



## Preparing multi-partite entanglement of photons and matter qubits<sup>♦</sup>

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# Preparing multi-partite entanglement of photons and matter qubits

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**Abstract.** We show how to make event-ready multi-partite entanglement between qubits which may be encoded on photons or matter systems. Entangled states of matter systems, which can also act as single photon sources, can be generated using the entangling operation presented in quant-ph/0408040. We show how to entangle such sources with photon qubits, which may be encoded in the dual rail, polarization or time-bin degrees of freedom. We subsequently demonstrate how projective measurements of the matter qubits can be used to create entangled states of the photons alone. The state of the matter qubits is inherited by the generated photons. Since the entangling operation can be used to generate cluster states of matter qubits for quantum computing, our procedure enables us to create any (entangled) photonic quantum state that can be written as the outcome of a quantum computer.

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## 1. Introduction

Single-photon sources are a very important resource in optical quantum communication and computation, and are currently at the forefront of the experimental effort in optical quantum information processing [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. In particular, sources that generate entangled multi-photon states are very useful, for example in cryptography [12], linear optical quantum computing [13], and Heisenberg-limited metrology and quantum lithography [14]. In addition, entanglement between qubits of a different physical nature (e.g., light and matter) is crucial for distributed quantum computing.

Here, we consider three alternative representations for encoding qubits in photonic states. Qubits can be encoded in the polarization, dual-rail, and time-bin degree of freedom. The first two are equivalent in that linear optical elements (polarization beam-splitters, polarization rotators) transform between the two deterministically. Here the qubit degree of freedom is the population of two distinct modes, either two orthogonal polarization modes or two spatial modes, so a maximally entangled two-qubit state

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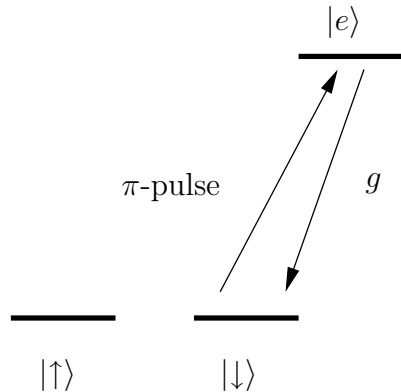
is written as  $|H, V\rangle + |V, H\rangle$  (polarization) or  $|0, 1; 1, 0\rangle + |1, 0; 0, 1\rangle$  (dual-rail). By contrast, the time-bin variable distinguishes between the arrival times (in one of two bins for a qubit) of the photon in the detector. This degree of freedom is particularly useful when the polarization is subject to decoherence (e.g., in long-distance quantum communication through optical fibers) [15].

In this paper, we present a general method for generating entangled multi-photon states, by first entangling the single photon *sources*. The sources we consider have an internal *matter qubit* degree of freedom (encoded, for example, in the spin degree of freedom of an electron), and can be entangled using the double-heralding entangling operation introduced by Barrett and Kok [16]. This entangling operation can be used to generate cluster states (of the matter qubits) for quantum computing, and hence can be used to create arbitrary multi-qubit entangled states [17, 18]. We describe how these entangled sources can generate photon states that inherit the entanglement properties of the matter qubits. Consequently, our method is capable of generating any photonic quantum state that can be written as the outcome of a quantum computer. We also show how to create ‘hetero-entanglement’ between matter qubits and photons. We note that related schemes for entangling single photon sources (comprising a double- $\Lambda$  energy level configuration), and mapping the entanglement onto polarization-encoded photonic qubits, have also recently been proposed [19, 20]. The scheme presented here uses an alternative energy level scheme, and is also naturally suited to creating time-bin encoded states.

