Weak force detection with superposed coherent states

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I. INTRODUCTION

Non classical states of light have received considerable attention in the field of quantum and atom optics. Many nonclassical states of light have been experimentally produced and characterized. These states include photon number states, squeezed states and certain entangled states. There are a number of suggested, and actual, applications of these states in quantum information processing including; quantum cryptography [1, 2], quantum teleportation[3–8], dense coding[9] and quantum communication[10–12] to name but a few. They have also been proposed for high precision measurements such as improving the sensitivity of Ramsey fringe interferometry[14] and the detection of weak tidal forces due to gravitational radiation. In this paper we consider how non classical states of simple harmonic oscillators may be used to improve the detection sensitivity of weak classical forces.

When a classical force, $F(t)$, acts for a fixed time on a simple harmonic oscillator, with resonance frequency ω and mass m, it displaces the complex amplitude of the oscillator in phase space with the amplitude and phase of the displacement determined by the time dependence of the force[15]. In an interaction picture rotating at the oscillator frequency, the action of the force is simply represented by the unitary displacement operator

$$D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a) \tag{1}$$

where $a, a^\dagger$ are the annihilation and creation operators for the single mode of the electromagnetic field satisfying $[a, a^\dagger] = 1$, and α is a complex amplitude which determines the average field amplitude, $\langle a\rangle = \alpha$. For simplicity we will assume that the force displaces the oscillator in a phase space direction that is orthogonal to the coherent amplitude of the initial state, which we take to be real with no loss of generality. The displacement is thus in the momentum quadrature, $\hat{Y} = -i(a - a^\dagger)$. To detect the force we would need to measure this quadrature. If the oscillator begins in a coherent state $|\alpha_0\rangle$, (α_0 is real) the displacement $D(i\epsilon)$ causes the coherent state to evolve to $e^{i\epsilon\alpha_0}|\alpha_0 + i\epsilon\rangle$. The signal is then measured to be $S = \langle\hat{Y}_{out}\rangle = 2\epsilon$, while the variance in the signal is given by $V = \langle\hat{Y}^2_{out}\rangle - \langle\hat{Y}_{out}\rangle^2 = 1$. The signal to noise ratio is hence

$$SNR = \frac{S}{\sqrt{V}} = 2\epsilon \tag{2}$$

which must be greater than unity to be resolved (the measured signal must be greater than the uncertainty of this quadrature in a coherent state). Thus we find a standard quantum limit for the weak force detection as

$$\epsilon_{SQL} \geq \frac{1}{2}\,. \tag{3}$$

II. WEAK FORCE DETECTION WITH SQUEEZED STATES.

It is well known[16] that this limit may be overcome if the oscillator is first prepared in a squeezed state (a uniquely quantum mechanical state) for which the uncertainty in the momentum quadrature is reduced below the coherent state level. For the case of an appropriately squeezed vacuum state

$$|\psi\rangle = \sqrt{1-|\lambda|^2}\sum_{n=0}^{\infty}\frac{\lambda^n\sqrt{(2n)!}}{n!}|2n\rangle \tag{4}$$

where the mean photon number is given by $\bar{n} = \lambda^2/(1-\lambda^2)$ and $\lambda = \tanh r$ with r being the squeezing parameter). A weak force causes a displacement $D(i\epsilon/2)$ on the squeezed vacuum. In this case the signal and variance for the measured momentum quadrature is given by[17]

$$\begin{aligned} S &= \langle\hat{Y}_{out}\rangle = 2\epsilon && (5)\\ V &= \langle\hat{Y}^2_{out}\rangle - \langle\hat{Y}_{out}\rangle^2 = e^{-2r} && (6)\end{aligned}$$

and hence a signal to noise ratio of $SNR = 2\epsilon e^r$ The minimum detectable force is given by[17]

$$\epsilon \geq \frac{1}{2e^r} \tag{7}$$

which for large squeezing corresponds to $\epsilon_{min} \geq 1/4\sqrt{\bar{n}}$. We see that squeezing provides an increased sensitivity that scales as $\bar{n}^{-1/2}$.

