Network Server Supply Chain at HP: A Case Study

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inventory management, supply chain management, stocking policies

This chapter describes an inventory management project conducted at Hewlett-Packard Laboratories for HP's Network Server Division (NSD). NSD manufactures a major subassembly of network servers in Singapore and ships it to four distribution centers (DCs) worldwide, where the assembly is completed due to customer specifications. Two modes of transportation are available (air and ocean) between the factory and DCs, and customer demand at the DCs is stochastic and non-stationary. The project goal was to compute inventory policies that reduce inventory- and shipment-related costs for this subassembly by enabling more cost-effective use of ocean shipments, all without compromising order fulfillment. We discuss the challenges this project presented within the context of recent trends in supply chain management. We also describe the approach developed at HP Laboratories, as well as qualitative and quantitative results from its implementation.

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Network Server Supply Chain at HP: A Case Study

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Abstract: This chapter describes an inventory management project conducted at Hewlett-Packard Laboratories for HP's Network Server Division (NSD). NSD manufactures a major subassembly of network servers in Singapore and ships it to four distribution centers (DCs) worldwide, where the assembly is completed due to customer specifications. Two modes of transportation are available (air and ocean) between the factory and DCs, and customer demand at the DCs is stochastic and non-stationary. The project goal was to compute inventory policies that reduce inventory- and shipment-related costs for this subassembly by enabling more cost-effective use of ocean shipments, all without compromising order fulfillment. We discuss the challenges this project presented within the context of recent trends in supply chain management. We also describe the approach developed at HP Laboratories, as well as qualitative and quantitative results from its implementation.

1 Introduction

The confluence of several trends in manufacturing organizations has created a climate rife with opportunity for supply chain management practitioners to apply their skills. These trends include the increasingly ubiquitous implementation of Advanced Planning Systems, the growing complexity of supply chains, and the recognition among managers of the importance of supply chain costs to profitability.

Advanced Planning Systems, or APS, is the general name given to a class of software solutions for supply chain management. Well-known products in this class include i2's Rhythm and SAP's Advanced Planner and Optimizer. Planning tools such as these, which provide a greater degree of visibility and coordination across supply chain entities than was previously available, are rapidly replacing more traditional planning methods. Though these software solutions are continually expanding in functionality, their core function is to determine coordinated procurement, production and shipment plans that meet demand, adhere to capacity and scheduling constraints, and honor inventory policies. Through expanded visibility of the supply chain, coordination across planning functions, explicit modeling of constraints, automation of complex planning rules, and search methods based on optimization and heuristics, these tools offer many advantages over traditional planning methods.

However, the core challenge in supply chain management, namely that of managing uncertainty, is not addressed by APS. These systems regard all inputs as deterministic, leaving uncertainty to be addressed outside of the scope of the tool's functionality. A planner hedges against demand and supply uncertainty through strategic placement of inventory. Practitioners of

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supply chain management know that determining effective inventory policies that trade off costs and order fulfillment goals can be extremely difficult for complex supply chains. Rather than determining inventory policies, APS software solutions require them as input. Without a systematic, analytical approach to determine effective inventory policies, planners cannot extract the purported benefits of costly APS implementations. It is this limitation of APS, coupled with their ubiquity, that creates an enormous opportunity for supply chain practitioners. The need for effective inventory management will follow the growth trajectory of the APS business.

Meanwhile, as the demand for inventory management expertise grows, so does the complexity of supply chains. Global differences in taxes, duties and labor costs have created incentives for companies to spread their supply chains out geographically. Furthermore, supply chains increasingly span several organizations, and so operational control may be distributed across supply chain entities rather than centralized. There are often multiple alternate shipment modes available between locations. These structural complexities are compounded with other factors such as short product life cycles to make inventory management difficult. These trends require supply chain practitioners to develop new, sophisticated tools, as simple single-location, stationary demand inventory models become inadequate to address the complexities in modern supply chains.

As the need for supply chain management expertise grows, receptivity among managers to adopt the needed sophistication in their planning methods is also on the rise. Supply chain costs become a critically important component of the bottom line as competition erodes margins. This trend is particularly apparent in high tech industries, with its endemic short product lifecycles and high demand variability. At the beginning of a product's life, insufficient inventory can compromise market share, whereas excess inventory at the end of the product's life can consume all of a product line's profits. These threats have further increased support among managers for supply chain management activities.

The trends that are creating opportunities for supply chain practitioners come hand in hand with challenges. Naturally, the growing complexity of supply chains calls for rich inventory models. Most existing models in the literature are stylized to lend insight into a particular supply chain feature and how that feature affects optimal inventory policies. For example, there is extensive literature on single location inventory management with two replenishment modes. There are few models, however, that address the simultaneous presence of multiple confounding factors, such as dual replenishment modes and non-stationary stochastic demand. Practitioners facing complex supply chains must draw upon the general principles and intuition derived from several models to create a useful approach to their multi-faceted problems.

Another difficult aspect of practical inventory management is finding strategies that are practical to implement. At least as important as the optimality or cost-effectiveness of a given policy is that it is compatible with existing information technology, the data supporting it is readily available and reliable, its computational complexity is low, and it is intuitive to planners. In many cases, these requirements create severe restrictions in the types of inventory strategies that can be selected. For example, if in-transit inventory data is unreliable, policies based on inventory position instead of on-hand inventory are difficult to execute with accuracy, and thus may be inappropriate.

To compound the difficulty in selecting an effective of inventory management policy, supply chain practitioners face the additional challenge of keeping pace with constant flux in business conditions and organizations. In this climate, the quest for optimality takes a back seat to timeliness of a solution. When sophisticated analytical tools are required, the successful implementation of these tools relies upon a strong collaboration between the ultimate users of the tool, who understand the business and the data, and the supply chain experts, who build an analytical model of the business. The lifecycle of a project should fall well within the tenure of each participant in his or her position. Moreover, the analytical model should be general enough to survive changes in business conditions. If these criteria are not met, the project's likelihood of being implemented is low.

This chapter describes a supply chain management project that was carried out at Hewlett-Packard Laboratories. This project came about due to the same forces that are creating widespread opportunities for supply chain practitioners. Managers at HP's Network Server Division (NSD) approached the authors in late 1997 to ask for help in managing inventory of a component of network servers. NSD was on the verge of implementing an APS called Red Pepper. In order to extract benefit from Red Pepper's planning engine, the division needed an effective strategy for managing the inventory of a component of network servers at their factory and distribution centers. In particular, they wanted to reduce their logistics costs while maintaining high availability at the distribution centers by finding the right balance of inventory in their supply chain and by using different shipment modes (air and ocean) efficiently. Several factors complicate the process of managing this inventory, including highly non-stationary demand with large random fluctuations, rapid depreciation, high risk of obsolescence, short product life-cycles, alternative shipment modes with different associated cost and lead times, and long supply lead times.

This project was replete with the challenges that plague modern supply chain management. First, while the many complicating factors in the NSD supply chain are addressed in the literature, no previous work addresses all of them simultaneously. Indeed, it is the combination of all of them that made this problem an exciting research challenge. The authors used an approach that synthesized the learnings from several simpler inventory models. Additional constraints were that the authors had to recommend inventory policies that could be computed quickly, implemented within Red Pepper, understood by planners, and driven by the data that was available. A final difficulty was that the window of opportunity was short; NSD needed a solution quickly. Thus, heuristics offering speed and tractability were favored over optimality. As a result of the aggressive project timeline, many interesting research questions have remained open since the immediate goal of the project was met.

The contents of this chapter are as follows. Section 2 describes the status of the NSD supply chain at the project's inception and the problem that the authors were asked to address. A summary of related literature is given in Section 3. In Section 4 we describe the HP Labs' model of the NSD supply chain. This model is the foundation of an approach used to find effective supply chain inventory policies, described in Section 5. The subject of Section 6 is the implementation of the approach and how it fit into existing information technology and planning processes. Section 7 describes the results obtained at NSD through the implementation of this project. We conclude in Section 8.

To protect NSD data, product names and numbers are omitted, and all dollar values are scaled.

2 Problem Description

2.1 Overview

NSD manufactures PC-based Windows NT® network servers. An important component of servers, called a MOD0 box, is pre-assembled in Singapore and then shipped to four distribution centers (DCs) throughout the world. The final assembly and configuration of servers is done according to re-sellers' specifications at the DCs just before the product is shipped to them. Shipments between the factory in Singapore and the distribution centers can be made by either air or ocean, with different transportation times and freight costs associated with each shipment mode. A central planning organization decides upon production and shipment quantities on a weekly basis.

NSD planners were faced with the perennial challenge of finding an inventory strategy to balance high service level objectives with supply chain costs. They wanted to guarantee a high level of off-the shelf availability of MOD0 boxes while minimizing freight and inventory–related costs. The goal of this project was to develop a decision support tool to help NSD planners determine effective inventory replenishment strategies for MOD0 boxes with respect to these objectives.

2.2 The Product

At the time the project commenced, NSD had ten active types of MOD0 boxes, which are subassemblies for approximately 30 product lines. MOD0's are a high-level subassembly of servers, and contain most of the non-configurable parts of a computer of a given product line, including the chassis, power supply, base motherboard (including minimal memory), control panel, and terminator. MOD0 boxes do not contain processor chips, additional memory, or disk drives; these are added to servers later at the DCs according to customer specifications. The cost of the MOD0 boxes constitutes about 50% of the cost of the finished product.