## 2. The single-photon source

The matter system that we consider here is illustrated in Fig. 1. The system consists of an atom-like system comprising three energy levels, with two low-lying levels denoted by  $|\uparrow\rangle$  and  $|\downarrow\rangle$  that are (near) degenerate, and one excited state  $|e\rangle$ , which is separated from the low-lying levels by an optical transition. We assume that the optical transition only couples the levels  $|\downarrow\rangle \leftrightarrow |e\rangle$ , and that the transition  $|\uparrow\rangle \leftrightarrow |e\rangle$  is forbidden, e.g. due to a selection rule. The low lying states  $|\uparrow\rangle$  and  $|\downarrow\rangle$  may be thought of as a qubit degree of freedom. We assume that arbitrary unitary operations can be performed on this qubit, and furthermore, that one can perform single shot measurements of the qubit in the computational basis,  $\{|\uparrow\rangle, |\downarrow\rangle\}$ .

The existence of the optical transition means that the matter system can also act as an on-demand single photon source. The three level system is placed in an optical cavity, such that the cavity mode couples to the  $|\downarrow\rangle \leftrightarrow |e\rangle$  transition. One of the cavity mirrors is assumed to be leaky, with leakage rate  $\kappa$ . Applying an optical  $\pi$ -pulse (e.g. using a classical laser field) to the system, tuned to the optical transition, induces the transformation  $|\downarrow\rangle \rightarrow |e\rangle$  and  $|\uparrow\rangle \rightarrow |\uparrow\rangle$ . Provided that spontaneous emission into modes other than the cavity mode can be neglected, the matter qubit-cavity system will emit a single photon into the desired mode, with an emission rate approximately given by  $\Gamma_{\text{slow}} = \kappa - \sqrt{\kappa^2 - g^2}$ , where  $g$  is the Jaynes-Cummings coupling between the  $|\downarrow\rangle \leftrightarrow |e\rangle$



**Figure 1.** The qubit system  $\{|\uparrow\rangle, |\downarrow\rangle\}$  with the excited state  $|e\rangle$ . The  $\pi$ -pulse affects only the transition  $|\downarrow\rangle \rightarrow |e\rangle$ , and the emission of a photon into the cavity mode brings the excited state back to the qubit state  $|\downarrow\rangle$ . The atom-cavity coupling is given by  $g$ . The cavity is sufficiently leaky in order to release the photon from the cavity into the freely propagating mode in the shortest possible time.

transition and the cavity mode [16]. We define a time  $t_{\text{wait}}$  which is the length of time that one should wait for the observation of a photon in a photo-detector after the optical  $\pi$ -pulse has been applied. Provided  $t_{\text{wait}} \gg \Gamma_{\text{slow}}^{-1}$ , and the initial state of the source was  $|\downarrow\rangle$ , a single photon wavepacket will be emitted into the desired mode with probability very close to one. Note that the emission of a photon from this system is conditional on the initial state of the qubit degree of freedom. In some physical implementations, it may be advantageous to use a fourth auxiliary level, such that the  $|\downarrow\rangle \leftrightarrow |e\rangle$  transition is replaced by a STIRAP process in a  $\Lambda$  configuration of three levels [11].

### 3. Entangling matter qubits

Matter qubits (with a computational basis  $\{|\uparrow\rangle, |\downarrow\rangle\}$ ) can be prepared in entangled states using the *double-heralding* technique described in [16]. We briefly review this technique here. We entangle two qubits by first preparing two atoms in separate cavities in the separable state  $(|\uparrow\rangle + |\downarrow\rangle)(|\uparrow\rangle + |\downarrow\rangle)$ . Subsequently, we apply an optical  $\pi$ -pulse to each atom, and wait for a time  $t_{\text{wait}}$ . This yields the total state

$$|\uparrow\uparrow\rangle|0, 0\rangle + |\uparrow\downarrow\rangle|0, 1\rangle + |\downarrow\uparrow\rangle|1, 0\rangle + |\downarrow\downarrow\rangle|1, 1\rangle,$$

where  $|0\rangle$  and  $|1\rangle$  respectively denote the vacuum and a single photon wavepacket in the freely propagating optical mode leaving the cavity. When these two modes interact in a 50:50 beam splitter, the total state becomes

$$\begin{aligned} &|\uparrow\uparrow\rangle|0, 0\rangle + \frac{1}{\sqrt{2}} \left[ (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)|0, 1\rangle \right. \\ &\quad \left. + (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)|1, 0\rangle + |\downarrow\downarrow\rangle(|2, 0\rangle - |0, 2\rangle) \right]. \end{aligned}$$

