A high-efficiency quantum non-demolition single photon number resolving detector

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A high-efficiency quantum non-demolition single photon number resolving detector

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In recent years we have seen signs of a new technological revolution in information processing, a revolution caused by a paradigm shift to information processing using the laws of quantum physics.[1] One natural architecture for realising quantum information processing technology would be to use states of light [2] as the information processing medium. There have been significance developments in all optical quantum information processing (QIP) following the recent discovery by Knill, Laflamme and Milburn that passive linear optics, photodetectors, and single photon sources can be used to create massive reversible nonlinearities.[3] Such nonlinearities are an essential requirement for optical quantum computation and many communication applications. These nonlinearities allow efficient gate operations to be performed. In principle, fundamental operations such as the nonlinear sign shift and CNOT gates have been demonstrated experimentally.[4] However, such operations are relatively inefficient (they have a probability of success significantly less than 50 percent) and hence are not scalable. This is primarily due to the current state of the art in single photon sources and detectors. Good progress is being made on the development of single photon sources [5] but current single photon detectors at visible wavelengths have efficiencies only up to 90% whilst in the microwave regime the efficiencies are much lower at approximately 30%. Before *true* optical universal quantum computation and information processing can be achieved, the efficiency of such detectors must be significantly improved. This is likely to require a drastic change in the approach to detection technology.[6]

In this letter we describe a new and novel single photon detection scheme based on the application of the giant Kerr nonlinearities achievable with electromagnetically induced transparency (EIT).[7] The scheme uses the giant Kerr nonlinearity to perform a photon number quantum non-demolition (QND) measurement on the signal mode, with only a few hundred EIT atoms and a weak pulse in the probe mode. The effect of the QND measurement in turn means that signal photons are not destroyed and can be reused if required. If the signal mode is in a superposition state (for instance a weak coherent state), then the QND measurement can project the signal mode into a definite number state.[8] Furthermore this effect can be used in junction with EIT's slow light effect to create a near deterministic on-demand single photon source.

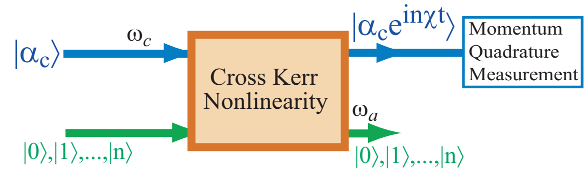


FIG. 1: Schematic diagram of a photon resolving detector based on a cross Kerr Nonlinearity. The two inputs are a Fock state $|n_a\rangle$ (with $n_a = 0, 1, ..$) in the signal mode a and a coherent state with real amplitude α_c in the probe mode c. The presence of photons in mode a causes a phase shift on the coherent state $|\alpha_c\rangle$ directly proportional to n_a which can be determined with a momentum quadrature measurement.

Before we begin our detailed discussion of the EIT detection scheme, we first consider the photon number QND measurement using a cross Kerr nonlinearity, which has a Hamiltonian of the form: $H_{QND} = \hbar\chi a^\dagger a c^\dagger c$ where the signal (probe) mode has the creation and destruction operators given by $a^\dagger, a$ ($c^\dagger, c$) respectively and χ is the strength of the nonlinearity. If the signal field contains n_a photons and the probe field is in an initial coherent state with amplitude α_c, the cross Kerr nonlinearity causes the combined system to evolve as

$$|\Psi(t)\rangle_{out} = e^{i\chi t a^\dagger a c^\dagger c}|n_a\rangle|\alpha_c\rangle = |n_a\rangle|\alpha_c e^{in_a\chi t}\rangle. \quad (1)$$

