Bidding By Empirical Bayesians In Sealed Bid First Price Auctions

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BIDDING BY EMPIRICAL BAYESIANS IN SEALED BID FIRST PRICE AUCTIONS

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1. INTRODUCTION

In Bayesian-Nash equilibrium formulations of auctions the bidders are assumed to know all the relevant structural elements of the game they are in. We investigate the empirical implications for bidding behavior of weakening this assumption in the context of sealed bid first price auctions. We study a model in which the bidders are assumed to estimate the unknown distributions that affect their payoff function by processing bids on same or similar items in previous auctions. We present Monte Carlo experimental results on the distribution of bidding strategies when a bidder substitutes the empirical analog of the unknown distribution of bids by the rivals. We also explore the performance of Bayesian-Nash equilibrium based empirical approach to estimate the latent distribution of bidder valuations when the proposed behavioral model rather than a Bayesian-Nash equilibrium process generates the observed bid data.

A typical game theoretic model of behavior in auctions formulates the strategic situation faced by bidders as a game of incomplete information. For surveys of the auction theory literature see McAfee and McMillan (1987), Wilson (1992) and Klemperer (1999). This approach postulates a world in which a few bidders each with private information on the value of the item for sale compete. The private valuations are represented by random variables. Privacy of information is then made operational by assuming that each bidder privately observes a realization of the random variable. The distribution of a bidder's valuation is typically assumed to be *common knowledge* – the statement '(each bidder knows that)^t bidder i's private information is drawn from the distribution $F(\cdot)$' is true for all values of t. An immediate question is then how the bidders came up with this knowledge in the first place. [1] In this note, we drop the assumption that the

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[1] In principle, the assumption that F () is common knowledge is one of convenience. All is needed to proceed is that common knowledge assumption holds at some level in an increasing hierarchy of models of the situation if not at the level of primitives usually stated. That is, for any game of incomplete information, there exists a large enough space of *types* the distribution of which is common knowledge among the players (Mertens and Zamir 1983, Tan and Werlang 1987).

distribution function $F(\cdot)$ is common knowledge and replace it with the assumption that the bidders have access to data that can be used to form an estimate of their payoff functions. Thus, in our approach, $F(\cdot)$ is not known by the bidders at all, let alone being common knowledge.

Behavioral principles of Bayesian-Nash equilibrium formulations may be decomposed into two sets: game theoretic equilibrium = decision theory + system consistency conditions. Decision theory rests on the assumption of expected utility maximization by players given some beliefs about the payoff generating process. System consistency conditions usually take the form of restrictions on beliefs or on conjectures about others' behavior. In this sense, Bayesian-Nash equilibrium may be seen as the limit case of a class of behavioral models featuring various levels of accuracy on the part of the players in guessing the payoff generating process they are facing.

The approach proposed here falls in the intersection of two recent and growing bodies of literature on the analysis of auctions. One strand of literature attempts to explore alternative theories of behavior in auctions with weaker and more realistic rationality restrictions than imposed by full-blown Bayesian-Nash equilibrium analysis. Some examples are McKelvey and Palfrey (1995), Battigalli and Siniscalchi (2000), Brainov and Sandholm (1999), Goeree, Holt and Palfrey (1999), Bajari (1997), Guler, Plott and Vuong (1994).

In the second strand, empirical implications of game theoretic approach to auctions are explored and structural econometric models are developed. See Laffont and Vuong (1996), Laffont (1997), Guerre, Laffont and Vuong (2000) for surveys and pioneering examples of this approach. A central research goal in this approach is to recover the unknown structural elements of the market environment, in particular, the distribution of bidder valuations, from observable bids. Since the empirical models are in this approach are based on equilibrium models of auctions, the knowledge assumptions of the underlying game models are maintained in the econometric specifications. In other words, the data is assumed to be generated by bidders who know the structural elements of the game they are in. These elements are unknown to the outside analyst. Instead we assume that the bidders, just like the outside analyst, have access to bid data in previous auctions and each bidder uses some standard estimation technique to estimate the empirical analog of his payoff function.

In section 2, the standard model of sealed bid first price auctions and the structural empirical models are reviewed. In section 3, we present a Monte Carlo experimental setup to explore the performance of equilibrium-based structural approach to recover the underlying latent distribution of bidder valuations. A number of experimental scenarios and results are described in section 4. Section 5 contains concluding remarks.

2. BIDDING IN SEALED BID FIRST PRICE AUCTIONS

We maintain the standard assumptions of the symmetric independent private values model with a fixed number of risk neutral bidders. A single indivisible item is auctioned using a sealed bid first price mechanism. The bidder valuations are independent identically distributed random variables drawn from a distribution $F(\cdot)$. In order to focus on the specific issue under investigation we take the number of bidders and the reserve price to be exogenously fixed.

The decision problem faced a by a typical bidder is one of selecting a bid, b, to maximize the expected value of the profit:

$$\pi(b,v) = (v-b) \text{ Prob}(b \text{ wins}) \tag{1}$$

In standard auction theory formulations the bidders are endowed with the knowledge of $F(\cdot)$ as well as a conjecture on (or knowledge of) the bidding strategy of a typical rival, $B(v)$. Letting $\phi(b)$ denote the inverse of the bidding function $B(v)$, the bidder's decision problem takes the form

$$\pi(v,b) = (v-b)F(\phi(b))^{n-1}. \tag{2}$$

A bidder who knows $F(v)$ and $\phi(b)$ can compose them to obtain $G(b) = F(\phi(b))$. $G(b)$ is the probability that a bid of b wins against a typical rival. So the assumption that $F(\cdot)$ is common knowledge together with the assumption that rival bidders follow a bidding strategy with inverse $\phi(b)$ reduces the decision problem faced by a bidder to one of selecting a bid b to maximize

$$\pi(v,b,G) = (v-b)G(b)^{n-1}. \tag{3}$$

Thus, the usual assumptions can then be replaced by the assumption that $G(b)$ is common knowledge among the bidders.

If the bidders know $G(b)$, optimal bid by a bidder with valuation v satisfies the first order condition

$$-G(b)^{n-1} + (v-b)(n-1)G(b)^{n-2}g(b) = 0 \tag{4}$$

or,

$$v = \{b + G(b)/(n-1)g(b)\}\ , \tag{5}$$

where $g(\cdot)$ is the derivative of $G(\cdot)$. The condition that the term in braces is monotone increasing in b is the standard second order condition.

The expression (5) characterizes the Nash equilibrium of the bidding game if and only if

$$G(b) = F(\phi(b)) \tag{6}$$

and

$$\phi(b) = b + \frac{G(b)}{(n-1)g(b)} \ . \tag{7}$$

That is, the assumed bid distribution is consistent with the valuation distribution function $F(\cdot)$ and the equilibrium behavior represented by the inverse bid function $\phi(b)$.

