Evaluating interface aesthetics: a measure of symmetry♦

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aesthetics, layout, measure of symmetry, visual symmetry

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ABSTRACT

Symmetry is one of the most fundamental principles in design. The choice between symmetry and asymmetry affects the layout and feeling of a design. A symmetrical page gives a feeling of permanence and stability, while informal or asymmetrical balance creates interest. The aim of this paper is to solve the problem of an automatic detection of axial and radial symmetry or lack of it in published documents. Previous approaches to this problem gave only a necessary condition for symmetry. We present a necessary and sufficient criterion for automatic symmetry detection and also introduce a Euclidean-type distance from any layout to the closest symmetrical one [3]. We present mathematical proof that the measure of symmetry we introduce is exact and accurate. It coincides with intuition and can be effectively calculated. Moreover, any other symmetry criterion will be a derivative of this measure.

Keywords: aesthetics, layout, measure of symmetry, visual symmetry

1. INTRODUCTION

Research and development of automatic layout engines for variable data printing raised an important question of automatically evaluating computer generated documents [6]. It is well established that highly customized and personalized documents present better value. However, this is only true if the quality of personalized documents is compatible with the quality of documents manually created by professional designers.

1.1. Symmetry of documents

Symmetry and in particular visual symmetry is one of the most fundamental principles in a design of a document. The choice between symmetry and asymmetry affects the layout and feeling of a page. A symmetrical layout of objects gives a feeling of permanence and stability to the page. Any symmetrical document content is likely to be more static and restful: it is used to advantage in advertisements emphasizing quality, and by businesses whose position in the community rests on trust. At the same time symmetric layout can easily become too boring, so while it can be preferable and desirable for some types of documents, it is best to be avoided for others. The aim of this paper is to solve the problem of an automatic detection of axial and radial symmetry or lack of it in published documents.

By symmetry we mean that the position and size of objects on one side of an axis of a page or a part of a page are duplicated exactly on the other side of the axis. The objects do not need to have the same content – one could be text and the other graphics for example. The exact duplication across the axis is not required either due to a limited resolution of the human eye. To distinguish from mathematically defined symmetry we call this visual or perceptual symmetry. The fact, that only visual symmetry is required, raises the necessity for a continuous measure with the possibility to introduce a threshold at some point where deviation from symmetry becomes noticeable or perception of a document changes to being rather non-symmetrical.

1.2. Prior art

The problem of symmetry detection for objects on the page is essentially non-linear. To be symmetrical two objects should have the same geometry, opposite orientation and at the same should be equidistant from the axis of symmetry. The prior art in symmetry detection, described in [1] and [2], provides only the necessary condition for symmetry to occur. The proposed criterion gives zero value for every symmetrical case and also for some non-symmetrical layouts. They proposed to sum sizes of objects, distances from their centers to the center of the frame separately in each quadrant on each side of the symmetry axis. Unfortunately, the drawback of this approach is that leaves the possibility for compensations, like for example in the layout shown in Figure 1. Separate sums of widths, heights, distances and other parameters are equal for the correspondent quadrants, thus falsely indicating symmetry with respect to the horizontal axis.

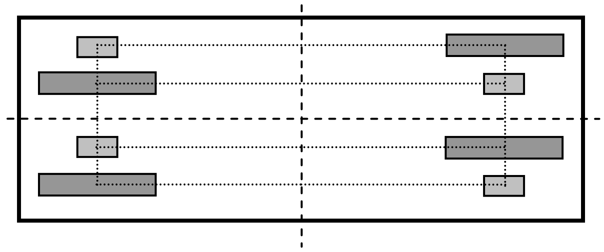


Figure 1

As a result, having a small value of the above measure one cannot possibly decide whether the considered layout is close to the symmetrical case (and thus to be accepted!) or just falls in the close vicinity of the "false" symmetry case.

2. MEASURE OF SYMMETRY

2.1. What measure is required

What properties should a good measure of symmetry observe? The first and obvious one for a measure of symmetry is to be invariant to any translations of the whole composition along the axis of symmetry. Translation of the composition shown in Figure 2 as well as shifting the origin of the axes along the axis of symmetry should not yield different symmetry readings.

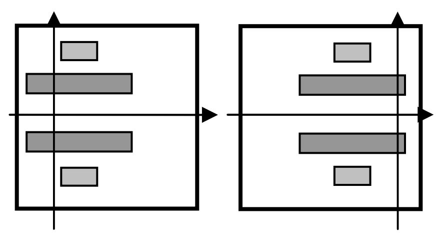


Figure 2

The other intuitive property is that scaling the whole composition up or down should not alter the degree of symmetry of the composition, and therefore should not change the computed value, as is shown in Figure 3.

A less intuitive property of scaling up or down in just one dimension either across or along the axis of symmetry is that it should not alter the computed value of symmetry either. Apart from intuitive observation shown in Figure 4, the necessity for this requirement is substantiated by the fact that perception of symmetry for a composition is independent from a scale selected on X or Y axes as is shown in Figure 5.

