# Efficient optical quantum information processing[♦]

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Quantum information offers the promise of being able to perform certain communication and computation tasks that cannot be done with conventional information technology (IT). Optical Quantum Information Processing (QIP) holds particular appeal, since it offers the prospect of communicating and computing with the same type of qubit. Linear optical techniques have been shown to be scalable, but the corresponding quantum computing circuits need many auxiliary resources. Here we present an alternative approach to optical QIP, based on the use of weak cross-Kerr nonlinearities and homodyne measurements. We show how this approach provides the fundamental building blocks for highly efficient non-absorbing single photon number resolving detectors, two qubit parity detectors, Bell state measurements and finally near deterministic control-not (CNOT) gates. These are essential QIP devices.

# Efficient optical quantum information processing

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**Abstract.** Quantum information offers the promise of being able to perform certain communication and computation tasks that cannot be done with conventional information technology (IT). Optical Quantum Information Processing (QIP) holds particular appeal, since it offers the prospect of communicating and computing with the same type of qubit. Linear optical techniques have been shown to be scalable, but the corresponding quantum computing circuits need many auxiliary resources. Here we present an alternative approach to optical QIP, based on the use of weak cross-Kerr nonlinearities and homodyne measurements. We show how this approach provides the fundamental building blocks for highly efficient non-absorbing single photon number resolving detectors, two qubit parity detectors, Bell state measurements and finally near deterministic control-not (CNOT) gates. These are essential QIP devices.

## 1. Introduction

It has been known for a number of years that processing information quantum mechanically enables certain communication and computation tasks that cannot be performed with conventional Information Technology (IT). The list of applications continues to expand, and there are extensive experimental efforts in many fields to realise the necessary building blocks for Quantum Information Processing (QIP) devices. One very appealing route (certainly in the short term when a pragmatic focus is on few-qubit applications) is that of optical QIP. In particular for quantum communication the qubits of choice are optical systems, since they can span long distances with minimal decoherence. In order to circumvent interconversion of the qubit species we need to process the optical qubits using optical circuits.

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Optical QIP is currently a very active research area, both theoretically and experimentally. The work of Knill, Laflamme and Milburn (KLM) has shown that in principle universal quantum computation is possible with linear optics [1], and there have been a number of recent experimental demonstrations of its gate components [2, 3, 4]. However, due to the probabilistic nature of gates in linear optical QIP, it is practically rather inefficient (in terms of photon resources) to implement [5, 6, 7].

Strong Kerr non-linearities are able to effectively mediate an interaction directly between photonic qubits. This would realise deterministic quantum gates and thus efficient optical QIP. In practice, however, such non-linearities are not available. On the other hand, much smaller non-linearities can be generated, for example, with electromagnetically induced transparencies (EIT)[8, 9, 10]. In this paper, we show that with modest additional optical resources these small non-linearities provide the building blocks for efficient optical QIP [11, 12, 13, 14, 15]. We present a highly efficient non-absorbing single photon number resolving detector, a nondestructive two qubit parity detector, a nondestructive Bell state measurement, and a near deterministic controlled-not (CNOT) gate.

## 2. Quantum non-demolition detectors

Before we discuss the construction of efficient quantum gates using weak non-linearities, let us first review the construction of a photon number quantum non-demolition (QND) measurement using a cross-Kerr non-linearity [16, 17, 10]. The cross-Kerr non-linearity has a Hamiltonian of the form:

$$H_{QND} = \hbar\chi a^\dagger a c^\dagger c \tag{1}$$

where the signal (probe) mode has the creation and annihilation operators given by $a^\dagger, a$ ($c^\dagger, c$) respectively and $\chi$ is the strength of the non-linearity. If the signal field contains $n_a$ photons and the probe field is in an initial coherent state with amplitude $\alpha_c$, the cross-Kerr non-linearity causes the combined system to evolve as

