Nonclassicality and information exchange in deterministic entanglement formation

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I. INTRODUCTION

Undoubtedly state entanglement of multicomponent systems is one of the most remarkable features of quantum mechanics [1], with both fundamental and practical implications. While performing a central role in discussions of nonlocal correlations [2], it is also the basis for quantum computation [3], quantum cryptography [4], teleportation [5] and dense coding [6]. Nowadays, the most accessible and controllable source of entanglement arises from the process of spontaneous parametric down-conversion in nonlinear crystals [1, 7]. Recently, experiments with trapped ions have also shown a high degree of control on entanglement generation [8]. Technological improvements on experiments involving atoms and cavity fields in both optic [9] and microwave [10] frequency regions will lead to new scenarios for entanglement experiments involving massive particles in the near future. Progress in this direction was already achieved as in the generation of atomic Eistein-Podolsky-Rosen (EPR) pairs [2, 11] and in the multiparticle entanglement engineering [12].

Recently the nonclassicality of a light beam state was identified as a necessary resource for entangling two light beams at the two arms of a beam splitter[13]. In fact the need of a nonclassical state in one of the beam splitter arms to tangle the fields has a dramatic consequence on the *principle of nonseparability* of quantum mechanics [14, 15]. As it is stated, state entanglement occurs in any multicomponent (multipartite) interacting system, meaning that each party can not be any more described in terms of an uncorrelated state vector. The dependence on nonclassicality adds the importance of initial conditions to establish quantum correlations. This dependence has no counterpart to establish classical (statistical) correlation. Thus deterministic generation of entanglement is (consistently with the nature of the quantum systems involved) dependent on their interaction and their initial state. Throughout this paper we call attention to these necessary conditions for deterministic entanglement generation, particularly to the importance of initial conditions. We begin in Sec. II with a clear definition of necessary quantum resources for deterministic entanglement generation and observe the surprisingly dependence on subsystems initial state for the nonseparability principle to be applied. As a specific application example, in Sec. III we focus our attention on a system with an infinite dimensional Hilbert space, a three mode linear bosonic system (linear equations of motion), corresponding to a three partite continuous variable system. In Sec. IV we discuss the role of nonclassicality for the entanglement of the three modes. We assume the modes initially uncorrelated. Classical states remain uncorrelated as time evolves. However with a truly quantum resource or state (a superposition state) in one of the modes we show how (near maximal) entangled states are formed (GHZ like). In fact it is also possible to transfer the quantum resource from one mode to the others entangling them in a Bell like state. The mode with the initial quantum resource is left unentangled. These examples are discussed in terms of quantum information transfer.

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II. INITIAL CONDITION AS QUANTUM RESOURCE FOR DETERMINISTIC ENTANGLEMENT FORMATION

For the deterministic (through dynamic evolution) generation of entanglement and superposition states, there are a few necessary conditions to be applied. A condition for entanglement formation is the evolution map itself - the interaction between involved parties. A second condition, to which we will pay more attention here, is the initial state considered. To begin our discussion consider a general system composed by two subsystems, initially decorrelated and described by the density operator ρ acting on $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$:

$$\rho = \rho^A \otimes \rho^B \tag{1}$$

Any allowed (completely-positive, linear and trace preserving) quantum map $\mathcal{L} : \rho \to U\rho U^{-1} = \sigma(t) \in \mathcal{H}$, as stated by the nonseparability principle [14, 15], lead their uncorrelated states to

$$\sigma(t) \ = \ (1-\epsilon)\sum_m p_m \sigma_m^A \otimes \sigma_m^B + \epsilon|\psi\rangle\langle\psi|_{AB}, \tag{2}$$

where ϵ is a number (as defined here it is actually a dynamic variable) such that $0 \leq \epsilon \leq 1$. p_m is the probability associated to each subset m such that $\sum_m p_m = 1$. If $\epsilon = 0$, the state σ is said to be separable as soon as it is described by mixtures of classical correlation origin [16]. The essential question posed here is to the entangling power of the unitary operation U [18] and to whether or not the initial condition ρ plays an important role for entanglement generation.