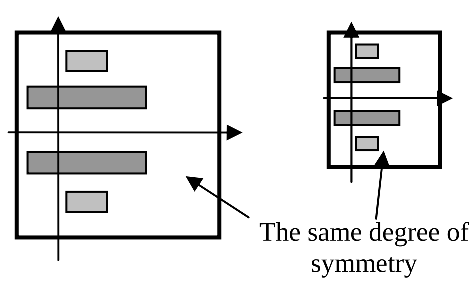


Figure 3

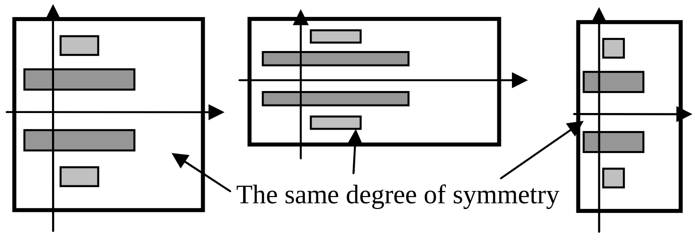


Figure 4

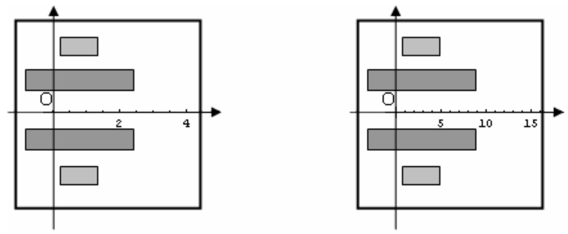


Figure 5

We are also looking for the measure that obeys the principle of "perceptually less noticeable": that is deviation from the symmetrical state caused by moving just a few points from their correspondent symmetrical positions is perceptually less noticeable for a bigger set of points than for a smaller one. If many points are nearly symmetrical we do not want to deal with accumulated or growth effect distance, but rather compute deviation of symmetry per point.

To address the issue of visual symmetry: deviations from exact symmetry positions that are unnoticed by the human eye and documents are still perceived as symmetrical, the measure of symmetry should be a continuous measure. Then the experimental threshold value can be introduced below which the layout can be accepted as symmetrical and above cannot. This experimental value can of cause depend on some other document properties, like for example, resolution. This will be discussed in more detail in Section 0 below.

2.2. Non-linearity of problem

The problem of axial symmetry for a given set of convex objects can be reduced to the problem of symmetry of their vertices, as the convex hull for a convex object is the same as the hull of its vertex point set.

It is a reasonably simple problem to establish symmetry for each pair of the correspondent points. While there is an infinite number of ways for two non-symmetrical points to be transferred to a symmetrical pair (see example on Figure 6, moving one of points to the reflection of the opposite point provides the minimal distance to the nearest symmetry position (as shown in Figure 6 b)).

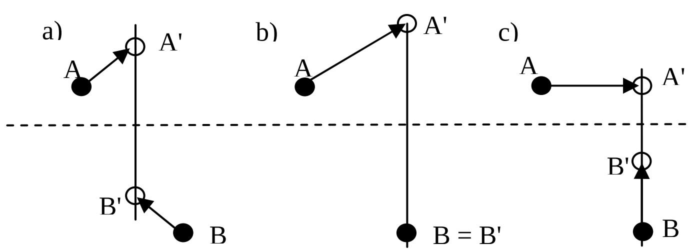


Figure 6

In the general case the problem of establishing pairing among *2n* points has n!-order of complexity. The space of symmetrical configurations is not a linear space as the sum of any two symmetrical configurations is not a symmetrical configuration generally. So, the distance from an arbitrary configuration to its nearest symmetrical state cannot be computed using the simple orthogonal projection.

2.3. Symmetry with respect to horizontal axis

As we already explained above the problem of axial symmetry for a given set of convex objects can be reduced to the problem of symmetry of their vertices. The question addressed in this section is how to establish axial symmetry for a set of points.

Let $\tilde{s} = \{\{\tilde{x}_1, \tilde{y}_1\},..., \{\tilde{x}_n, \tilde{y}_n\}\}$ be a set of n feature points (e.g. vertices, and for non-convex objects also centers of mass) of the original set of objects and axis of symmetry is given by $\tilde{y} = \tilde{y}_A$.
Introduce new translated and scaled coordinates by the following formulas:

$$x_j = \frac{\tilde{x}_j - \tilde{x}_c}{\max_j\{|\tilde{x}_j - \tilde{x}_c|\}},$$

$$y_j = \frac{\tilde{y}_j - \tilde{y}_A}{\max_j\{|\tilde{y}_j - \tilde{y}_A|\}}, \quad j = 1,..., n$$