MOD0 boxes stand out among all components of network servers due to their bulk and their relatively small dollar value per unit of volume. In addition, although prone to depreciation like all electronics products, MOD0 boxes do not depreciate as fast as hard drives or processors. These properties make them good candidates for ocean shipments.

2.3 The Supply Chain

As mentioned above, MOD0 boxes are assembled at the Singapore Factory (Warehouse).[2] Components are procured from external and internal suppliers and stored in the Singapore component inventory. Procurement leadtimes for components vary but can be as long as 8 weeks. MOD0 boxes are assembled in the Singapore factory and then shipped to one of the four distribution centers: Roseville, California; Grenoble, France; Guadalajara, Mexico and Singapore. Assembly takes only a few hours, and assembly capacity is usually available when needed.

The Singapore DC is located in the same complex as the factory and therefore there is no shipment leadtime nor any choice of shipment modes required. Shipments to the remaining three DCs can either be made by air or sea. Air shipments to all the DCs take about one week. Ocean

[2] Throughout the paper we will use Factory and Warehouse synonymously. Since we do not treat capacity constraints at the Factory, it functions in our model as an ordinary central storage location that in the inventory literature is usually called Warehouse.

shipments take, including loading, unloading, and subsequent land transportation to the DC, about four weeks to Grenoble and Roseville and five weeks to Guadalajara.

Occasionally there are transshipments between DCs. They are only made under extraordinary circumstances, such as significant overstocking in one DC and significant understocking in another, and are almost always executed by air. In total they amount to less than 2% of total shipments.

Each DC serves demand in a pre-assigned region only (North America, Asia Pacific, Europe, Latin America). For example, European customers cannot receive shipments directly from the Roseville DC.

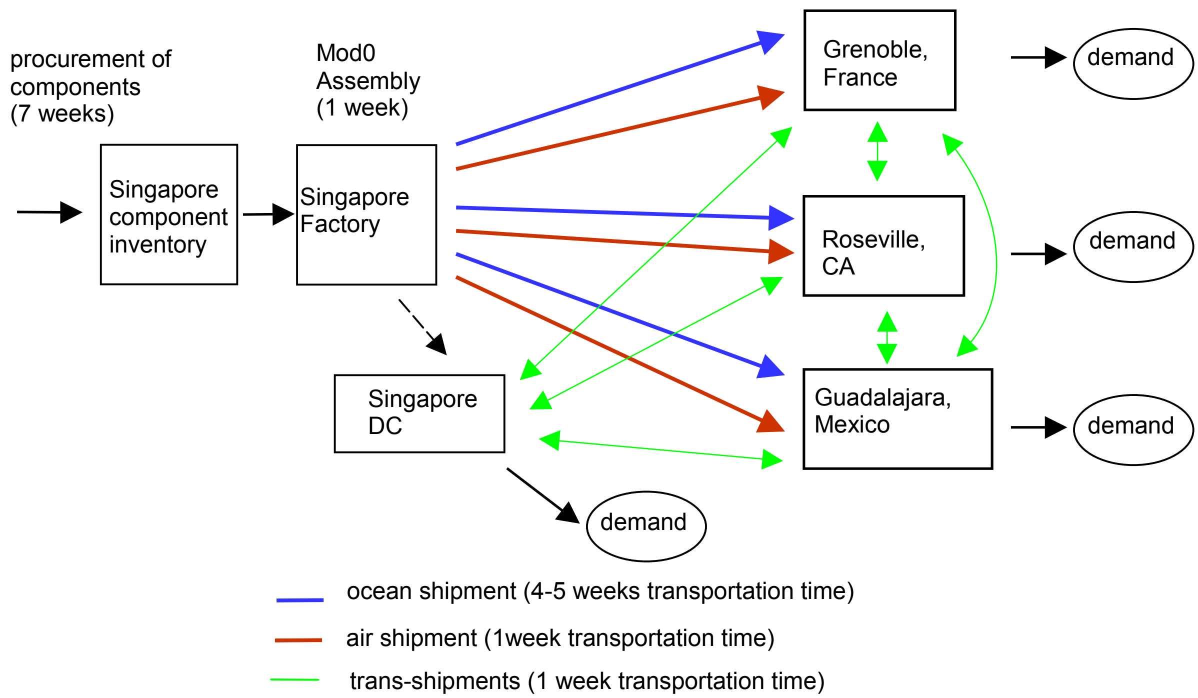


Figure 1. The NSD MOD0 Supply Chain

2.4 Costs and Performance Measures

The four types of costs associated with the supply chain operation are inventory holding costs, depreciation, obsolescence costs, and transportation costs.

Because inventory in the supply chain must be financed, holding costs are applied to inventory at any location or in transit between nodes. These finance charges are proportional to the cost of inventory held and expressed as a percentage of the standard material cost per year.

If a server cannot be sold before the end of the product life cycle, revenue loss is substantial. The server can then only sold in “fire sales”, cannibalized for its parts or, in the worst case, written off. Components, on the other hand, are often used in subsequent products, and so are less vulnerable to obsolescence. A given type of MOD0 box is typically used for multiple products within its product line, so its own life cycle is longer than that of any particular product, currently about 20-24 months. When an assembled MOD0 box does become obsolete, only a small fraction of its standard material cost can be recovered. Therefore, obsolescence cost equal to the difference of the original standard material cost and the recovered cost is incurred for every assembled MOD0 box left at the end of its life cycle. Obsolescence cost for lower-level component inventory is much lower than obsolescence cost for assembled boxes since many of the parts can be used in successor products.

Prices for the components needed to build a MOD0 box decline rapidly over the course of its lifecycle. Therefore, every MOD0 box held as inventory from one week to the next could have been built one week later at a lower price. The price difference is expressed in a depreciation cost applied to each unit of inventory in each period and location.

Transportation costs include freight cost, insurance, and costs of loading and unloading. Ocean shipment costs are charged per container shipped, whereas air shipment cost is charged per pallet. Ocean freight costs are about one-fifth of air freight costs. This cost advantage must be weighed against the higher inventory, depreciation and obsolescence cost associated with ocean shipments, as well as diminished agility as compared to air shipments.

The performance of the supply chain is currently measured by the off-the-shelf availability for finished servers (the percentage of orders filled within a week or less) and the total cost. Availability of components is currently not used of a performance measure.

2.5 Supply Chain Operations

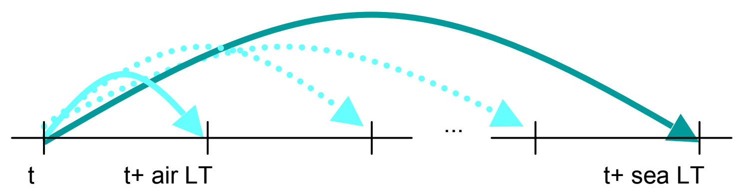
At the project's inception, planners in the NSD World Wide Planning division in Santa Clara created weekly production and shipment plans using rules that had been programmed into spreadsheets. Production orders and shipments were based on on-hand inventory targets for all the DCs and the Singapore factory. When projected inventory on hand fell below the on-hand target, an order was triggered. Planners generally chose air as the shipment mode; only when a MOD0 box was in surplus due to repeated high forecasts was an ocean shipment made. As a result, the majority of shipments (over 65%) were made by air.

At that time, inventory targets were determined by using independent single-location, single-replenishment-mode inventory models at each location. A myopic order-up-to policy was computed based on the required service level. These calculations did not take into account non-stationarity of the demand, interactions between the DCs and the factory, nor potential savings from utilizing different shipment modes appropriately. The end of the product life cycle was handled on an manual exception basis. Inventory targets were reduced during this phase, creating lower availability (or longer response times) in order to avoid excessive obsolescence cost caused by leftover inventory.

At the same time, NSD was in the process of implementing Red Pepper® as its planning engine. This system would automate the production and shipment decisions that planners were using spreadsheets to make. This automation would include the following rule for determining the allocation of air and sea shipments. First, let *airLT* and *seaLT* respectively denote the air and sea shipment leadtimes for a given DC. For a given period t and this DC, Red Pepper compares the projected inventory (based on on-hand and in-transit inventory and forecasts) in period $t + airLT$ with the DC's target inventory for that period. Red Pepper triggers an air shipment to the DC if the projected inventory falls short of the target. Next, it considers making a sea shipment to the DC. To avoid making a sea shipment now that will adversely impact the ability to make air shipments that would arrive sooner, Red Pepper first "reserves" inventory at the factory that is needed to make air shipments to all DCs that would arrive before the sea shipments in this period. If factory inventory remains after reserving for air shipments, then a sea order is triggered to bring projected inventory in period *t+seaLT* up to the target for that period. Whenever there is a shortage of inventory at the factory, it is allocated to the DCs in proportion to the size of their requested shipments. This shipment mode selection rule is illustrated in Figure 2.

This selection rule, coupled with the relative magnitude of factory and DC targets, determines the fraction of shipments that are made by air. Thus, shipment decisions were not directly within the scope of the decision tool developed at HP Labs. Instead, these decisions were to be a by-product of the recommended inventory strategy.