We observe immediately that the Fock state $|n_a\rangle$ is unaffected by the interaction but the coherent state $|\alpha_c\rangle$ picks up a phase shift directly proportional to the number of photons n_a in the $|n_a\rangle$ state. If we measure this phase shift we can infer the number of photons in the signal mode a. Such a measurement can be achieved simply with a homodyne measurement (depicted schematically in Figure (1)).[9] The homodyne apparatus allows measurement of the quadrature operator $x(\theta) = ce^{i\theta} + c^\dagger e^{-i\theta}$

with an expected result

$$\langle x(\theta)\rangle \ = \ 2\mathrm{Re}\left[\alpha_c\right]\cos\delta + i2\mathrm{Im}\left[\alpha_c\right]\sin\delta \qquad (2)$$

where $\delta = \theta + n_a\chi t$. For a real initial α_c, a highly efficient homodyne measurement of the position $X = a + a^\dagger$ or momentum $iY = a - a^\dagger$ quadratures would yield signals

$$\langle X\rangle \ = \ 2\alpha_c\cos\left(n_a\chi t\right) \qquad (3)$$
$$\langle Y\rangle \ = \ 2\alpha_c\sin\left(n_a\chi t\right) \qquad (4)$$

with unit variance. For the momentum quadrature this gives a signal-to-noise ratio $\mathrm{SNR}_Y = 2\alpha_c\sin\left(n_a\chi t\right)$ which should be much greater than unity for the different n_a inputs to be distinguished. In more detail, if the inputs in mode a are the Fock state $|0\rangle$ or $|1\rangle$, the respective outputs of the probe mode c are the coherent states $|\alpha_c\rangle$ or $|\alpha_c e^{i\chi t}\rangle$. The probability of misidentifying these states is then given by

$$P_{\mathrm{error}} = \left|\langle\alpha_c|\alpha_c e^{i\chi t}\rangle\right|^2 = \exp\left[-4\alpha_c^2\sin^2\left(\chi t\right)\right]. \qquad (5)$$

which can be written as $P_{\mathrm{error}} = \exp\left[-\mathrm{SNR}_Y^2\right]$. A signal to noise ratio of $\mathrm{SNR}_Y = 3$ would thus give $P_{\mathrm{error}} \sim 10^{-4}$. To achieve the necessary phase shift we require $\alpha_c\sin\left(\chi t\right) \approx 3$, which can be achieved in a number of ways dependent upon the range of values available for α_c and χt. For example, we could choose $\alpha_c \gg 3$ with χt small and satisfy the above inequality; alternatively we could choose $\chi t = \pi/2$ with $\alpha_c = 3$. The particular regime chosen depends on the strength of the Kerr nonlinearity achievable in the physical system.

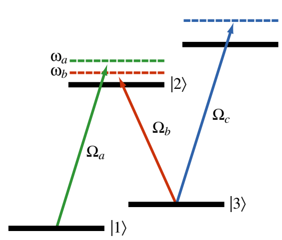


FIG. 2: Schematic diagram of the interaction between a four-level $\mathcal{N}$ atom and a nearly resonant three-frequency electromagnetic field. We note that the annihilation of a photon of frequency ω_k is represented by the complex number Ω_k.

We now address the generation of the large nonlinearity required to perform the QND measurement. We consider a model (depicted in Fig. 2) of the nonlinear electric dipole interaction between three quantum electromagnetic radiation fields with angular frequencies ω_a, ω_b, ω_c and a corresponding four-level $\mathcal{N}$ atomic system.[10] Here we define the effective vacuum Rabi frequency of each interacting field as

$$\left|\Omega_k\right|^2 = \frac{1}{8\pi}\,\frac{\sigma_k}{\mathcal{A}}\,A_k\,\Delta\omega_k, \qquad (6)$$

