Nonparametric Estimation of Asymmetric First Price Auctions: A Simplified Approach

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Abstract

We present an approach to non-parametric estimation of pseudo-values in asymmetric affiliated private values (APV) models for first-price auctions that eliminates the ‘curse of dimensionality.’ We show that pseudo-values can be estimated by using two-dimensional functions of bids for APV models regardless of the form asymmetry.

Keywords: Affiliated Private Values, First Price Auctions, Asymmetric Auctions, Structural Econometrics, Nonparametric Estimation

JEL codes: C14, D44

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1. Introduction

Structural econometrics analysis of auctions treats observed bid data as the Bayesian-Nash equilibrium of a game-theoretic auction model. An important breakthrough in the structural econometrics of first price auctions was Guerre, Perrigne & Vuong (2000)'s (GPV in the sequel) procedure that uses the equilibrium strategies to express a bidder's unobserved valuation in terms of his bid and the distribution and density functions of observed bids. (See Perrigne and Vuong (1999) for a survey of recent contributions to the structural econometrics of first price auctions.) GPV procedure is a two-step estimation procedure. In the first step, a sample of pseudo-values is constructed using the equilibrium inverse bid function. In the second step, the sample of pseudo-values is used to estimate the underlying density of bidders' valuations.

More recently, Campo, Perrigne & Vuong (2003) applied the same method to estimate asymmetric affiliated private values (APV) model of first-price auctions. Campo et al.'s approach requires estimation of high dimensional functions of bids to construct the pseudo-values. As is well known sample sizes required by the kernel methods to achieve a given quality of estimation (e.g. mean-square error) increase dramatically as the dimensionality of the function to be estimated increases[1]. Moreover, as the number of bidder segments increases, the computational complexity of functions to be estimated increases considerably.

[1] For example, Silverman (1998) shows that the sample sizes needed to estimate a multivariate normal density at the origin with a mean-square error less than 0.1 for 1, 2, 3, 4, and 7-dimensional densities are 4, 19, 67, 223, and 10700, respectively.

In this paper, we show that one only needs to estimate two-dimensional functions of bids to generate a sample of pseudo-values for the APV models regardless of the form of asymmetry. The proposed approach reduces the sample size requirements considerably and it is much simpler computationally.

In the next section we introduce the affiliated private values model and the results of Campo *et al*. In section 3 we present our simplified approach that reduces the dimensionality of the functions to be estimated to at most two.

2. An asymmetric APV model of first price auctions

In this section we introduce the asymmetric APV model and the method of construction of pseudo-values used in Campo *et al*. A single and indivisible item is auctioned to n risk neutral bidders. The affiliated private value model with risk neutral bidders is defined by an n-dimensional cumulative distribution function $F(\cdot)$. The vector of private valuations $(v_1, \ldots, v_n)$ is a realization of a random vector with joint distribution $F(\cdot)$.

For $i = 1, ..., n$, denote by $b_i = s_i(v_i)$ and $\phi_i(b_i) = s_i^{-1}(b_i)$ the equilibrium bidding strategy of bidder i, and its inverse, respectively. In equilibrium, the joint distribution of valuations, $F(\cdot)$, and that of bids, $G(\cdot)$, are related by $G(b_1, ..., b_n) = F(\phi_1(b_1), ..., \phi_n(b_n))$.

Ex ante asymmetries among bidders are represented by appropriate restrictions on the distribution $F(\cdot)$. Campo *et al.* use a model of asymmetry with two bidder segments with symmetric bidders in each segment. It is assumed that segment 0 contains n_0 bidders and segment 1 contains n_1 bidders, with $n_0 + n_1 = n$. The joint distribution of bidder valuations, $F(v_{0,1}, ..., v_{0,n_0}, v_{11}, ..., v_{1,n_1})$, is exchangeable within the first n_0 and the last n_1 variables. For $k = 0,1$ and $j = 1, ..., n_k$, denote by $b_{k,j} = s_k(v_{k,j})$ and $\phi_{k,j} = s_k^{-1}(b_{k,j})$ the equilibrium bidding strategy of bidder j in segment k, and its inverse, respectively.

