Locally Causal Quantum Theory and the Collapse Locality Loophole

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Locally causal quantum theory is an umbrella term for ordinary quantum theory modified by two hypotheses: state vector reduction is a well-defined process, and strict local causality applies. The first of these holds in some versions of Copenhagen quantum theory and need not necessarily imply practically testable deviations from ordinary quantum theory. The second implies that measurement events which are spacelike separated have no non-local correlations. To test this prediction, which sharply differs from standard quantum theory, requires a precise definition of state vector reduction. Formally speaking, any precise version of locally causal quantum theory defines a local hidden variable theory. However, locally causal quantum theory is most naturally seen as a variant of standard quantum theory. For that reason it seems a more serious rival to standard quantum theory than local hidden variable models relying on the locality or detector efficiency loopholes. Some plausible versions of locally causal quantum theory are not refuted by any Bell experiments to date, nor is it evident that they are inconsistent with other experiments. They evade refutation via a neglected loophole in Bell experiments - the *collapse locality loophole* - which exists because of the possible time lag between a particle entering a measurement device and a collapse taking place. Fairly definitive tests of locally causal versus standard quantum theory could be made by observing entangled particles separated by ~ 0.1 light seconds.

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I. INTRODUCTION

The subtle relationship between quantum theory and relativity raises questions fundamental to our understanding of nature. Entanglement was first identified as a potential source of tension between the two theories by Einstein, Podolsky and Rosen [1], while Bell's work [2,3] in the early 1960s made precise the sense in which classical intuitions based on the principles of special relativity conflict with quantum theory. Theoretical and experimental investigations have continued ever since.

After much careful analysis, a strong consensus emerged and has held firm over the last two decades. On this consensus view, insofar as special relativity inspires us to consider alternatives to standard quantum theory, those alternatives are characterised by Bell's definition of local hidden variable theories. However, the experimental evidence very strongly favours quantum theory against local hidden variable theories. The hypothesis of local hidden variables can only be maintained by supposing that a local hidden variable theory somehow exploits one or more loopholes arising from our inability to construct perfect experimental tests.

Only two loopholes — the detector efficiency [4] and locality loopholes — have generally been considered worth serious attention, and even they are not generally thought to be plausible mechanisms for reconciling local hidden variables with experiment. Indeed, one recent experiment [5] has succeeded in closing the detector efficiency loophole. And, although in principle the locality loophole can never be completely closed, it has been substantially closed by another recent experiment [6], which implies that local hidden variable theories which use the locality loophole would have to correlate the states of quantum random number generators with those of the entangled particles being measured.

Admittedly, no experiment to date has succeeded in simultaneously closing both loopholes, and there is a serious case for attempting still more stringent experiments (see e.g. [7,8] for discussions). Nonetheless, the general consensus is that a local hidden variable mechanism which exploits either or both loopholes in a way which would not have shown up in experiments to date would require a theory so perversely conspiratorial as to be almost incredible.

However, one or two gaps in this analysis have lately been noted. Altering standard causation, either by directly postulating reverse causation [9] or by considering statistically based configuration space models [10] allows alternatives to local hidden variable theories that are consistent with relativity and not excluded by Bell's theorem. Also, a previously neglected loophole in Bell experiments — the memory loophole — has lately been identified [11,12]. However, no way of actually reproducing quantum predictions within non-standard causation models has been identified apart from ad hoc constructions [10] that again appear perversely conspiratorial. As for the memory loophole, its potential effect, though real, is negligible when large numbers of entangled particle pairs are tested [12,13]. Moreover, analysing the experimental data in a nonstandard but natural way can eliminate the effect entirely [12,13].

The memory loophole thus seems more like an interesting footnote to the main line of argument than a serious challenge. Its late discovery should, though, at least disturb the general confidence that absolutely everything was sorted out by Bell's and Clauser et al.'s analyses [2,3,14] and subsequent experiments (for example [15,16,6]).

Non-standard causation models, too, have at least one virtue: they illustrate that considering new physical principles can suggest new ways of thinking about non-local correlations. One can easily fall into the habit of caricaturing local hidden variable theories as involving small classical particles flying from source to measuring device, carrying tables of instructions telling them what to do when measured. For theories that exploit the locality loophole, the caricature version has little signalling devices sitting in the experimental apparatus, sending signals to something like a radio receiver attached to the particles, to inform them of prematurely made random choices. When the detector efficiency loophole is used, the caricature version equips the particles with probes which identify the detector, calculate its efficiency, and adjust the instructions table accordingly. These pictures are indeed fantastically conspiratorial. But nothing in the mathematical analysis of non-local correlations implies that hidden variables theories have to be like this. Despite their admitted defects, proposals like reverse causation [9] or statistical configuration models defined by local weightings [10] do at least illustrate the possibility of a different sort of story.