where $\tilde{x}_c = \frac{1}{n}\sum_j \tilde{x}_j$ is x-coordinate of a center mass of the given composition. Note, that new coordinates satisfy $-1 \le x_j \le 1$ and $-1 \le y_j \le 1$, the axis of symmetry now coincides with the X-axis and the Y-axis contains the center of mass of $\tilde{s}$. By such selection of the Y-axis we ensure that the measure of symmetry is invariant to translations of the whole composition along the axis of symmetry. By scaling coordinates we eliminate the dependence of measure from composition scaling.
Next, map this set of points $s = \{\{x_j, y_j\}\}$ into the complex plane: $S = \{z_1,..., z_n\}$, where $z_j = x_j + \mathrm{I} y_j, 1 \le j \le n$. Our problem in new coordinates is to give a criterion for symmetry of S with respect to the real axis in the complex plane. Recall that the complex conjugate of a complex number $z = x + \mathrm{I}\, y$ is defined by $\bar{z} = x - \mathrm{I}\, y$. This means that the complex conjugate numbers are symmetrical with respect to the real axis in the complex plane. So, symmetry of S with respect to the real axis means that S is a set of real numbers and pairs of complex conjugate numbers only.
Let us identify all such S's with a subset of a factor space $\mathbb{C}^n/S_n$, $\mathbb{C}^n$ is a n-dimensional linear complex space and S_n is a permutation group of n elements. Denote the set of all symmetric configurations of n points by Sym_n. Since Sym_n is not a linear space, the problem of finding distance from an arbitrary point $Z \in \mathbb{C}^n/S_n$ to Sym_n is a difficult one.
To overcome this difficulty we need to find another representation of $\mathbb{C}^n/S_n$ and Sym_n, in which the problem becomes linear. The Fundamental Theorem of Algebra (see for example [4], [7]) says that a polynomial of degree n with complex coefficients (which includes real coefficients as a special case, of course) has exactly n complex roots, counting multiplicities. Using this theorem one can introduce the following one-to-one correspondence between $\mathbb{C}^n/S_n$ and the space of complex polynomials of order n with a unitary leading coefficient:

$$F : (z_1,..., z_n) \mapsto z^n + a_{n-1}z^{n-1} + ... + a_1 z + a_0$$

$$P_n(z) = \prod_{j=1}^{n}(z - z_j) = \sum_{j=0}^{n} a_j z^j,$$

where $a_n = 1$ and $a_j = a_j(z_1,...,z_n)$, $1 \le j < n$ are defined by the Vieta formulas.

One of the fundamental results of complex analysis (see for example [7]) says that a polynomial with a unitary leading coefficient (a_n =1) has real or complex conjugated roots if and only if all its coefficients are real ($a_i \in$ P). This means that the space of all real polynomials is an image of Sym_n under F. The condition for coefficients of the polynomial (built on original set of points as roots) to be real numbers is necessary and sufficient condition for the original set of points to be symmetrical with respect to the real axis. So, after such linearization we are in a situation of finding the distance from an arbitrary polynomial to the real linear subspace of real polynomials. Since any distance can be constructed from the standard Euclidian distance we use it as a main function to measure a distance from Z to Sym_n:

$$D(z_1,..., z_n) = \frac{1}{n}\sqrt{\sum_{j=0}^{n-1}(\mathrm{Im}\, a_j)^2},$$

where coefficients a_j are given by Vieta formulas

$$a_{n-m} = (-1)^m \cdot \sum_{0<j_1<j_2<...<j_m\le n} z_{j_1} \cdot z_{j_2} \cdot ... \cdot z_{j_m}$$

The proposed measure $D(z_1,...,z_n)$ is equal to zero if and only if a set of points $\{z_1,...,z_n\}$ in the complex plane is symmetric with respect to the real axis. The further away a layout is from its nearest symmetrical one, the bigger the value of $D(z_1,...,z_n)$. This will be demonstrated by examples given in Section 3. Dividing by the number of points in a set we are computing the average deviation per point of the set, and thus responding to the principle of perceptually less noticeable, described in Section 2.1.

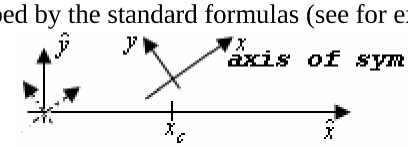
The only problem with using Vieta formulas is that they are computationally expensive and better ways to compute $D(z_1,...,z_n)$ are presented in Section 0 below.

2.4. Symmetry with respect to other axis and radial symmetry

Symmetry with respect to the horizontal axis has been considered so far. However, the above method can address symmetry with respect to any axis of symmetry: vertical or even inclined. Suppose the angle between the axis of symmetry and the original $\hat{x}$ axis is a. Applying the rotation described by the standard formulas (see for example [5]):

$$\tilde{x} = \hat{x}\cos[\theta] - \hat{y}\sin[\theta]$$

$$\tilde{y} = \hat{x}\sin[\theta] + \hat{y}\cos[\theta], \quad \text{where} \quad \theta = \tan^{-1} a$$



the original axis $\hat{x}$ is transformed into $\tilde{x}$ that is collinear to the axis of symmetry and thus the problem is reduced to the one already addressed in Section 2.3. In particular, for symmetry with respect to the vertical axis ($a = 90^\circ$) the following simple coordinate transformation $\tilde{x} = -\hat{y}$ and $\tilde{y} = \hat{x}$ reduces the problem to the question of symmetry with respect to the horizontal axis.

