Quantum non-demolition measurements: a new resource for making linear logic scalable

Kae Nemoto[1], William J. Munro, Timothy P. Spiller,
R.G. Beausoleil[2]
Trusted Systems Laboratory
HP Laboratories Bristol
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E-mail: firstname.lastname@hp.com

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We discuss a new and novel approach to the problem of creating a photon number resolving detector using the giant Kerr nonlinearities available in electromagnetically induced transparency. Our scheme can implement a photon number quantum non-demolition measurement with high efficiency (>99%) which can distinguish 0, 1 and 2 photons. We then show how it is possible to construct a near deterministic CNOT using several single photons sources, linear optics, photon number resolving quantum non-demolition detectors and feed-forward.

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[1] National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan
[2] HP Laboratories, 13837 175th Pl., NE, Redmond, WA 98052-2180, USA
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Kae Nemoto*, W. J. Munro†*, T. P. Spiller† and R. G. Beausoleil**

*National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan
†Hewlett-Packard Laboratories, Filton Road, Stoke Gifford, Bristol BS34 8QZ, United Kingdom
**Hewlett-Packard Laboratories, 13837 175th Pl. NE, Redmond, WA 98052–2180, USA

Abstract. We discuss a new and novel approach to the problem of creating a photon number resolving detector using the giant Kerr nonlinearities available in electromagnetically induced transparency. Our scheme can implement a photon number quantum non-demolition measurement with high efficiency (>99%) which can distinguish 0, 1 and 2 photons. We then show how it is possible to construct a near deterministic CNOT using several single photons sources, linear optics, photon number resolving quantum non-demolition detectors and feed-forward.

INTRODUCTION

In recent years we have seen signs of a new technological revolution in information processing, a revolution caused by a paradigm shift to information processing using the laws of quantum physics. One natural architecture for realising quantum information processing technology would be to use states of light as the information processing medium. There have been significance developments in all optical quantum information processing (QIP) following the recent discovery by Knill, Laflamme and Milburn[1] that passive linear optics, photo-detectors, and single photon sources can be used to create massive reversible nonlinearities. In principle, fundamental operations such as the CNOT gate have been demonstrated experimentally[2, 3, 4]. However, such operations are relatively inefficient and hence are not currently scalable. This is primarily due to the current state of the art in single photon sources and detectors. Good progress is being made on the development of single photon sources but current single photon detectors at visible wavelengths have efficiencies only up to 90%. Before *true* optical universal quantum computation and information processing can be achieved, the efficiency of such detectors must be significantly improved.

In this article we describe a new and novel single photon detection scheme based on the application of the giant Kerr nonlinearities achievable with electromagnetically induced transparency (EIT)[5]. The scheme uses the giant Kerr nonlinearity to perform a photon number quantum non-demolition (QND) measurement on the signal mode, with only a few hundred EIT atoms and a weak pulse in the probe mode[6]. The effect of the QND measurement in turn means that signal photons are not destroyed and can be reused if required. If the signal mode is in a superposition state (for instance a weak coherent state), then the QND measurement can project the signal mode into a definite number state. This has enormous implications for optical quantum computation and in fact allows a near deterministic CNOT gate to be constructed.

SINGLE PHOTON QND DETECTION

Let us begin with a brief description of how the photon number QND measurement can be achieved using a cross Kerr nonlinearity[7, 8]. The scheme is depicted schematically in Figure 1. The cross Kerr nonlinearity has a Hamiltonian of the form $H_{QND} = \hbar\chi a_s^\dagger a_s c_p^\dagger c_p$ where the signal (probe) mode has the creation and destruction operators given by $a_s^\dagger, a_s$ ($c_p^\dagger, c_p$) respectively, with χ being the strength of the nonlinearity. If we consider the signal state $|\psi\rangle = c_0|0\rangle_a + c_1|1\rangle_a + c_2|2\rangle_a$, with the probe beam initially in a coherent state $|\alpha\rangle_p$, the cross-Kerr interaction causes the combined signal/probe system to evolve as

$$U_{ck}|\psi\rangle_a|\alpha\rangle_p = e^{iH_{QND}t/\hbar}\left[c_0|0\rangle_a + c_1|1\rangle_a + c_2|2\rangle_a\right]|\alpha\rangle_p = c_0|0\rangle_a|\alpha\rangle_p + c_1|1\rangle_a|\alpha e^{i\theta}\rangle_p + c_2|2\rangle_a|\alpha e^{2i\theta}\rangle_p \quad (1)$$