The ith bidder in segment 1 maximizes his expected payoff

$$\max_{b_{1,i}} \Pi_1(b_{1,i}, v_{1,i}) = \max_{b_{1,i}} [(v_{1,i} - b_{1,i}) * \Pr(y_{1,i}^* < \phi_1(b_{1,i}) \quad and \quad y_{0,i}^* < \phi_0(b_{1,i}) \mid v_{1,i})],$$

where $y^*_{1,i} = \max\{v_{1,j} : j = 1,\ldots,n_1 \;\&\; i \neq j\}$ and $y^*_{0,i} = \max\{v_{0,j} : j = 1,2,\ldots,n_0\}$. The objective function of a typical bidder in segment 0 is defined similarly.

We refer the reader to Campo *et al*. (2003) for the details of the derivations and state their results. Campo *et al*. show that the equilibrium inverse bid functions satisfy:

$$v_{1,i} = \mathsf{f}_1(b_{1,i}) = b_{1,i} + \frac{G_{B^*_1,B^*_0|b_i}(b_{1,i}, b_{1,i} \mid b_{1,i})}{dG_{B^*_1,B^*_0||b_i}(b_{1,i}, b_{1,i} \mid b_{1,i})/dX}, \tag{1}$$

where $B^*_{1,i} = \max\{b_{1,j} : j = 1,\ldots,n_1 \;\&\; i \neq j\}$ and $B^*_{0,i} = \max\{b_{0,j} : j = 1,2,\ldots,n_0\}$.

The expression (1) involves functions of bids only and is the basis of estimating the latent valuations from observed bids. Campo *et al*. eliminate the conditioning on b_1 in the second term by noting that this ratio can be interpreted as

$$\frac{\Pr(B^*_1 \le b_1, B^*_0 \le b_1, b_1 = b_1)}{\Pr(B^*_1 = b_1, B^*_0 \le b_1, b_1 = b_1) + \Pr(B^*_1 \le b_1, B^*_0 = b_1, b_1 = b_1)} = \frac{G_{B^*_1,B^*_0,b_1}(b_1, b_1, b_1)}{D_{11}(b_1, b_1, b_1) + D_{12}(b_1, b_1, b_1)}\;. \tag{2}$$

The ratio in (2) is then estimated by substituting kernel estimates for the corresponding terms.

The complexity of expressions in (2) and their empirical counterparts increase considerably for cases with more than two bidder segments. When there are *K* bidder segments, for each of the segments the denominator in the ratio would have *K* functions each with *K+1* arguments, requiring (K+1)-dimensional kernel estimation.

3. A Simplified Approach

Our simplified approach is based on working with the *i*th bidder's payoff function expressed in terms of equilibrium distribution of rival bids conditioned on the *i*th bidder's valuation. Let $B_i = \max_{j \neq i}(b_j)$, the maximum rival bid for bidder *i*. The *i*th bidder's expected payoff from participating in the auction with a bid b_i is given by

$$\Pi_i(b_i, v_i) = (v_i - b_i) * \text{Prob}(B_i < b_i \mid v_i).$$

The *i*th bidder's optimal bid b_i^* should be the solution of the following equation

$$d\Pi_i(b_i^*, v_i)\big/db_i = -\text{Prob}(B_i < b_i^* \mid v_i) + (v_i - b_i^*) * \left[\frac{\text{dProb}(B_i < b_i \mid v_i)}{\text{d}b_i}\right]_{b_i = b_i^*} = 0\,. \qquad (3)$$

Rearranging the terms in (3), we get a relation between the valuation and the corresponding optimal bid:

$$v_i = b_i^* + \frac{\text{Prob}(B_i < b_i^* \mid v_i)}{\left[\text{dProb}(B_i < b_i \mid v_i)\big/db_i\right]_{b_i = b_i^*}}\,. \qquad (4)$$