Note, however, that even without the equilibrium restrictions (5) has a behavioral content in that it does represent the optimal bidding behavior of a bidder who believes that a typical rival's bid is drawn from $G(b)$. This belief may or may not be based on further reasoning and analysis. Note also that alternative weaker versions of this consistency may be imposed. For example, $G(b)$ may be required to approach $F(\phi(b))$ asymptotically as increasingly larger samples become available to form such beliefs.

A natural candidate model is based on the assumption that bidder decisions are best responses to estimated distribution of rival bids. That is, a typical bidder may not know $G(b)$ and needs to estimate $G(b)$ from a sample (possibly a subsample) of bids in previous auctions. If all bidders plug in an estimate $G_t(b)$ based on all auctions observed up to time *t* for $G(b)$ in (3) and then choose a bid that maximizes the point estimate of their payoff function, then bids in successive periods will be generated by the empirical analog of the first-order condition in (5). A bidder participating in auction in period *t* with valuation v_{it} will select a bid b_{it} that is a best response to the estimated distribution of bids by rivals. This best response solves

$$v_{it} = b + \frac{G_t(b)}{(n-1)g_t(b)} \ . \tag{8}$$

We will refer to the bidding model in (8) as *empirical best response model*.

In a stationary environment, successive cohorts of *n* bidders each independently drawing a valuation from a fixed distribution $F(\cdot)$ will generate a sequence of bid distributions. Unlike the Bayesian-Nash equilibrium bid function that solves (5), the bidding behavior implied by (8) will exhibit randomness due to sampling variation in $G_t(\cdot)$. The process is initialized with an initial belief $G_0(\cdot)$ that is shared by all bidders, or alternatively, a prior sample of bids with distribution $G_0(\cdot)$.

3. ESTIMATION OF VALUATION DISTRIBUTIONS

From the point of view of an outside analyst, the task is to make inferences about the unknown structural element $F(\cdot)$ based on observable bid data. A recent breakthrough in the structural econometrics of auctions was the observation that (6) can be used as a basis for estimating the unknown (to the analyst) distribution function $F(\cdot)$. This is achieved by replacing the unknown $G(b)/g(b)$ with an estimate $\hat{G}(b)/\hat{g}(b)$ based on the observed sample of bids (Guerre et al 2000).

The two models (3) and (8) correspond to two alternative data generating processes. The conditions in (6) and (7) can be combined to express Bayesian-Nash equilibrium in terms of the bid distribution it implies. A bid distribution $G(b)$ is the Bayesian-Nash equilibrium bid distribution for the value distribution $F(\cdot)$ if and only if it solves the functional equation

$$G(b) = F[b + G(b)/(n-1)g(b)] \tag{9}$$

Therefore, under the Bayesian-Nash equilibrium model, observed bid data is a sample from a static data generating process $G(b)$ that solves (9). An estimate of the unknown function $F(\cdot)$ can be obtained using estimates of $G(b)$ and $g(b)$ using the bids data.

The data generating process implied the empirical best response model (8) is dynamic in nature. The observed data under this model is generated by a sequence $\{G_t(\cdot)\}$ that satisfies

$$G_{t+1}(b) = F[b + G_t(b)\big/(n-1)g_t(b)] \tag{10}$$

If available data includes dates of the auctions as well as bids, then one can estimate the sequence of functions $\{G_t(\cdot), g_t(\cdot)\}$, given an initial belief $G_0(\cdot)$. The resulting estimates can then be used in (10) to get estimate of the unknown distribution of valuations $F(\cdot)$.

In the next section we explore the following questions in the context of a Monte Carlo experiment: how is the resulting distribution of bid functions based on (8) is related to the Bayesian-Nash equilibrium bid function? How does this relation change with increasing sample sizes? Finally, how good is the Bayesian-Nash equilibrium based structural econometric approach in recovering the latent value distribution $F(\cdot)$ when the observations are generated by the process (10) rather than (9)?

4. A MONTE CARLO EXPERIMENT

In this section we describe a Monte Carlo experiment to generate data based on the proposed behavioral model of bidding in auctions.

Inputs:

1. A distribution function, $F(\cdot)$.
2. Number of bidders, *n*.
3. Number of auctions in the initial sample of bids, *L*.
4. Distribution of bids in the initial sample, $G_0(\cdot)$.
5. Number of iterations, *T*.
6. Window size, *W*.
7. Number of replications, *R*.

Data generating process:

Step 0: Generate a sample, B_0, of *nL* bids from the distribution $G_0(\cdot)$.

For *r=1* to *R,* generate a samples, $\mathcal{B}_r$, $\mathcal{V}_r$ and $\widehat{\mathcal{V}}_r$ of *nT* bids, valuations and pseudo-valuations, respectively, by performing the following steps:

Step *r.a*: For *t* = 1, to *T* perform the following steps:

Step *t.a*: Form a sample of bids, D_{t-1}, by taking the most recent *W* rows of the sample of bids, B_{t-1}.

Step *t.b*: Use the sample D_{t-1} of bids to get estimates $\widehat{G}_t(\cdot)$ and $\widehat{g}_t(\cdot)$ of the bid distribution, $G_t(\cdot)$, and its density, $g_t(\cdot)$, respectively.

Step *t.b*: Generate *n* independent valuations, $(v_{1t},...,v_{nt})$, from the distribution $F(\cdot)$.

Step *t.c*: Generate *n* bids, $(b_{1t},...,b_{nt})$ by solving

$$v_{it} = b + \frac{\widehat{G}_t(b)}{(n-1)\widehat{g}_t(b)}$$

Step *t.d*: Append the vector $(b_{1t},...,b_{nt})$ to the sample B_{t-1} to get the new sample of bids B_t.

Step *t.e:* Append the vector $(v_{1t},...,v_{nt})$ to the sample V_{t-1} to get the new sample of bids V_t.

Step *r.b:* Save the replication samples $\mathcal{B}_r = B_T$ and $\mathcal{V}_r = V_T$.

Step *r.c:* Use the sample $\mathcal{B}_r$ to get estimates $\widehat{H}_t(\cdot)$ and $\widehat{h}_t(\cdot)$ of the bid distribution and its density, H_r and h_r, respectively.

Step *r.d:* Generate a sample $\hat{\mathcal{V}}_r$ of *nT* valuations, $(\widehat{v}_{1t},...,\widehat{v}_{nt})$ for *t=1* through *T*, by solving

$$\widehat{v}_{it} = b_{it} + \frac{\widehat{H}_t(b_{it})}{(n-1)\widehat{h}_t(b_{it})}$$

5. EXPERIMENTAL SCENARIOS AND RESULTS

We consider a total of 16 scenarios obtained by all combinations of the four scenario elements. In each scenario, we use the total sample of bids generated by the scenario and apply Bayesian-Nash equilibrium based method (Guerre et al 2000) to estimate the unknowns. We compute the sample mean and standard deviation (in a sample of 40 replications) of the bid function, the bid distribution and inferred distribution of bidder valuations.