For special classes of evolution maps the initial state of each party plays an essential role in the establishment of entanglement. Such is the case for the special class of Linear Bogoliubov transformation maps- transformations whose generator form a Lie algebra of the $\mathbf{SU}(2)$ group such as beam-splitter operations [13, 19] for bosonic fields. These transformation cannot map a gaussian (classical) state into a nongaussian (nonclassical) one. If in a bosonic system whose the transformation generators (Hamiltonian) are as such, the only way to entangle the state of the parties is if another quantum resource is present, as the parties initial states [13, 19]. To prove this assertion let the two uncorrelated bosonic fields A and B, correspondently prepared in coherent states, evolve under a linear map whose generators are of the form

$$\frac{1}{2}\left(ab^\dagger + a^\dagger b\right), \ \ \frac{i}{2}\left(ab^\dagger - a^\dagger b\right), \tag{3}$$

which are the generators of the $\mathbf{SU}(2)$ group. It is direct to prove that any coherent state under this transformation evolves coherently and so the states of the two parties A and B evolve uncorrelated. However, if one of the states of A or B is a legitimate quantum state, like Fock states or superposition of coherent states, it will evolve in an entangled state [13, 19]. The same is not valid for linear Bogoliubov transformations of the form

$$\frac{1}{2}\left(a^\dagger b^\dagger + ab\right), \ \ \frac{i}{2}\left(a^\dagger b^\dagger - ab\right), \tag{4}$$

which together with (3) form the algebra of the sympletic group $\mathbf{Sp}(4,R)$[20]. Hamiltonians of the kind of Eq.(4) are well known to generate scaling transforms - stretching and contractions in such a way to conserve the volume of the phase space, know in quantum optics as squeezing [21]. It is straightforward to show that this transformation map is able to entangle any initial classical state. On the other hand the generators of the $\mathbf{SU}(2)$ group generates rotations in the system phase space. The central result of this discussion is that contrarily to classical correlation (statistical) generation present in any past time interacting classical multipartite systems, quantum correlations are fully dependent not only on the parties interaction nature, but also on the quantum nature of their initial states.

III. ENTANGLING POWER OF BILINEAR OPERATIONS

If it is impossible to entangle systems prepared in classical states, through linear Hamiltonians of the (3), it is still possible, to determine conditions for the dynamic generation of entanglement when one of the parties is prepared in a special quantum state. This is quite useful as it allows one to understand how a initial one-party quantum state is transformed in a N-parties quantum state. An applicative example of the above statement is easily found in quantum optical systems. Let us consider a system composed of three bosonic modes, which are coupled bilinearly. There are a number of physical systems that this could represent. For instance it essentially describes the interaction of three electromagnetic fields in a nonlinear crystal [22]. It can also describes the interaction between different polarisation

of light fields [23], the interaction of light fields and the motion of trapped atoms as in the proposal of Parkins and Kimble [24], or even the interaction between the ionic motional degrees of freedom, let us say, in the *x, y and z* directions, of a trapped ion [25]. Here the important element is the bilinear coupling of the (3) kind.

More specifically, the Hamiltonian describing the interaction between three modes of EM field in the presence of strong classical pumping can be written as [22, 26]

$$H = H_{\text{free}} + H_I \tag{5}$$

where

$$H_{\text{free}} = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b + \hbar\omega_c c^\dagger c \tag{6}$$

$$H_I = \hbar\lambda\left[e^{-i(\nu t-\phi)}a^\dagger b + e^{i(\nu t-\phi)}ab^\dagger\right]$$
$$+ \hbar\kappa\left[e^{-i(\mu t-\theta)}a^\dagger c + e^{i(\mu t-\theta)}ac^\dagger\right] \tag{7}$$

Here the coupling parameters λ and κ are the product of the pump amplitudes and the coupling constant between the EM-field and the crystal. ν and μ are the pump field frequencies. The operators a, b, c and $a^\dagger$, $b^\dagger$, $c^\dagger$ are the annihilation and creation of the field quanta respectively. This rotating wave interaction is pictorially useful as it allows, as presented in [27] for two modes, not only the transference of energy, but the fully transference of state between the modes.