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Following early work by Bollinger et al. [13], Heulga et al.[14] have shown that quantum entangled states can be used to improve the sensitivity of frequency estimation using Ramsey fringe interferometry. Can entanglement be used to improve the sensitivity for force detection? To begin, let us consider an entangled state of two harmonic oscillators, the two mode squeezed state,

$$|\psi\rangle = \sqrt{1-\lambda^2}\sum_{n=0}^{\infty}\lambda^n|n,n\rangle \tag{8}$$

where $|n,n\rangle = |n\rangle_1 \otimes |n\rangle_2$. The entanglement in this state can be seen in a variety of ways. Most obviously it is an eigenstate of the number difference operator $a_1^\dagger a_1 - a_2^\dagger a_2$, between the two modes, and in the limit of large squeezing, $\lambda \to 1$, a near eigenstate of phase sum[18]. Alternatively we can consider the correlations between quadrature phase operators. In the limit of large squeezing ($\lambda \to 1$), the state approaches a simultaneous eigenstate of both $\hat{X}_1 - \hat{X}_2$ and $\hat{Y}_1 + \hat{Y}_2$, which is the kind of state considered by Einstein Podolsky and Rosen[19]. This kind of correlation has been exploited by Furasawa *et al.*[20] to realize an experimental teleportation protocol. With two oscillators, we need to specify how the weak force acts. We will specify that the force acts independently on each oscillator. To detect the force, consider a measurement of the joint physical quantity described by the operator $\hat{Y}_1 + \hat{Y}_2$. It is then straightforward to show that the signal and variance of the measured result, after the displacement, are given by

$$\begin{aligned} S &= \langle \hat{Y}_1 + \hat{Y}_2\rangle = 4\epsilon && (9)\\ V &= \langle\left(\hat{Y}_1 + \hat{Y}_2\right)^2\rangle - \langle \hat{Y}_1 + \hat{Y}_2\rangle^2 = 2e^{-2r} && (10)\end{aligned}$$

which gives a a signal to noise ratio of $SNR = 2\sqrt{2}\epsilon e^r$. The minimum detectable force is then $\epsilon \geq 1/(2\sqrt{2}e^r)$ which is a $\sqrt{2}$ improvement over the single mode squeezed state. For large squeezing the minimum detectable force can be expressed in terms of the total mean photon number for both modes. In this limit $\epsilon_{min} \approx 1/(4\sqrt{\bar{n}_{tot}})$. This is the same scaling as we found for a single mode squeezed state. The apparent improvement due to entanglement is simply a reflection of the fact that we have a two mode resource with double the mean photon number.

For the two mode squeezed state, there is simple way to understand this result. The entangled two mode squeezed state,(8), is easily disentangled by the application of a unitary operator of the form $U = \exp(-i\pi(a_1^\dagger a_2 + a_1 a_2^\dagger)/4)$, which does not change the total energy. We will refer to this unitary transformation as the beam splitter transformation as in the case that the two oscillator modes correspond to optical fields modes, this transformation describes the scattering matrix of an optical beam splitter. The resulting state becomes a (disentangled) product state of two single mode squeezed states (as in Eq.(4)). The weak force now acts to displace each of the single mode squeezed states, each of which may be used to achieve the squeezed state limit for displacement detection. As there are two realisations of the measurement scheme there will be an additional $1/\sqrt{2}$ improvement in sensitivity simply from classical statistics. It is thus inaccurate to attribute the improved force sensitivity of a two mode squeezed state to entanglement. In assessing the limits to force detection using entangled states of N harmonic oscillators we thus need to consider if any apparent improvement could have been achieved simply by using N copies of an appropriate non classical state of a single harmonic oscillator.

