedge\wedge b}, \text{domain}, \text{SymProp})$            | $(\overline{b_{11}}, \text{domain}, \overline{b_{10}})$           | $(\text{complmntOf}, \text{domain}, \text{Thing})$                |
| $(\text{"true"}^{\wedge\wedge b}, \text{domain}, \text{Thing})$              | $(\overline{b_{11}}, \text{domain}, \overline{\text{owl:Class}})$ | $(\text{different}, \text{domain}, \text{Thing})$                 |
| $(\text{"true"}^{\wedge\wedge b}, \text{domain}, \text{TransProp})$          | $(\overline{b_{11}}, \text{domain}, \text{Thing})$                | $(\text{disjoint}, \text{domain}, \overline{\text{owl:Class}})$   |
| $(\text{"true"}^{\wedge\wedge b}, \text{domain}, \text{Literal})$            | $(\overline{b_{11}}, \text{domain}, \text{List})$                 | $(\text{disjoint}, \text{domain}, \text{Thing})$                  |
| $(\text{"true"}^{\wedge\wedge b}, \text{domain}, \text{boolean})$            | $(b_{12}, \text{domain}, b_9)$                                    | $(\text{dstnctMems}, \text{domain}, \text{AllDiff})$              |
| $(b_8, \text{domain}, \overline{\text{owl:Class}})$                          | $(b_{12}, \text{domain}, \overline{\text{owl:Class}})$            | $(\text{dstnctMems}, \text{domain}, \text{Thing})$                |
| $(b_8, \text{domain}, \text{Thing})$   | $(b_{12}, \text{domain}, \text{Thing})$                           | $(\text{equivC}, \text{domain}, \overline{\text{owl:Class}})$     |