$$|\Psi(t)\rangle_{out} = e^{i\chi t a^\dagger a c^\dagger c}|n_a\rangle|\alpha_c\rangle = |n_a\rangle|\alpha_c e^{in_a\theta}\rangle. \tag{2}$$

where $\theta = \chi t$ with $t$ being the interaction time for the signal and probe modes with the non-linear material. The Fock state $|n_a\rangle$ is unaffected by the interaction with the cross-Kerr non-linearity but the coherent state $|\alpha_c\rangle$ picks up a phase shift directly proportional to the number of photons $n_a$ in the signal $|n_a\rangle$ state. If we could measure this phase shift we could then infer the number of photons in the signal mode $a$. This can be achieved simply with a homodyne measurement (depicted schematically in figure (1)). The homodyne apparatus allows measurement of the quadrature operator $x(\phi) = ce^{i\phi} + c^\dagger e^{-i\phi}$ with expectation value

$$\langle x(\phi)\rangle = 2\text{Re}\,[\alpha_c]\cos\delta + i2\text{Im}\,[\alpha_c]\sin\delta \tag{3}$$

where $\delta = \phi + n_a\theta$ and $\phi$ the phase of the local oscillator. For a real initial $\alpha_c$, a highly efficient homodyne measurement of the position $X = a + a^\dagger$ or momentum $iY = a - a^\dagger$

quadratures yield the expectation values

$$\langle X \rangle = 2\alpha_c \cos(n_a\theta) \qquad\qquad \langle Y \rangle = 2\alpha_c \sin(n_a\theta) \qquad\qquad (4)$$

with a unit variance. For the momentum quadrature this gives a signal-to-noise ratio $\text{SNR}_Y = 2\alpha_c \sin(n_a\theta)$ which should be much greater than unity for the different $n_a$ inputs to be distinguished. In more detail, if the inputs in mode $a$ are the Fock state $|0\rangle$ or $|1\rangle$, the respective outputs of the probe mode $c$ are the coherent states $|\alpha_c\rangle$ or $|\alpha_c e^{i\theta}\rangle$. The probability of misidentifying these states is given by

$$P_{\text{error}} = \frac{1}{2}\text{Erfc}\left[\alpha_c \sin\theta/\sqrt{2}\right] = \frac{1}{2}\text{Erfc}\left[\text{SNR}_Y/2\sqrt{2}\right]. \qquad (5)$$

A signal to noise ratio of $\text{SNR}_Y = 6$ would thus give $P_{\text{error}} \sim 10^{-3}$. To achieve the necessary phase shift we require $\alpha_c \sin\theta \approx 3$ which can be achieved with a small non-linearity $\theta$ as long as the probe beam is intense enough. These results so far indicate that with a weak cross-Kerr non-linearity it is possible to build a high efficiency photon number resolving detector that does not absorb the photon from the signal mode. In reference [10] we discuss an example of how this level of non-linearity could be achieved through electromagnetically induced transparency (EIT), and give some details for this approach with NV-diamond systems.
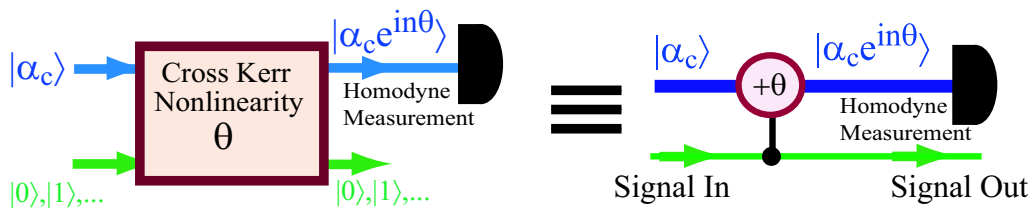


**Figure 1.** Schematic diagram of a photon resolving detector based on a cross-Kerr Non-linearity and a homodyne measurement. The two inputs are a Fock state $|n_a\rangle$ (with $n_a = 0, 1, ..$) in the signal mode $a$ and a coherent state with real amplitude $\alpha_c$ in the probe mode $c$. The presence of photons in mode $a$ causes a phase shift on the coherent state $|\alpha_c\rangle$ directly proportional to $n_a$ which can be determined with a momentum quadrature measurement.