Using the notation

$$H^{(i)}_{B_i|v_i}(b \mid v_i) = \text{Prob}(B_i < b \mid v_i)$$

and $$h^{(i)}_{B_i|v_i}(b \mid v_i) = \text{dProb}(B_i < b \mid v_i)\big/db\,,$$

we can rewrite (4) as

$$v_i = b_i^* + H^{(i)}_{B_i|v_i}(b_i^* \mid v_i)\big/h^{(i)}_{B_i|v_i}(b_i^* \mid v_i)\,. \qquad (5)$$

Given the assumption that bid functions are strictly monotone, the conditioning on valuations can be replaced by conditioning on bids. Defining

$$G^{(i)}_{B_i|b_i}(b \mid b_i) = H^{(i)}_{B_i|v_i}(b \mid \text{f}_i(b_i))$$

and $$g^{(i)}_{B_i|b_i}(b \mid b_i) = h^{(i)}_{B_i|v_i}(b \mid \text{f}_i(b_i))\,,$$

one can rewrite (5) as

$$v_i = b_i^* + G^{(i)}_{B_i|b_i}(b_i^* \mid b_i^*)\big/g^{(i)}_{B_i|b_i}(b_i^* \mid b_i^*)\,. \qquad (6)$$

The expression (6) without the bidder superscript is the same as the formula derived by Li *et al.* (2002) for the symmetric APV model. An important difference is that now (6) applies without the symmetry assumption.[2]

For estimation we assume that a sample of L independent auctions from the structure F is available. As in Li *et al.* (2002) we estimate $G^{(i)}_{B_i|b_i}(b_i^* \mid b_i^*) / g^{(i)}_{B_i|b_i}(b_i^* \mid b_i^*)$ by $\hat{G}^{(i)}_{B_i|b_i}(b_i^*, b_i^*) / \hat{g}^{(i)}_{B_i|b_i}(b_i^*, b_i^*)$, where

[2] Note that although independent private values models can be estimated using one-dimensional kernels regardless of bidder asymmetries, APV models require at least two-dimensional functionals of bids.

$$\hat{G}^{(i)}_{B_i|b_i}(B,b) = \frac{1}{Lh_G}\sum_{l=1}^{L} 1(B_{i,l} \le B) K_G\left(\frac{b-b_{i,l}}{h_G}\right), \tag{7}$$

$$\hat{g}^{(i)}_{B_i|b_i}(B,b) = \frac{1}{Lh_g^2}\sum_{l=1}^{L} K_g\left(\frac{B-B_{i,l}}{h_g}, \frac{b-b_{i,l}}{h_g}\right), \tag{8}$$

K_G is a one-dimensional kernel with bandwidth h_G, and K_g is a two dimensional kernel with bandwidth h_g.[3]

In addition to greatly simplifying the estimation of asymmetric APV models, the proposed approach has several advantages[4]. First, one need not impose a priori restrictions on the form of asymmetry among the bidders. Valuation distributions can be estimated separately for each bidder using (6). Furthermore, the estimated distributions can be used to test the existence and the nature of asymmetry among the bidders. Finally, to the extent that the set of participating bidders is not endogenous, one need not assume a fixed set of bidders in the sample. One can construct pseudo-values for each bidder in the sample by replacing L in (7) and (8) with L_i - the subsample of auctions in which bidder *i* submits a bid.

4. Conclusions

We have shown that one can avoid the curse of dimensionality in estimation of pseudo-values for asymmetric APV model of first price auctions. Regardless of the number of asymmetric bidder segments, one needs no more than a two-dimensional function of bids to estimate the pseudo-values in the first step of the two-step GPV procedure.

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[3] The details on how to choose the kernels and the bandwidths are available in general density estimation literature and in Li *et al.* (2002).
[4] We thank an anonymous referee for bringing these points to our attention.

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