The total inventory of MOD0 boxes in the NSD supply chain, including in-transit between the factory and DCs, was approximately 8 weeks of supply when the project began. The off-the-shelf availability was approximately 85% for MOD0 boxes and 75% for servers. NSD hoped to ultimately satisfy 95% of demand for the finished product within one week response



1. Make air shipments in period t. For each DC, determine desired air shipment using projected inventory and on-hand target in t+air LT. Make DC air shipments by allocating warehouse inventory in proportion to DC desired air shipments.

2. Plan for air shipments in t+1, …, t+seaLT-airLT-1. For each DC, reserve factory inventory for air shipments that will arrive to meet DC targets in periods t+airLT+1, …, t+seaLT-1. Account for shipments that will arrive at the factory in time to make these planned air shipments.

3. Make sea shipments in period t. For each DC, Determine desired sea shipments to meet on-hand target in t+seaLT. Allocate unreserved factory inventory in proportion to DC desired sea shipments to make shipments.

Figure 2. Selection of Shipment Mode

time. Such an improvement would require, among other things outside the scope of this project, a dramatic increase of MOD0 box availability.

2.6 Anticipated Project Benefits

There were three ways in which the project team hoped they could reduce NSD's supply chain costs without compromising availability. The first was in redistribution of inventory. They hoped that they might yield improved availability per unit of inventory by rebalancing inventory between the factory and DCs. The second way they hoped to improve operations was to reduce overall supply chain inventory. A reduction of total MOD0 box inventory by 20% would save an equal percentage in inventory and depreciation cost annually. The third goal was increased used of ocean shipments. By shifting the proportion of shipments from 65% air and 35% ocean to the reverse proportions, NSD anticipated savings of about 30% of freight costs. Of course, this savings would be partly offset by higher inventory, depreciation and obsolescence cost; this tradeoff would have to be considered by the model.

Naturally, the potential cost savings was inversely related with service level goals. Thus, we wanted to develop an approach that would quantify the trade off between service level goals and cost, to enable NSD managers to strike the balance they desired.

The customers (re-sellers) have the right to return a certain percentage of the products they previously bought to the DC. Returns play a particularly important role close to the end of the life cycle of the product since they contribute to a potential excess inventory and can incur high obsolescence cost for HP. On the other hand, even if inventory at the resellers is not returned to the DC, price protection cost, which is based on the reseller inventory, constitutes a big contribution to NSD's supply chain cost. An increase in service (shorter and more reliable response times) can impact both problems positively. If the DCs are more responsive resellers do not need to hold large inventory and returns as well as price protection expenses may drop.

2.7 The Data

NSD World Wide Planning receives demand forecast on a monthly basis from NSD Marketing. These monthly forecasts are broken down into weekly "buckets" based on an empirical distribution of historical orders over the month. Using the bill of material, the component demand is determined. Historical data of actual demand back to 1996 is available. Forecasts have a relatively high error; coefficients of variation of 30-60% are not uncommon.

Cost data for freight and capital cost are available and relatively reliable. Depreciation cost and obsolescence cost can be estimated from historical data and specific business knowledge available from the planners and the marketing department.

3 Related Literature

Three features of the NSD supply chain problem complicate finding an optimal inventory policy. The first is the existence of two replenishment modes. The second is the fact that the supply chain in question is a two-echelon distribution system. The third is the non-stationarity of demand. Each of these features has been extensively studied independently. In what follows, we briefly summarize the previous contributions in each of those areas.

A number of authors have considered single location inventory models with two supply modes. One setting in which the form of the optimal inventory control policy has been characterized is under periodic review, with the critical assumption that the two modes' leadtimes differ by exactly one period. Many authors, notably Fukuda (1960), Bulinskaya (1964a,b), Daniel (1962), Neuts (1964), Veinott (1966), Wright (1968), and Whittmore and Saunders (1977) have presented results for this case. In those papers, it is shown that the optimal policy is a generalization of the traditional order-up-to type policy, in which there are two order-up-to levels in each period, for the short and long leadtime mode inventory positions, respectively.

Without the assumption that the leadtimes differ by one period, the form of the optimal policy is not known. Today's shipment decision is certain to depend on the amount of inventory on hand as well as the quantities due to arrive in each period between the short and long leadtimes from now. Moreover, it can be seen from a dynamic programming formulation of the problem than an optimal strategy will not only depend on the total on-hand and pipeline inventory (inventory position) but also on the period in which what fraction of the pipeline inventory is going to arrive. Work in this more general setting, both in periodic and continuous review, has been done by authors such as Allen and D'Esopo (1968), Moinzadeh and Nahmias (1988), Moinzadeh and Schmidt (1991), Aggarwal and Moinzadeh (1994), Pyke and Cohen (1994), and Chaing and Gutierrez (1996). These papers propose heuristic policies, the latter two in a multi-echelon setting. The work presented in this chapter also takes the approach of proposing a form of heuristic policy that is more practical to implement than an optimal policy would be, and searching for the best among such heuristic policies.

There are numerous papers on multi-echelon inventory systems with the same supply chain layout as NSD's. Optimal policies have not been characterized in this setting, but instead, practical policies have been proposed and evaluated. One exception is in serial systems, where Clark and Scarf (1960) gave the form of the optimal policy. Others such as Federgruen and Zipkin (1984), Jackson (1988), and Graves (1996) propose heuristics for the periodic review case. Authors who consider continuous review (R,Q)-type policies in the multi-echelon setting include Sherbrooke (1968), Deuermeyer and Schwarz (1981), Graves (1985), Moinzadeh and Lee (1986), Lee and Moinzadeh (1987), Svoronos & Zipkin (1988), and Axsater (1993). This list is not exhaustive, but is hopefully representative of the literature in this area. Like previous approaches, our tactic is to consider a class of inventory policies that we expect to be effective and practical to implement, and to search for the best policy within this class.

Finally, there have been many different models of nonstationary demand in single location inventory models. The papers most relevant to NSD's business and most related to the approach taken here are those of Karlin (1960) and Veinott (1965). Karlin showed that order-up-to policies are optimal in simple one node inventory systems even when demand is nonstationary, but these order-up-to levels aren't easy to calculate in general. Veinott showed that if the demands were stochastically nondecreasing, then the optimal order-up-to levels were in fact the myopic ones (the newsvendor solution) and so are easy to compute.

4 The Model

In this section we describe the model used to represent NSD's supply chain. In order to make the model tractable, some simplifying assumptions were made. We felt that these assumptions were general enough reflect the main features of the original supply chain closely, so that the recommendations made would remain valid.

Due to the relatively high capacity for assembly at the Singapore factory and the linear cost structure, different products do not interact in our model. For that reason we can treat each of the products in a separate one-product model.

4.1 Product Flow

Since transshipments between DCs are rare and NSD does not wish to plan for them, we assume that transshipments are not available as replenishment mechanisms. The supply chain resulting from this assumption is a distribution system. Customer demand is filled by the DC assigned for the region, DCs receive their shipments from the Singapore factory, which in turn uses the components from the Singapore inventory. Figure 3 shows the simplified supply chain layout.

Each inventory location in the supply chain is characterized by its replenishment leadtimes, i.e. the time between the instant a replenishment order is placed until the ordered amount arrives in the inventory given the entity the order is demanded from has stock available.

For the DCs in France, the US and in Mexico, these leadtimes are the air and sea transportation times, which are inputs to the model. For the Singapore DC we assume the leadtime to be zero due to its proximity to the factory. The leadtime for the Singapore component inventory is taken to be the component procurement time plus the assembly time. This absorption of the assembly time into the leadtime requires that assembly capacity is not restrictive.

4.2 Demand

Demand occurs only at the DCs. We assume that the weekly demands can be described as sequences of independent non-negative random variables. Weekly demands are not assumed to be identically distributed; indeed they have been historically been highly non-stationary, due to life cycle issues.

Non-negativity of demand is a technical assumption needed to make the mathematical analysis of the model more tractable. This assumption is not very restrictive for most of the product's life where actual demands vastly exceed returns. In fact, if demand is determined as sell-through+to[3], non-negativity is virtually guaranteed.