The problem of radial symmetry can be effectively reduced to axial symmetry if we recall that radial symmetry transformation can be presented as a composition of the horizontal and vertical transformations applied sequentially, as shown in Figure 7: left (A) and bottom (C) images are symmetrical with respect to the origin (radial symmetry), while top images (A and B) are vertically symmetrical and right images (B and C) are horizontally symmetrical.

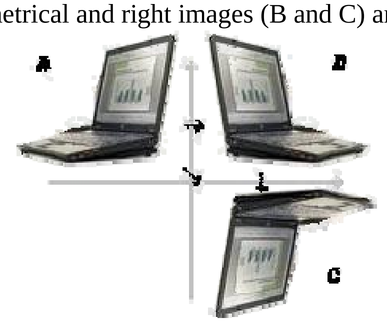


Figure 7

Any line passing through the center of symmetry can be selected as the X-axis. Probably selecting the horizontal line is preferable as it results in simpler coordinate transformations. The Y-axis is, as usual, selected to be orthogonal to X axis. Then by applying the following mirror transformation to the upper half of the plane:

$$x_i = \begin{cases} -\hat{x}_i, \text{if } \hat{y}_i > 0 \\ \hat{x}_i, \text{if } \hat{y}_i \le 0 \end{cases}$$

$$y_i = \hat{y}_i$$

the problem of radial symmetry is effectively reduced to that already addressed in Section 2.3.

Better ways to compute

The Vieta formulas referred to earlier are optimal for obtaining any individual coefficient of a polynomial, however, as all coefficients of the polynomial are required, using the Vieta formulas renders too much computational redundancy and as a result complexity. The better way to build the final polynomial by sequential multiplications of monomials $\left(z - z_{j+1}\right)$ is schematically shown in Figure 8.

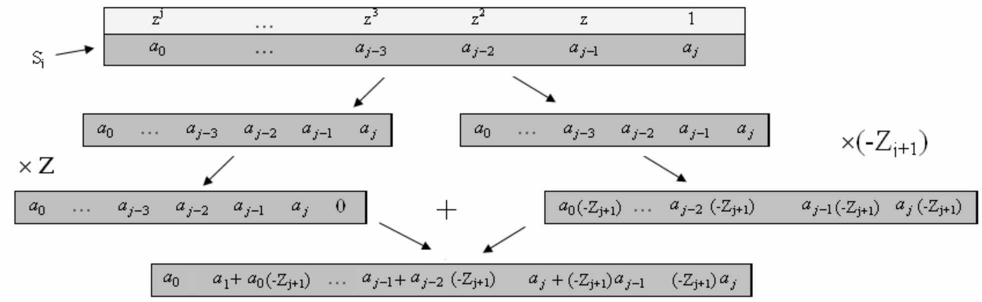


Figure 8

Any polynomial of degree $P_j(z) = \sum_{k=0}^{j} a_{j-k} z^k$ can be presented as a string of its coefficients: $S_j = \{a_0, ..., a_j\}$.

Multiplying polynomial $P_j(z)$ by a monomial $\left(z - z_{j+1}\right)$ means:

A. shifting string S_j one position left $\{a_0, ..., a_j, 0\}$ (the left side of the diagram in Figure 8),

B. multiplying string S_j by $\left(-z_{j+1}\right)$: $\{a_0\left(-z_{j+1}\right), ..., a_j\left(-z_{j+1}\right)\}$ (the right side of the diagram in Figure 8),

C. adding results of A and B: $S_{j+1} = \{a_0, a_1 - a_0 z_{j+1}, ..., a_{j-k+1} - a_{j-k} z_{j+1}, ..., -a_j z_{j+1}\}$ (the bottom of the diagram in Figure 8).

Starting from $\left(z - z_0\right)$, that is represented by its string of coefficients $S_0 = \left(1, -z_0\right)$, and sequentially multiplying by monomials correspondent to other roots requires only $O\left(n^2\right)$ operations to compute all coefficients for the polynomial by its roots.

Of course, other expressions could be employed to derive a value indicating how far the imaginary parts of the coefficients deviate from the ideal zero value. A suitable equivalent distance within the space can be determined by selecting n different real value points and calculating the value of the polynomial $P_n(z)$ in these points. If all the coefficients of the polynomial are real, the value of the polynomial for each of n points will also be real. A distance D^* can be calculated using the formula

$$D^*(z_1, ... z_n) = \frac{1}{n} \sqrt{\sum_{j=0}^{n} \left(\operatorname{Im} P_n(j)\right)^2} \ .$$

3. TEST CASES

The following examples illustrate the distance-like behavior of the proposed measure. A symmetric layout should always result in $D = 0$. If a layout deviates away from its symmetrical state, then the correspondent measure should monotonically increase. Starting from a simple symmetrical layout of two objects, we will continuously change a parameter of one of the objects and compute the symmetry measure as a function of this changing parameter.