Constants:

1. *n = 3*.
2. *L = 20.*
3. *R = 40.*

Scenarios:

1. $F(x) \in \{x, x^3\}$
2. $G_0(b) \in \{uniform[0,1],\ G_{NASH,F(\cdot)}(b)\}$
3. $W \in \{20, all\}$
4. $T \in \{20, 80\}$

In all the scenarios we consider, the Bayesian-Nash equilibrium is a linear function. The estimation method we used is the one proposed in Guerre et al (2000) with a boundary correction (Silverman 1998).

Results:

The results of the Monte Carlo analysis are reported in Figures 1 through 8. In Figures 1 through 4, the initialization sample of bids is generated by the Nash equilibrium bid function that for the value distribution $F(\cdot)$ that underlies the scenario. Figures 5 through 8 present the results for the scenarios in which the initialization sample of bids is generated using a uniform distribution. In odd numbered figures, the bidders are assumed to use all available bid data when they estimate their payoff functions. In even numbered figures, only the most recent 20 observations are used. In all figures, the panels in the left and right columns depict the results for sample sizes 20 and 80, respectively. Again, in all figures, the panels in the top (respectively, middle, bottom) rows present the results for bid functions (respectively, bid distributions, valuation distributions).

Bid Functions:

In each figure, panels in the top row exhibit three bid functions. The green line depicts theoretical Bayesian-Nash equilibrium bid function. The solid red line is the mean (over 40 replications) of the bid function based on inferred valuations using Bayesian-Nash specification. The solid blue line depicts the mean (over 40 replications) of the bid function based on the model of empirical payoffs proposed in this paper. Finally, the dashed lines represent one-standard deviation confidence bounds for the respective bid function.

Equilibrium bid function in all the scenarios we consider is linear in valuation. The bid function generated by the empirical best response process, however, seems to be concave in the upper range of valuations. This is a consequence of the bootstrap sample averaging and the randomness of bid function under the empirical best response model – i.e. bids submitted by bidders with a given valuation v exhibits randomness due to sampling variation in the empirical probability of winning.

Bid Distributions:

The panels in the middle row present the means and one-standard deviation bounds for the bid distributions. The solid green line is the mean (over 40 replications) of the bid distribution that would be observed if bidders followed the Bayesian-Nash equilibrium bidding strategy for the realization of bidder valuations. The solid red line is the mean (over 40 replications) bid distribution corresponding to the actual data generating process used in the scenario.

A visual inspection of the figures suggests that if bidders' initial beliefs are consistent with the equilibrium bid distribution then observed bids data generated by the empirical-best-response process can be treated as if they are generated by a static Bayesian-Nash equilibrium process. (Figures 1 through 6). If, on the other hand, initial beliefs deviate significantly from the equilibrium bid distribution (Figures 7 and 8), then bid distributions generated by the empirical best response process and Bayesian-Nash equilibrium process are significantly different. The figures also suggest that this pattern is generally independent of whether the bidders use data from the latest 20 auctions (W=20) or all past auctions for their current period bidding decisions. The observed pattern is also independent of the sample size (T=20 or 80).

Value Distributions:

Finally, the bottom panels depict the mean (over 40 replications) distribution of bidder valuations. The solid green line is the mean of the empirical valuation distribution estimates based on actual realizations of valuations. The solid red line is the same based on pseudo-value estimates using the procedure of Guerre, Perrigne and Vuong (2000).

The general conclusion that seems to emerge from a visual inspection of the figures is that if bidders' initial beliefs are consistent with the equilibrium bid distribution then observed bids data generated by the empirical-best-response process can be treated as if they are generated by a static Bayesian-Nash equilibrium process without a substantial difference in estimated latent valuation distributions (Figures 1 through 6). If, on the other hand, initial beliefs deviate significantly from the equilibrium bid distribution (Figures 7 and 8), then valuation distribution estimated using the equilibrium model differ substantially from the valuation distribution estimated using the correct model of the data generating process. The figures also suggest that this pattern is generally independent of whether the bidders use data from the latest 20 auctions (W=20) or all past auctions for their current period bidding decisions. The observed pattern is also independent of the sample size (T=20 or 80).

6. DIRECTIONS FOR FUTURE EXTENSIONS

We have looked at a limited set of scenarios to explore the effect of deviations from Bayesian-Nash equilibrium behavior on inferences about the latent distribution of bidder valuations. There remain a number of interesting questions that can be explored in the framework proposed in this paper. Among such questions is the role of scenario elements that are kept fixed in this study, such as the number of bidders and the reserve price. Additional questions include: What if different bidders have different data, different sample sizes, use different estimation techniques. In particular, does a bidder have incentives to be more sophisticated in his bidding decision than postulated by the empirical best response model? We explore some of these questions in a companion paper (Guler and Zhang 2002). We intend to investigate a more comprehensive set of scenarios including ones with non-linear equilibrium bid functions and affiliated bidder valuations to evaluate the robustness of equilibrium-based empirical specifications to variations in the data generating process. We also intend to investigate the same set of questions when the Bayesian–Nash equilibrium generates the observed bids but the analyst uses the empirical-best-response model to estimate the unknown distribution of values.

Another set of interesting questions is related to characterization of environments for which empirical best response process converges to the Nash equilibrium. Specifically, given initial beliefs, $G_o(b)$, say a diffuse prior on a bounded interval, what are the properties of environments such that if an environment, $(F(\cdot), n)$, has those properties the empirical best response process starting with $G_o(b)$ converges to the Nash equilibrium bid distribution of that environment. Alternatively, given a fixed environment, $(F(\cdot), n)$, how 'close' should initial beliefs be to the Nash equilibrium of $(F(\cdot), n)$ for the empirical best response process to converge to it. We intend to use the framework of this paper to explore this set of questions.

We have not explored the finer details of the sampling distribution of bids and inferred valuations when the empirical best response process generates the observed bids. For example, under this process observed data is likely to exhibit both cross-sectional and temporal correlation even when the latent structure is one of independent private valuations. We intend to investigate the sample correlation structure in the observed bids and inferred valuations in future work.

Finally, a seller who makes a decision on any auction parameter (say, reserve price, entry fees, auction format) faces a similar problem in formulating an objective function to optimize. We intend to explore the same problem from the seller's point of view in future work. Several interesting sets of variations arise as the seller may make his decisions: (a) when bidders know $G(b)$ but the seller has to use data to estimate, and (b) when both the bidders and the seller has to estimate the unknown distribution from available bid data.