where $\theta = \chi t$ with t being the interaction time. We observe immediately that the Fock state $|n_a\rangle$ is unaffected by the interaction but the coherent state $|\alpha_c\rangle$ picks up a phase shift directly proportional to the number of photons n_a in the $|n_a\rangle$ state. Also it is clear that the signal and probe fields have become entangled and hence a highly efficient homodyne/heterodyne measurement of one of the specific phase shifted coherent states will project the signal mode into a definite number state $|n\rangle_a$ or superposition of number states. For this measurement to have the photon number resolving ability we need for QIP applications we require $\alpha\theta \gg 1$. The requirement $\alpha\theta \gg 1$ is interesting as it tells us that a large nonlinearity θ is not required to distinguish different $|n_a\rangle$, even for zero, one and two photon states. We can have θ small but then require α, the amplitude of the probe beam, large. This is entirely possible and means that we can operate in the regime $\theta \ll 1$ compared with $\theta = \pi$ for a direct single photon-single photon CNOT gate. This is the versatility these devices have for QIP applications.

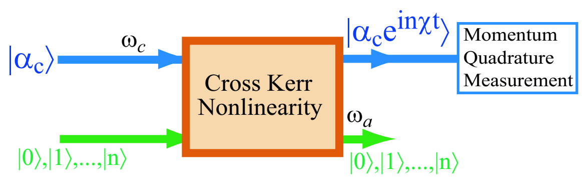

FIGURE 1. Schematic diagram of a photon resolving detector based on a cross Kerr Nonlinearity. The two inputs are a Fock state $|n_a\rangle$ (with $n_a = 0, 1, ..$) in the signal mode a and a coherent state with real amplitude α_c in the probe mode c. The presence of photons in mode a causes a phase shift on the coherent state $|\alpha_c\rangle$ directly proportional to n_a which can be determined with a momentum quadrature measurement.

THE GENERATION OF CROSS-KERR NONLINEARITIES

We now address the generation of the weak nonlinearities required to perform the QND measurement. We will consider an EIT model (depicted in Fig. 2) of the nonlinear electric dipole interaction between three quantum electromagnetic radiation fields with angular frequencies ω_a, ω_b, ω_c and a corresponding four-level $\mathscr{N}$ atomic system. Here we define the effective vacuum Rabi frequency of each interacting field as $|\Omega_k|^2 = \sigma_k A_k \Delta\omega_k / 8\pi\mathscr{A}$, where $\sigma_k \equiv 3\lambda_k^2/2\pi$ is the resonant atomic absorption cross section at wavelength $\lambda_k \cong 2\pi c/\omega_k$, $\mathscr{A}$ is the effective laser mode cross-sectional area, A_k is the spontaneous emission rate between the two corresponding atomic levels, and $\Delta\omega_k$ is the bandwidth of the profile function describing the adiabatic interaction of a pulsed laser field with a stationary atom. We consider a number

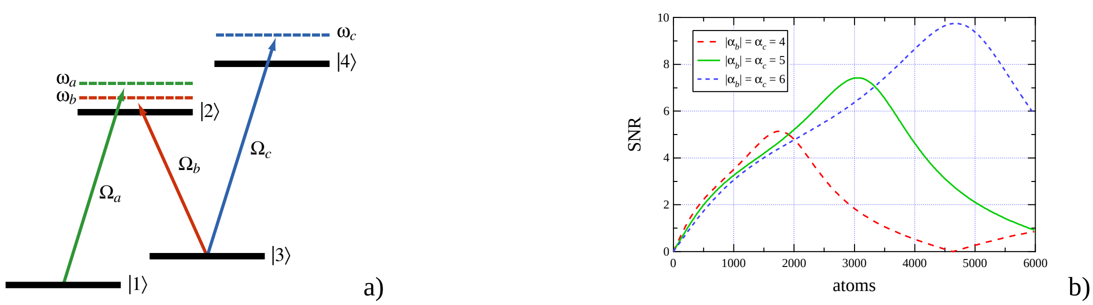

FIGURE 2. In a) Schematic diagram of the interaction between a four-level $\mathscr{N}$ atom and a nearly resonant three-frequency electromagnetic field. We note that the annihilation of a photon of frequency ω_k is represented by the complex number Ω_k. In b) Plot of the signal-to-noise ratio as a function of the number of atoms localized in the interaction region for $\nu_c/\gamma_2 = 30$ with $|\alpha_b| = \alpha_c = 4, 5, 6$.