3.1. Response of measure due to change in sizes of one object

The first two examples below show the response of the measure to one object of being shrunk from the symmetry state. The static bottom object in both examples is described by the position of its lower left corner $\{-1, -1\}$ and by its width and height that are 1 and 0.5 respectively. The deformable object in the first scenario is squeezed from the right and in the second case from both sides simultaneously. The intuitive expectation is that while deviating from the same symmetrical position the measure of symmetry should rise sharper for the second layout.

The next example demonstrates the dependence on the change of the orthogonal to the axis of symmetry coordinate: the height of one of the objects is now changed thus deviating the whole composition away from symmetry.

3.1.1. Squeezing from one side

This example shows the behavior of the symmetry measure when the top object is squeezed from the right. The shrinking top object is described by the position of its lower right corner $\{1-\mathrm{p}, 0.5\}$ moving to the left as the value of p changes from 0 to 2. As a result of this transformation the width of this object shrinks as $1-\mathrm{p}$ while its height stays unchanged. The sequential positions are shown in Figure 11 with parameter $0 \le p \le 2$ with objects (left) and feature points (right). Note that X axis coincides with axis of symmetry and Y axis is passing through the center of mass of the composition.

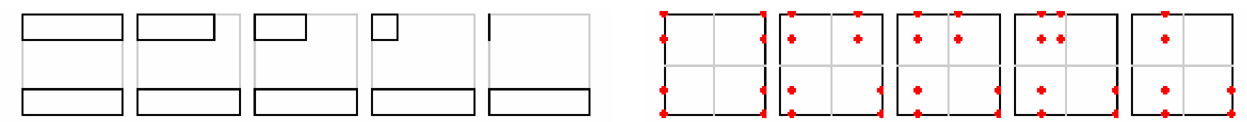


Figure 9

The resulting measure of symmetry shown in

Figure 10 reflects the expected behavior where the symmetry measure is monotonically growing from zero as the correspondent layout deviates from the symmetry state.

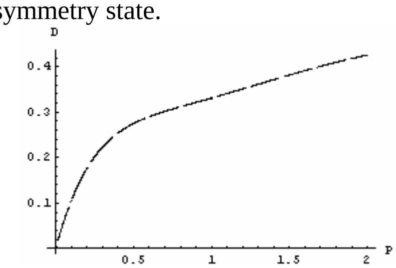


Figure 10

3.1.2. Squeezing from both sides

This example shows the behavior of the symmetry measure when the top object is squeezed from both sides simultaneously. The shrinking top object is described by the position of its lower left corner $\{-1+p, 0.5\}$ moving to the right and lower right corner $\{1-p, 0.5\}$ moving to the left as value of p changes from 0 to 1. As a result of this transformation the width of this object shrinks as $1-2p$ while its height stays unchanged. The sequential positions of objects (left diagram) and feature points (right diagram) are shown in Figure 11 with parameter p changing $0 \le p \le 1$.

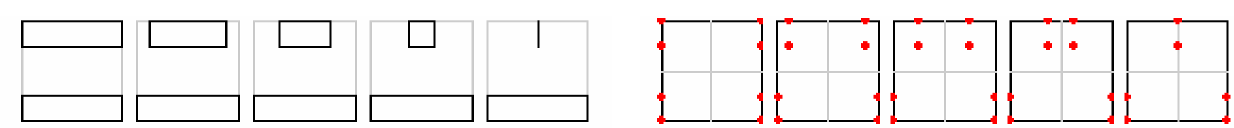


Figure 11

The resulting graph as expected reflects the situation that the layout starts from the symmetrical one and with the rise of parameter p deviates away from symmetry. When the top object is squeezed into a segment $p = 1$ intuitively the farthest away from symmetry point is reached and the measure reaches its maximum value for the given sets of layouts. The result and the comparison with the previous example graphs are shown in Figure 12. Intuitive expectations that the second case should deviate from symmetry faster is captured correctly.

Figure 12

3.1.3. Changing height

In the first two examples the response of the symmetry measure to a change in width was investigated, i.e. a change in coordinate that is parallel to the axis of symmetry. Now we investigate the dependence on varying height (orthogonal to the axis of symmetry parameter) as it is shown in Figure 14:

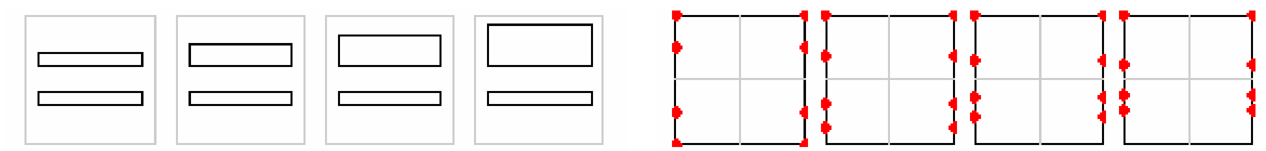


Figure 13

The observed results, shown in Figure 14, match the intuitive expectation that the behavior should be similar to one with altered width.