In many optical quantum computation tasks our information is not encoded in photon number but polarization instead. When our information encoding is polarization based there are two separate detection tasks that we need to be able to perform. The first and simplest is just to determine for instance whether the polarization is in one of the basis states $|H\rangle$ or $|V\rangle$. This can be achieved by converting the polarization information to "which path" information on a polarizing beam-splitter. The "which path" information is photon number encoded in each path and hence a QND photon number measurement of each path will determine which polarization basis state the photon was originally in. The second task (one that is critically important for error correction codes in optics) is to determine whether our single-photon polarization-encoded qubit is present or not. That is, for the optical field under consideration we want to determine whether it contains a photon or not. If it does contain a photon, we

do not want to destroy the information in its polarization state. This can be achieved by first converting the polarization qubit to a "which path" qubit. Each path then interacts with a weak cross-Kerr non-linearity $\theta$ with the same shared probe beam (Figure 2). If the photon is present in either path of this signal beam it induces a phase shift $\theta$ on the probe beam; however, with this configuration it is not possible to determine which path induced the phase shift. This allows the preservation of the "which path" and hence polarization information,
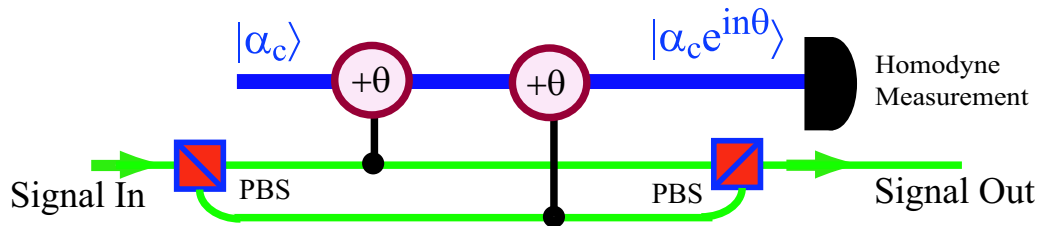


**Figure 2.** Schematic diagram of a polarization-preserving photon number quantum non-demolition detector based on a pair of identical cross-Kerr optical non-linearities. The signal mode is a Fock state with an unknown polarization is converted into which path qubits by a polarizing beam splitter (square box). The phase shift applied to the probe mode is proportional to $n_a$, independent of the polarization of the signal mode.

## 3. A Two Qubit Parity Gate

Now that we have discussed the basic operation of a single photon quantum non-demolition detector, it is worthwhile asking whether this detection concept can be applied to several qubits. Basically if we want to perform a more "generalized" type of measurement between different photonic qubits, we could delay the homodyne measurement, instead having the probe beam interact with several cross-Kerr non-linearities where the signal mode is different in each case. The different signal modes could be from separate photonic qubits. The probe beam measurement then occurs after all these interactions in a collective way which could for instance allow a nondestructive detection that distinguishes superpositions and mixtures of the states $|HH\rangle$ and $|VV\rangle$ from $|HV\rangle$ and $|VH\rangle$. The key here is that we could have no net phase shift on the $|HH\rangle$ and $|VV\rangle$ terms while having a phase shift on the $|HV\rangle$ and $|VH\rangle$ terms. We will call this generalization a *two qubit polarization parity QND gate*.