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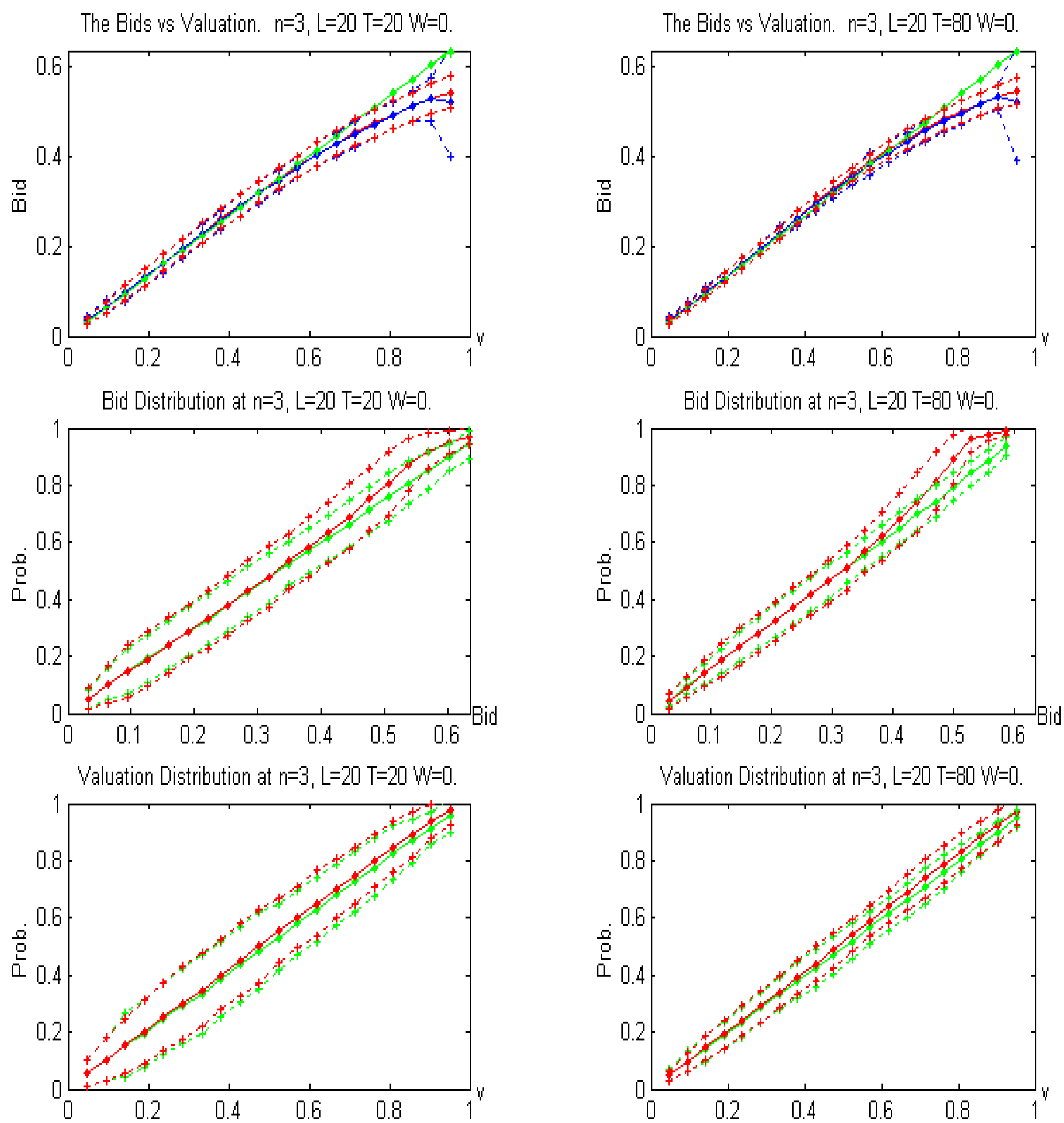

Figure 1: Scenario: F(v) =v, $G_0(b) = G_{NASH,F}$ (b), window= all past bids
Top panels: Nash equilibrium bid function (----), Bid vs. inferred valuation (-----)
Bid vs. actual valuation (-----)
Middle panels: Nash equilibrium bid distribution (----), actual bid distribution (---)
Bottom panels: Actual valuation distribution (---), inferred valuation distribution (---)