The Heisenberg equations of motion for the field operators are given by

$$\dot{a} = i\omega_a a - i\lambda e^{i(\Omega t+\phi)}b - i\kappa e^{i(\Gamma t+\theta)}c$$
$$\dot{b} = i\omega_b b - i\lambda e^{-i(\Omega t+\phi)}a \tag{8}$$
$$\dot{c} = i\omega_c c - i\kappa e^{-i(\Gamma t+\theta)}a,$$

where $\Omega \equiv \omega_a - \omega_b - \nu$ and $\Gamma \equiv \omega_a - \omega_c - \mu$. The equations of motion (8) are linear and hence easily solvable. Choosing $\Omega = \Gamma$ ($\omega_b + \nu = \omega_c + \mu$) the solution for $a(t)$, $b(t)$ and $c(t)$ are

$$a(t) = e^{-i\omega_a t}\left[u_1(t)a(0) + v_1(t)b(0) + w_1(t)c(0)\right] \tag{9}$$
$$b(t) = e^{-i\omega_b t}\left[u_2(t)a(0) + v_2(t)b(0) + w_2(t)c(0)\right] \tag{10}$$
$$c(t) = e^{-i\omega_c t}\left[u_3(t)a(0) + v_3(t)b(0) + w_3(t)c(0)\right] \tag{11}$$

where $a(0)$, $b(0)$ and $c(0)$ are the initial conditions for the three modes. The time-dependent coefficients in the above expression are given by

$$u_1(t) = e^{i\Omega t/2}f^*(t) \tag{12}$$
$$v_1(t) = -i\frac{\lambda e^{i\phi}e^{i\Omega t/2}}{A}\sin(At) \tag{13}$$
$$w_1(t) = -i\frac{\kappa e^{i\theta}e^{i\Omega t/2}}{A}\sin(At) \tag{14}$$
$$u_2(t) = -i\frac{\lambda e^{-i\phi}e^{-i\Omega t/2}}{A}\sin(At) \tag{15}$$
$$v_2(t) = 1 + \frac{\lambda^2}{\kappa^2+\lambda^2}\left[e^{-i\Omega t/2}f(t) - 1\right] \tag{16}$$
$$w_2(t) = \frac{\lambda\kappa e^{i(\theta-\phi)}}{\kappa^2+\lambda^2}\left[e^{-i\Omega t/2}f(t) - 1\right] \tag{17}$$
$$u_3(t) = -i\frac{\kappa e^{-i\theta}e^{-i\Omega t/2}}{A}\sin(At) \tag{18}$$
$$v_3(t) = \frac{\lambda\kappa e^{-i(\theta-\phi)}}{\kappa^2+\lambda^2}\left[e^{-i\Omega t/2}f(t) - 1\right] \tag{19}$$
$$w_3(t) = 1 + \frac{\kappa^2}{\kappa^2+\lambda^2}\left[e^{-i\Omega t/2}f(t) - 1\right] \tag{20}$$

where

$$f(t) = \left[\cos(At) + \frac{i\Omega}{2A}\sin(At)\right], \tag{21}$$
$$A = \sqrt{\Omega^2 + 4(\kappa^2+\lambda^2)}/2. \tag{22}$$

The oscillatory character of these solutions is predicted by the quantum recurrence theorem[28].

Using these expressions for the time-dependent Heisenberg operators, it is possible via the *characteristic functions*[29] method to find expressions for the state vectors. The symmetric form of the three modes characteristic function are given by

$$
\begin{aligned}
\chi_S(\eta, \zeta, \xi, t) \ = \ & \mathrm{Tr}_{ABC}\left\{\rho_{ABC}(0)\ \exp\left[\eta a^\dagger(t) + \zeta b^\dagger(t)\right.\right. \\
& \left.\left. +\ \xi c^\dagger(t) - \eta^* a(t) - \zeta^* b(t) - \xi^* c(t)\right]\right\}
\end{aligned}
\tag{23}
$$

where the RHS of the first (second) line stands for the Schrödinger (Heisenberg) picture and ρ_{ABC} is the density operator for the three-mode system. As the field operators $a(t)$, $b(t)$ and $c(t)$ (Heisenberg picture) depend linearly on $a(0)$, $b(0)$ and $c(0)$ (Schrödinger picture) we now define for convenience new time-dependent functions