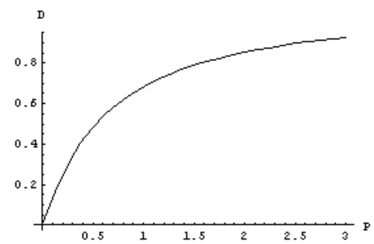


Figure 14

3.2. Response of the measure due to a change in position

In the two following examples we investigate the response of a symmetry measure to the translation of objects parallel to the axis of symmetry and in the orthogonal direction.

3.2.1. Direction orthogonal to the axis of symmetry

First, run the set of experiments, where deviation from an initially symmetrical layout is due to the movement in the direction perpendicular to the axis of symmetry as shown in Figure 15

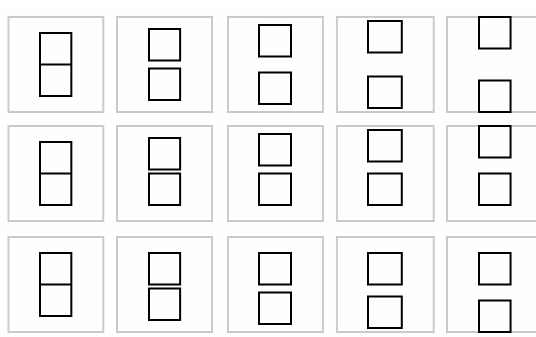


Figure 15

The top row reflects the simultaneous movement of both rectangles that results in symmetry preservation, so the correct measure should be zero. The computed results are shown in Figure 16: the dark grey line, corresponding to the top row of Figure 15, coincides with $D = 0$. The middle and bottom rows of Figure 15 show loss of symmetry that is due to one object moving away. The rise in the symmetry measure should be observed and the intuitive expectation is that the rise for these two cases should be identical: one case can be obtained from the other by a 180^{o} rotation that should not affect the symmetry readings. Light grey and dashed black curves (shown in Figure 16), corresponding to the middle and bottom rows of Figure 15, coincide with each other, as expected.

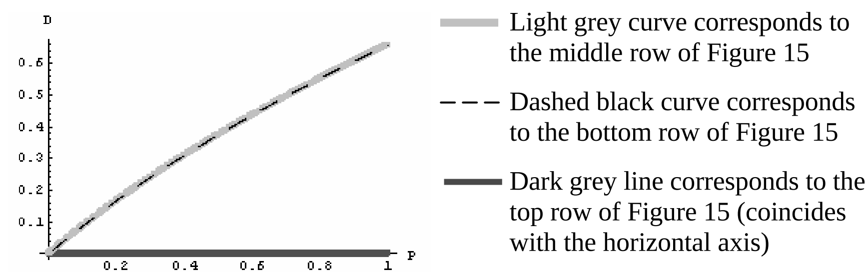


Figure 16

3.2.2. Translation parallel to the axis of symmetry

The computed test case is shown in Figure 17: the top object is moving to the right while the bottom rectangle is stationary.

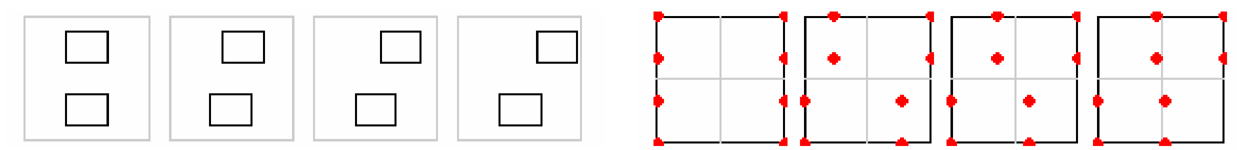


Figure 17

The symmetry measure behavior shown in Figure 18 matches our intuition: starting from the symmetrical case with $D = 0$ the measure monotonically grows as layout deviates further away from the symmetry case.

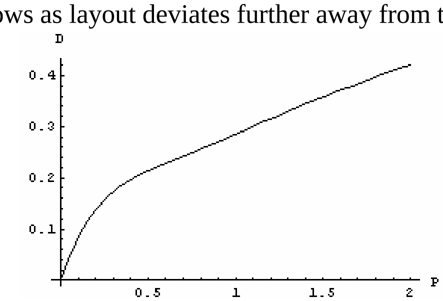


Figure 18

3.3. Dual symmetry example

Consider a more complicated example with two objects moving simultaneously: the grey square moves from position 1 to position 2 and the black square -- simultaneously from 3 to 4 as it is shown in Figure 19. The overall layout changes from one symmetry position {1,3} to another {2,4}. This is a so-called dual symmetry example. Two objects start from one the symmetrical position with respect to the X axis as is shown on the most left picture in Figure 19) and then move towards the other symmetrical position (shown on the rightmost picture in Figure 19). The movement of the centers of these two squares is described by a parameter p, $0 \le p \le 3$.