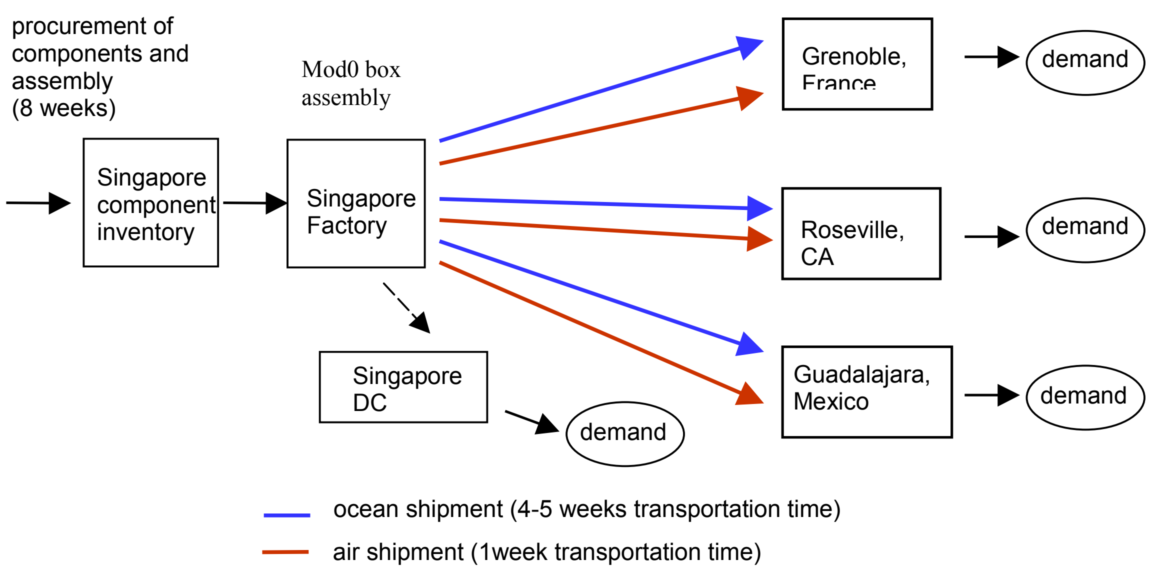


Figure 3. The Simplified Supply Chain

Earlier investigations at NSD showed that Weibull distributions fit historical demand data reasonably well, and so we used them to model demand. The distribution parameters are determined by using the forecast as the distribution's mean and the empirical coefficient of variation (obtained from historical data) as the distribution's coefficient of variation. Our analysis does not crucially depend on this particular choice of distribution type. In fact, the software that was developed gives the user a choice of demand distributions to be specified in the input data.

[3] Slightly simplistic, sell-through + to can be thought of as end-customer demand.

4.3 Cost and Performance Measures

Although our analysis allows time- and location-dependent rates for holding and depreciation, NSD chose to apply constant, universal rates to all inventories and products in transit. In particular, NSD lacked the data required to support a nonlinear depreciation rate model. Both costs are expressed as a constant percentage of the standard material cost.

A one-time obsolescence cost is applied to the products in inventory at the end of the planning horizon. Currently the obsolescence cost is taken to be a fixed percentage of standard material cost for all products, and it is only applied to DCs, since inventory there represents assembled MOD0s whereas factory inventory represents more versatile components. These choices are not required for the analysis. In practice, the actual value that can be recovered at the end of a product's life will vary from product to product, but it is difficult to estimate these numbers. We use the historical average for similar products.

Transportation cost is assumed to be proportionate to the amount shipped. The rate can vary across product, location and shipment mode. The proportionality assumption ignores the effects of batching. This assumption is justified by the fact that NSD combines several different products in one container, making batch sizes small compared to total volume shipped. This practice enables them to ship full containers only.

The performance of the system is measured by a type II service level for each of the products across all DCs. A type II service level is the mean of the fraction of demand which is satisfied off-the-shelf:

$$\lambda^{\text{Product}} = \mathrm{E}\left(\frac{\text{Demand Satisfied from Stock on Hand}}{\text{Total Demand for the Product}}\right). \tag{1}$$

Our goal is to minimize the total cost incurred over the life cycle while guaranteeing a minimum service level for each product.

5 Approach

In what follows we describe the approach we used to find cost-effective inventory management policies. This approach, based on the model assumptions outlined above, is comprised of three major components. The first component is a refinement of the class of inventory strategies that we consider, and a parameterization of this class of strategies to afford efficient search among them. This is described in Section 5.1. The second element of our approach is a simulation engine that emulates and evaluates how policies in our candidate class perform with respect to the metrics we have established. The simulation is described in Section 5.2 The third component, the subject of Section 5.3, is a search procedure that relies on the parameterization of strategies and the simulation results to select the best inventory strategy.

5.1 Candidate Inventory Replenishment Strategies

5.1.1 Implementable Strategies

In every time period the inventory manager makes decisions about how much to order for the factory and how much to ship to each of the DCs. All order and shipment quantities are non-negative. Naturally, the decision must be based only on the information available at the time it is made; such decisions are called non-anticipative. In any period, the state of the supply chain is completely described by the on-hand and in-transit quantities for each DC and the factory. Since demand is assumed to be independent between periods, there is an optimal feedback policy, i.e. a policy that only depends on the current state rather than the complete history of the system.

Unfortunately, the optimal strategy is unlikely to have a simple structure, like that of a order-up-to or a two-bin policy. Because there are two shipment modes with different lead times, the optimal replenishment decisions at the DCs will depend on the inventory on hand as well as shipment quantities due to arrive in several different periods. The limitations of the Red Pepper system at that time prohibited implementing such complex rules.

Computing an optimal policy may also be difficult. To integrate our policy computation into a decision support tool that would interface with the Red Pepper planning engine, lengthy computation times would be unacceptable.

For these reasons, and because of the urgency of getting a working solution, we decided to restrict our attention to the class of order-up-to policies. These policies are described by an on-hand inventory target for each location and each period. This choice is in accordance with current practice at NSD, and seemed most fitting to ensure a timely implementation of the project. Since the class of order-up-to policies does not, in general, contain the optimal policy, we plan to consider more general policies at a later stage of the project.

5.1.2 Further Reduction of the Class of Strategies

A general order-up-to strategy requires inventory targets for the factory and each DC for each period. For the current supply chain layout with one factory and four DCs, and a typical planning horizon of 52 weeks, we would need 260 parameters to describe a order-up-to strategy completely. To reduce complexity of the optimization problem, we propose a heuristic approach that reduces the number of decision variables while still including a broad and sensible class of strategies. We deem a order-up-to strategy sensible if it meets the following requirements:

- Targets should reflect changes in magnitude and variability of demand over time.[4]
- Service should be balanced over all DCs. A strategy that achieves good overall service level in terms of Equation (1) but exhibits a significant imbalance in service between DCs is not acceptable.
- Service should be balanced over time. A strategy that achieves good overall service level but exhibits a significant imbalance in service over time[5] is not acceptable. A controlled decline in service at the end of the life of a product may be desirable to avoid expensive excess inventory, see Section 5.1.3.

In what follows we describe a class of sensible strategies parameterized by two numbers, instead of one decision variable for each location and time period. As we will illustrate, these two numbers correspond respectively to factory and DC service levels in independent single location inventory models. To motivate this parameterization, recall that in a single location inventory model with one replenishment mode, the optimal inventory targets are determined by the service level desired. These targets reflect the changing magnitude of demand as well as the changing variability, expressed as its standard deviation, over time. The parameterization of policies allows us to construct a sensible inventory strategy for the entire supply chain from targets derived from simple, single location models.

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[4] Restricting the search to stationary strategies, e.g., would simplify the analysis but seems like too strong a restriction for highly non-stationary demand of products like computers.

To illustrate this approach, consider the following class of strategies, described by two parameters δ^{DC} and δ^{WH}, each between zero and one:

- For each DC, use δ^{DC} as a type I service level[6] for a single location inventory with only the longer leadtime mode available (ocean) to determine a sequence of inventory targets for each period of the horizon of the problem ($S_1^{DCi}, S_2^{DCi}, \ldots, S_N^{DCi}$), $i = 1, \ldots, M$. For details on how the inventory targets are calculated, see Section 5.1.4. Figure 4 illustrates how different values δ^{DC} of produce different sets of inventory targets at a particular DC.
- For the warehouse, use δ^{WH} as a service level for a single node inventory model with a fixed leadtime (the warehouse replenishment leadtime) to determine a sequence of inventory targets for each period of the horizon of the problem ($S_1^{WH}, S_2^{WH}, \ldots, S_N^{WH}$). The warehouse demand used for this calculation is the sum of the DC demands offset by the DCs ocean leadtime.

For each pair of parameters (δ^{DC}, δ^{WH}), the above calculations yield a sequence inventory targets for each of the DC and the warehouse and each period. We denote this strategy by

$$\boldsymbol{S}(\delta^{DC}, \delta^{WH}) = \left((S_1^{DCi}(\delta^{DC}), \ldots, S_N^{DCi}(\delta^{DC})), (S_1^{WH}(\delta^{WH}), \ldots, S_N^{WH}(\delta^{WH}))\right)$$

and call δ^{DC} and δ^{WH} the *shape parameters* of the strategy. This strategy, used as described in the previous section by Red Pepper® and applied to a particular stream of realizations of demand, determines shipments and replenishment orders.