II. LOCALLY CAUSAL QUANTUM THEORY AND THE COLLAPSE LOCALITY LOOPHOLE

This paper considers another gap in the analysis of entanglement and non-local correlations — one which seems at least as serious as any of the loopholes previously considered. This is the possibility that state reduction is a well-defined physical process, localised in space-time, and that, once this definition is taken into account, strict local causality (in Bell's sense [17,18]) holds. *Locally causal quantum theory* is a useful umbrella term for the class of theories that arise in this way, modulo various possible definitions of state reduction.

The strict local causality hypothesis implies in particular that, if $P_{\text{lc qt}}(A|\Lambda_P;\psi(-\infty))$ is the probability of a state reduction A taking place at a point P in space-time, given all the state reduction events in the past light cone Λ_P of P and the initial state at $t=-\infty$, and if B is any collection of state reduction events taking place at points spacelike separated from P, then

$$P_{\text{lc qt}}(A|\Lambda_P;\psi(-\infty)) = P_{\text{lc qt}}(A|\Lambda_P;\psi(-\infty);B)\,. \tag{1}$$

In other words, and contrary to standard quantum theory, state reduction involves no non-local correlations. On the other hand, local state reduction probabilities themselves, conditioned on past light cone events, should agree with those predicted by standard quantum theory, after perhaps allowing for some slight modification arising from introducing a precise definition of state reduction:

$$P_{\text{lc qt}}(A|\Lambda_P;\psi(-\infty)) \approx P_{\text{standard qt}}(A|\Lambda_P;\psi(-\infty))\,. \tag{2}$$

Consider for example two widely separated particles prepared in a state close to (but, for reasons which will become apparent later, not precisely equal to) a singlet:

$$|\psi\rangle \approx \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B)\,. \tag{3}$$

According to locally causal quantum theory, if the two particles are measured in the $|0\rangle, |1\rangle$ basis, and *if the state reductions corresponding to the measurements are completed at space-like separations*, the joint outcome probabilities are

$$P_{\text{lc qt}}(0_A,0_B) \approx P_{\text{lc qt}}(0_A,1_B) \approx P_{\text{lc qt}}(1_A,0_B) \approx P_{\text{lc qt}}(1_A,1_B) \approx 1/4\,, \tag{4}$$

whereas

$$P_{\text{standard qt}}(0_A,1_B) \approx P_{\text{standard qt}}(1_A,0_B) \approx 1/2\,, \qquad P_{\text{standard qt}}(0_A,0_B) \approx P_{\text{standard qt}}(1_A,1_B) \approx 0\,. \tag{5}$$

It seems that this gross discrepancy should show up immediately in experiments on entangled particles: there is no need even to vary the measurement choices at A and B in order to see the difference between the two theories. Given the impressive confirmation of quantum theory in Bell experiments to date, what is the point in considering locally causal quantum theory?

The loophole is in the italicized qualification. We can't be sure that state reductions occur at space-like separations, without knowing precisely when they take place — in other words, without a precise theory of state reduction. And if the state reduction at A in fact takes place in the past light cone of that at B, or vice versa, then

$$P_{\text{lc qt}}(0_A, 1_B) \approx P_{\text{lc qt}}(1_A, 0_B) \approx 1/2\,, \qquad P_{\text{lc qt}}(0_A, 0_B) \approx P_{\text{lc qt}}(1_A, 1_B) \approx 0\,. \tag{6}$$

in agreement with standard quantum theory. More generally, locally causal quantum theory will agree with standard quantum theory in any Bell experiment, so long as the state reductions for particles A and B in any given pair are timelike separated.

It is worth emphasizing that the loophole locally causal quantum theory exploits is quite distinct from the locality loophole, mentioned above. The locality loophole relies on the fact that it is difficult to arrange a Bell experiment so that the measurements at A and B are chosen randomly and independently for each pair, in such a way that the random choices are themselves made at points which are space-like separated from one another and from the point at which the measured singlet is created. Much experimental ingenuity has been devoted to closing this loophole as far as possible, from the famous experiments of Aspect et al. [15] utilising the quasi-randomness of high frequency waves, to recent experiments using fast quantum random number generators [6].