Of course it may not always be so obvious to transform an entangled state to a product of non classical states. Consider an entangled state of the form

$$|\Psi\rangle = \sum_{n=0}^{\infty} c_n|n\rangle|n\rangle \tag{11}$$

This state is correlated in number, but unlike the two mode squeezed state, it is not necessarily a near eigenstate of phase sum. If we consider a measurement of $Y_1 + Y_2$ as previously, the signal and variance after the displacement are

$$\begin{aligned} S &= 4\epsilon && (12)\\ V &= 2\left(1 + \langle a^\dagger a + b^\dagger b\rangle - \langle a^\dagger b^\dagger + ab\rangle\right) && (13)\end{aligned}$$

which gives an improvement in the signal to noise ratio when $\langle a^\dagger a + b^\dagger b\rangle < \langle a^\dagger b^\dagger + ab\rangle$. A state like this, with correlated photon number, is the pair-coherent (or "circle") state given by[21, 22]

$$|\text{circle}\rangle_m = \mathcal{N}\int_0^{2\pi} |\alpha e^{i\varsigma}\rangle_a |\alpha e^{-i\varsigma}\rangle_b d\varsigma \tag{14}$$

where $|\ldots\rangle_a$ and $|\ldots\rangle_b$ represent coherent states in the modes $\hat{a}$ and $\hat{b}$. $\mathcal{N}$ is a normalization coefficient and α the amplitude of the coherent state. This state can be written in the form (11) with

$$c_n = \frac{1}{\sqrt{I_0(2\alpha)}}\frac{\alpha^n}{n!}. \tag{15}$$

Here I_0 is a zeroth order modified Bessel function. It is found that the minimum detectable force occurs when

$$\epsilon_{min} = \frac{1}{2}\sqrt{\frac{1}{2} + \bar{n} - \alpha} \tag{16}$$

with the mean photon number being given by $\bar{n} = \alpha I_1(2\alpha)/I_0(2\alpha)$. A small improvement is seen for all α, with the minimum occurring at $\alpha = 0.85$ ($\epsilon_{min} = 0.221108$). As $\alpha \to \infty$ we have $\epsilon_{min} \to 0.25$. In this optimal region the mean photon number is small. The measurement of $Y_1 + Y_2$ is not necessary optimal however because it is not a near eigenstate. We should also point out that the circle state cannot be separated into a product state via the beam splitter transformation.

III. WEAK FORCE DETECTION WITH CAT STATES.

Let us now turn our attention to a less straightforward example. In the previous example two entangled harmonic modes, the two mode squeezed state, gave an improvement in the signal to noise ratio (compared to a single mode) of $\frac{1}{\sqrt{2}}$. With an entangled state comprised of more modes, an even better improvement may be achievable. However there is no simple way to generalise the two mode squeezed state to give an entangled state of many modes. We now consider another class of non classical states, based on a coherent superposition of coherent states, which can be entangled over N modes. In this case we again find that the apparent improvement in force sensitivity could have been achieved by using N copies of a particular single mode non classical state, the 'cat state'. We do not know if this equivalence is always possible for an arbitrary entangled states of N harmonic modes.

Consider N harmonic oscillators prepared in the cat state

$$|\psi\rangle_N = \mathcal{N}_+(|\alpha,\alpha,\ldots,\alpha\rangle + |-\alpha,-\alpha,\ldots,-\alpha\rangle) \quad (17)$$

where

$$|\alpha,\alpha,\ldots,\alpha\rangle = \Pi_k^{\otimes N}|\alpha\rangle_k \quad (18)$$

is tensor product of coherent states and $\mathcal{N}$ is the normalization constant given by

$$\mathcal{N} = \frac{1}{\sqrt{2+2e^{-N|\alpha|^2}}} \quad (19)$$

We take α to be real for convenience. For $\alpha >> 1$ this normalization constant approaches $1/\sqrt{2}$, and we henceforward make this assumption. Parkins and Larsabal[23] recently suggested how this highly entangled state might be formed in the context of cavity QED and quantized motion of a trapped atom or ion.