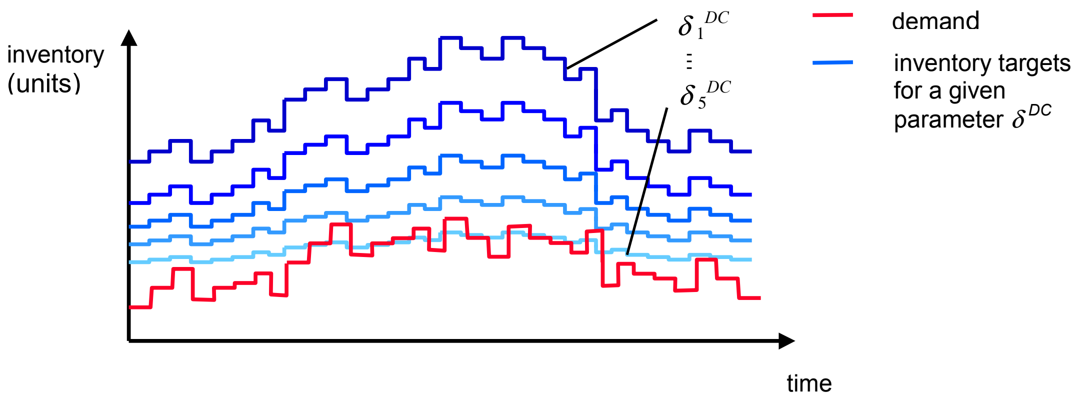

Figure 4. Inventory Targets at one DC for different values of δ^{DC}

The class of strategies $\boldsymbol{\Sigma}=\{\boldsymbol{S}(\delta^{DC}, \delta^{WH}), 0\leq\delta^{DC}<1, 0\leq\delta^{WH}<1\}$ is certainly less general then the class of all possible combinations of on-hand inventory targets in the DCs and the warehouse and, therefore, the best among strategies in $\boldsymbol{\Sigma}$ may not include the best of all possible order-up-to strategies. On the other hand, the strategies within this class are sensible according to the criteria outlined above. Moreover, changing the parameters δ^{DC} and δ^{WH} changes the total amount of supply chain inventory, the balance between warehouse and DC inventory, and in turn the ratio of air to ocean shipments. While hard to prove, we feel that the generality of the

[6] A type I service level is the probability of not stocking out in a given period.

strategies in **Σ** ensures that the best strategy in **Σ** will perform closely to the overall best strategy in terms of cost. In subsequent sections we will demonstrate how we select a strategy in **Σ.**

5.1.3 Changing Service Level Requirements

Up to this point, we assumed the shape parameters of a particular strategy to be constant over the whole horizon. This assumption corresponds to a constant level of service for all periods. Under some circumstances it may be desirable to "distribute" service over the horizon unequally. For example, in the phase of new product introduction availability of the product might be crucial and one may want to achieve a higher level of service (lower probability of stock-outs) during this phase. On the other hand, at the end of life one might be willing to sacrifice service in order to clear the supply chain of the product and minimize excess inventory. We accommodate these requirements by introducing a service level shape that can be specified by the user. This service level shape is a multiplicative parameter for each period. If the service level shape for period t is α_τ and for period t' it is $\alpha_{t'}$ then the required service level in period t' is $\alpha_{t'}/\alpha_\tau$ times the required service level in period t. For a given sequence of service level shape parameters $(\alpha_t)_{t=1,\ldots,N}$ we modify the class of candidate policy to

$$\mathbf{\Sigma}^\alpha=\{\boldsymbol{S}^\alpha(\delta^{DC},\ \delta^{WH}),\ 0\leq\delta^{DC}<1,\ 0\leq\delta^{WH}<1\}$$

with

$$\boldsymbol{S}^\alpha(\delta^{DC},\ \delta^{WH}) =\big((S_1^{DCi}(\alpha_1\delta^{DC}),\ \ldots,\ S_N^{DCi}(\alpha_N\delta^{DC})),\ (S_1^{WH}(\delta^{WH}),\ \ldots,\ S_N^{WH}(\delta^{WH}))\big).$$

We apply the service level shape to the DC targets only for two reasons. The first reason is that NSD incurs obsolescence cost only for excess inventory at the DCs. The second is that the level of service at the DCs is a major management objective while the service at the factory is merely a means to an end. Figure 5 depicts a typical service level shape for an end-of-life situation.

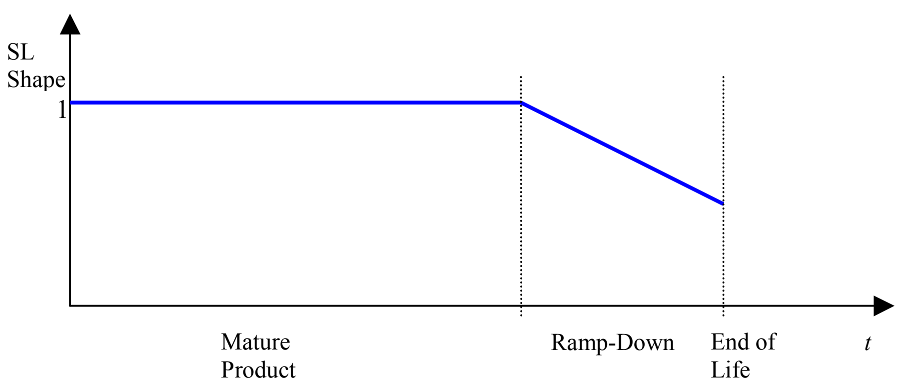

Figure 5. Typical Service Level Shape

5.1.4 Calculating Inventory Policies in Σ

In this section we show how the inventory targets for a given shape parameter δ^{DC} (respectively, δ^{WH}) are calculated. Since the calculations are essentially the same for each DC and the warehouse, we restrict our attention to one particular DC. As mentioned earlier, we derive the targets from a single-location, single-shipment-mode inventory model with leadtime equal to the ocean shipment leadtime. Denote this leadtime by *seaLT*. Let y_t be the inventory position in period t after demand but before an order is placed. Thus, y_t equals the inventory on hand after demand in period plus the inventory in transit arriving on or before t+*seaLT*. In the context of this single-mode model, the period t+*seaLT* is the first period whose on hand inventory can be influenced by decisions in period t. The on hand inventory after demand in period t+*seaLT* is equal to

$$y_t - D_{t+1} - \ldots - D_{t+seaLT}$$

where D_k denotes the demand in period k. The parameter δ^{DC} represents a type I service level goal applied in each period. This means that in each period, the probability of being able to fulfill all demand must be at least δ^{DC}. In period t+seaLT this constraint is written as

$$\mathrm{P}(y_t - D_{t+1} - \ldots - D_{t+seaLT} > 0) \geq \delta^{DC}.$$

Note that the left-hand side of the above inequality is non-decreasing in y_t. Therefore, the smallest y_t that satisfies the inequality satisfies the equality

$$\mathrm{P}(y_t - D_{t+1} - \ldots - D_{t+seaLT} > 0) = \delta^{DC}. \qquad (2)$$

From the probability distributions of demands, we can solve this equation in y_t and obtain the minimum inventory position y_t^* after ordering in period t in order to meet the service level δ^{DC} in period t + *seaLT*. In Appendix A. we describe our approach of solving Equation (2).

If the inventory position after ordering in period t is equal to y_t^* then the expected on-hand inventory after demand in period t+*seaLT* is equal to

$$S_{t+seaLT} = y_t^* - \mathrm{E}(D_{t+1}) - \ldots - \mathrm{E}(D_{t+seaLT}).$$

Since we assume the forecast to be unbiased, we can replace the means in the above equations by the forecast values for the respective periods. The values $S_{t+seaLT}$, $t = 0, \ldots, N - seaLT$, are the on-hand inventory targets for these periods. In order to obtain targets for periods 1, …, *seaLT*-1 we add *seaLT*-1 "dummy" periods with zero demand at the beginning of the horizon.

To determine warehouse inventory targets, we use essentially the same approach. The major difference lies in the structure of the demand. Warehouse demand is determined by the shipment requests from the DCs. Using DC demands as approximations for DC shipment requests (a reasonable assumption if inventory targets are reasonably stable from week to week) and assuming DCs request only ocean shipments, we obtain for the warehouse demand in period t

$$D_t^{WH} = \Sigma_i\, D^{\mathrm{DCi}}_{t+seaLTi.}$$

The warehouse inventory position targets y_t^{WH*} are the solution of

$$\mathrm{P}(y_t^{WH} - D^{WH}_{t+1} - \ldots - D^{WH}_{t+whLT} > 0) = \delta^{WH}.$$

Again, the on-hand targets are obtained by subtracting the mean leadtime demand (forecast):

$$S^{WH}_{t+whLT} = y_t^{WH*} - \mathrm{E}(D^{WH}_{t+1}) - \ldots - \mathrm{E}(D^{WH}_{t+\mathrm{whLT}}).$$

5.2 Simulating Inventory Policies

The second major component of the approach developed at HP Labs is a simulation engine. The simulation takes, as input, an inventory strategy characterized by an order-up-to level at each location and time period in the horizon. (For example, this strategy could be one calculated as in Section 5.1.4 for a given pair of parameters (δ^{DC}, δ^{WH})). It also requires demand distribution information (forecast, coefficient of variation, and distribution type) for each period and location, as well as structural supply chain data and costs. From this input, the program simulates how Red Pepper® would make shipment decisions given random demand drawn from the specified distributions. In the process, it measures the costs associated with a policy and its overall service level (the percentage of demand satisfied immediately from stock) for a given demand outcome. This is repeated for many random demand scenarios, and the resulting costs and service levels for all runs are averaged. These averages are an approximation for the expected cost and service level of the given set of inventory targets. Confidence intervals for these approximations are also computed.

5.3 Searching for Optimal Policies In Σ

Section 5.2 describes how we evaluate the expected cost and order fulfillment performance of an arbitrary set of inventory targets in the NSD supply chain. In particular, the simulation can be used to evaluate strategies in the class parameterized by δ^{DC} and δ^{WH}. In this section we discuss how we use the simulation to guide a search for the (δ^{DC}, δ^{WH}) expected to perform best according to NSD's cost and while meeting the overall service level criterion. To that end, define a point (δ^{DC}, δ^{WH}) in the grid as infeasible if its associated policy has an expected service level below the desired one.