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| (equivC, domain, Thing)                      | ( <u>priorVers</u> , domain, $b_8$ )               | ( <u>priorVers</u> , domain, OntProp)                |
| (equivProp, domain, <u>ObjProp</u> )         | ( <u>priorVers</u> , domain, $\overline{b_{10}}$ ) | ( <u>priorVers</u> , domain, SymProp)                |
| (equivProp, domain, Thing)                   | ( <u>priorVers</u> , domain, eg:c)                 | ( <u>priorVers</u> , domain, Thing)                  |
| (intersect, domain, <u>owl:Class</u> )       | ( <u>priorVers</u> , domain, AllDiff)              | ( <u>priorVers</u> , domain, TransProp)              |
| (intersect, domain, Thing)                   | ( <u>priorVers</u> , domain, AnnProp)              | ( <u>priorVers</u> , domain, priorVers)              |
| (inverseOf, domain, <u>ObjProp</u> )         | ( <u>priorVers</u> , domain, <u>owl:Class</u> )    | ( <u>priorVers</u> , domain, List)                   |
| (inverseOf, domain, Thing)                   | ( <u>priorVers</u> , domain, FunProp)              | ( <u>priorVers</u> , domain, CMemProp)               |
| (oneOf, domain, <u>owl:Class</u> )           | ( <u>priorVers</u> , domain, InvFunProp)           | ( <u>priorVers</u> , domain, Datatype)               |
| (oneOf, domain, Thing)                       | ( <u>priorVers</u> , domain, <u>Nothing</u> )      | ( <u>priorVers</u> , domain, Literal)                |
| ( <u>priorVers</u> , domain, $b_3$ )         | ( <u>priorVers</u> , domain, <u>ObjProp</u> )      | ( <u>priorVers</u> , domain, boolean)                |
| (sameAs, domain, Thing)                      | (subClass, domain, owl:Class)                      | ("true"^^b, range, Literal)                          |
| (unionOf, domain, <u>owl:Class</u> )         | (subClass, domain, Thing)                          | ("true"^^b, range, boolean)                          |
| (unionOf, domain, Thing)                     | (subPropOf, domain, ObjProp)                       | ( $b_8$ , range, Thing)                              |
| (first, domain, Thing)                       | (subPropOf, domain, Thing)                         | ( $b_8$ , range, List)                               |
| (rest, domain, Thing)                        | ("true"^^b, range, FunProp)                        | ( $b_{10}$ , range, Thing)                           |
| (type, domain, Thing)                        | ("true"^^b, range, InvFunProp)                     | ( $b_{10}$ , range, List)                            |
| (domain, domain, ObjProp)                    | ("true"^^b, range, <u>ObjProp</u> )                | ( $\overline{b_{11}}$ , range, $\overline{b_{10}}$ ) |
| (domain, domain, Thing)                      | ("true"^^b, range, SymProp)                        | ( $\overline{b_{11}}$ , range, owl:Class)            |
| (range, domain, ObjProp)                     | ("true"^^b, range, Thing)                          | ( $\overline{b_{11}}$ , range, Thing)                |
| (range, domain, Thing)                       | ("true"^^b, range, TransProp)                      | ( $\overline{b_{11}}$ , range, List)                 |
| ( $b_{12}$ , range, $b_8$ )                  | (disjoint, range, <u>owl:Class</u> )               | (inverseOf, range, <u>ObjProp</u> )                  |
| ( $b_{12}$ , range, $\overline{b_{10}}$ )    | (disjoint, range, Thing)                           | (inverseOf, range, Thing)                            |
| ( $b_{12}$ , range, owl:Class)               | (dstnctMems, range, Thing)                         | (oneOf, range, Thing)                                |
| ( $b_{12}$ , range, Thing)                   | (dstnctMems, range, List)                          | (oneOf, range, List)                                 |
| ( $b_{12}$ , range, List)                    | (equivC, range, owl:Class)                         | ( <u>priorVers</u> , range, $b_3$ )                  |
| (eg:p, range, Thing)                         | (equivC, range, Thing)                             | ( <u>priorVers</u> , range, $b_8$ )                  |
| (eg:p, range, Literal)                       | (equivProp, range, <u>ObjProp</u> )                | ( <u>priorVers</u> , range, $\overline{b_{10}}$ )    |
| (complmntOf, range, owl:Class)               | (equivProp, range, Thing)                          | ( <u>priorVers</u> , range, eg:c)                    |
| (complmntOf, range, Thing)                   | (intersect, range, Thing)                          | ( <u>priorVers</u> , range, AllDiff)                 |
| (different, range, Thing)                    | (intersect, range, List)                           | ( <u>priorVers</u> , range, AnnProp)                 |
| ( <u>priorVers</u> , range, owl:Class)       | ( <u>priorVers</u> , range, List)                  | (rest, range, Thing)                                 |
| ( <u>priorVers</u> , range, FunProp)         | ( <u>priorVers</u> , range, CMemProp)              | (type, range, owl:Class)                             |
| ( <u>priorVers</u> , range, InvFunProp)      | ( <u>priorVers</u> , range, Datatype)              | (type, range, Thing)                                 |
| ( <u>priorVers</u> , range, <u>Nothing</u> ) | ( <u>priorVers</u> , range, Literal)               | (domain, range, owl:Class)                           |
| ( <u>priorVers</u> , range, <u>ObjProp</u> ) | ( <u>priorVers</u> , range, boolean)               | (domain, range, Thing)                               |
| ( <u>priorVers</u> , range, OntProp)         | (sameAs, range, Thing)                             | (range, range, owl:Class)                            |
| ( <u>priorVers</u> , range, SymProp)         | (unionOf, range, Thing)                            | (range, range, Thing)                                |
| ( <u>priorVers</u> , range, Thing)           | (unionOf, range, List)                             | (subClass, range, owl:Class)                         |
| ( <u>priorVers</u> , range, TransProp)       | (first, range, owl:Class)                          | (subClass, range, Thing)                             |
| ( <u>priorVers</u> , range, priorVers)       | (first, range, Thing)                              | (subPropOf, range, ObjProp)                          |
| (subPropOf, range, Thing)                    |  |  |

## 18 $H_9$

|  |  |                                   |
|--|--|-----------------------------------|
| ("true"^^b, equivProp, "true"^^b)                        | (equivProp, equivProp, equivProp)                  | (domain, equivProp, domain)       |
| ( $b_{10}$ , equivProp, $b_{10}$ )                       | (intersect, equivProp, intersect)                  | (range, equivProp, range)         |
| ( $\overline{b_{11}}$ , equivProp, $\overline{b_{11}}$ ) | (inverseOf, equivProp, inverseOf)                  | (subClass, equivProp, subClass)   |
| ( $b_{12}$ , equivProp, $b_{12}$ )                       | (oneOf, equivProp, oneOf)                          | (subPropOf, equivProp, subPropOf) |
| (eg:p, equivProp, eg:p)                                  | ( <u>priorVers</u> , equivProp, <u>priorVers</u> ) | ("true"^^b, subPropOf, equivProp) |
| (complmntOf, equivProp, complmntOf)                      | (sameAs, equivProp, sameAs)                        | ("true"^^b, subPropOf, inverseOf) |
| (different, equivProp, different)                        | (unionOf, equivProp, unionOf)                      | ("true"^^b, subPropOf, sameAs)    |
| (disjoint, equivProp, disjoint)                          | (first, equivProp, first)                          | ("true"^^b, subPropOf, subPropOf) |
| (dstnctMems, equivProp, dstnctMems)                      | (rest, equivProp, rest)                            | ( $b_8$ , subPropOf, different)   |
| (equivC, equivProp, equivC)                              | (type, equivProp, type)                            | ( $b_9$ , subPropOf, $b_{11}$ )   |