To begin our consideration of these states, let us consider the case of a single oscillator ($N = 1$),

$$|\phi\rangle = \frac{1}{\sqrt{2}}\left(|\alpha\rangle + |-\alpha\rangle\right) \quad (20)$$

where the mean photon number is given by $\bar{n} = |\alpha|^2$. When a weak classical force acts on the state in Eq.(20) it is displaced by

$$\begin{aligned}|\phi\rangle_{out} &= \frac{1}{\sqrt{2}}\left(e^{-iIm(\alpha\beta^*)}|\alpha+\beta\rangle + e^{iIm(\alpha\beta^*)}|-\alpha+\beta\rangle\right)\\ &\approx \frac{1}{\sqrt{2}}\left(e^{i\theta}|\alpha\rangle + e^{-i\theta}|-\alpha\rangle\right)\\ &= \cos\theta|+\rangle + i\sin\theta|-\rangle \quad (21)\end{aligned}$$

where $\theta = -Im(\alpha\beta^*)$ and we have defined the even ($|+\rangle$) and odd parity ($|-\rangle$) eigenstates

$$|\pm\rangle = \frac{1}{\sqrt{2}}\left(|\alpha\rangle \pm |-\alpha\rangle\right) \quad (22)$$

Our problem is thus reduced to finding the optimal readout for the rotation parameter θ for a two dimensional sub-manifold of parity eigenstates. The rotation is described by the unitary transformation

$$U(\theta) = \exp\left(i\theta\hat{\sigma}_x\right) \quad (23)$$

where $\hat{\sigma}_x = |+\rangle\langle-| + |-\rangle\langle+|$ is a Pauli matrix.

The objective is now to find an optimal measurement scheme to estimate the rotation parameter, θ, and thus the force parameter, ϵ. The maximum sensitivity will occur when $\theta = -Im(\alpha\beta^*)$ is maximised for a given displacement. Having chosen α real, θ is maximised by choosing β purely imaginary. This corresponds to a displacement $D(\beta)$ entirely in the momentum quadrature. Setting $\beta = i\epsilon$, we have $\theta = \epsilon\alpha$. The theory of optimal parameter estimation[24] indicates that the limit on the precision with which the rotation parameter can be determined is

$$(\delta\theta)^2 \geq \frac{1}{Var(\hat{\sigma}_x)_{in}} \quad (24)$$

where $Var(\hat{\sigma}_x)_{in}$ is the variance in the generator of the rotation in the input state $|+\rangle$, which is simply unity. Thus we find that uncertainty on the force parameter is bounded below by $\delta\epsilon \geq 1/(2\alpha)$. It thus follows that the minimum detectable force is $\epsilon_{min} \geq 1/(2\alpha)$, which may be written in terms of the total mean excitation number of the input state as

$$\epsilon \geq \frac{1}{2\sqrt{\bar{n}}} \quad (25)$$

where the mean photon number $\bar{n} = |\alpha|^2$. Comparison with the result for the single mode squeezed state shows a similar dependence on the mean excitation number however the squeezed state sensitivity is better by a factor 1/2.

We consider a two mode entangled cat state.

$$|\psi\rangle_1 = \mathcal{N}(|\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle) \quad (26)$$

However this state is easily disentangled with the unitary transformation

$$U(\pi/2) = \exp[-i\frac{\pi}{2}(a_1^\dagger a_2 + a_1 a_2^\dagger)] \quad (27)$$

(for a quantum optical realisation this is a 50:50 beamspilter) to produce the separable state

$$|\tilde{\psi}\rangle_1 = \mathcal{N}_1\mathcal{N}_2\left(|\alpha\rangle_1 + |-\alpha\rangle_1\right)\otimes\left(|\alpha\rangle_2 + |-\alpha\rangle_2\right) \quad (28)$$

As in the case for squeezed states, we only need consider the force detection sensitivity for the state of a single oscillator. The minimum detectable force is given by

$$\epsilon \geq \frac{1}{\sqrt{2\bar{n}}} \quad (29)$$

Here we see the $\sqrt{2}$ improvement from classical averaging. For the N mode state given by eqn (17) a linear transformation also exists to transform the N mode entangled state to a product state of single mode cat states. In this case the minimum detectable force using N modes, each prepared in cat state with amplitude α, is

$$\epsilon_{min} > \frac{1}{\sqrt{N\bar{n}}} \tag{30}$$

As each mode has a mean photon number given by $\bar{n} = \alpha^2$, the total mean photon number used in the entire experiment is $\bar{n}_{tot} = N\alpha^2$, the minimum detectable force can be written as $\epsilon_{min} > \bar{n}_{tot}^{-1/2}$, which has the same scaling dependence on $\bar{n}$ as the squeezed state case.