$$
\bar{\eta} \ \equiv \ \eta u_1^* + \zeta u_2^* + \xi u_3^*, \tag{24}
$$

$$
\bar{\zeta} \ \equiv \ \eta v_1^* + \zeta v_2^* + \xi v_3^*, \tag{25}
$$

$$
\bar{\xi} \ \equiv \ \eta w_1^* + \zeta w_2^* + \xi w_3^*, \tag{26}
$$

IV.   INITIAL CONDITION FOR THE ENTANGLEMENT OF A THREE-PARTITE BOSONIC SYSTEM

Before we continue with our discussion about the initial condition role for deterministic entanglement formation we give a brief review of three partite entanglement classification as given by Dür *et al* [17], which we use extensively in what follows. Let us write down four possible state configuration to which we will address later

$$
\rho(t) \ = \ \sum_i |a_i\rangle_A\, \langle a_i| \otimes |b_i\rangle_B\, \langle b_i| \otimes |c_i\rangle_C\, \langle c_i| \tag{27}
$$

$$
\rho(t) \ = \ \sum_i |a_i\rangle_A\, \langle a_i| \otimes |\psi_i\rangle_{BC}\, \langle \psi_i| \tag{28}
$$

$$
\rho(t) \ = \ \sum_i |b_i\rangle_B\, \langle b_i| \otimes |\psi_i\rangle_{AC}\, \langle \psi_i| \tag{29}
$$

$$
\rho(t) \ = \ \sum_i |c_i\rangle_B\, \langle c_i| \otimes |\psi_i\rangle_{AB}\, \langle \psi_i| \tag{30}
$$

where,$|a_i\rangle$, $|b_i\rangle$ and $|c_i\rangle$ are (unnormalized) states of systems A, B, and C, respectively, and $|\psi_i\rangle$ are states of two systems. One can classify a three-partite state as: The **Class 1** (*Fully inseparable states*) is the one where the states cannot be written in any of the above forms- when the sate can be written in a state vector form. An obvious example is the GHZ $(|000\rangle + |111\rangle)/\sqrt{2}$, which is a maximally entangled state of three qubits. The **Class 2** (*1-qubit biseparable states*) constitutes the one where biseparable states with respect to one subsystem are states that are separable with respect to one subsystem, but non-separable with respect to the other two. That is, states that can be written in the form (28) or (29) or (30), but not in both of them, i e, exclusively. The **Class 3** (*2-qubit biseparable states*) relates to biseparable states with respect only to two of the subsystems. Those are states that can be written in two and only two of the forms (28), (29) and (30). The **Class 4** (*3-qubit separable states*) are those that can be written in all of the forms (28), (29) and (30), but not as (27). Finally the **Class 5** (*fully separable states*) is related to states that can be written in the form (27).

Assuming that the initial joint density operator is fully separable (**class 5**)[17],

$$
\rho_{ABC}(0) = \rho_A(0) \otimes \rho_B(0) \otimes \rho_C(0)\,, \tag{31}
$$

then the normal joint characteristic function (23) factorises as

$$
\begin{aligned}
\chi_N(\eta, \zeta, \xi; t) \ = \ & e^{\frac{1}{2}\left(|\eta|^2 + |\zeta|^2 + |\xi|^2\right)} \chi_S(\eta, \zeta, \xi, t) \\
= \ & e^{\frac{1}{2}\left(|\eta|^2 + |\zeta|^2 + |\xi|^2\right)} \chi_S(\bar{\eta}, \bar{\zeta}, \bar{\xi}, 0) \\
= \ & \chi_N^A(\bar{\eta}; 0)\, \chi_N^B(\bar{\zeta}; 0)\, \chi_N^C(\bar{\xi}; 0)
\end{aligned}
\tag{32}
$$

In the Fourier space the modes evolve in an apparent uncorrelated fashion. The actual correlations are due to the temporal functions $\bar{\eta}$, $\bar{\zeta}$ and $\bar{\xi}$. To calculate the evolved joint quantum state we must assume specific initial conditions.