Let us now discuss the operation of this parity QND gate. Consider a general two qubit state which can be written as $|\Psi_{12}\rangle = \beta_0|HH\rangle + \beta_1|HV\rangle + \beta_2|VH\rangle + \beta_3|VV\rangle$. This may be separable or entangled depending on the choices of $\beta_i$. As shown in Figure (3), these qubits are individually split on polarizing beam-splitters (PBS) into spatial encoded qubits which then interact with separate weak cross-Kerr non-linearities. The action of the PBS's and cross-Kerr non-linearities evolves the combined system of photonic qubits and probe beam to

$$|\psi\rangle_T = [\beta_0|HH\rangle + \beta_3|VV\rangle]|\alpha_c\rangle_p + \beta_1|HV\rangle|\alpha_c e^{i\theta}\rangle_p + \beta_2|VH\rangle|\alpha_c e^{-i\theta}\rangle_p$$
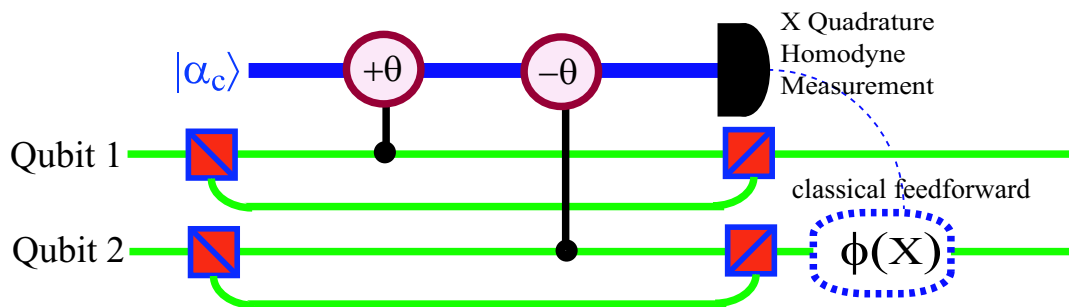
**Figure 3.** Schematic diagram of a two qubit polarization QND detector that distinguishes superpositions and mixtures of the states $|HH\rangle$ and $|VV\rangle$ from $|HV\rangle$ and $|VH\rangle$ using several cross-Kerr non-linearities and a coherent laser probe beam $|\alpha\rangle$. The scheme works by first splitting each polarization qubit into a which path qubit on a polarizing beam-splitter. The action of the first cross-Kerr non-linearity puts a phase shift $\theta$ on the probe beam only if a photon was present in that mode. The second cross-Kerr non-linearity put a phase shift $-\theta$ on the probe beam only if a photon was present in that mode. After the non-linear interactions the which path qubit are converted back to polarization encoded qubits. The probe beam only picks up a phase shift if the states $|HV\rangle$ and/or $|VH\rangle$ were present and hence the appropriate homodyne measurement allows the states $|HH\rangle$ and $|VV\rangle$ to be distinguished from $|HV\rangle$ and $|VH\rangle$. The two qubit polarization QND gate thus acts like a parity checking device. If we consider that the input state of the two polarization qubit is $|HH\rangle + |HV\rangle + |VH\rangle + |VV\rangle$ then after the parity gate we have conditioned on an $X$ homodyne measurement either to the state $|HH\rangle + |VV\rangle$ or to $e^{i\phi(X)}|HV\rangle + e^{-i\phi(X)}|VH\rangle$ where $\phi(X)$ is a phase shift dependent on the exact result of the homodyne measurement. A simple phase shift achieved via classical feed-forward then allows this second state to be transformed to the first if we wish.

It is now obvious that the $|HH\rangle$ and $|VV\rangle$ terms pick up no phase shift and remain coherent with respect to each other while the $|HV\rangle$ and $|VH\rangle$ pick up opposite sign phase shift $\theta$ which could allow them to be distinguished by a general homodyne/heterodyne measurement. We thus need to perform a measurement that does not allow the sign of the phase shift to be determined. With $\alpha_c$ real an $X$ homodyne measurement achieves this by projecting the probe beam to the position quadrature eigenstate $|X\rangle\langle X|$[13]. The resulting two photonic qubit state is then

$$|\psi_X\rangle_T = f(X, \alpha_c) \left[\beta_0|HH\rangle + \beta_3|VV\rangle\right] \qquad (6)$$
$$+ f(X, \alpha_c \cos\theta) \left[\beta_1 e^{i\phi(X)}|HV\rangle + \beta_2 e^{-i\phi(X)}|VH\rangle\right]$$

where

$$f(x, \beta) = \exp\left[-\frac{1}{4}(x - 2\beta)^2\right] / (2\pi)^{1/4} \qquad (7)$$

$$\phi(X) = \alpha_c \sin\theta(x - 2\alpha_c \cos\theta) \bmod 2\pi . \qquad (8)$$