where $\sigma_k \equiv 3\lambda_k^2/2\pi$ is the resonant atomic absorption cross section at wavelength $\lambda_k \cong 2\pi c/\omega_k$,[11] $\mathcal{A}$ is the effective laser mode cross-sectional area, A_k is the spontaneous emission rate between the two corresponding atomic levels, and $\Delta\omega_k$ is the bandwidth of the profile function describing the adiabatic interaction of a pulsed laser field with a stationary atom.[12–14] We consider a number N of $\mathcal{N}$ atoms, fixed and stationary in a volume that is small compared to the optical wavelengths, and that the three frequency channels of the resonant four-level manifold of the quantum system are driven by Fock states containing n_a, n_b, and n_c photons, respectively. Then, if the durations of the three pulse envelope functions are long compared to the lifetime of atomic level $|2\rangle$, the evolution of the unperturbed number eigenstate $|1, n_a, n_b, n_c\rangle$ is simply given by

$$|1, n_a, n_b, n_c\rangle \longrightarrow e^{-iWt}\,|1, n_a, n_b, n_c\rangle\,. \qquad (7)$$

If we assume that the laser frequencies ω_a and ω_b are both precisely tuned to the corresponding atomic transition frequencies, that dephasing is negligible, and that the spontaneous emission branching ratios from atomic levels $|2\rangle$ and $|4\rangle$ are approximately unity, then W is given by

$$W = \frac{N\,\left|\Omega_a\right|^2\left|\Omega_c\right|^2 n_a n_c}{\nu_c\left|\Omega_b\right|^2 n_b + i\left(\gamma_4\left|\Omega_b\right|^2 n_b + \gamma_2\left|\Omega_c\right|^2 n_c\right)}. \qquad (8)$$

where $\nu_c \equiv \omega_c - \omega_{43}$, $\gamma_2 \approx A_{21}$, and $\gamma_4 \approx A_{43}$. We see immediately from (8) that W is complex in nature indicating potential absorption of the photons in $|n_a\rangle$. However in the parameter regime where the inequality

$$\frac{\left|\Omega_b\right|^2\left|\alpha_b\right|^2}{\gamma_2}\,\frac{\nu_c}{\gamma_4} \gg \frac{\left|\Omega_b\right|^2\left|\alpha_b\right|^2}{\gamma_2} + \frac{\left|\Omega_c\right|^2\left|\alpha_c\right|^2}{\gamma_4} \qquad (9)$$

is satisfied, the probability that a single photon in channel a will be scattered by one of the atoms becomes vanishingly small. (This condition is equivalent for the simplified case where $\left|\Omega_b\right|^2\left|\alpha_b\right|^2/\gamma_2 \approx \left|\Omega_c\right|^2\left|\alpha_c\right|^2/\gamma_4$ to the assumption $\nu_c/\gamma_4 \gg 1$). Working in this regime, the ground state $|1, n_a, n_b, n_c\rangle$ acquires a phase-shift for the nonlinear mechanism. It is this phase shift that is the basis of our high efficiency nondestructive detector.

We now consider the evolution of an N-atom quantum state during an interaction with an n_a-photon Fock state in the a channel, and weak coherent states parametrized by α_b and α_c in the b and c channels, respectively. It is straightforward to calculate the evolution of the atom-field state by evaluating the sum over the Fock states representing each coherent state.[10] With an initial state of $|\psi\left(n_a\right)\rangle_{in} = |1, n_a, \alpha_b, \alpha_c\rangle$, after a time t we find

$$|\psi\left(n_a\right)\rangle_{out} = e^{-\frac{1}{2}|\alpha_b|^2}\sum_{n_b=0}^{\infty}\frac{\alpha_b^{n_b}}{\sqrt{n_b!}}\left|1, n_a, n_b, \alpha_c e^{-i\,n_a\phi\,\frac{|\alpha_b|^2}{n_b}}\right\rangle \qquad (10)$$

where the angle ϕ is defined by

$$\phi \equiv \frac{N\left|\Omega_a\right|^2\left|\Omega_c\right|^2}{\nu_c\left|\Omega_b\right|^2\left|\alpha_b\right|^2}t. \tag{11}$$