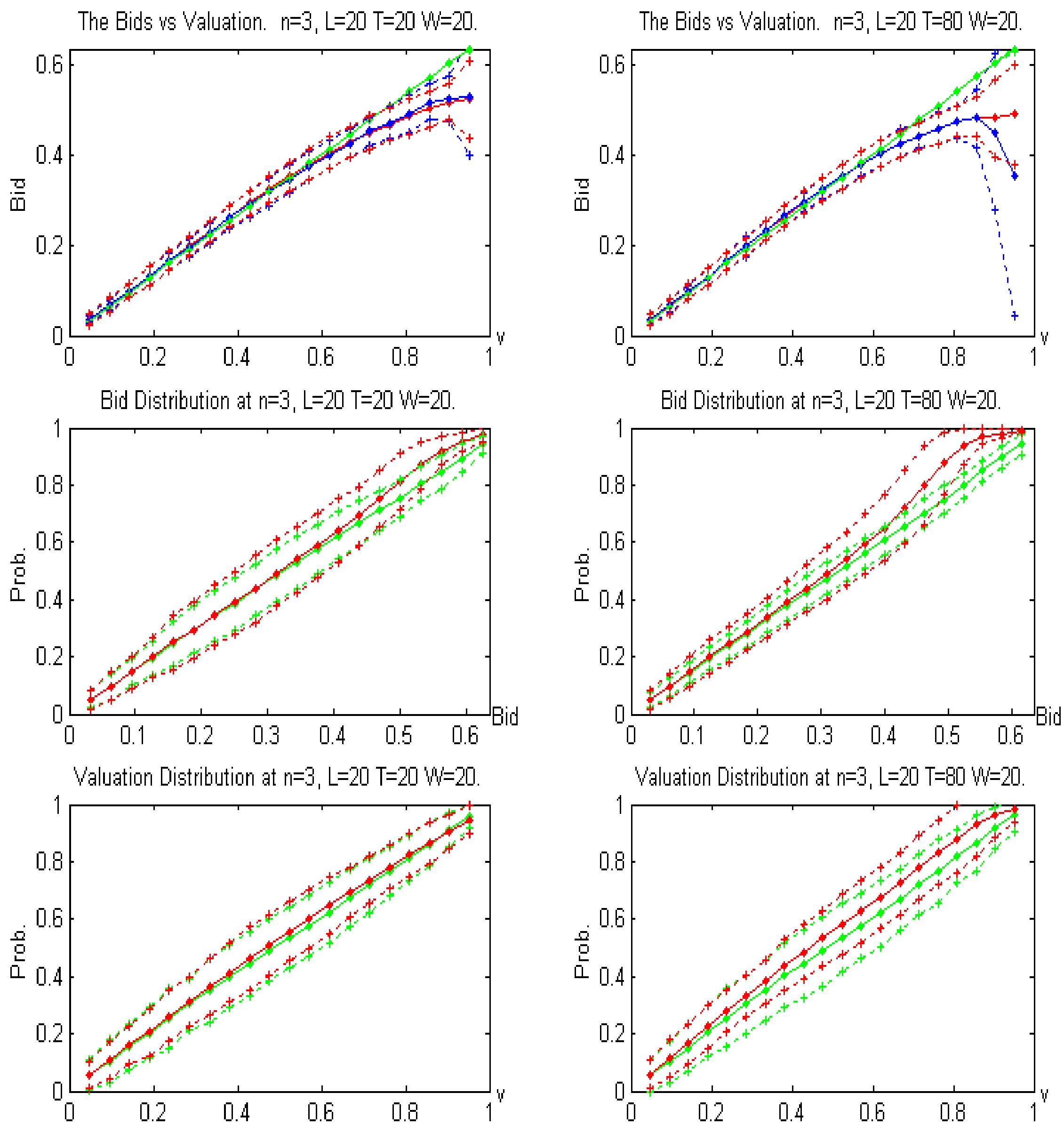

Figure 2: Scenario: $F(v) = v$, $G_0(b) = G_{NASH,F}(b)$, window= last 20 auctions
Top panels: Nash equilibrium bid function (----), Bid vs. inferred valuation (-----)
Bid vs. actual valuation (-----)
Middle panels: Nash equilibrium bid distribution (----), actual bid distribution (---)
Bottom panels: Actual valuation distribution (---), inferred valuation distribution (---)

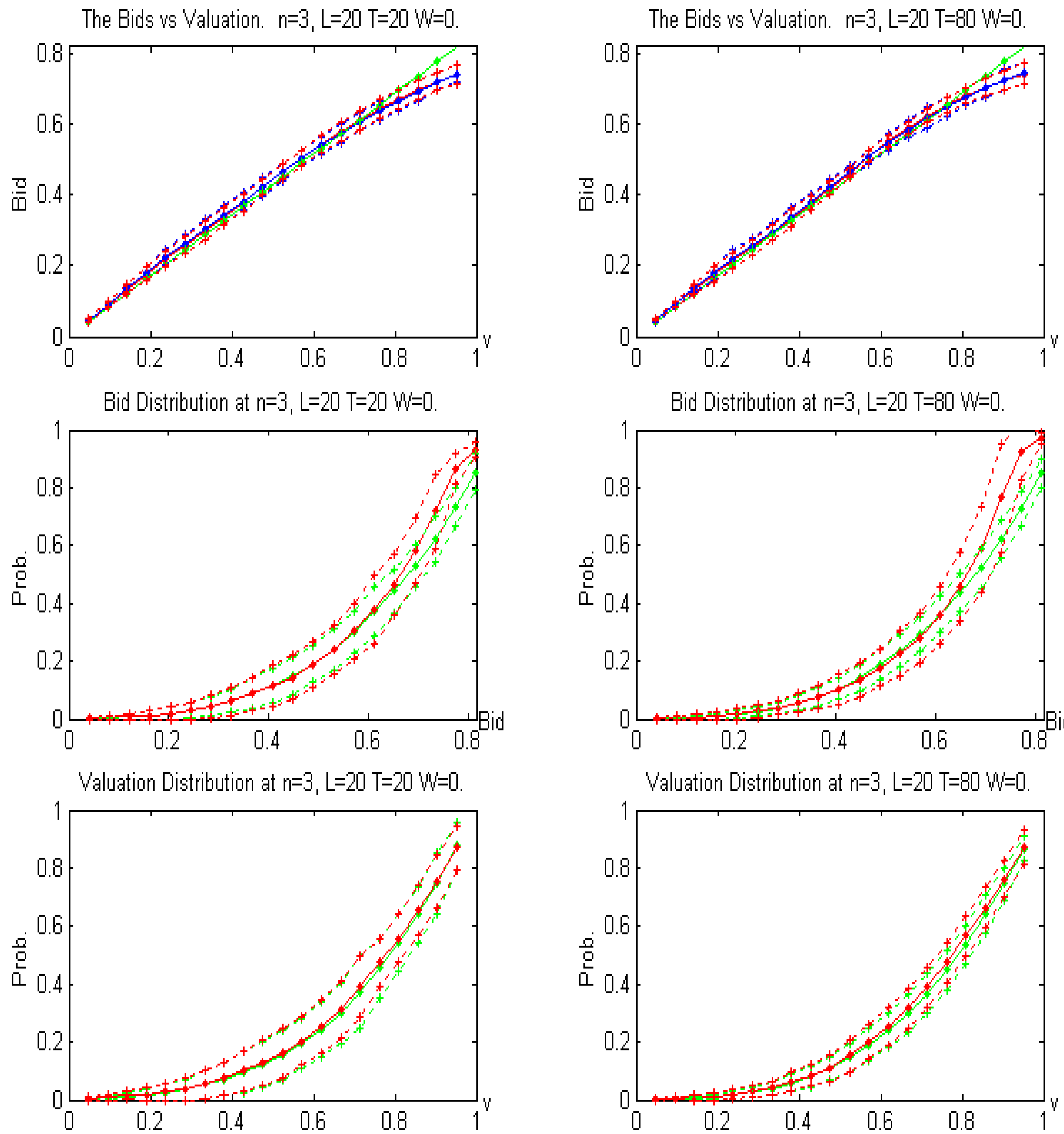

Figure 3: Scenario: $F(v) = v^3$, $G_0(b) = G_{NASH,F}(b)$, window= all past bids

Top panels: Nash equilibrium bid function (----), Bid vs. inferred valuation (-----)

Bid vs. actual valuation (-----)

Middle panels: Nash equilibrium bid distribution (----), actual bid distribution (---)

Bottom panels: Actual valuation distribution (---), inferred valuation distribution (---)