We observe that $f(X, \alpha)$ and $f(X, \alpha \cos\theta)$ are two Gaussian curves with the mid point between the peaks located at $X_0 = \alpha_c [1 + \cos\theta]$ and the peaks separated by a distance $X_d = 2\alpha_c [1 - \cos\theta]$. As long as this difference is large $X_d \sim \alpha_c \theta^2 \gg 1$, then there is little overlap between these curves. For an $X$ homodyne $X > X_0$ our solution (6)

collapses to

$$|\psi_{X>X_0}\rangle_T \sim \beta_0|HH\rangle + \beta_3|VV\rangle \tag{9}$$

while for $X < X_0$ we have

$$|\psi_{X<X_0}\rangle_T \sim \beta_1 e^{i\phi(X)}|HV\rangle + \beta_2 e^{-i\phi(X)}|VH\rangle \tag{10}$$

The action of this two mode polarization non-demolition parity gate is clear: It splits the even parity terms (9) nearly deterministically from the odd parity cases (10).

Above we have chosen to call the even parity state $\{|HH\rangle, |VV\rangle\}$ and the odd parity states $\{|HV\rangle, |VH\rangle\}$, but this is an arbitrary choice primarily dependent on the form/type of PBS used to convert the polarization encoded qubits to which path encoded qubits. Any other choice is also acceptable and it does not have to be symmetric between the two qubits.

Our solution in Eqn (10) depends on the value of the measured quadrature $X$. Simple local rotations using phase shifters dependent on the measurement result $X$ can be performed via a feed forward process to transform this state to $\beta_1|H\rangle_a|V\rangle_b + \beta_2|V\rangle_a|H\rangle_b$ which is independent of $X$. This does mean our homodyne measurement must be accurate enough such that we can determine $\phi(X)$ precisely, otherwise this unwanted phase factor cannot be undone. By this we mean that the uncertainty in the $X$ quadrature homodyne measurement must be much less than $2\pi/\alpha_c \sin\theta$ and this can generally be achieved by ensuring that the strength of the local oscillator is much more intense than the probe mode.

In the above solutions (9) and (10) we have used the the approximate symbol $\sim$ as there is a small but finite probability that the state (9) can occur for $X < X_0$ and vice versa. The probability of this error occurring is given by

$$P_{\text{error}} = \frac{1}{2}\text{Erfc}[X_d/2\sqrt{2}] \tag{11}$$

which is less than $10^{-4}$ when the distance $X_d \sim \alpha_c\theta^2 > 8$. This shows that it is possible to operate in the regime of weak cross-Kerr non-linearities ($\theta \ll \pi$) and achieve an effectively deterministic parity measurement.

## 4. Bell state measurements

These effectively deterministic nondestructive parity measurements are critically important in optical quantum information processing as they naturally allow an efficient and deterministic Bell state measurement to be implemented. Bell state measurements are known to be one of the tools and mechanism required in quantum computation and communication. The four Bell states can be written as

$$|\Psi^{\pm}\rangle \equiv \frac{1}{\sqrt{2}}\left(|H,V\rangle \pm |V,H\rangle\right) \qquad |\Phi^{\pm}\rangle \equiv |\frac{1}{\sqrt{2}}\left(|H,H\rangle \pm |V,V\rangle\right) \tag{12}$$

and we can now see why the parity gate can form the basis of a Bell state detector. The parity gate distinguishes states within the even parity $|H,H\rangle$ and $|V,V\rangle$ subspace from the odd parity $|H,V\rangle$ and $|V,H\rangle$ subspace. Hence one application of the parity detector