A. Coherent state

The first and simplest situation to consider is the case where all the modes are initially prepared in coherent states. Let us consider the three modes to be given by the density operators

$$\rho_A(0) = |\alpha\rangle \langle\alpha|, \; \rho_B(0) = |\beta\rangle \langle\beta|, \; \rho_C(0) = |\gamma\rangle \langle\gamma| \tag{33}$$

where α, β and γ are the amplitudes of the respective coherent states. The normal ordered characteristic function (32) can be written as

$$\chi_N(\eta, \zeta, \xi; t) = e^{\eta x^* - \eta^* x + \zeta y^* - \zeta^* y + \xi z^* - \xi^* z}, \tag{34}$$

where

$$
\begin{aligned}
x &= u_1\alpha + v_1\beta + w_1\gamma \\
y &= u_2\alpha + v_2\beta + w_2\gamma \\
z &= u_3\alpha + v_3\beta + w_3\gamma.
\end{aligned}
\tag{35}
$$

Comparing Eq. (34) with the expression for the normal ordered characteristic function in the Schrödinger picture, it is straightforward to write the density operator for the combined system as

$$\rho_{ABC}(t) = |\Psi_{ABC}(t)\rangle \langle\Psi_{ABC}(t)|, \tag{36}$$

where the joint state vector $|\Psi_{ABC}(t)\rangle$ is given by

$$|\Psi_{ABC}(t)\rangle = |x\rangle_A \otimes |y\rangle_B \otimes |z\rangle_C. \tag{37}$$

This state vector simply shows that the three modes will never become entangled. In fact they evolve as separate coherent states. We started with a fully separable quantum state and it evolved fully separable. The amplitude of each of the coherent states does change, but not its quantum signature. While these states may develop classical correlations due to their interaction, to observe quantum correlations such as entanglement requires special conditions on the prepared states.

B. Superposition states and quantum resources transfer

This example raises several interesting questions. In what follows, we still assume that the three modes are initially uncorrelated but what occurs if the initial state of the mode A is a actually a true quantum state - a superposition of coherent states. Does the three modes remain uncorrelated as the total system evolves? How is the information available in its state distributed over the other modes? To begin this investigation let us describe the initial states as

$$\rho_A(0) = |\psi_A\rangle \langle\psi_A|, \rho_B(0) = |\beta\rangle \langle\beta|, \rho_C(0) = |\gamma\rangle \langle\gamma| \tag{38}$$

where mode A is in the coherent superposition state

$$|\psi_A\rangle = \frac{1}{N^\alpha} \left(|\alpha\rangle + e^{i\Phi} |-\alpha\rangle\right), \tag{39}$$

with the normalisation factor $N^\alpha = \sqrt{2 + 2\cos\Phi \, e^{-2|\alpha|^2}}$. For $\Phi = 0, \pi/2, \pi$ the superposition state given by (39) is known as the *even cat state* (even coherent state) [30], *Yurke-Stoler cat state* [31] and *odd cat state* (odd coherent state) [30], respectively.

Now following the same steps as previously the final joint state of the three interacting modes is given by

$$
\begin{aligned}
|\Psi_{ABC}(t)\rangle = \; & \frac{1}{N^\alpha} \left(|X_1\rangle_A \otimes |Y_1\rangle_B \otimes |Z_1\rangle_C \right. \\
& + \left. e^{i\Phi} |X_2\rangle_A \otimes |Y_2\rangle_B \otimes |Z_2\rangle_C\right)
\end{aligned}
\tag{40}
$$

where

$$
\begin{aligned}
X_1 &= u_1\alpha + v_1\beta + w_1\gamma, \; X_2 = -u_1\alpha + v_1\beta + w_1\gamma \\
Y_1 &= u_2\alpha + v_2\beta + w_2\gamma, \; Y_2 = -u_2\alpha + v_2\beta + w_2\gamma \\
Z_1 &= u_3\alpha + v_3\beta + w_3\gamma, \; Z_2 = -u_3\alpha + v_3\beta + w_3\gamma
\end{aligned}
$$

In certain parameter regimes (namely $|\alpha| \gg 1$) we have $\langle X_1|X_2\rangle \approx 0$, $\langle Y_1|Y_2\rangle \approx 0$ and $\langle Z_1|Z_2\rangle \approx 0$. It can then be seen that (40) is of the form of an approximate GHZ state, and so it is also very close to be maximally entangled (**class 1**)[17]. Such characteristics come from the initial superposition nature of the mode A state.