Note that the beam splitter must be placed in such a way that the spatio-temporal photon modes overlap at the beam splitter, in order to erase the ‘which path’

information. Detecting both the outgoing modes of the beam splitter, each with a realistic detector (i.e. a detector with finite efficiency, and which cannot discriminate between optical states with one or more photons), gives the following state of the qubits (given just a single detector click):

$$\rho = f(\eta)|\Psi_{\pm}\rangle\langle\Psi_{\pm}| + (1 - f(\eta))|\downarrow\downarrow\rangle\langle\downarrow\downarrow|, \quad (1)$$

where  $|\Psi_{\pm}\rangle = (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)/\sqrt{2}$  and  $f(\eta) \leq 1$  is a function of the combined collection and detection efficiency,  $\eta$ . The relative phase in  $|\Psi_{\pm}\rangle$  is determined by the detection signature, (“click”, “no click”) or (“no click”, “click”) for the two detectors.

The state in Eq. (1) is an incoherent mixture of a maximally entangled state and the separable state  $|\downarrow\downarrow\rangle\langle\downarrow\downarrow|$ . However, we can remove this separable part by first applying a bit flip operation  $|\uparrow\rangle \leftrightarrow |\downarrow\rangle$  to both matter qubits. We subsequently apply a second  $\pi$ -pulse to each matter system. The separable part cannot generate photons. Thus, conditional on observing another single detector click, the final two qubit state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle) \quad (2)$$

is obtained. The total success probability of this procedure is  $\eta^2/2$ . Note that we can also make *any* pure two-qubit state by using the initial states  $\mu_j|\uparrow\rangle + \nu_j|\downarrow\rangle$  (rather than  $(|\uparrow\rangle + |\downarrow\rangle)$ ), performing the double heralding steps described above, and subsequently performing single-qubit operations. Furthermore, using this entangling operation, combined with single qubit unitaries and measurements, we can efficiently produce cluster states of *many* matter qubits [16]. This is an extremely useful result, since it is known that many other interesting multi-qubit entangled states, such as GHZ states, can be generated by performing single qubit measurements on cluster states [17]. But most importantly, cluster states together with single qubit unitary operations and measurements can be used to implement any quantum algorithm [18]. We can therefore efficiently produce any state that may be the output of a quantum computer.

It is worth discussing here the effect of experimental imperfections on the state given in Eq. (2), which of course represents an idealization of the state of the two qubit system after the *double-heralding* operation. In real systems, physical imperfections can reduce the fidelity of the state. Perhaps the most important such imperfection is that of photon loss, for example by emission into unwanted modes, absorption in the optical elements, or detector inefficiency. It turns out that such losses do not affect the fidelity of our entangled output states, but merely reduce the success probability of the protocol [16]. Mismatch in the cavity parameters  $g$  and  $\kappa$  between different cavities can reduce the fidelity. However this scheme is reasonably robust to such losses: a mismatch of a few percent leads to a reduction in fidelity of less than  $10^{-3}$  [16]. Decoherence of the qubit degrees of freedom can also reduce fidelity. However this can be mitigated by choosing a system whose intrinsic decoherence time is long compared to  $t_{\text{wait}}$ ; examples of such systems were given in [16].