distinguishes two of the Bell states $|\Phi^{\pm}\rangle$ from the $|\Psi^{\pm}\rangle$ ones without destroying them. Similarly if we replace the polarizing beam-splitter in the parity gates with 45 degree PBS then the parity gate will allow us to distinguish the $|\Phi^{+}\rangle, |\Psi^{+}\rangle$ Bell states from $|\Phi^{-}\rangle, |\Psi^{-}\rangle$ ones.
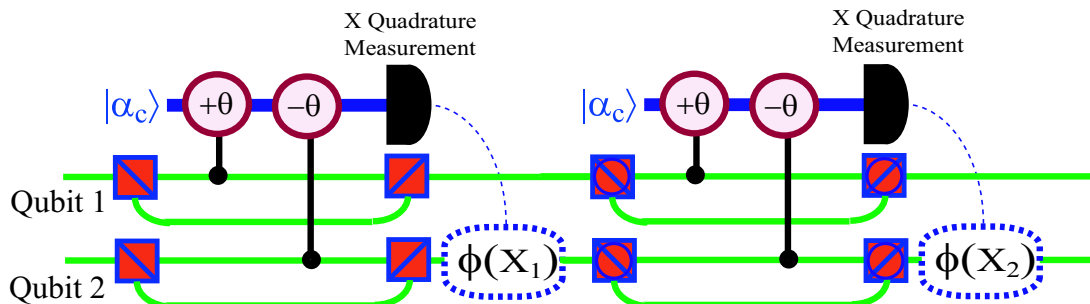


**Figure 4.** Schematic diagram of a non-destructive Bell state measurement composed of two QND parity detectors. The first parity gates uses the standard PBS and distinguishes the $|\Phi^{\pm}\rangle$ Bell states from the $|\Psi^{\pm}\rangle$ ones. An even parity results for this first parity gate indicates the present of the $|\Phi^{\pm}\rangle$ Bell states while an odd parity result indicates the present of the $|\Psi^{\pm}\rangle$ states. For this odd parity result a local operation on the second qubit is required to remove the $\phi(X_1)$ phase shifted induced from the measurement. Once this correction is done the second parity gate can be applied. This gate is similar to the first one but has 45 deg PBS's (square box with circle inside) instead of the normal PBS's. The 45 deg PBS's operate in the {H+V,H-V} basis. An even parity result indicates the presence of the $|\Phi^{+}\rangle, |\Psi^{+}\rangle$ Bell states while an odd parity result indicates the presence of the $|\Phi^{-}\rangle, |\Psi^{-}\rangle$ Bell states. Again a phase correction $\phi(X_2)$ in the {H+V,H-V} basis is needed for the odd parity result to remove the unwanted phase shift.

Since both of these detectors are nondestructive on the qubits and select different pairs of Bell states, they allow the natural construction of a Bell state detector (depicted in Fig (4)). From each parity measurements we get one of information indicating whether the parity was even or odd and so from both parity measurements we end up with four possible results (even, even), (even, odd), (odd, even) and (odd, odd). This is enough to uniquely identify all the Bell states as the $|\Phi^{+}\rangle$ gives the result (even, even), $|\Phi^{-}\rangle$ (even, odd), $|\Psi^{+}\rangle$ (odd, even) and $|\Psi^{-}\rangle$ (odd, odd). It is important that after an odd parity measurement result that we remove the unwanted phase factors that have arisen. This need to be done in the same basis as the PBS in the particular parity gate. For instance for an odd parity results giving $X = X_1$ on the first parity gate a phase shift $\phi(X_1)$ needs to be removed in the PBS $\{H, V\}$ basis. Similarly for a odd parity results giving $X = X_2$ on the second parity gate a phase shift $\phi(X_2)$ needs to be removed in the PBS $\{H + V, H - V\}$ basis

So far we have shown how it is possible using linear elements, weak cross-Kerr non-linearities and homodyne measurements to create a wide range of high efficiency quantum detectors and gates that can perform task ranging from photon number discrimination to Bell state measurements. This is all achieved non-destructively on the photonic qubits and hence provides a critical set of tools extremely useful for single

photon quantum computation and communication. With these tools universal quantum computation can be achieved using the ideas and techniques originally proposed by KLM.