The pair (δ^{DC}, δ^{WH}) lies in the square [0,1) x [0,1). We begin by discretizing this region into a grid of arbitrarily small grid size. We use three observations to enable efficient search of points in this grid. The first observation is that if a given point (δ^{DC}, δ^{WH}) is infeasible, then any point smaller in both coordinates is also infeasible. This follows from the fact that decreasing either parameter corresponds to decreasing inventory targets uniformly at all locations. (see Figure 6). The next two observations were made based on empirical evidence rather than proven analytically. The second is that for a fixed δ^{WH}, the cost is empirically observed to be strictly increasing in the range of feasible δ^{DC}. So the least feasible δ^{DC} is best for a fixed δ^{WH}. The third observation is that for a fixed δ^{DC}, the cost is quasi-convex in δ^{WH}. This means that as you increase δ^{WH} the cost never decreases after it increases.

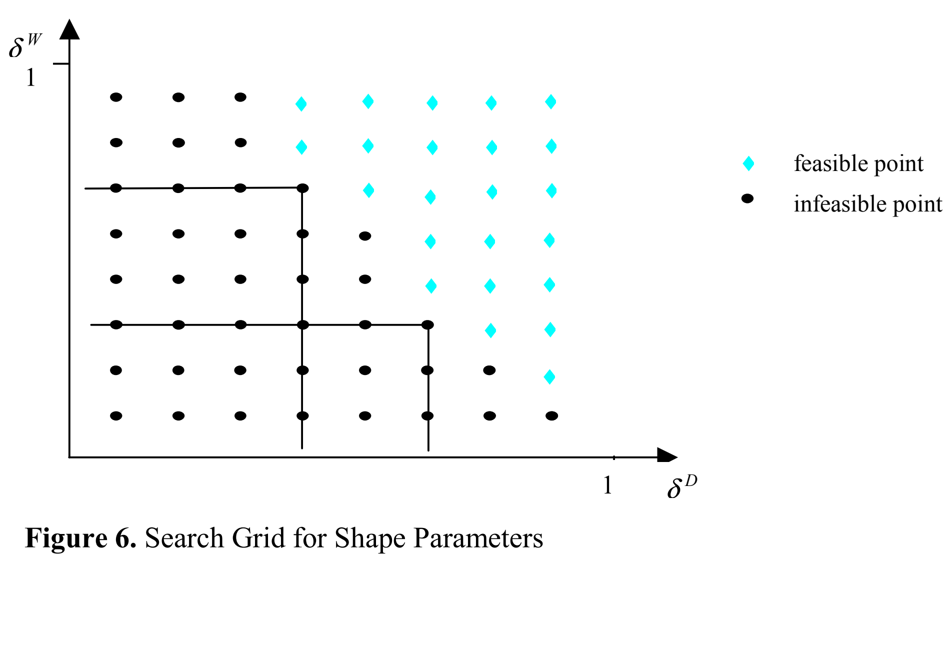

Figure 6. Search Grid for Shape Parameters

These three observations lead to the following efficient search algorithm. Let N represent our discretization granularity, so that the points (δ^{DC}, δ^{WH}) can be expressed as $\delta^{DC} = k/\mathrm{N}$, $\delta^{WH}=l/\mathrm{N}$ for k ,l = 1,2,…,N-1.

Step 0. Start in the lower right corner of the grid, with $\delta^{DC} = (N\text{-}1)/N$ and $\delta^{WH} = 0$. Let *is_feasible* = false, *best_cost* = infinity, and *best_*δ^{DC}*_cost* = infinity.

Step 1. Keeping δ^{DC} fixed, increase δ^{WH} by the increment $1/N$ until we reach the first feasible δ^{WH} for this δ^{DC}. When we reach such a point, let *is_feasible* = true and record the value of δ^{WH} at this point. If no $\delta^{WH} \leq (N\text{-}1)/N$ is feasible, then there is no feasible point in the grid for this or any smaller value of δ^{DC}; go to step 2. Otherwise, continue increasing δ^{WH} as long as costs are improving and $\delta^{WH} < (N\text{-}1)/N$. The last point before costs stop improving is the best δ^{WH} for this δ^{DC}. Let *best_*δ^{DC}*_cost* be the cost at this point.

- If *best_*δ^{DC}*_cost* > *best_cost*, then this value of δ^{DC} offered no improvement over the previous value. Go to step 2.
- Otherwise, *best_*δ^{DC}*_cost* ≤ *best_cost*. In this case, let *best_*δ^{DC}*_cost* = *best_cost*, *best_*δ^{DC} $=\delta^{DC}$, and *best_*$\delta^{WH}=\delta^{WH}$. Also let δ^{WH} equal the least value that was feasible for this δ^{DC}. If $\delta^{DC} > 0$, decrement it by $1/N$ and return to step 1. If $\delta^{DC} = 0$, go to step 2.

Step 2. If *is_feasible* = true, report the pair (best_δ^{DC} , best_δ^{WH}). Otherwise, there is no feasible pair in the grid. In either case, terminate.

This search procedure uses the three observations about the cost function to fathom sections of the grid before they have been evaluated. In practice it evaluates only about 6% of the grid before terminating with its recommended policy. It should be noted that even if all of these observations had been proven analytically, our approach would still be a heuristic method for finding the optimal (δ^{DC}, δ^{WH}) that is feasible. The reason is that estimates of expected cost and service level, rather than exact values, guide the search. By adjusting the search to account for confidence intervals, we could improve the likelihood that it finds an optimal (δ^{DC}, δ^{WH}) under the condition whenever the observations are true. Even without this adjustment, because we use many simulation trials and produce very small confidence intervals, we are confident that this procedure finds optimal solution in the vast majority of cases.

6 The Software Tool

As described in Section 5, our approach combines three elements: the calculation of inventory targets for each given pair of parameters (δ^{DC}, δ^{WH}), the simulation of how these inventory targets would be used by Red Pepper®, and a grid search to find the most cost effective parameter combination such that a minimum service level is still met. In this section we illustrate how these components are synthesized into a software tool that produces optimal inventory targets among those in Σ, we show how the tool is positioned within NSD planning process, and we discuss details of its implementation.

6.1 Program Flow

The flow of the program, depicted in Figure 7, is as follows:

(a) Initialization

The program reads the input data from files and uses the command line parameter options. These data include:

- supply chain characteristics (names and number of DCs, available shipment modes, leadtimes)
- demand characteristics (forecasts for each DC and each period, demand distribution type, forecast error)
- cost parameters (holding cost, depreciation cost, transportation cost for each DC and each mode, obsolescence cost for each DC)
- information about number of periods and the minimum overall service level required

The program then discretizes the demand distributions and produces a predefined number of pseudo-random demand sequences (the length of each sequence is the number of periods of the horizon) for each DC following the pre-specified distributions (type, forecast and error). These sequences are used later on to simulate demand.

(b) Determining Inventory Targets

As outlined in Section 5.1.4, inventory targets are computed for each (δ^{DC}, δ^{WH}) by using standard single node/single mode inventory models. Solving equations of the form of (1) involve calculating convolutions of the leadtime demand. This is done efficiently using Fast Fourier Transforms.

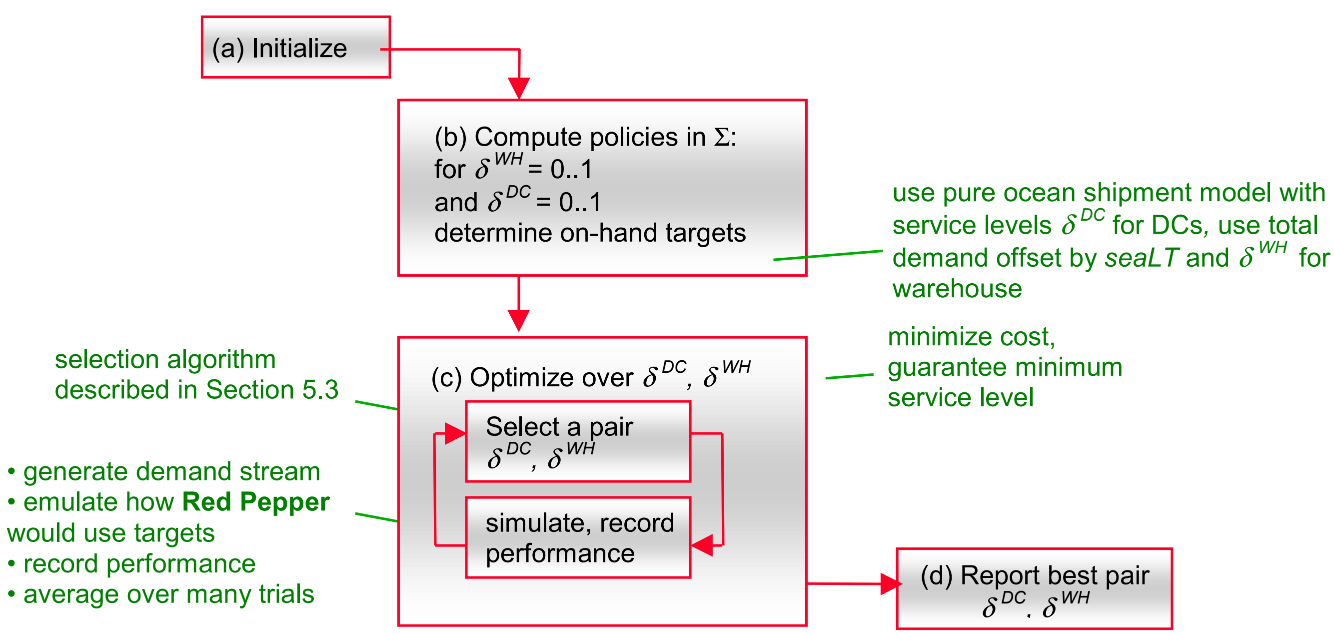

Figure 7. Program Flow

(c) Simulation
For each demand sequence generated in part (a), we simulate how the targets associated with a given (δ^{DC}, δ^{WH}) are would be used by Red Pepper's planning engine, as discussed in Section 5.2. Service level, cost, and any other desired performance metrics are reported.
(d) Grid Search
Steps (b) and (c) are repeated for values of δ^{DC} and δ^{WH} in a grid of values between zero and one with pre-specified increments. The process for determining the sequence of (δ^{DC}, δ^{WH}) pairs is described in Section 5.3.