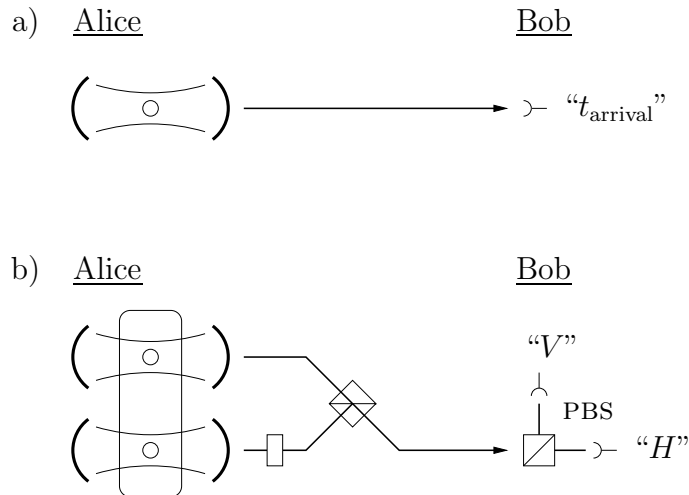
#### 4. Entanglement between matter qubits and photons

We can now create entangled states between the matter qubits and photons by applying more  $\pi$ -pulses (and bit flips). This type of ‘hetero-entanglement’ (entanglement between systems of a different physical class) can be very useful, for example in quantum key distribution via the Ekert protocol [12]. In that case, it is sufficient for Alice and Bob to share *any* maximally entangled two-qubit state, such as  $|\uparrow\rangle|H\rangle + |\downarrow\rangle|V\rangle$  (where  $H$  and  $V$  denote polarized photons). Alice can measure her matter qubit in a spin basis of her choice, while Bob uses photo-detection in a polarization basis that he chooses.

Alice and Bob can share hetero-entanglement using time-bin photons, polarized photons, or dual rail photons. To generate time-bin entanglement, let Alice hold a cavity with a matter qubit in the state  $|\uparrow\rangle + |\downarrow\rangle$ . After applying an optical  $\pi$ -pulse, waiting for a time  $t_{\text{wait}}$ , applying a bit flip, applying another optical  $\pi$ -pulse, and waiting for a second time window  $t_{\text{wait}}$ , the total state will be given by  $|\uparrow\rangle|E\rangle + |\downarrow\rangle|L\rangle$ , where  $|E\rangle$  and  $|L\rangle$  denote photon wavepackets localized in the ‘early’ and ‘late’ spatio-temporal modes, respectively. The optical mode that supports the photon is detected by Bob (see Fig. 2a). Note that, when qubits are represented by such time-bin photons, Alice and Bob must share some classical timing reference information, such that both parties agree on the definition of ‘early’ and ‘late’ time bins. This information could be provided by, for example, a shared classical laser pulse or electronic signal.

Alternatively, we can entangle the qubit with the polarization degree of freedom of a photon. This variant requires two matter qubits: Alice first prepares the two matter qubits in the maximally entangled state  $|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle$ . This is formally equivalent to a *new* qubit  $|\tilde{0}\rangle + |\tilde{1}\rangle$ , with  $|\tilde{0}\rangle \equiv |\downarrow\uparrow\rangle$  and  $|\tilde{1}\rangle \equiv |\uparrow\downarrow\rangle$ . The next step in generating hetero-entanglement is then to apply a  $\pi$ -pulse to the two matter qubits, and waiting for a time  $t_{\text{wait}}$  for a photon to be emitted. By construction, a photon will be emitted by one and only one of the matter systems. At this point we have generated hetero-entanglement between the composite qubit system and a dual-rail photon. Assuming that the outgoing photon has a definite polarization (e.g., horizontal), one of the modes undergoes a polarization rotation, and the two modes are combined with a polarization beam splitter (see Fig. 2b). The total system will be in the state  $|\tilde{0}\rangle|H\rangle + |\tilde{1}\rangle|V\rangle$ .