## 5. A resource efficient CNOT gate

The parity gate and Bell state detector has shown how versatile the weak non-linearities and homodyne conditioning measurements are. Both of these gates/detectors can be used to induce two qubit operations and hence are all they are necessary with single qubit operation and single photon measurements to perform universal quantum computation. The parity and Bell state gates are not the typical two qubits gates that one generally considers in the standard quantum computational models. The typical two qubit gate generally considered is the CNOT gate. This can be constructed from two parity gates (like the Bell state detector) but it also requires an ancilla qubit. This CNOT gate is depicted schematically shown in Fig (5)) and operates as follows.
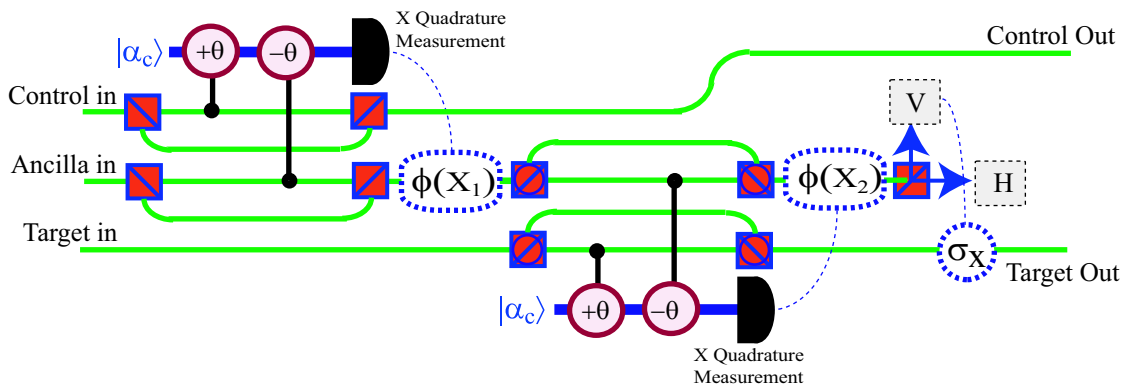


**Figure 5.** Schematic diagram of a near deterministic CNOT composed of two parity gates (one with PBS in the {H,V} basis and one with PBS in the {H+V,H-V} basis), one ancilla qubit prepared initially as $|H\rangle + |V\rangle$, a polarisation determining photon number QND measurement and classical feed-forward elements.

Assume that our control and target qubits are initially prepared as $c_0|H\rangle_c + c_1|V\rangle_c$ and $d_0|H\rangle_t + d_1|V\rangle_t$. With an ancilla qubit prepared as $|H\rangle_a + |V\rangle_a$ the action of the first parity gate on the control and ancilla qubits (with appropriate phase corrections for the odd parity result) conditions the system to

$$[c_0|HH\rangle_{ca} + c_1|VV\rangle_{ca}] \otimes [d_0|H\rangle_t + d_1|V\rangle_t] \tag{13}$$