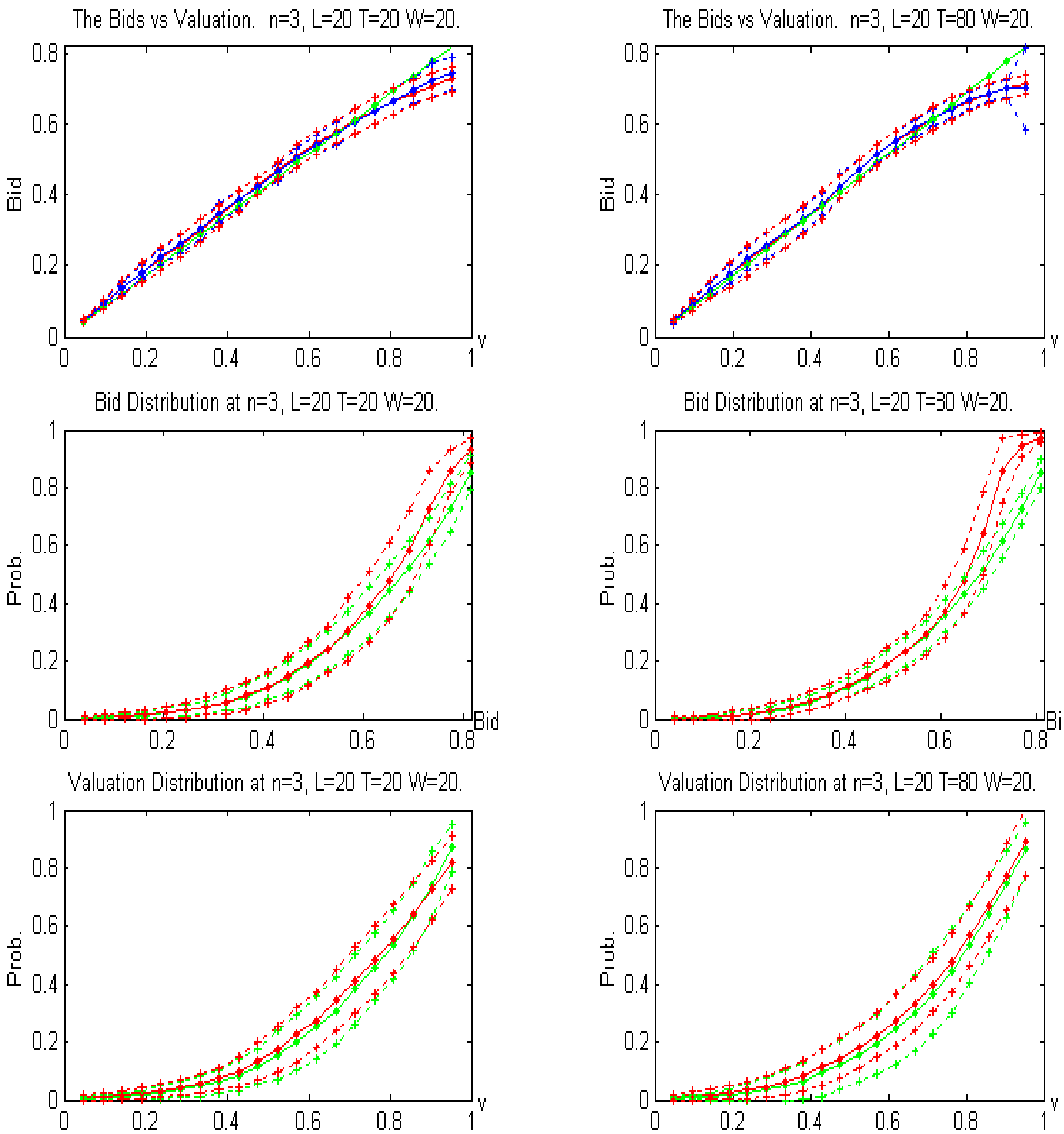

Figure 4: Scenario: $F(v) = v^3$, $G_0(b) = G_{NASH,F}(b)$, window= past 20 auctions
Top panels: Nash equilibrium bid function (----), Bid vs. inferred valuation (-----)
Bid vs. actual valuation (-----)
Middle panels: Nash equilibrium bid distribution (----), actual bid distribution (---)
Bottom panels: Actual valuation distribution (---), inferred valuation distribution (---)

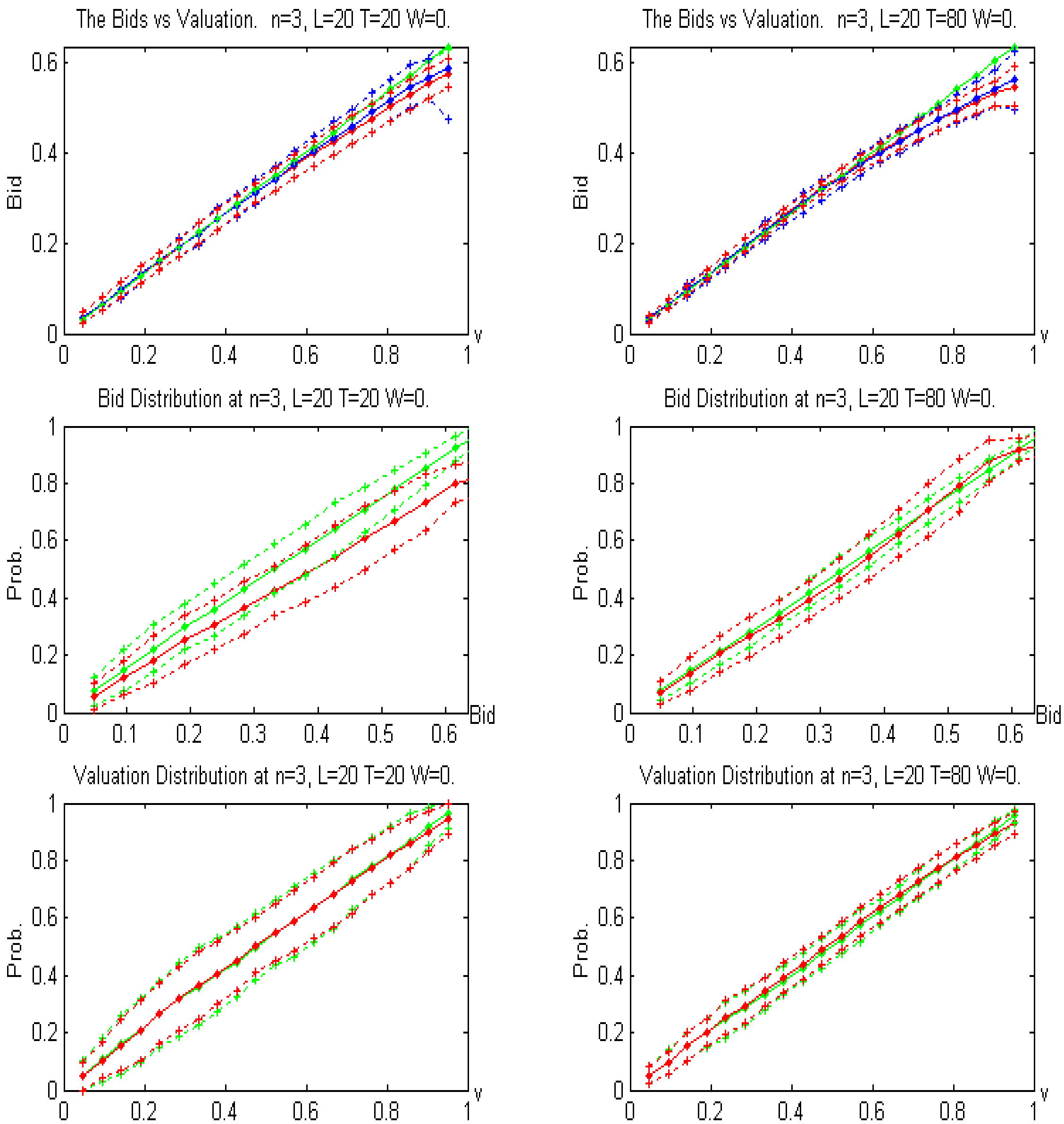

Figure 5: Scenario: F(v) =v, $G_0(b)$ = b, window= all past bids

Top panels: Nash equilibrium bid function (----), Bid vs. inferred valuation (-----)

