Superconducting circuits for quantum computing*

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Abstract

Superconducting circuits provide one of the numerous current candidate routes for the realization of quantum computing (QC) technology. In this paper I review the basic physics behind this candidature and outline some superconducting QC proposals currently on the table, demonstrating their common origins. More detailed discussion of the proposals follows, assessing how they measure up to the "DiVincenzo checklist" for QC realizations and the current status of experiments.

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1 Introduction

Quantum algorithms, such as those for factoring [1] or searching [2], demonstrate that technology which stores and processes information according to the laws of quantum physics will be capable of computational tasks infeasible with any conventional information technology. Whilst it is likely that more applications and algorithms than those we have to date will be needed for a large scale *quantum information technology* industry in the future, there is clearly great potential in and need for further research—in quantum hardware and software. Most quantum computing work to date has focussed on quantum bits—two-state quantum systems, or qubits (although in principle larger individual Hilbert spaces could be used).

Over the last few years, a number of two-level systems have been examined (theoretically and/or experimentally) as candidates for qubits and quantum computing. These include ions in an electromagnetic trap [3, 4, 5], atoms in beams interacting with cavities at optical [6] or microwave frequencies [7], photons interacting with photons [8], electronic [9] and spin [10] states in quantum dots, nuclear spins in a molecule in solution [11, 12] or in solid state [13], charge (single Cooper pair) states of nanometer-scale superconductors [14, 15, 16], flux states of superconducting circuits [18, 19] , quantum Hall systems [20] and states of electrons on superfluid helium [21]. Unlike the case of quantum cryptography, where there is a clear favourite realization—photons (or at least weak light pulses), down optical fibres or possibly free space—there is currently no clear favourite quantum computing realization out of all the candidates.[1] Indeed, this is why it's worth a whole special issue of a journal to discuss them all! My job in this paper is to discuss the superconducting candidates.

2 Quantum circuits

The foundations for superconducting qubits are found in basic quantum electrodynamics. Although not very fashionable, it is perfectly possible to quantize the electromagnetic field using the physical (and obviously gauge invariant) electric and magnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$ [28]. The fields do not commute, for example (in cylindrical polar coordinates)

$$[B_z(\boldsymbol{r}',t), E_\theta(\boldsymbol{r},t)] = \frac{i\hbar}{\epsilon_0}\frac{\partial}{\partial\rho}\delta^3(\boldsymbol{r}-\boldsymbol{r}') \ . \tag{1}$$

For low frequency[2] circuit applications, it is adequate to work with integrated variables. Integrating the commutator eq. (1) over two intersecting surfaces, such as S_1 and S_2 shown in figure 1 (i.e. $\int_{S_1} \boldsymbol{dS} \int_{S_2} \boldsymbol{dS}'$), reveals that the magnetic flux Φ threading S_2 and the electric flux Q (which has the dimensions of electric charge) threading S_1 do not commute [29],

$$[\Phi, Q] = i\hbar \ . \tag{2}$$

[1] A useful factoring or searching quantum computer will require many qubits (e.g. a few thousand qubits for useful—say 200 digit—factoring [22] rising to $\sim 10^5$ with error correction [23, 24, 25, 26, 27]), although other applications, demonstrations or simulations may be realised with rather fewer. At present, even the most experimentally advanced QC candidates are only at the single-digit qubit level.

[2] Low means $c/\omega \gg \mathit{circuit\ \ dimensions}$, so spatial variations of fields across circuits are negligible.

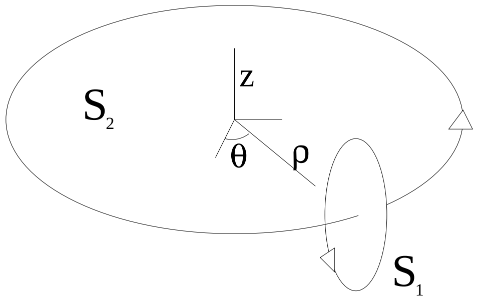

Figure 1: Intersecting surfaces S_1 and S_2.

So, (magnetic) flux and (electric) charge for circuits are conjugate quantum variables.[3] However, as we all know, everyday electromagnetic circuits don't show any quantum behaviour. There are essentially two barriers which have to be overcome to turn a common, all-garden circuit into one capable of exhibiting quantum behaviour.

1. Pure states: The first is to realize pure (or decent fidelity) quantum states. The problem with ordinary macroscopic circuits is that they contain a vast (maybe 10^{21}, $\sim 10^{-3}$.Avogadro's number) number of electrons at a finite temperature. Given the coupling between these condensed matter degress of freedom and the electromagnetic field, there is just no chance of producing a nice pure state of the field. Simply cooling the system will not suffice either; there will still be an awful lot of degrees of freedom to disentangle. Clearly one solution is to make the circuit very small (a nanostructure) and to reduce the number of electrons down to the order of one! Such (normal, i.e. non-superconducting, material) "circuits" are more familiarly termed quantum dots, boxes, wires etc.. These systems certainly can exhibit quantum behaviour. However, their potential for quantum computing will be discussed elsewhere. My interest is with an alternative solution, which also, in effect, vastly reduces the number of condensed matter degrees of freedom to which the electromagnetic field is coupled.

   In superconductors, the conduction electrons pair up (so the fundamental unit of charge is $2e$) and these Cooper pairs form a macroscopic quantum condensate[4] [30]. Excited microscopic (quasiparticle) degrees of freedom exist, but there is an energy gap for their creation.[5] Quasiparticles aside, if an electromagnetic circuit is constructed from superconductor, the field essentially couples to one degree of freedom, the condensate. It is then clearly much easier to produce something approximating a pure state of the field, compared to when the circuit electrons are not condensed. However, just making things superconducting doesn't guarantee that they exhibit intrinsically quantum phenomena...