The loophole considered here also involves locality, but it involves the problem of ensuring that the state reduction *events* associated with measurements are spacelike separated, rather than ensuring that randomly made measurement *choices* are. Let us call it the *collapse locality loophole.*

III. IS LOCALLY CAUSAL QUANTUM THEORY SELF-CONSISTENT?

The basic features of locally causal quantum theory are best illustrated in the idealised model of quantum states and measurements, commonly used in quantum information theory, in which subsystems are treated as effectively pointlike and measurements are carried out at a definite point in space and time. To simplify the notation further, we can also assume that the subsystems are stationary relative to one another, and that the hamiltonian is zero. Since the predictions of locally causal quantum theory depend on past events, we need to specify a state history as well as a state. It is simplest to assume that we are given a state which has been undisturbed for long enough that locally causal and standard quantum theory initially agree. Suppose thus that at $t = 0$ we have a system in a state

$$|\Psi(0)\rangle = \sum_{i_1 \ldots i_n} a_{i_1 \ldots i_n} |i_1\rangle_1 \ldots |i_n\rangle_n\,, \tag{7}$$

where $1, \ldots, n$ denote distinct fixed points $x_1, \ldots, x_n$ in space and $i_1, \ldots, i_n$ label internal degrees of freedom. Suppose too that the system has been undisturbed — in particular, no measurement event has taken place — for a time long compared to any of the subsystem separations.

Now suppose that we have some definite theory of state reduction which tells us precisely when a measurement takes place on any subsystem, and characterises the nature of the measurement. Its prescriptions take the following form: a state reduction takes place at $(\underline{x_i}, t_i)$, defined by a set of operators $\{A_j\}$ which obey

$$\sum_j (A_j)^\dagger A_j = I\,, \tag{8}$$

and which act on the Hilbert space corresponding to the internal degrees of freedom of particle j. We define $\mathcal{A}_j$ to be the corresponding operator on the tensor product Hilbert space: that is,

$$\mathcal{A}_j = I \otimes \ldots \otimes I \otimes A_j \otimes I \otimes \ldots \otimes I\,, \tag{9}$$

where A_j is the j-th term in the product.

In standard quantum theory, whenever such a reduction takes place on a state $|\Psi\rangle$, we get outcome i with probability

$$\text{Tr}((\mathcal{A}_i)^\dagger \mathcal{A}_i \rho_\Psi)\,, \tag{10}$$

where $\rho_\Psi = |\Psi\rangle\langle\Psi|$. After this reduction, the state becomes

$$\frac{\mathcal{A}_i |\Psi\rangle}{(\text{Tr}((\mathcal{A}_i)^\dagger \mathcal{A}_i \rho_\Psi))^{1/2}}\,. \tag{11}$$

In between collapses, since the hamiltonian is zero, the state remains constant.

In the corresponding version of locally causal quantum theory, to calculate outcome probabilities for a measurement taking place at $P_i = (x_i, t_i)$, we first need to calculate a causally defined version of the standard quantum state — let us call it the *local state* — of subsystem i at P_i. This is obtained by starting from $|\Psi(0)\rangle$ and then applying (11) for each measurement within the past light cone of P_i, but no others. Equation (10), applied to the local state, then defines the outcome probabilities for the measurement at P_i.

But are these rules well-defined? Suppose for example that we begin with two subsystems in a singlet state

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1|1\rangle_2 - |1\rangle_1|0\rangle_2)\,. \tag{12}$$

Suppose that at time t, reductions take place at *both* x_1 and x_2 and that in both cases the reduction operators are $\{P_0, P_1\}$, the projections onto $|0\rangle$ and $|1\rangle$. According to our rules, with probability 1/4, the outcome in both cases will be P_0. If no further reductions take place before time $T = t + |x_1 - x_2|$, then at that point our rules suggest both local states become

$$\frac{P_0 \otimes P_0|\Psi(0)\rangle}{|P_0 \otimes P_0|\Psi(0)\rangle|}\,, \tag{13}$$

which is undefined. After this point, we thus have no rule for predicting future measurement outcomes.

There are two attitudes one can take to this. One is to conclude that locally causal quantum theory is not a properly defined theory, and deserves no further attention. The other is to note that, in practice and even in principle, the theory can be saved quite easily.

The practical counter-argument to the above is that the singlet state is never precisely realised in nature. A more realistic version of the above discussion would begin with

$$|\Psi(0)\rangle = \sum_{ij} a_{ij}|i\rangle_1|j\rangle_2\,,$$

where $a_{01} \approx 1/\sqrt{2} \approx a_{10}$ and $a_{00} \approx 0 \approx a_{11}$, but neither of the last two terms are precisely zero. A still more realistic version would take the initial state to be a mixture, dominated by states of approximately this form. Either way, the local states after measurement are, by virtue of the correction terms, well-defined.