Note that single qubit operations on *physical* qubits are not sufficient to perform arbitrary operations in the *encoded* basis  $\{|\tilde{0}\rangle, |\tilde{1}\rangle\}$ , as may be required for some applications. However, it is possible to remove one of the physical qubits by performing a measurement in the  $\{|+\rangle, |-\rangle\}$  basis, where  $|\pm\rangle = |\uparrow\rangle \pm |\downarrow\rangle$ . Such a measurement can be implemented by first performing a Hadamard operation on the relevant physical qubit, and subsequently performing a measurement in the computational basis. The resulting state is of the form  $|\downarrow\rangle|H\rangle \pm |\uparrow\rangle|V\rangle$ . Note that this state contains only one matter qubit, which can be manipulated using single qubit operations. The relative phase between the terms is determined by the outcome of the physical qubit measurement. This conditional phase can be corrected using a single qubit operation on either the matter qubit or the photon, provided sufficiently fast classical switching is available. Alternatively, it is



**Figure 2.** Creating entanglement between the qubit in the cavity and a freely propagating photon: a) for time-bin-qubit entanglement we apply a  $\pi$ -pulse, wait  $t_{\text{wait}}$  (the early bin), apply a bit flip and a second  $\pi$ -pulse, and wait  $t_{\text{wait}}$  (the late bin); b) for polarization-qubit entanglement we apply a single  $\pi$ -pulse. In b) a polarization rotation (bit flip) is applied to one mode before they are combined on a polarization beam-splitter (PBS) for transmission. Bob uses a PBS at his end to separate the polarizations for detection.

sufficient in many applications just to keep a (classical) record of the relative phase, and take it into account when interpreting the results of individual measurements of the entangled qubits.

## 5. Entangled states of light

The entangled states of matter qubits can also be used to make entangled states of light. Suppose we want to make time-bin entanglement of the form  $|E, L\rangle + |L, E\rangle$ . We start with the two-qubit state  $|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle$  and apply a  $\pi$ -pulse to the two cavities. After a time  $t_{\text{wait}}$  the total state is then

$$|\downarrow\uparrow\rangle|E, 0\rangle + |\uparrow\downarrow\rangle|0, E\rangle .$$

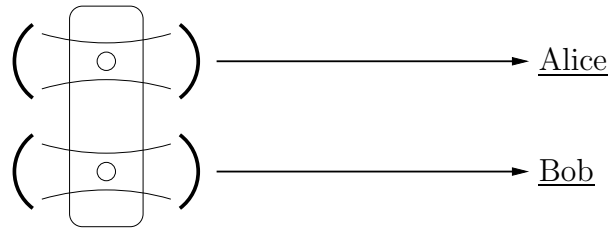
That is, the early photon wavepacket  $|E\rangle$  has left the cavities. We then perform a bit-flip on the qubits, after which we apply the second  $\pi$ -pulse to both cavities. After a further time  $t_{\text{wait}}$ , this yields the highly entangled state

$$|\uparrow\downarrow\rangle|E, L\rangle + |\downarrow\uparrow\rangle|L, E\rangle .$$

This may be thought of as four-party hetero-entanglement.

When we want two-photon entanglement, we have to *disentangle* the photons from the matter qubits. To this end, we apply a Hadamard operation to both matter qubits. The total state then becomes

$$|\Psi\rangle = \frac{1}{2\sqrt{2}}|\uparrow\uparrow\rangle(|E, L\rangle + |L, E\rangle)$$



**Figure 3.** Creating two time-bin entangled photons. We start with the two matter qubits in the entangled state  $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ . The first  $\pi$ -pulse in the cavity system yields the early photon. After a bit flip on both qubits, the second  $\pi$ -pulse yields the late photon.

$$\begin{aligned}
 & + \frac{1}{2\sqrt{2}}|\uparrow\downarrow\rangle(-|E, L\rangle + |L, E\rangle) \\
 & + \frac{1}{2\sqrt{2}}|\downarrow\uparrow\rangle(|E, L\rangle - |L, E\rangle) \\
 & - \frac{1}{2\sqrt{2}}|\downarrow\downarrow\rangle(|E, L\rangle + |L, E\rangle) .
 \end{aligned} \tag{3}$$