N of $\mathscr{N}$ atoms, fixed and stationary in a volume that is small compared to the optical wavelengths, and that the three frequency channels of the resonant four-level manifold of the quantum system are driven by Fock states containing n_a, n_b, and n_c photons, respectively. Then, if the durations of the three pulse envelope functions are long compared to the lifetime of atomic level $|2\rangle$ and if we assume that the laser frequencies ω_a and ω_b are both precisely tuned to the corresponding atomic transition frequencies, that de-phasing is negligible, and that the spontaneous emission branching ratios from atomic levels $|2\rangle$ and $|4\rangle$ are approximately unity we get the following effective Hamiltonian for the interaction between the optical fields

$$H_{eff} = \hbar \frac{N\,|\Omega_a|^2\,|\Omega_c|^2\,\bar{n}_a\,\bar{n}_c}{\nu_c\,|\Omega_b|^2\,\bar{n}_b + i\left(\gamma_4\,|\Omega_b|^2\,\bar{n}_b + \gamma_2\,|\Omega_c|^2\,\bar{n}_c\right)}. \tag{2}$$

where $\nu_c \equiv \omega_c - \omega_{43}$, $\gamma_2 \approx A_{21}$, and $\gamma_4 \approx A_{43}$. The probability that a single photon in channel a will be scattered by one of the atoms becomes vanishingly small when $\nu_c/\gamma_4 \gg 1$ with $|\Omega_b|^2\,|\alpha_b|^2/\gamma_2 \approx |\Omega_c|^2\,|\alpha_c|^2/\gamma_4$. We now assume that the interaction region is encapsulated within a waveguide that has an effective cross-sectional area approximately equal to $3\lambda_a^2/2\pi$, and that the pulses have weak super-Gaussian profiles so that the bandwidth-interaction time product is $\Delta\omega_k t = 8$. Thus $|\Omega_a|^2 t \approx \gamma_2/\pi$, and — if inequality is satisfied — we can obtain a phase shift θ given by $\theta \approx N\gamma_2/\pi\,\nu_c\,|\alpha_b|^2$. This now allows us to determine the number of atoms necessary to give a large enough phase shift so that the phase shifted coherent states are distinguishable from the original ones. We consider the specific case where our signal mode has a single photon in it, $n_a = 1$, with the probe field α_c real and use the signal-to-noise ratio given by $\mathrm{SNR}_Y = (\langle Y(1)\rangle - \langle Y(0)\rangle)/\sqrt{\langle Y^2(1)\rangle - \langle Y(1)\rangle^2}$ to quantity the performance of the measurement in inferring the presence of the photon in the a mode. Here $Y(n_a)$ represent a Y-quadrature homodyne measurement where there was initially n_a photons in the signal mode. Now the higher the signal to noise ratio the better the inference that a single photon was present in the signal. We generally want $\mathrm{SNR}_Y > 2$ for a good interference. In Fig (2 b) we plot the signal-to-noise ratio as a function of the number of atoms localized in the interaction region for a detuning of $\nu_c/\gamma_2 = 30$. If we had a perfect cross-Kerr nonlinearity, each curve in Fig (2 b) would be given by $2\,|\alpha_b|\sin\theta$ and would exhibit a peak at $N = 15\pi^2\,|\alpha_b|^2$ atoms. Instead, the peaks correspond to phase shifts smaller than $\pi/2$ because of the dependence for the c mode on n_b. With approximately 570 atoms, a phase shift of 0.24 radians corresponding to a SNR value of 2.19 is achievable. This shows the potential of using EIT for constructing QND detectors.

ENTANGLING GATES

In this fock state detection model described above we measure the phase of the probe beam immediately after it has interacted with the weak cross-Kerr nonlinearity. This is the regime where the QND detector functions like the standard single photon detector. However, if we want to do a more ”generalised” type of measurement between different signal beams, we could delay the measurement of the probe beam instead having the probe beam interacts with several cross-Kerr nonlinearities where the signal beam is different in each case. The probe beam measurement then occurs after all these interactions which could allow for multi-qubit conditioning. We may not be able to tell which qubit or components of that qubit caused the phase shift.

Consider for instance two polarisation qubits initially prepared in the states $|\Psi\rangle_a = c_0|H\rangle_a + c_1|V\rangle_a$ and $|\Psi\rangle_b = d_0|H\rangle_b + d_1|V\rangle_b$ where the a, b label the first and second qubits respectively. These qubits are split individually on polarizing beam-splitters (PBS) and interacts with cross Kerr nonlinearities as shown in Figure (3). Assuming that $\theta^2\alpha \gg 1$ an X quadrature measurement projects the signal modes into either

$$|\psi\rangle_T \sim c_0 d_0|HH\rangle + c_1 d_1|VV\rangle \tag{3}$$

$$|\psi\rangle_T \sim c_0 d_1 e^{i\phi(X)}|HV\rangle + c_1 d_0 e^{-i\phi(X)}|VH\rangle \tag{4}$$