Bid vs. actual valuation (-----)

Middle panels: Nash equilibrium bid distribution (----), actual bid distribution (---)

Bottom panels: Actual valuation distribution (---), inferred valuation distribution (---)

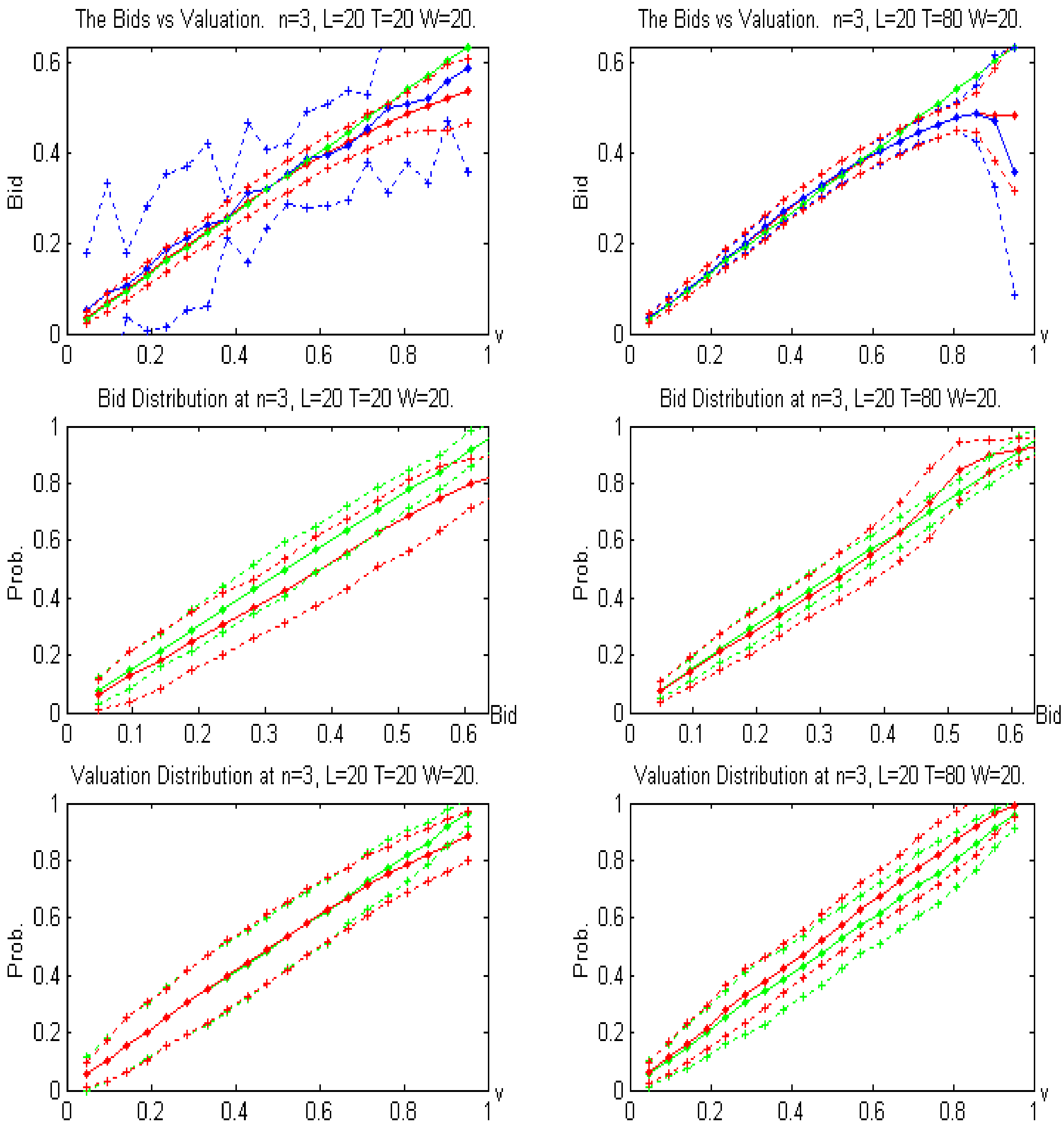

Figure 6: Scenario: F(v) =v, $G_0(b)$ = b, window= past 20 auctions

Top panels: Nash equilibrium bid function (----), Bid vs. inferred valuation (-----)

Bid vs. actual valuation (-----)

Middle panels: Nash equilibrium bid distribution (----), actual bid distribution (---)

Bottom panels: Actual valuation distribution (---), inferred valuation distribution (---)