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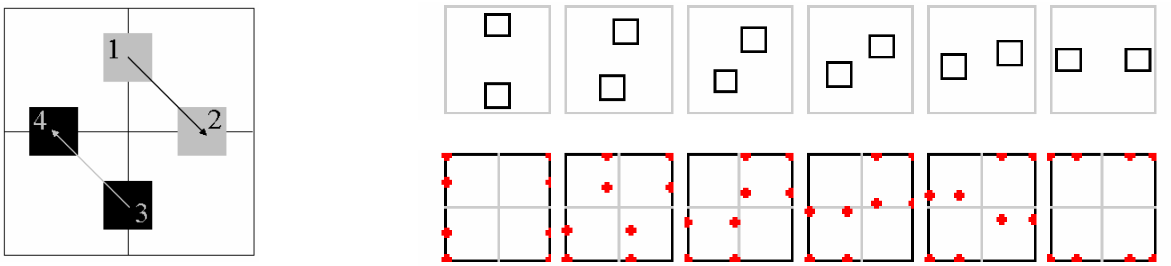

Figure 19

grey object: top → right

$$x = p$$
$$y = 3 - p$$

black object: bottom → left

$$x = -p$$
$$y = -3 + p$$

The expected behavior of the symmetry measure in this case is to be a continuous function with two zeroes $p = 0$ and $p = 3$ and with at least one maximum point that will correspond to the most remote point from the symmetry state position. The corresponding graph (Figure 21) shows the dependence of the symmetry measure on parameter p.

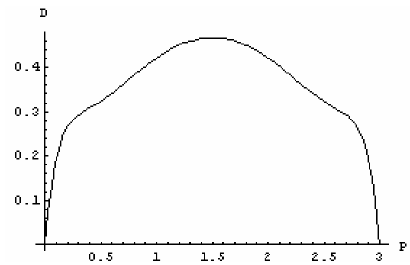

Figure 20

3.4. Prior art example

Let us now return back to the example shown in Figure 1, where the “false” symmetry was detected by the prior art. To investigate the behavior of the symmetry measure near this layout we continuously deform it by expanding small boxes until it reaches the symmetrical state and beyond. The subsequent fragments of this deformation are shown Figure 21. The leftmost picture in Figure 21 shows the original layout and the third from the left fragment corresponds to the exact symmetrical state.

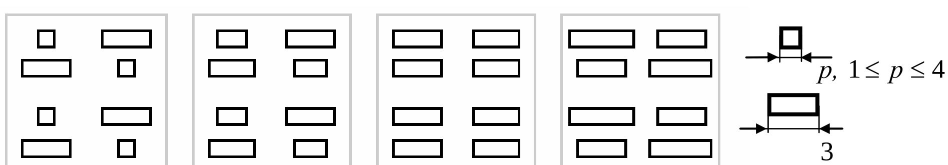

Figure 21

The widths of wide and small rectangles in the original layout (the left fragment in Figure 21) are 3 and 1 correspondingly. The layout is non-symmetrical with respect to the horizontal axis and the measure of symmetry for this layout is equal to 0.351918. The computed measure of symmetry as a function of the width p of small rectangles is shown in Figure 22. By increasing the widths of the small rectangles from 1 to 3 ($1 \le p \le 3$) the layout approaches the symmetrical case and this is reflected by decreasing the value of the symmetry measure. By the time $p = 3$ the originally small and wide rectangles become equal and exact symmetry is reached. The value of the symmetry measure at this point is equal to zero.

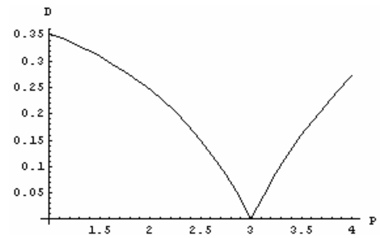

Figure 22

Expanding former small rectangles even further $3 \leq p \leq 4$ results in the deviation from the symmetrical state with the measure of symmetry growing as can be seen in Figure 22: the fragment of the curve for $p > 3$. Thus, the behavior of the proposed measure of symmetry around this layout accurately detects the exact symmetry and matches our intuitive perception of symmetry for the near symmetrical cases.

3.5. Non-rectangular objects and non-convex objects

The proposed measure of symmetry is not limited to rectangular objects: we used them so far as they are perceptually most intuitive whilst the least obscure. Also they are primary objects in layouts. However, when dealing with non-convex objects, their centers of mass should be added to the set of feature points (as we already mentioned in Section 2.3). Also, it is often acceptable to replace the non-convex objects by their convex hull, thus treating their concavity as the internal content that is not required to be repeated on the other side of the axis as it is explained in Section 1.1.