We see from here that there is no real advantage in using entangled states, as the improvement is only the standard statistical improvement that one gets from multiple copies of a single mode cat state produced by disentangling the state. When the total cost of resources are taken into account we get the same result as would be obtained for a single cat state. If you can make the cat state at all, mixing multiple copies to get a massively entangled cat state does not offer any advantage. The quantum resource we are using is already present in a single cat, in the same way that a single squeezed state by itself already provides for all the improvement in force sensitivity that the quantum nature of the state allows. On the other hand it may be more natural in some contexts to produce the entangled cat state directly. We have shown that such a state yields the same overall sensitivity for force detection as multiple copies of single cat states. In both cases the sensitivity is not quite as good as can be achieved using squeezed states with the same total mean excitation number.

IV. GENERALIZED CAT STATES.

In the example just discussed, maximum sensitivity required the classical force to displace the cat states in a direction orthogonal to the phase of the superposed coherent amplitudes. In general there is no way to arrange this before hand, as the phase of the displacement depends on an unknown time dependence of the classical force. However a simple generalization of the previous cat states can be used to relax this constraint. Note that the cat states are parity eigenstates and are thus the conditional states resulting from a measurement of the number operator modulo 2, $\hat{n}_N = a^\dagger a \bmod 2$, on an input state $|\alpha\rangle$ with α real. We are thus led to consider the conditional states for measurements of $\hat{n}_K = a^\dagger a \bmod K$. Such states have previously been considered by Schneider et al.[25]. Given a result $\nu = 0, 1, \ldots, K-1$ for such a measurement, the conditional (unnormalized) states are

$$|K, \nu\rangle = \sum_{\mu=0}^{K-1} \exp\left[\frac{2\pi i\mu\nu}{K}\right] |\alpha e^{2\pi i\mu/K}\rangle \tag{31}$$

which are eigenstates of $e^{i2\pi a^\dagger a/K}$ with eigenvalues $e^{-i2\pi\nu/K}$.

The case of $K = 4$ has recently been considered by Zurek[26] in the context of decoherence and quantum chaos. Assume that the oscillator is initially prepared in the state

$$|\psi\rangle_{in} = |4, 0\rangle = |\alpha\rangle + |i\alpha\rangle + |-i\alpha\rangle + |-\alpha\rangle \tag{32}$$

with α real. Under the action of a weak force characterized by a complex amplitude displacement β, the output state is

$$|\psi\rangle_{out} = e^{i\theta}|\alpha\rangle + e^{i\phi}|i\alpha\rangle + e^{-i\phi}|-i\alpha\rangle + e^{-i\theta}|-\alpha\rangle \tag{33}$$

where $\theta = \alpha\mathrm{Im}(\beta)$ and $\phi = \alpha\mathrm{Re}(\beta)$. The state now carries information on both the real and imaginary components of the displacement due to the force which may be extracted by measuring the projection operator onto the initial state. In the limit that $K >> |\alpha|^2 >> 1$, the initial conditional state is simply the vacuum state and we recover the usual standard quantum limit for force detection by number measurement[16].

V. DISCUSSION AND CONCLUSION.

We now compare our results to the study of Ramsey fringe interferometry introduced by Bollinger et al. [13] and discussed by Heulga et al.[14]. In Ramsey fringe interferometry the objective is to detect the relative phase difference between two superposed states, $\{|0\rangle, |1\rangle\}$. that form a basis for a two dimensional Hilbert space. These states could be the ground and excited states of an electronic dipole transition. The problem reduces to a quantum parameter estimation problem. The unitary transformation which induces a relative phase in the specified basis is $U(\theta) = \exp[i\theta\hat{Z}]$ where $\hat{Z} = |1\rangle\langle 1| - |0\rangle\langle 0|$. We are free to choose the input state $|\psi\rangle_i$ and the measurement we make on the output state, which is described by an appropriate positive operator valued measure (POVM).