We note that the output state $\left|\psi\left(n_a\right)\right\rangle_{out}$ from the interaction with the four level atoms is no longer a simple tensor product of a Fock state and two coherent state unless $|\alpha_b| \gg 1$, in which case $\left|\psi\left(n_a\right)\right\rangle_{out} \cong \left|1, n_a, \alpha_b, \alpha_c e^{-i\,n_a\phi}\right\rangle$, in agreement with (1). Therefore, only when the coupling field driving channel b is a classical field does the EIT mechanism provide a true cross-Kerr nonlinearity. For weak coherent pulses only an approximate Kerr nonlinearity is generated; the adiabatic elimination of this control field is not permitted in this case. However, this quasi-Kerr nonlinearity can still be used to implement the required detection protocol.

Using the full expression for $\left|\psi\left(n_a\right)\right\rangle_{out}$ it is straightforward to calculate the various moments of the quadrature homodyne operator $\hat{x}(\theta) = ce^{i\theta} + c^\dagger e^{-i\theta}$ on mode c. The first and second quadrature moments are given by

$$\langle\hat{x}(\theta)\rangle = \frac{\sqrt{2}e^{-\left|\alpha_b\right|^2}\left|\alpha_b\right|^{2n_b}}{n_b!}\,\mathrm{Re}\left[\alpha_c\, e^{-i\left(\frac{n_a}{n_b}\phi\left|\alpha_b\right|^2+\theta\right)}\right] \tag{12}$$

$$\left\langle\hat{x}^2(\theta)\right\rangle = \frac{1}{2} + \left|\alpha_c\right|^2 + e^{-\left|\alpha_b\right|^2} \times \sum_{n_b=0}^{\infty}\frac{\left|\alpha_b\right|^{2n_b}}{n_b!}\,\mathrm{Re}\left[\alpha_c^2\, e^{-i2\left(\frac{n_a}{n_b}\phi\left|\alpha_b\right|^2+\theta\right)}\right]. \tag{13}$$

Using these moments we can obtain an estimate of whether the present of a photon in $|n_a\rangle$ is distinguishable from the no photon case. For convenience we again choose α_c real. In this case the probability our detector will register a false positive count for $n_a = 1$ is given by

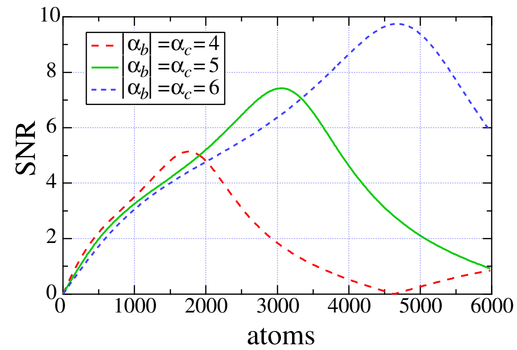
$$\mathrm{SNR}_Y = \frac{\langle Y(1)\rangle - \langle Y(0)\rangle}{\sqrt{\left\langle Y^2(1)\right\rangle - \left\langle Y(1)\right\rangle^2}} \tag{14}$$

where we have defined $Y\left(n_a\right) \equiv \hat{x}\left(\theta = \pi/2, n_a\right)$.

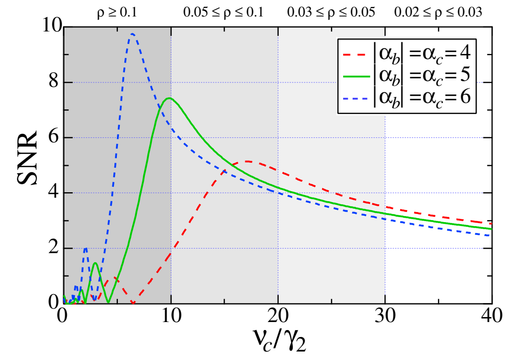
What values of SNR_Y are achievable? To establish an estimate we need to make several assumptions about the physical system and its geometry. We assume that the interaction region (where the light and $\mathcal{N}$ atoms interact) is encapsulated within a waveguide that has an effective cross-sectional area approximately equal to $3\lambda_a^2/2\pi$,[14] and that the pulses have weak super-Gaussian profiles so that the bandwidth-interaction time product is $\Delta\omega_k t = 8$. Thus $\left|\Omega_a\right|^2 t \approx \gamma_2/\pi$, and — if inequality (9) is satisfied — we can obtain a phase shift ϕ given by