The first situations occurs for measurement results X where $X > \alpha(1+\cos 2\theta)$ and the second for $X < \alpha(1+\cos 2\theta)$. We observe that this second case is dependent on the measurement result X, however simple local rotations using phase shifters can be performed to transform this second state to $c_0 d_1|H\rangle_a|V\rangle_b + c_1 d_0|V\rangle_a|H\rangle_b$ which is independent of X. It is now very clear the action of this two mode gate; it splits the even parity terms nearly deterministically from the odd parity cases. This is really the power enabled by non-demolition measurements and why we can engineer strong nonlinear interactions using weak cross Kerr effects. From a different perceptive our two mode gate is acting like a polarizing beam-splitter but that does not allow the photon bouncing effects. These transformations are very interesting as it seems possible with the appropriate choice of c_0, c_1 and d_0, d_1 to create arbitrary entangled states near deterministically from separable ones.

There are a number of ways that we can use this entangling gate. For instance we can use these entangling gates at the core of Franson parity and destructive CNOT gates. Here our entangling gate replace Franson PBS's. Such a

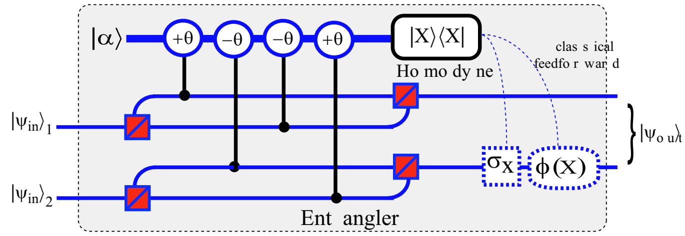

FIGURE 3. Schematic diagram of a two qubit polarisation QND detector that distinguishes superpositions and mixtures of the states $|HH\rangle$ and $|VV\rangle$ from $|HV\rangle$ and $|VH\rangle$ using several cross Kerr nonlinearities nonlinearities and a coherent laser probe beam $|\alpha\rangle$. The scheme works by first splitting each polarisation qubit into a which path qubit; the $|H\rangle$ qubit is transformed to $|10\rangle$ while the $|V\rangle$ transforms to $|01\rangle$. Thus the two polarisation encoded qubits can be encoded into four spatial modes, $|HH\rangle \rightarrow |1010\rangle$, $|HV\rangle \rightarrow |1001\rangle$, $|VH\rangle \rightarrow |0110\rangle$ an $|VV\rangle \rightarrow |0101\rangle$. The action of the first (and fourth) cross Kerr nonlinearity put a phase shift θ on to the probe beam only if a photon was present in that mode. The second (and third) cross Kerr nonlinearity put a phase shift $-\theta$ on to the probe beam only if a photon was present in that mode. After the four nonlinear interactions the which path qubit are converted back to polarisation encoded qubits. The probe beam only picks up a phase shift if the states $|HV\rangle$ and/or $|VH\rangle$ were present and hence the appropriate homodyne measurement allows the states $|HH\rangle$ and $|VV\rangle$ to be distinguished from $|HV\rangle$ and $|VH\rangle$. If we consider that the input state of the two polarisation qubit is $|HH\rangle + |HV\rangle + |VH\rangle + |VV\rangle$ then after the parity gate we have conditioned on an X homodyne measurement either the state $|HH\rangle + |VV\rangle$ or $e^{i\phi(X)}|HV\rangle + e^{-i\phi(X)}|VH\rangle$ where $\phi(X)$ is a phase shift dependent on the result of the homodyne measurement. A simple phase shift achieved via classical feed-forward then allows this second state to be transformed to the first and thus provides a near deterministic entangling gate

change allows us to construct a non-destructive version of Franson four photon CNOT[9] near deterministically. This represents a huge saving in the physical resources to implement single photon quantum logic. Alternatively it is well known that any entangling two-qubit gate is universal for quantum computation, when assisted by one-qubit gates[10]. We have such an entangling gate here and hence with trivial single qubit rotations a universal set of gates.

DISCUSSION

To summarize, we have shown a scheme for a highly efficient single photon number resolving detector based on the cross Kerr nonlinearity produced by an EIT system with several hundred atoms. Our detection scheme is based on a photon number QND measurement, where the phase of a probe beam is altered in proportion to the number of photons in the signal beam. The scheme does not destroy photons present in the signal mode, and it allows conditioning of their evolution. This has immediate applications for linear logic. Finally, we show how it is possible using weak cross-Kerr nonlinearities to construct a near deterministic CNOT gate with far fewer physical resources that for other all linear optical schemes. The strength of the nonlinearities are orders of magnitude weaker than those required to perform CNOT gates naturally between the single photons.

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