The example below investigates the behavior of the symmetry measure for non-rectangular objects with respect to the vertical axis. First, the transformation described in Section 2.4 is applied to reduce the problem to the symmetry with respect to the horizontal axis. One of the vertices of the pentagon shown in Figure 23(1) is being moved along some arbitrary selected line whilst the symmetry readings are recorded The movement outwards from the pentagon corresponds to the positive value of the parameter p and vice versa as shown in Figure 23(2). Note, that the object is concave for $p < 0$, so the center mass of this object is added as its feature point.

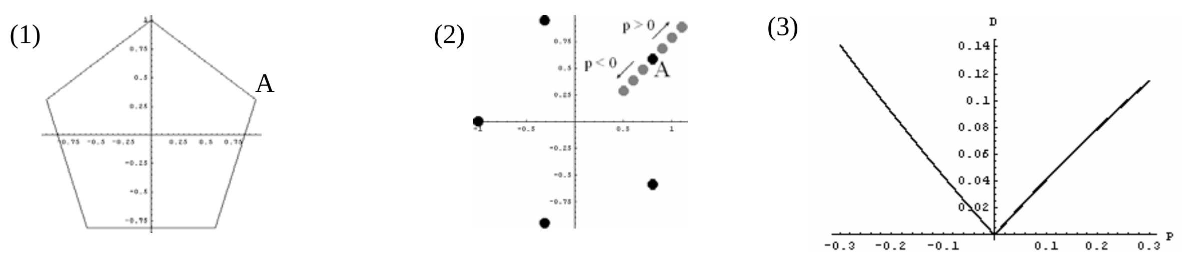

Figure 23

The correspondent measure of symmetry is shown in Figure 23(3). As expected the measure of symmetry is zero for the original pentagon as it is symmetrical with respect to the vertical axis and then grows as the shape deviates from symmetry in either direction.

4. VISUAL SYMMETRY

4.1. Setting a threshold

Let us look through the examples considered in Section 3 and try to establish a threshold, below which the deviation from symmetry is unnoticeable. Some of three layouts shown in Figure 24 are not actually symmetrical; however deviations from symmetry are small and hardly (if at all) noticeable. Could you guess which ones are symmetrical? The correct answer is neither of these layouts is symmetrical. In group A the upper rectangle is squeezed from both sides (as in Section 3.1.2), the top rectangle in the group B is squeezed from the right (as in Section 3.1.1), and in the group C the bottom rectangle is taller than the top one (a mirrored example from Section 3.1.3)

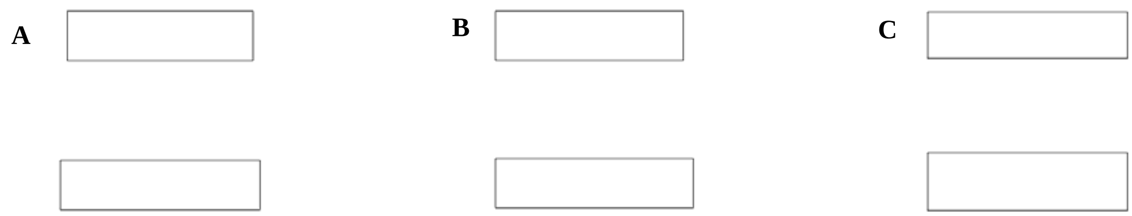

Figure 24

The correspondent values for measure of symmetry are $D_A = 0.13$, $D_B = 0.11$ and $D_C = 0.07$. The visual symmetry for non-rectangular objects can tolerate even higher thresholds.

4.2. Other factors to consider

There cannot be a single number for a visual symmetry threshold on every occasion. Factors like correlation with alignment and resolution will always influence whether a particular document is perceived as symmetrical or not.

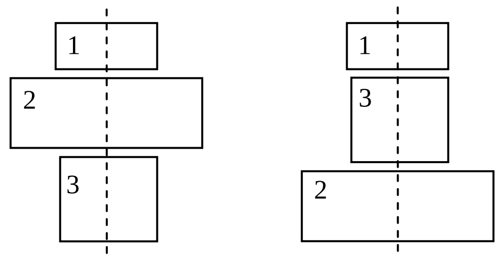

Figure 25

The high sensitivity of the human eye to even small misalignments of near-located objects may result in earlier than otherwise expected detection of visually broken symmetry. Two sets of nearly symmetrical rectangles are presented in Figure 25**.** Broken symmetry for rectangle 3 on the right is observed due to a misalignment problem with rectangle 1. For the same set of objects on the left in Figure 25 the symmetry is visually observed as rectangles 1 and 3 are spatially distributed.

5. CONCLUSIONS

We presented a necessary and sufficient criterion for the automatic detection of symmetry and also introduced a Euclidean-type distance measure from any layout to the closest symmetrical one. We proved that the introduced measure of symmetry is exact and accurate. It coincides with intuition and can be effectively calculated.

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