More generally, whatever measurement operators arise in a theory of reduction, one would not ever expect to find in nature — or to be able to create artificially — a state that is *precisely* a zero eigenstate of a tensor product of non-zero operators.

A more principled way of avoiding the difficulty is to require that the theory of state reduction should involve only measurement operators which have no zero eigenvalues. (So, in particular, it cannot include projections.) The Ghirardi-Rimini-Weber spontaneous localisation model [19] is an example of such a theory. (Note, though, that in current non-relativistic versions of the GRW model, the measurement operators are not perfectly localised, so that the probability of a collapse event centred at a point P is not determined by events in the past light cone of P.) In such theories, although the cumulative effect measurements can asympotically tend to the action of a projection, no collection of measurement events ever completely annihilates the component of the state in any given subspace. In particular, whatever the initial state, the calculations in the locally causal version of such theories will never produce a zero value for the unnormalised local state, and the local states are always defined.

Put succinctly, the worry about locally causal quantum theory was that it might imply combinations of measurement outcomes that are impossible in standard quantum theory — and that when that happens, locally causal quantum theory breaks down. The way around this is to notice that in practice the measurement outcomes which occur in locally causal quantum theory will almost surely never actually be impossible in standard quantum theory: if one prefers, by slightly restricting the theory of reduction one can ensure this is always true. However, unless the details of the reduction theory somehow prevent long-range entanglement, combinations of outcomes which are extremely unlikely according to standard theory can be expected to be fairly common in locally causal quantum theory.

IV. TESTING LOCALLY CAUSAL QUANTUM THEORY

As noted above, locally causal quantum theory and standard quantum theory predict different outcome probabilities for separated measurements on entangled states, so that Bell experiments are the first obvious place to look for a refutation of locally causal quantum theory. The greatest separation over which apparently non-local correlations have so far been demonstrated was in the experiments of Tittel et al. [16], who have demonstrated the violation of Bell inequalities, and the confirmation of quantum predictions, by entangled photons separated by $\approx$ 10km, or

$\approx 3 \times 10^{-5}$ light seconds. Tittel et al.'s experimental arrangement was somewhat asymmetric, but let us suppose, generously, that such an experiment can be carried out with effectively identical apparatus on both sides, such that entangled photons on both sides enter detectors at precisely the same time, and that the results still confirm standard quantum theory. If we could be sure that any sensible theory of state reduction implies that reduction takes place in the detectors within 3×10^{-5} seconds — or even merely that the times at which it takes place in the two detectors are separated by $< 3 \times 10^{-5}$ seconds — then we could conclude from these results that locally causal quantum theory was definitely refuted.

But can we be so sure? All ideas about theories of state reduction are speculative, but among them, at least three have been taken seriously from time to time by thoughtful people: Wigner's suggestion [20] that state reduction could be somehow caused by conscious minds, Penrose's suggestion [21] that state reduction takes place when required to prevent a superposition of macroscopically distinct gravitational fields, and Ghirardi-Rimini-Weber type theories in which state reduction results from a spontaneous localisation process occurring (for example) at rates proportional to particle number or mass.

Insofar as these ideas can be made precise at all, none of them seems necessarily to imply state reductions in photodetectors that are necessarily separated by times short compared to 3×10^{-5}sec. Indeed, if Wigner's suggestion were right, reduction wouldn't occur at all until experimenters look at the data. The other two cases cannot properly be analysed without a detailed description of the apparatus. However, given that the reduction of a superposition in the GRW and Penrose theories depends on the extent to which it involves macroscopically distinct separations of massive particles in position space, it would be surprising if very tight bounds on the reduction time in these experiments could be derived. Even in an experiment with perfect symmetry between the two wings, in which the photons enter the photodetectors at the same time t in the experimental rest frame, it need not necessarily be the case that the collapse events also take place at the same time $t + \delta$ — the time δ before collapse could, as in the GRW theory, be stochastically determined, with independent stochastic processes associated to space-like separated points on the two wings.

Still, if one takes the idea of a state reduction theory seriously in the first place then, whatever one thinks of Wigner's suggestion, there is a good reason to assume that reduction *does* take place (at the very latest) not long after the impression of a measurement result registers in a human observer's brain — namely, our own experience. When we watch an apparatus carrying out measurements, it seems to us as though each measurement produces a definite result, and it seems to us that these results are accessible to us rather soon after the point at which the signal reaches our eye or ear.

Of course, this does not logically imply that a state reduction has taken place. It could conceivably be that we enter a superposition state, entangled with the apparatus and the measured system, at least for some time, but that the properties of our consciousness are such that it constructs for us the impression of quickly accessible definite results before reduction takes place. But once one entertains *this* hypothesis, there seems no reason to postulate state reduction at all. One might as well go all the way, and follow Everettians in assuming that there is only unitary evolution, but that the properties of consciousness are such that we perceive things according to one component of the universal state vector, in which definite measurement results took place and were observed by us. (See e.g. Ref. [22] for a recent discussion advocating this view.)