(e) Report
Having found the most cost effective feasible grid point, the program reports the associated inventory targets, the achieved service level as well as the expected cost for this strategy. A vast array supply chain performance metrics can also be reported, such as the percentage of shipments expected to be made by air, expected backlogs, or decomposition of the expected total cost into inventory, transportation, and obsolescence cost for each location.

6.2 Positioning of the Tool

Figure 8 shows the proposed positioning of our tool within the NSD inventory management process. As mentioned before, the purpose of the tool is to provide inventory targets, measured in weeks of supply[7] (WOS). These targets will serve as inputs to the Red Pepper® inventory management tool.

While an automated data interchange between the HPL software tool and NSD's implementation of Red Pepper® is possible, we opted instead for web-based interfaces for input

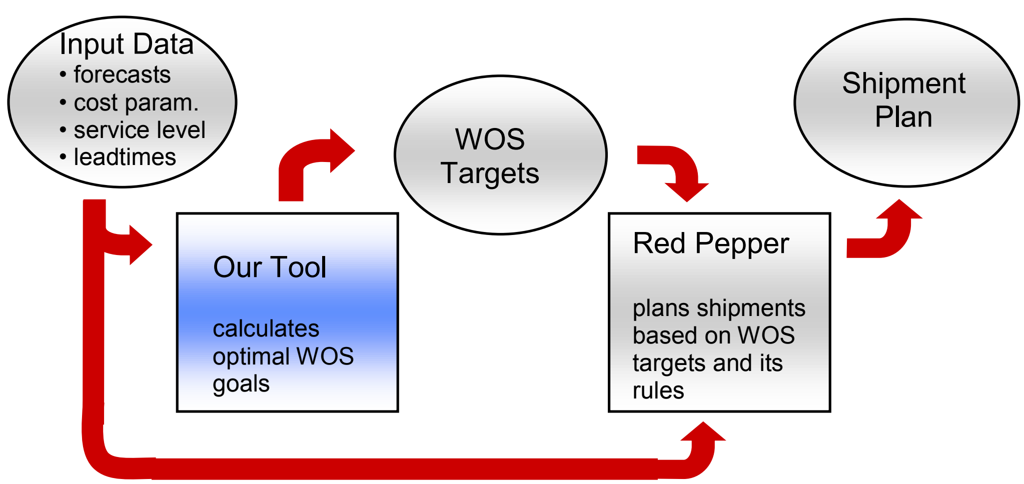

Figure 8. Positioning of the Tool

[7] Weeks of Supply (WOS) is a common measure of inventory targets and safety stock in inventory management practice. At NSD, 1WOS is measured as next month's demand divided by 4.3. Its purpose is to decouple targets from ever-changing forecasts and to give a sense of how long it is expected to take to deplete the inventory if no subsequent deliveries would occur. From a theoretical standpoint, in a non-stationary demand environment it would be more appropriate to express targets as a fraction of leadtime demand.

and output. This gives planners more control over how the tool's recommended targets are used, and facilitates scenario analyses with respect to all input parameters of the program. The interface was developed by a group in HP called Supply Chain Information Systems (SCIS)

6.3 Implementation

The program itself is written in C and runs on HPUX. It allows for batching several scenarios, taking advantage of the fact that certain calculations are common across scenarios. Several global options can be set through command line parameters. These include the availability of only one shipment mode, whether or not the end of life of the product coincides with the end of the model horizon or whether or not a particular service level shape should be applied.

7 Sample Results

This section contains a representative sample of results obtained for one of the MOD0 boxes based on actual NSD data. The horizon of the model is 47 weeks. Two of the DCs have sea shipments available (Roseville, CA and Grenoble, France). The others use only one shipment mode. The results of the optimization and simulation with our tool are compared to the previous inventory targets used at NSD.

Figure 9 compares the cost for different minimum service levels, the current NSD strategy (NSD Strategy) and the best strategy found by our tool that achieves the same service level as the NSD strategy (NSD Service). Using our strategy, the simulation predicts a 27% cost improvement while maintaining the current service level.

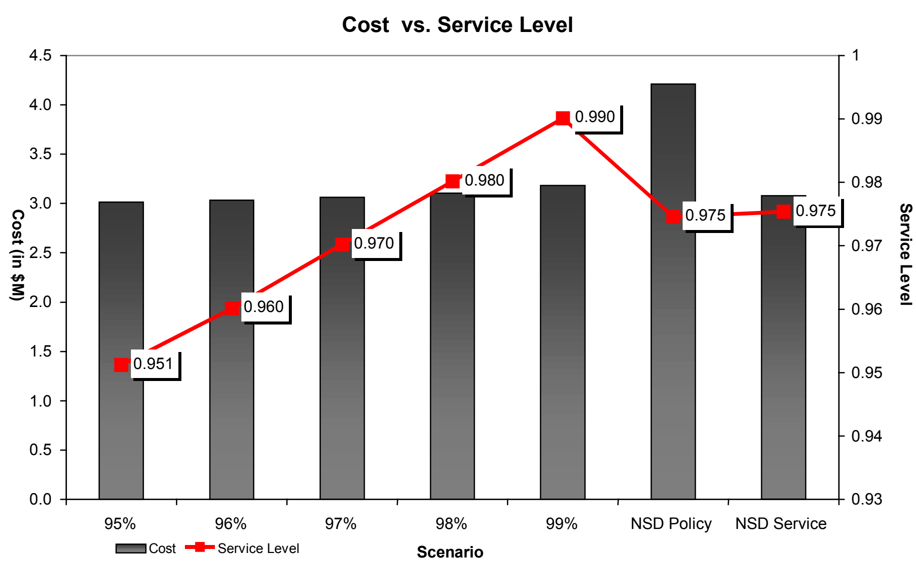

Figure 9. Cost for Dfferent Scenarios

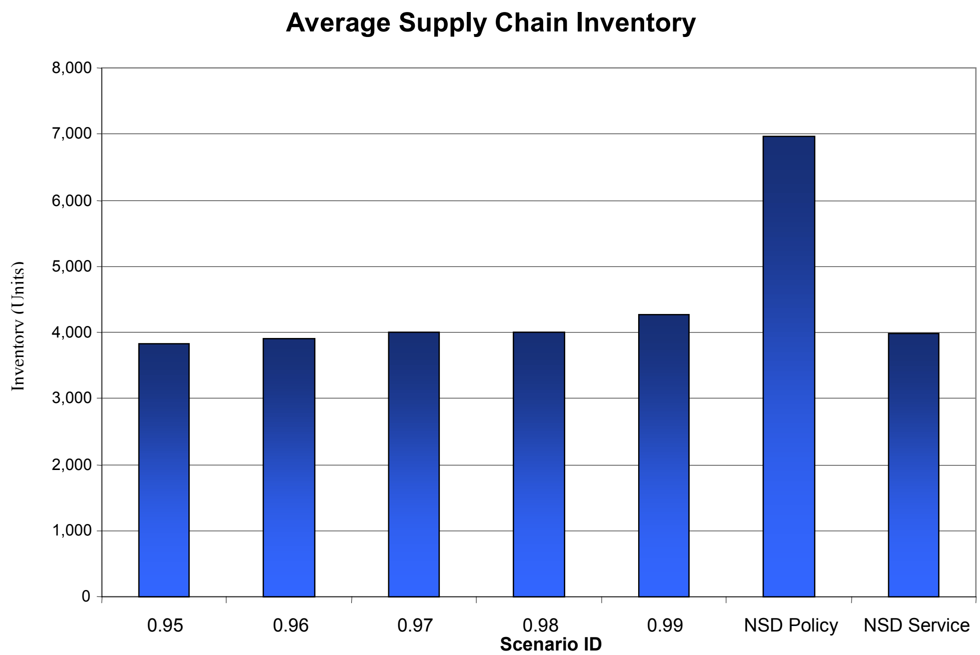

Figure 10. Average of Total Supply Chain Inventory over Time

As Figure 10 shows, this cost reduction goes hand in hand with a reduction of average total supply chain inventory.[8] The fact that inventory is reduced while maintaining the original overall level of service shows the strength of the non-stationary multi-node model. This reduction is achieved by a efficient reallocation of inventory over time and location.