The theory of quantum parameter estimation[24] indicates in this case that we should choose the input state as $|\psi\rangle_i = (|0\rangle + |1\rangle)/\sqrt{2}$ and the optimal measurement is a projective measurement in the basis $|\pm\rangle = |0\rangle \pm |1\rangle$. The probability to obtain the result $+$ is $P(+|\theta) = \cos^2\theta$. In N repetitions of the measurement the uncertainty in the inferred parameter is

$$\delta\theta = \frac{1}{\sqrt{N}} \tag{34}$$

which achieves the lower bound for quantum parameter estimation. Repeating the measurement N times is equivalent to a single product POVM on the initial product state $\prod_{i=1}^{N} \otimes(|0\rangle_i + |1\rangle_i)/\sqrt{2}$. However it was first noted by Bollinger et al.[13] that a more effective way to use the n two level systems is to first prepare them in the maximally entangled state

$$|psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2 \ldots |0\rangle_N + |1\rangle_1|1\rangle_2 \ldots |1\rangle_N) \tag{35}$$

and subjecting the entire state to the unitary transformation $U(\theta) = \prod_{i=1}^{N} \exp(-i\theta \hat{Z}_i)$, the uncertainty in the parameter estimation then achieves the Heisenberg lower bound of

$$\delta\theta = \frac{1}{N} \tag{36}$$

We now show that the entangled state in Eq.(35) is in fact a cat state for a collective operator algebra.

The Hilbert space of N two level systems is the tensor product space of dimension 2^N. The entangled state in Eq.(35) however resides in a lower dimensional subspace of permutation symmetric states[27]. These states constitute an $N+1$ dimensional irreducible representation of SU(2) with infinitesimal generators defined by

$$\hat{J}_z = \frac{1}{2}\sum_{i=1}^{N} \hat{Z}_i, \;\; \hat{J}_y = \frac{1}{2}\sum_{i=1}^{N} \hat{Y}_i, \;\; \hat{J}_x = \frac{1}{2}\sum_{i=1}^{N} \hat{X}_i \tag{37}$$

where $\hat{Z}_i = |1\rangle_i\langle 1| - |0\rangle_i\langle 0|$, $\hat{X}_i = |1\rangle_i\langle 0| + |0\rangle_i\langle 1|$, $\hat{Z}_i = i|1\rangle_i\langle 0| - i|0\rangle_i\langle 1|$. The Casimir invariant is $\hat{J}^2 = \frac{1}{4}(\hat{J}_x^2 + \hat{J}_y^2 + \hat{J}_z^2)$ with eigenvalue $\frac{N}{2}(\frac{N}{2}+1)$. The operator $\hat{J}_z$ has eigenvalues $m = -N/2, -N/2+1, \ldots, N/2$ which is one half the difference between the number of zeros and ones in an eigenstate. It is more convenient to use of the eigenstates, $|m\rangle_{N/2}$, of these commuting operators as basis states in the permutation symmetric subspace. In this notation the entangled state defined in Eq.(35) may be written

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|-N/2\rangle_{N/2} + |N/2\rangle_{N/2}) \tag{38}$$

In this form we can regard the state as an SU(2) 'cat state' for N two-level atoms.

A closer atomic analogy to a single mode cat state would be a cat state for a single N level electronic system. For example we could consider the unnormalized state defined on a hyperfine manifold with quantum number F, $|F\rangle_F + |-F\rangle_F$. Such states have been considered in reference [28]. A similar state could also be generated for the large magnetic molecular systems considered in references [29–31]. In this case one might have thought that an even greater advantage would be gained by entangling the state of many single molecule N level systems[31], however the analysis of this paper suggests that this would offer no advantage over simply using each of the molecules separately provided that the internal state of each molecule could be prepared in a cat state.

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