On this reasoning, any state reduction theory worth taking seriously should imply that reduction ordinarily would take place within ≈ 0.1sec — roughly the timescale over which we can discriminate events — of the signal from a measurement apparatus reaching us. Given this, a fairly definitive test of locally causal quantum theory could be carried out by allowing observers separated by ≈ 0.1 light seconds to carry out synchronized measurements on entangled particles and directly observe the results, before later comparing them.

V. CONCLUSIONS

The standard case for studying loopholes in Bell experiments is that quantum non-locality has such fundamental significance that it is worth demonstrating as rigorously as possible. Even highly implausible alternative explanations are worth analysing and, if possible, refuting.

A more practical motivation has also recently been suggested [23,24]. It may be crucial for future users of quantum cryptography and quantum communication systems to guard against fakery or sabotage by testing that states involving allegedly entangled separated subsystems genuinely are entangled states of the correct form. In principle, Bell experiments can do this. But, again in principle, a saboteur might make use of any Bell experiment loophole to produce apparent, but unreliable, evidence of entanglement.

While these are certainly sufficient motivations for considering the collapse locality loophole, I think there are stronger reasons. For there is a principled case for taking seriously both the hypotheses which define locally causal quantum theory.

Take first the idea that there is an explicit physical theory of state reduction. This is not, by any means, everyone's preferred solution to the measurement problem — but it is a natural solution which has often been advocated. Indeed, almost everyone who has studied the Copenhagen interpretation must have wondered whether one could not replace the projection postulate, with its vague reference to measurement, with a precise physical law. Granted, devising explicit collapse models is a project fraught with difficulties. It seems hard to find satisfactorily relativistic versions of GRW's models (see e.g. Ref. [25] for a recent attempt). It is also hard to precisely formulate Penrose's idea, let alone Wigner's. These are very serious worries. But so far *every* proposed solution to the measurement problem is fraught with difficulties, and yet presumably there must be a solution.

As for the idea that strict local causality should hold: this is, obviously, inspired by special relativity. Of course, we have learned that quantum theory respects Minkowski causality more subtly. But strict local causality remains a natural hypothesis — albeit, unless nature really is exploiting the collapse locality loophole, an incorrect one.

Against locally causal quantum theory, it might be argued there is something decidedly strange about a theory which — a harsh critic could say — maintains consistency only by relying on the existence of small errors or by its own inability ever to give completely definite answers. Maybe: but then many features of standard quantum theory seem strange until, perhaps, familiarity breeds acceptance.

A stronger argument, perhaps, is that, if locally causal quantum theory were right, there should be visible consequences other than in Bell-like experiments on entangled states. For instance, particles whose wave functions have extended support ought, so to speak, to be observable in two or more places at once. For the reasons already discussed, this need not lead to contradiction. For example, one particle could turn out to derive from some other source, or one measurement record could turn out to be false, in ways that would be unlikely in standard quantum theory. But one might think it ought to have observable consequences, in cosmology and elsewhere. Very possibly it does: the question deserves careful thought.

Another fair argument is that it seems that, for locally causal quantum theory to be right, and yet not to have been detected in Bell experiments to date, the relevant parameters in the hypothetical explicit collapse model would have to be relatively fine-tuned. Bell experiments with separations of $> 10^{-5}$ light seconds have been performed, with no detectable deviation from standard quantum theory; an experiment with separation of $\approx 10^{-1}$ light seconds should show dramatic deviations from quantum theory if locally causal quantum theory were correct. It would be a bit of a quirk of fate for the critical separation to lie in a range covered by fewer than four orders of magnitude. Of course, the type of measurement carried out in the experiments is crucial. The argument for the latter experiment sufficing relies on direct observation of the results by human observers, rather than the photodetectors used in the former. But, at least from a GRW or Penrosean perspective, it isn't obvious that the human brain should be hugely better at inducing collapse.

At the moment, though, these counterarguments don't seem completely compelling. Although the arguments against local hidden variable theories exploiting the detector efficiency and (standard) locality loopholes are stronger, experimentalists rightly continue to work towards more definitive tests. The Earth is large enough to allow almost, if not absolutely, conclusive experimental tests of locally causal versus standard quantum theory; if one part of the experiment were carried out on a short manned space flight, a completely definitive test could be made. Once technology allows long distance distribution of entanglement, the experiments should be done.

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