$$\phi \approx \frac{N\,\gamma_2}{\pi\,\nu_c\left|\alpha_b\right|^2}. \tag{15}$$

Therefore, given typical values for the relative detuning ν_c/γ_2 and $\langle n_b\rangle = \left|\alpha_b\right|^2$, we can now determine the number of atoms needed to provide a given phase shift.



(a)



(b)

FIG. 3: Plot of the signal-to-noise ratio given by (14) as a function of: (a) the number of atoms localized in the interaction region for $\nu_c/\gamma_2 = 30$ with $|\alpha_b| = \alpha_c = 4, 5, 6$; (b) ν_c/γ_2 with 1000 atoms localized in the interaction region again with $|\alpha_b| = \alpha_c = 4, 5, 6$.

Fig (3 a) shows the signal-to-noise ratio as a function of the number of atoms localized in the interaction region for a detuning of $\nu_c/\gamma_2 = 30$ for three different values of $|\alpha_b| = \alpha_c$. If the state defined by (10) was indeed a coherent state, each curve in Fig (3) would be given by $2\left|\alpha_b\right|\sin\left(\phi\right)$ and would exhibit a peak at $N = 15\,\pi^2\left|\alpha_b\right|^2$ atoms. Instead, the peaks correspond to phase shifts smaller than $\pi/2$ because of the dependence for the c mode in (10) on n_b. In practice, we must choose a value of $|\alpha_b|$ that creates a sufficiently large transparency window in the a channel;[10] for the parameters chosen here, we must have $\left|\alpha_b\right|^2 > 8\pi \approx 25$. Thus, from Fig. (3) and (14), we can determine the number of atoms needed to provide a sufficiently low probability of a false positive detection. With approximately 570 atoms, a phase shift of 0.24 radians corresponding to a SNR value of 2.19 is achievable. This leads to a false positive detection error probability of approximately 1% with a 0.8% probability of the absorption of the photon in the $1 \longrightarrow 2$ transition. There is a wide range of reasonable parameters which leads to similarly low error rates. For example, to decrease the false positive detection error and absorption rates by an order of magnitude, a detuning of 160, 6900 atoms, and $\alpha_b = \alpha_c = 10$ give a phase shift of 0.137, which leads to a SNR of 2.66 (a false detection probability of 0.08%) and an absorption rate of 0.08%. Generally an increase in the SNR requires an increase in the number of atoms.

However, Fig (3 a) also shows that for a given value of ν_c/γ_2 and α_c, the effect of increasing N eventually leads to a decrease in the SNR. This is simply explained by the fact that we have created too large a phase. If such numbers of atoms are to be used there is a natural correction method, the decrease ν_c/γ_2. In Fig (3 b) we plot the SNR as a function of ν_c/γ_2 for $\alpha_c = 4, 5, 6$ with $N = 1000$. This figure clearly shows an excellent operating window within which the detector can be used. This flexibility is one of the hallmarks of our proposal.