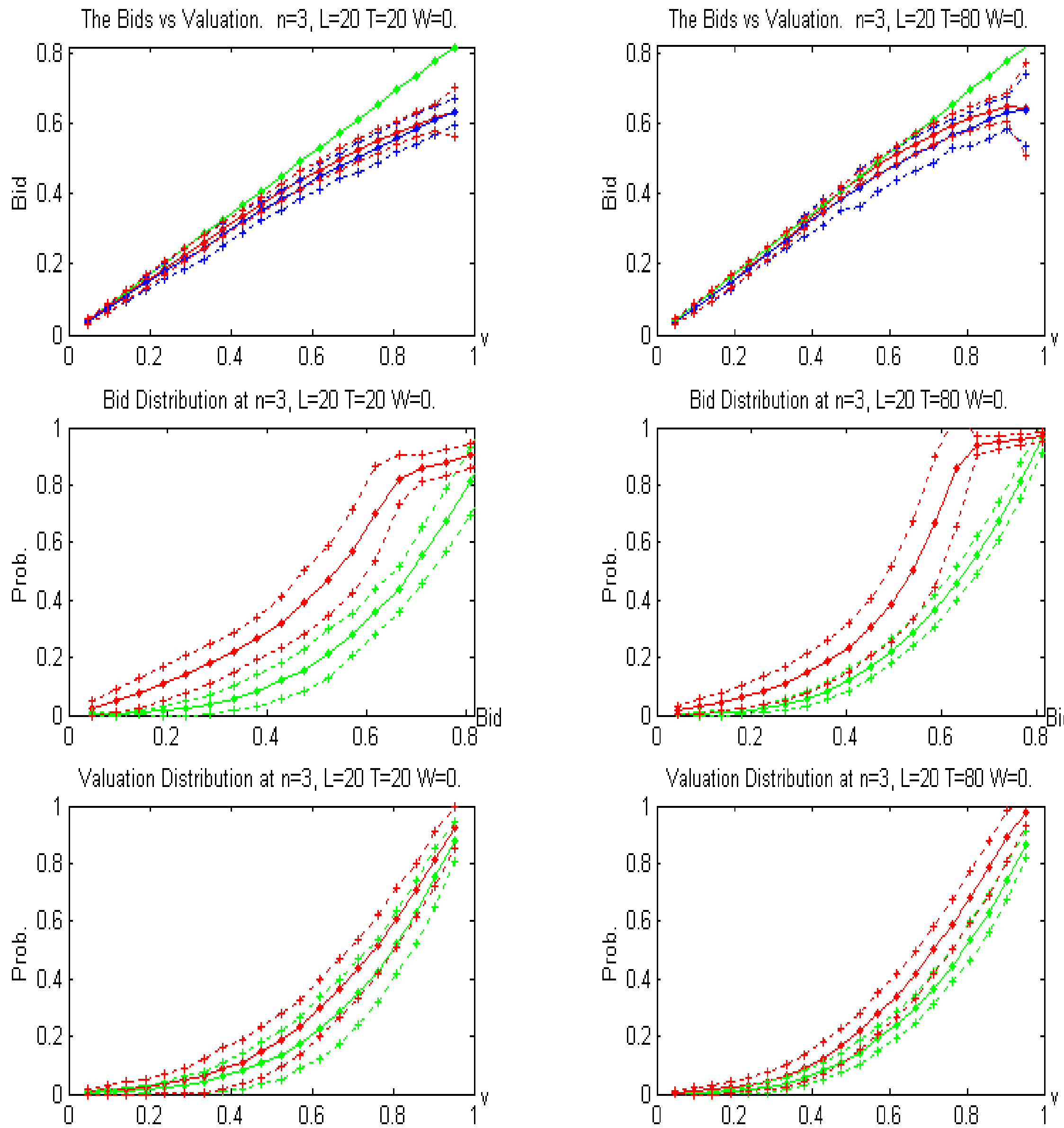

Figure 7: Scenario: $F(v) = v^3$, $G_0(b) = b$, window= all past bids
Top panels: Nash equilibrium bid function (----), Bid vs. inferred valuation (-----)
Bid vs. actual valuation (-----)
Middle panels: Nash equilibrium bid distribution (----), actual bid distribution (---)
Bottom panels: Actual valuation distribution (---), inferred valuation distribution (---)

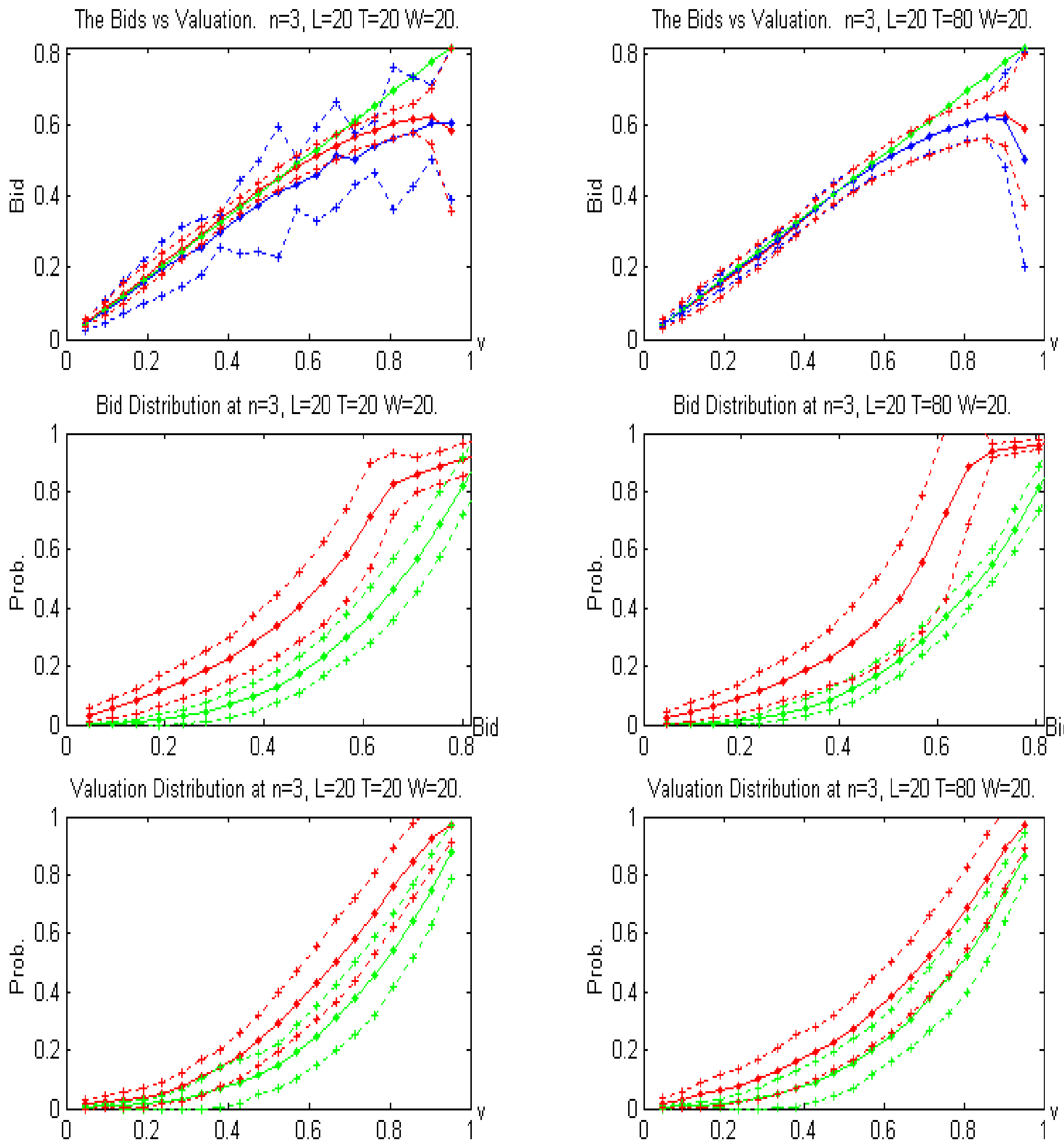

Figure 8: Scenario: $F(v) = v^3$, $G_0(b) = b$, window= past 20 auctions
Top panels: Nash equilibrium bid function (----), Bid vs. inferred valuation (-----)
Bid vs. actual valuation (-----)
Middle panels: Nash equilibrium bid distribution (----), actual bid distribution (---)
Bottom panels: Actual valuation distribution (---), inferred valuation distribution (---)