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| $(b_{10}, \text{subPropOf}, \text{different})$                                 | $(\text{complmntOf}, \text{subPropOf}, \text{different})$            | $(\overline{\text{priorVers}}, \text{subPropOf}, b_{12})$            |
| $(b_{11}, \text{subPropOf}, b_9)$  | $(\text{complmntOf}, \text{subPropOf}, \text{disjoint})$             | $(\overline{\text{priorVers}}, \text{subPropOf}, \text{eg:p})$       |
| $(\overline{b_{11}}, \text{subPropOf}, \text{different})$                      | $(\text{dstnctMems}, \text{subPropOf}, \text{different})$            | $(\overline{\text{priorVers}}, \text{subPropOf}, \text{complmntOf})$ |
| $(\overline{b_{11}}, \text{subPropOf}, \text{disjoint})$                       | $(\text{equivC}, \text{subPropOf}, \text{subClass})$                 | $(\overline{\text{priorVers}}, \text{subPropOf}, \text{different})$  |
| $(\overline{b_{11}}, \text{subPropOf}, \text{subClass})$                       | $(\text{equivProp}, \text{subPropOf}, \text{subPropOf})$             | $(\overline{\text{priorVers}}, \text{subPropOf}, \text{disjoint})$   |
| $(b_{12}, \text{subPropOf}, b_8)$  | $(\text{intersect}, \text{subPropOf}, \text{different})$             | $(\overline{\text{priorVers}}, \text{subPropOf}, \text{dstnctMems})$ |
| $(b_{12}, \text{subPropOf}, \text{different})$                                 | $(\overline{\text{priorVers}}, \text{subPropOf}, \text{"true"^^b})$  | $(\overline{\text{priorVers}}, \text{subPropOf}, \text{equivC})$     |
| $(b_{12}, \text{subPropOf}, \text{disjoint})$                                  | $(\overline{\text{priorVers}}, \text{subPropOf}, b_8)$               | $(\overline{\text{priorVers}}, \text{subPropOf}, \text{equivProp})$  |
| $(b_{12}, \text{subPropOf}, \text{oneOf})$                                     | $(\overline{\text{priorVers}}, \text{subPropOf}, b_{10})$            | $(\overline{\text{priorVers}}, \text{subPropOf}, \text{intersect})$  |
| $(\text{eg:p}, \text{subPropOf}, \text{different})$                            | $(\overline{\text{priorVers}}, \text{subPropOf}, \overline{b_{11}})$ | $(\overline{\text{priorVers}}, \text{subPropOf}, \text{inverseOf})$  |
| $(\overline{\text{priorVers}}, \text{subPropOf}, \text{oneOf})$                | $(\overline{\text{priorVers}}, \text{subPropOf}, \text{rest})$       | $(\overline{\text{priorVers}}, \text{subPropOf}, \text{subPropOf})$  |
| $(\overline{\text{priorVers}}, \text{subPropOf}, \overline{\text{priorVers}})$ | $(\overline{\text{priorVers}}, \text{subPropOf}, \text{type})$       | $(\text{unionOf}, \text{subPropOf}, \text{different})$               |
| $(\overline{\text{priorVers}}, \text{subPropOf}, \text{sameAs})$               | $(\overline{\text{priorVers}}, \text{subPropOf}, \text{domain})$     | $(\text{first}, \text{subPropOf}, \text{different})$                 |
| $(\overline{\text{priorVers}}, \text{subPropOf}, \text{unionOf})$              | $(\overline{\text{priorVers}}, \text{subPropOf}, \text{range})$      | $(\text{rest}, \text{subPropOf}, \text{different})$                  |
| $(\overline{\text{priorVers}}, \text{subPropOf}, \text{first})$                | $(\overline{\text{priorVers}}, \text{subPropOf}, \text{subClass})$   |  |

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