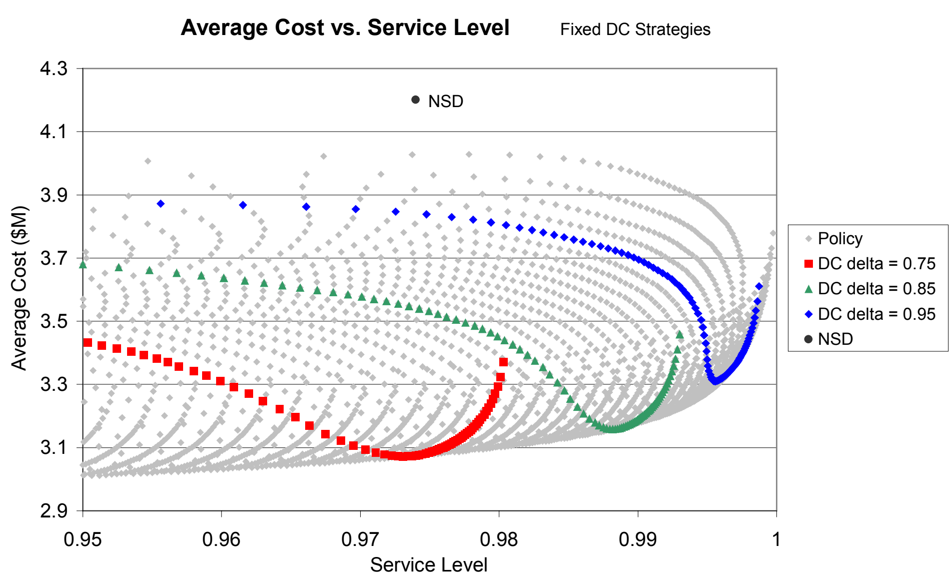

Figure 11. The Effect of Changing Factory Strategies

[8] Total average supply chain inventory is computed as the average over all periods of inventory on hand and in transit for all locations.

Figures 11 and 12 depict total cost and service level for a number of policies in Σ. To generate these figures we used a grid for (δ^{DC}, δ^{WH}) with δ^{DC} = k/100, δ^{WH}=l/100 for k ,l = 1,2,…,99. If the strategy $\boldsymbol{S}(\delta^{DC}, \delta^{WH})$ produces a service level higher than .95, its cost is plotted over the service level. The lower envelope of the curve corresponds to the best strategy in Σ for a given service level. It is an 'efficient frontier' in the sense that for all points along it, increasing service requires increasing cost. For comparison we also plotted NSD's current policy.

In Figure 11 we highlighted sets of policies that keep δ^{WH} constant and, therefore all use the same targets at the Factory. From left to right δ^{DC} increases, i.e., DC targets increase. As a result, service improves at the expense of monotonically increasing cost. The use of different shipment modes is primarily dependent on the responsiveness of the factory. which is kept constant. Therefore, increased targets at the DC's increase the total supply chain inventory and with it the cost. An interesting observation is that δ^{WH}=0.65 appears to be close to optimal for a wide range of desired service levels for this particular product. Since δ^{WH} is the main driver for the percentage of shipments made by sea, this percentage is almost constant with respect to the required service level and is approximately equal to 30%.

By fixing δ^{DC} and varying δ^{WH} as done in Figure 12, we observe a different behavior. In general, increasing δ^{WH} will increase responsiveness of the Factory and therefore decrease shipment cost. On the other hand, it will increase total supply chain inventory and with it inventory and possibly obsolescence cost. For a small δ^{WH}, i.e. low Factory targets, DCs are starved of inventory, and a high percentage of the products has to be shipped by air as emergency shipments. High air shipment cost dominates the total cost. Increasing δ^{WH} will reduce the fraction of air shipments made and in doing so increase the in-transit inventory. Also, higher Factory targets mean higher inventory at the factory. This explains why for increasing δ^{WH} total cost first decreases and then increases.

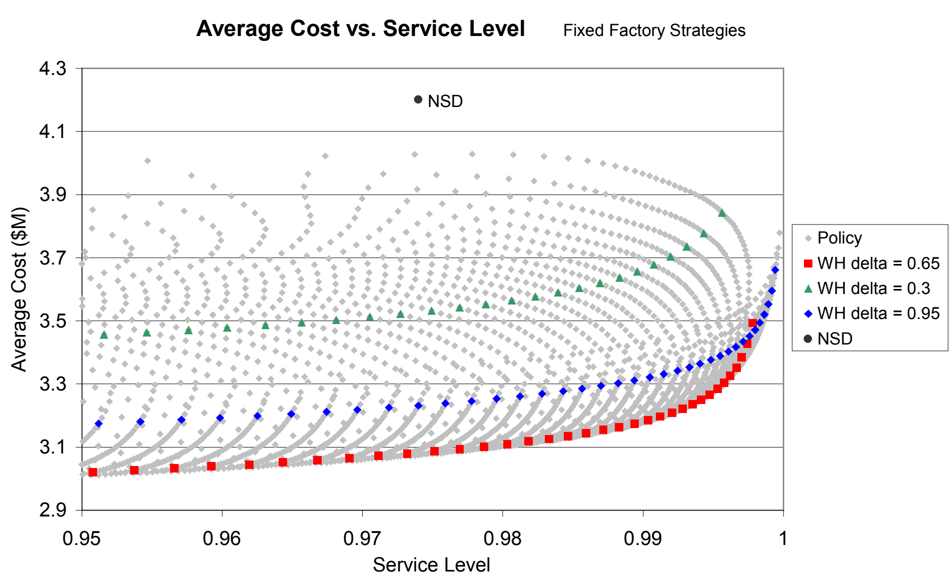

Figure 10. The Effect of Changing DC Strategies

Since the inception of this project our results have been implemented at NSD. Total supply chain inventory has been reduced significantly without jeopardizing off-the-shelf availability. Transportation cost has been reduced by more extensive use of ocean shipments.

8 Conclusion

Prior to the commencement of this project, NSD used simple single-location, single-supply-mode inventory models to aid in the determination of inventory targets. These models also assume demand to be stationary. This chapter describes an approach that models the true two-tier, two-mode nature of the NSD supply chain and the interaction between these tiers. In particular, it takes into account the effects of shortages at the warehouse and the competition between DCs for warehouse supply. The model also accommodated product life-cycles by not requiring demand or service level goals to be stationary over time.

Our results demonstrate that this approach produced more effective inventory strategies than their previous method. This can explained by the fact that both the amount of supply chain inventory and its distribution between the factory and the DCs are critically important in determining overall costs and level of service. NSD's previous approach to finding inventory strategies was not general enough to address these factors.

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Appendix A. Solving Equation (2)

We rewrite Equation (2) as

$$\mathrm{P}(y_t > D_t + D_{t+1} + \ldots + D_{i+seaLT}) = \delta^{DC}. \tag{3}$$

Let $\Phi(x)$ be the cumulative distribution function of the sum $D_t + \ldots + D_{t+seaLT}$. Then we can write (3) as

$$1\text{-}\Phi(y_t) = \delta^{DC} \tag{4}$$

or equivalently

$$y_t = \Phi^{-1}(\delta^{DC}). \tag{5}$$

Let $\Phi_t(x), \ldots, \Phi_{t+seaLT}(x)$ be the cumulative distribution functions of $D_t, D_{t+1}, \ldots, D_{t+seaLT}$, respectively. Then, the function $\Phi(x)$ is the convolution[9] of these functions:

$$\Phi(x) = \Phi_t(x) * \Phi_{t+1}(x) * \ldots * \Phi_{t+seaLT}(x). \tag{6}$$

While Equation (6) defines the function $\Phi(x)$, using only the definition of the convolution it is rather difficult to compute it. We use the following well-known theorem:

THEOREM 1. Let $\mathrm{F}(f)$, $\mathrm{F}(g)$ denote the Fourier Transform[10] of the functions f and g. Then

$$\mathrm{F}(f*g) = \mathrm{F}(f)\,\mathrm{F}(g).$$

Using Theorem 1 we can write (5) as

$$\Phi(x) = \mathrm{F}^{-1}\left(\mathrm{F}(\Phi_t(x))\mathrm{F}(\Phi_{t+1}(x)) \times \ldots \times \mathrm{F}(\Phi_{t+seaLT}(x))\right). \tag{7}$$

Using readily available C code for Fast Fourier Transforms (FFT) and its inverse the function $\Phi(x)$ can be computed relatively quickly. Applying (4) yields the desired order-up-to-levels.

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[9] The convolution $\Psi(x)=\Phi(x)*\Theta(x)$ of two (differentiable) cumulative distribution functions $\Phi(x)$ and $\Theta(x)$ is defined as

$$\Psi(x) := \int_0^x \Phi(x-z)\Theta'(z)\,\mathrm{d}z.$$

[10] The Fourier Transform $\mathrm{F}(f)$ is defined as

$$\mathrm{F}(f)(t) := \int_{-\infty}^{\infty} e^{-itx} f(x)\,\mathrm{d}x$$