We turn now to the versatility of these devices and their use as detectors for QIP applications. In the discussions so far we have only considered the situation with zero or one photon in the signal mode a. What happens with a more general input? For instance, if we have a Fock state superposition can we use the detection process to condition the evolution of the system, if the appropriate phase shift is observed? For calculational simplicity we revert back to using the ideal cross Kerr nonlinearity rather than the approximate form generated by the EIT mechanism (the same physical results occur). For the superposition $c_0|0_a\rangle + c_1|1_a\rangle$, it is straightforward to show that the joint signal and probe modes evolve according to

$$|\Psi\rangle_{out} = c_0|0_a\rangle|\alpha_c\rangle + c_1|1_a\rangle|\alpha_c e^{i\chi t}\rangle. \tag{16}$$

We see immediately that the signal and probe fields have become entangled assuming $\chi t \neq 2n\pi$ (with n a non negative integer). This entanglement can be used to condition the evolution of the system. With probability $|c_1|^2$, the homodyne measurement will detect a significant phase shift (a phase shift that can not be attributed to the $|\alpha_c\rangle$ component) and the system (16) will be projected into the state $|1_a\rangle|\alpha_c e^{i\chi t}\rangle$. For the more general superposition $|\psi\rangle = \sum c_n|n_a\rangle$, the total system after the Kerr interaction is

$$|\Psi\rangle_{out} = \sum_{n_a} c_{n_a}|n_a\rangle|\alpha_c e^{in_a\chi t}\rangle. \tag{17}$$

Using successive (single or dual) homodyne measurements it is possible to project the system into a specific number state [8]. If our input superposition state were composed of zero, one and two photons, then the detector could project system into one of the specific Fock states and could therefore distinguish between 0, 1 and 2 photons. Also the projection for $|\psi\rangle$ could be onto a series of number states. For instance with $\chi t = \pi$ the state (17) can be written as

$$|\Psi\rangle_{out} = \sum_{n_a\ even} c_{n_a}|n_a\rangle|\alpha_c\rangle + \sum_{n_a\ odd} c_{n_a}|n_a\rangle|-\alpha_c\rangle \tag{18}$$

A dual homodyne (heterodyne) measurement can then be used to project (18) into either the even or odd photon number basis depending on whether the probe mode is measured to be either $|\alpha_c\rangle$ or $|-\alpha_c\rangle$. The probability of failure, that is not being able to assign the measurement result to either $|\pm\alpha_c\rangle$ with high confidence, is small for $\alpha_c \gg 1$. This thus allows the implementation of a parity measurement. More generally, the use of dual homodyne (heterodyne) measurements gives considerable flexibility for the conditioning of the system.

The considerations here have been idealized in that we have not discussed the effects of noise and dissipation. Generally, the use of EIT devices for QIP gates requires very low noise and dissipation.[10] However, for this detector arrangement the bounds are not as strict. One of the chief sources of error is dephasing on the c transition. Its effect on the coherent state $|\alpha_c\rangle$ is to introduce a random phase shift ϕ_r resulting in the state $|\alpha_c e^{i\phi_r}\rangle$. With a general superposition state $c_0|0\rangle + c_1|1\rangle$ as input on the signal channel a, the Y momentum quadrature in the probe mode is given by

$$\langle Y_c\rangle = 2\sum_{n_a}^{1} |c_{n_a}|\alpha_c \sin(\phi_r + n_a\chi t) \tag{19}$$

which depends on the random phase ϕ_r. The distribution for ϕ_r depends on the exact dephasing mechanism present for the EIT system. However, it is clear that for small dephasing ($\phi_r \ll \chi t \leq \pi$) the presence of a large Y_c implies that the state $c_0|0\rangle + c_1|1\rangle$ is still projected to a Fock state with good accuracy.

To summarize, we have shown in this letter a scheme for a highly efficient single photon number resolving detector based on the cross Kerr nonlinearity produced by an EIT system with a few hundred atoms. Our detection scheme is based on a photon number QND measurement, where the phase of a probe beam is altered in proportion to the number of photons in the signal beam. The scheme does not destroy photons present in the signal mode, and it allows conditioning of their evolution. Finally, this detection approach could clearly be combined with the inherent storage ability of EIT systems to turn a non-deterministic single photon source into an on-demand source.

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