A measurement of the matter qubits in the computational basis will then reveal which particular form of entanglement we have prepared. As in the case of hetero-entanglement between matter qubits and polarization qubits, when the measurement outcome is  $\uparrow\downarrow$  or  $\downarrow\uparrow$ , the relative minus sign can be directly corrected using a fast-switching phase shift on one of the qubits, or by taking account of the relative phase when interpreting subsequent measurement outcomes. When the output of the two cavities is sent to Alice and Bob, respectively, they will share maximal two-photon time-bin entanglement (see Fig. 3).

In order to make maximal dual rail and polarization entanglement, we need four matter qubits in four separate leaky cavities (see Fig. 4). A  $\pi$ -pulse applied to each of these systems will yield the following transformation, after a time  $t_{\text{wait}}$ :

$$\begin{aligned}
 |\tilde{0}\rangle \otimes |0, 0\rangle & \rightarrow |\tilde{0}\rangle \otimes |1, 0\rangle \\
 |\tilde{1}\rangle \otimes |0, 0\rangle & \rightarrow |\tilde{1}\rangle \otimes |0, 1\rangle .
 \end{aligned} \tag{4}$$

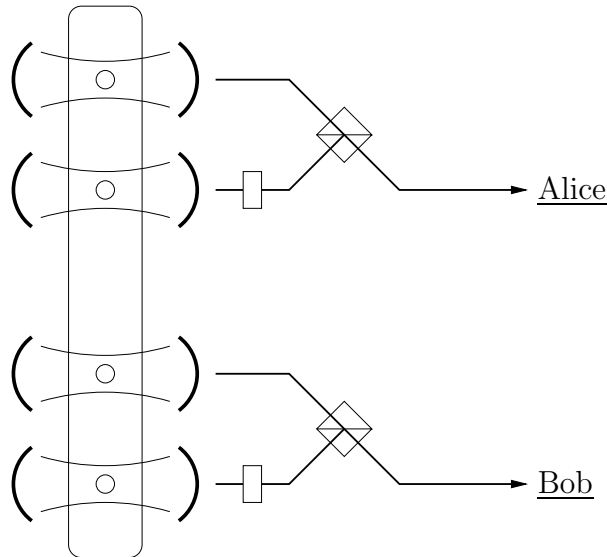
where we have used the composite qubit systems  $|\tilde{0}\rangle$  and  $|\tilde{1}\rangle$ . The states  $|0, 1\rangle$  and  $|1, 0\rangle$  denote a single photon with a dual-rail degree of freedom.

The four matter qubits must first be prepared in the state  $|\tilde{0}, \tilde{1}\rangle + |\tilde{1}, \tilde{0}\rangle$ . This state corresponds to a GHZ state of four matter qubits which, as noted above, may be generated by first preparing a larger cluster state and subsequently performing single qubit measurements. Subsequently, applying a  $\pi$ -pulse to each matter qubit generates a highly entangled state

$$|\tilde{0}, \tilde{1}\rangle|1, 0; 0, 1\rangle + |\tilde{1}, \tilde{0}\rangle|0, 1; 1, 0\rangle .$$

Again, we have to transform each of the four matter qubits with a Hadamard gate, and perform a measurement of the four matter qubits in the computational basis. Up to correctable relative phases, we have obtained the required entangled state.





**Figure 4.** Creating polarization entanglement using four entangled matter qubits. These four qubits are prepared in the GHZ state  $|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle$ . This will create dual-rail entangled photons after a  $\pi$ -pulse to the four-qubit system. As in Fig. 2b), one photon qubit mode undergoes a polarization rotation before the modes are combined at a PBS for transmission.

There is a duality between the dual-rail representation and the time-bin representation. In the dual-rail (or polarization) representation, we need  $2N$  matter qubits in a suitable state to create an  $N$ -photon entangled state  $|\psi\rangle$ . On the other hand, in the time-bin representation we need only  $N$  matter qubits, but we need to apply the  $\pi$ -pulse twice (with an intermediary bit flip on all the qubits). In their respective computational bases, these representations are thus completely equivalent.