The action of the second parity gate (using 45 deg PBS's instead of normal PBS's) on the ancilla qubit and target qubit conditions the three qubit system to

$$\{c_0|H\rangle_c - c_1|V\rangle_c\}(d_0 - d_1)|\bar{D}, \bar{D}\rangle_{at} + \{c_0|H\rangle_c + c_1|V\rangle_c\}(d_0 + d_1)|D, D\rangle_{at} \tag{14}$$

where $|D\rangle = |H\rangle + |V\rangle$, $|\bar{D}\rangle = |H\rangle - |V\rangle$ and for the odd parity measurement result $X < X_0$ the usual phase correction is applied. Also for this odd parity result a bit flip

is applied to the ancilla qubit and a sign flip $|V\rangle_c \rightarrow -|V\rangle_c$ on the control qubit. Once this operation have been performed the ancilla mode is measured in the $\{H, V\}$ basis using QND photon number resolving detectors. The output state of the control and target qubits is then final state from these interactions and feed forward

$$c_0 d_0 |HH\rangle_{ct} + c_0 d_1 |HV\rangle_{ct} + c_1 d_0 |VV\rangle_{ct} + c_1 d_1 |VH\rangle_{ct}, \tag{15}$$

where an additional bit flip was applied to the target qubit if the ancilla photons state was $|V\rangle$. This final state is the state that one will expect after a CNOT gate is applied to the initial control and target qubits. This really shows that our QND-based parity gates have performed a near deterministic CNOT operation utilizing only one ancilla qubit (which is not destroyed at the end of the gate). This represents a huge saving in the physical resources to implement single photon quantum logic gates.

## 6. Concluding Discussions

We have shown how it is possible to create near deterministic two qubit gates (parity, bell and CNOT) without a huge overhead in ancilla resources. In fact, an ancilla photon is required only for the CNOT gate. The key addition to the general linear optical resources are weak cross-Kerr nonlinearities and efficient homodyne measurements. Homodyne measurements are a well established technique frequently used in the continuous variable quantum information processing community. However weak cross-Kerr nonlinearities are not commonly used elements within optical quantum computational devices and as such it a discussion of the source and strength of such elements is required. We will start with a discussion of the strength of the nonlinearity as this constraints the possible physical realisations, however before this we really need to define what we mean by *weak* or weak compared with what. Basically it is well known that deterministic two qubit gates can be performed if one has access to a cross-Kerr nonlinearity that can induce a $\pi$ phase shift directly between single photon. This leads to a natural definition of weak nonlinearities, that is, the use of nonlinear cross-Kerr materials (when all are taken into account) that can not directly induce a phase shift within an order of magnitude or several orders of magnitude of $\pi$. This seems to give an acceptable functional definition.

For the parity based gates discussed previously we have established that the nonlinearity $\theta$ must satisfy the constraint $\alpha_c \theta^2 \sim 8$ where just to re-emphasise $\alpha_c$ is the amplitude of the probe beam. Thus due to the weak nature of the nonlinearity $\theta \ll 1$ we must choose $\alpha_c \sim 10/\theta^2$, so for instance if $\theta \sim 10^{-2}$ then $\alpha_c \geq 10^5$ (which corresponds to a probe beam with mean photon number $10^{10}$). For a smaller $\theta$ we need a much larger $\alpha_c$. This puts a natural constraints on $\theta$, since $\alpha_c$ can not be made arbitrarily large in practice.

This leads to the question of a mechanism to achieve the weak cross-Kerr nonlinearity. Natural $\chi^3$ materials have small nonlinearities on the order of $10^{-18}$ [18] which would require lasers with $\alpha_c \sim 10^{37}$ which is physically unrealistic. However, systems such as optical fibers [19], silica whispering-gallery microresonators [20] and

cavity QED systems [11, 21], and EIT [8] are capable of producing much much larger nonlinearities. For instance calculations for EIT systems in NV diamond [10] have shown potential phase shifts of order of magnitude of $\theta = 0.01$. With $\theta = 0.01$ the probe beam must have an amplitude of at least $10^5$ which is physically reasonable with current technology.

Finally, by using these weak cross-Kerr nonlinearities to aid in the construction of near deterministic two qubit gates we can build quantum circuits with far fewer resources than is known for the current corresponding linear optical only approaches.It is straightforward to show in principle that an $n$ qubit computation requires only of order $n$ single photons sources. This has enormous implications for the development of single photon quantum computing and information processing devices and truly indicates the power of a little nonlinearity.

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