The evolution of these modes is also very interesting in other parameter regimes and at specific times. For simplicity we are going to restrict ourselves to the case in which $\Omega = 0$ ($\nu = \omega_a - \omega_b$ and $\mu = \omega_a - \omega_c$). For $t = 2n\pi/A$, where $n = 0, 1, 2...$, it is easily shown that the joint state recurs to its initial uncorrelated form. Mode A returns to a superposition state, while the modes B and C are coherent states. No entanglement is seen for such times. However for times given by

$$t = \left(n - \frac{1}{2}\right)\frac{\pi}{A} \qquad \text{n} = 1, 2, 3, .. \tag{41}$$

the final joint state of the interacting modes is given by

$$
\begin{aligned}
|\Psi_{ABC}(t)\rangle &= \frac{1}{N^\alpha}|\bar{X}\rangle_A \\
&\otimes \left(|Y_1\rangle_B \otimes |Z_1\rangle_C + e^{i\Phi}|Y_2\rangle_B \otimes |Z_2\rangle_C\right)
\end{aligned} \tag{42}
$$

where $\bar{X} = (X_1 + X_2)/2$. Here it is clear that mode A is in a coherent state and is definitely not entangled with the modes B+C. The initial superposition state of the mode A is completely transferred to the couple BC in form of a *1-qubit biseparable* (**class 2**) entangled state [17]. More specifically, when the modes B and C are prepared in vacuum state and with $\Phi = 0$, the state of Eq. (42) is given by

$$
\begin{aligned}
|\Psi_{ABC}(t)\rangle &= \frac{1}{2N_+^\alpha}|0\rangle_A \otimes \left(N_+^{\lambda\alpha/A}N_+^{\kappa\alpha/A}|+\rangle_B \otimes |+\rangle_C\right. \\
&\left. + N_-^{\lambda\alpha/A}N_-^{\kappa\alpha/A}|-\rangle_B \otimes |-\rangle_C\right)
\end{aligned} \tag{43}
$$

where

$$|\pm\rangle_B = \frac{1}{N_\pm^{\lambda\alpha/A}}\left(|\lambda\alpha/A\rangle \pm |-\lambda\alpha/A\rangle\right) \tag{44}$$

$$|\pm\rangle_C = \frac{1}{N_\pm^{\kappa\alpha/A}}\left(|\kappa\alpha/A\rangle \pm |-\kappa\alpha/A\rangle\right). \tag{45}$$

The couple B-C is in a truly inseparable entanglement of orthogonal states, independently of the magnitude of α. In fact when mode A is traced out from Eq. (43), the partite B+C system is left in a Bell state.

A related inverse problem is the generation of superposition states. It is straightforward to deduce from the above discussion that the generation of a superposition (nonclassical) state in the mode A is possible, in principle, whenever this mode interacts by the Hamiltonian here discussed and if the two other modes B and C are prepared in a entangled Bell state. It is interesting to notice how these two intrinsic characteristic of quantum systems, entanglement and superposition states are so related, and that the conservation of quantum information is the strong bond that relates them. The generation of these kind of states through the Hamiltonian (7) is only possible when a quantum resource is available.

V. CONCLUSION

In summary, we have discussed in this article the importance of initial conditions (states) as quantum resources in the deterministic generation of multiparticle entangled systems. We have explicitly considered three bilinearly coupled modes of the electromagnetic field and showed that without the initial presence of a quantum resource for one of the three parties the resulting system remains completely unentangled (but may become classically correlated). If however one of the parties is initially prepared in a Schrödinger cat state (a nonclassical state), then this quantum resource can be transferred to the other parties. In fact a maximally entangled GHZ (**class 1**) state for the three parties can be generated. As a central result the superposition resource has been converted into an entanglement resource. More interesting however is that this superposition resource for party A can be transferred completely to an entanglement resource for parties B and C only generating a **class 2** state. Parties B and C are left in a maximally entangled Bell state while party A is now an unentangled coherent state. A slightly different parameter regime leaves parties B and C strongly entangled to each other but weakly entangled to A while other parameter regimes leave the

total system completely unentangled (**class 5**). This results indicate the importance of the nature of the initial state when the modes are bilinearly coupled and how the nonclassical nature of one parties initial state can be shared or exchanged with other parties. In this article we have considered only superposition of coherent states as our quantum resource but any other genuine nonclassical states (for instance Fock states) generate similar effects.

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