## 6. Multi-photon entanglement

The definition of  $|\tilde{0}\rangle$  and  $|\tilde{1}\rangle$  is particularly fruitful, in that it immediately suggests a generalization of the schemes presented here. We consider the generation of polarized photonic states. Let  $|\mathcal{P}_k\rangle$  be a string of  $N$  photonic qubit states in the computational basis (for example, when  $N = 2$ , the four strings  $|\mathcal{P}_k\rangle$  for polarized photons are  $|H, H\rangle$ ,  $|H, V\rangle$ ,  $|V, H\rangle$ , and  $|V, V\rangle$ ). Any pure (entangled) state can then be written as

$$|\psi\rangle = \sum_{k=1}^{2^N} \alpha_k |\mathcal{P}_k\rangle$$

In order to create such a state, we need to create a  $2N$ -qubit state

$$|\Psi\rangle = \sum_{k=1}^{2^N} \alpha_k |\mathcal{S}_k\rangle \otimes |0\rangle_P, \quad (5)$$

where  $\mathcal{S}_k$  is a string of  $N$  bipartite qubits  $|\tilde{0}\rangle$  and  $|\tilde{1}\rangle$  and  $|0\rangle_P$  the vacuum of  $N$  spatial modes. If such a state can be obtained with a quantum computer, we can use the technique in [16] to create the appropriate cluster state and do the computation. The final state is then given by Eq. (5).

A  $\pi$ -pulse into all  $2N$  cavities will then create the photons, which are still heavily entangled with the qubits:

$$|\Psi\rangle \rightarrow \sum_{k=1}^{2^N} \alpha_k |\mathcal{S}_k\rangle \otimes |\mathcal{P}_k\rangle_P.$$

Applying local Hadamard operations to all matter qubits and measuring them in the computational basis then yields the desired state up to local single-photon transformations.

$$|\psi_\ell\rangle_P = \text{Tr}_S \left[ H^{\otimes 2N} |\Psi\rangle \langle \Psi| H^{\otimes 2N} \hat{D}_\ell \right],$$

where  $D_\ell$  is the detector signature of all  $2N$  matter qubits, and  $H$  is the Hadamard transform of a single matter qubit. There are  $4^N$  possible detector outcomes (indexed by  $\ell$ ), and the output state  $|\psi_\ell\rangle_P$  can be transformed into  $|\psi\rangle$  with phase shifters and polarization rotations. The procedures described above for generating dual rail or time-bin entanglement can similarly be generalized to the multi-qubit case.

It is worth noting briefly the effect that photon loss has on the scalability of the scheme. Photon loss can, ultimately, be detected, since in many applications, all the photons can be detected at the end of the experiment. Thus photon loss in itself need not degrade the fidelity of the resulting entangled states. However, the total success probability (i.e. the probability of detecting all  $N$  photons) scales exponentially with  $N$ . This will ultimately place an upper limit on  $N$  in a practical experiment. Nevertheless, even relatively small numbers of entangled photonic qubits can be a useful resource in quantum information processing tasks such as cryptography [12] and Heisenberg-limited metrology [14]. Furthermore, they can be used as a resource in linear optical quantum computing schemes [13], which are in principle scalable.

## 7. Conclusion

In conclusion, we have shown how the double-heralded entangling operation introduced by Barrett and Kok [16] can be used to create multi-partite photonic entanglement. In particular, we can create any (entangled) state of photonic qubits that can be written as the outcome of a universal quantum computer. This works in the polarization and dual rail basis, as well as in the time-bin degree of freedom. In addition, we can generate hetero-entanglement between (many) qubits of a different physical nature. This is crucial for distributed quantum computing, where local clusters of matter qubits can be connected via optical (flying) qubits.

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