[3]Note that for the arrangement of figure 1 the operators Φ and Q both have continuous spectra on the real line $[-\infty, +\infty]$, just like x and p for a particle in 1D.

[4]At least for traditional metal and alloy superconductors—there is still debate about the mechanism for the more complex compound high-T_c superconductors.

[5]This gap is of the order of the superconducting transition temperature T_c, so at temperatures well below this the excitation probability is suppressed by a Boltzmann factor.

2. Appropriate states: Consider, for example, a superconducting microwave resonator (for which equations of the form eq. (1) rather than the integrated versions eq. (2) are probably more appropriate). If the only states of this system which get utilized are coherent[6] states, it will behave like a very good (high quality factor) classical oscillator. One needs to play with other states, such as the lowest few Fock states, or squeezed ones, to reveal intrinsically quantum phenomena which have potential application for quantum computing.

   For example, the lowest two Fock states—zero or one quantum present—of a superconducting microwave cavity mode can be used as an effective qubit. Such superconducting cavity qubits will be discussed elsewhere, as they are used in conjunction with beams of Rydberg atoms in atom-cavity approaches to quantum computing. My interest here is with other forms of superconducting circuit, which generate different quantum states of the electromagnetic field, also employable as qubits.

3 Charge and flux states and the Josephson effect

Charge and flux quantization effects both arise in superconducting circuits and, as will be seen, both have potential for use as qubits. In the commutator eq. (2), both variables Φ and Q arise from fields integrated over open surfaces (see figure 1) and so generally have continuous spectra. Under certain conditions this changes. (In addition to [29], further background on quantum circuits can be found in [31, 32, 33].)

3.1 Charge states

If the superconducting circuit consists of two or more distinct pieces of superconductor[7] separated by insulating (e.g. oxide) layers, it is possible to envisage closing the surface S_1 around one of these pieces.[8] In this case, the electric flux through the surface equals the enclosed charge,[9] and so the variable Q has a discrete spectrum, integer (N) multiples of the Cooper pair charge $2e$. Correspondingly, it is no longer possible to close the contour which defines the surface S_2 to define a magnetic flux. The contour then simply runs from the charged piece of superconductor to another (where the image charge resides) outside S_1, and so the conjugate variable to Q is the (gauge invariant) phase difference θ between the superconducting condensates. The charge unit $2e$ sets the periodicity of Φ (the phase variable in units of magnetic flux) to be $h/2e$ ($\equiv \Phi_0$, the superconducting flux quantum), so $\theta = 2\pi\Phi/\Phi_0$.

The integer charge states of such a circuit are not degenerate. The capacitive energy

[6] In the $|c\rangle_{coh} = \sum_{n=0}^{\infty} c^n (n!)^{-1/2} \exp(-|c|^2/2)|n\rangle$ sense, where the sum is over the Fock states of some mode.

[7] These might be grains of material, or sections of a fabricated circuit or nanostructure.

[8] More precisely, the closed surface S_1 should lie everywhere in free space, or at least space devoid of free/conduction charges (i.e. where their probability density is vanishingly small). For example, a conducting wire cannot puncture S_1.

[9] Any pieces of the electric field which are divergenceless within S_1 do not contribute to the flux.

stored is quadratic in the (electric) field, with the magnitude set by the effective capacitance C of the device. If, in addition to the charge $Q = 2Ne$, an external polarization charge[10] Q_x is applied to the system, the Hamiltonian is

$$H_C = \frac{(Q - Q_x)^2}{2C} \ . \tag{3}$$

The distinction between the integer and continuous spectra cases for Q is very important and has been much discussed in the literature. (See, for example [34, 35] and references therein.) In the case of an electrically isolated capacitor, for which surface S_1 can be closed around one of the "plates" and $Q = 2Ne$ has an integer spectrum, the conjugate variable θ is angular.[11] Electrical connection to the capacitor necessarily prevents closure of S_1; this is thus a physically distinct system from the isolated case, describable again by eq. (2) with continuous spectra Φ and Q. The breaking of the angular symmetry is obvious when an external current source I_x is fed to the system as this generates a term $-I_x\Phi$ in the Hamiltonian. The physics of the isolated system is described by periodic angular eigenfunctions, whereas the connected one needs the full set of Bloch eigenfunctions [35].

3.2 Flux states

There is not complete symmetry between charge and flux; this would be between electric and magnetic charge and there appears to be a distinct dearth of the latter, at least in our part of the universe. Thus the argument for quantized magnetic flux, or vortex, states does not involve closing surface S_2; it follows for the layout of figure 1. I don't reproduce the proper argument here.[12] Instead, and rather handwavingly, consider a fundamental unit of charge $2e$ transported around the closed contour defining the surface S_2. If the physical system is completely invariant under this process, the electric flux Q is equivalent to $Q + 2e$ and so the conjugate magnetic flux variable Φ is quantized[13] in units of Φ_0. The invariance can only occur if the contour avoids regions of current flow, fields or dissipative influences and so it must lie everywhere within bulk superconductor. If the whole of the *surface* S_2 lies within superconductor the flux has to be zero, but if the circuit is topologically equivalent to a doughnut/bagel, other integer n multiples of Φ_0 are allowed.

The integer flux states of such a circuit are not degenerate. The inductive energy stored is quadratic in the (magnetic) field, with the magnitude set by the effective inductance L of the device. Associated with each flux state there is a quantized circulating current,[14] but its value is not fundamental as it scales with the inductance. If part of the quantized flux Φ is

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[10] This is an externally controllable parameter in the Hamiltonian, a continuous classical variable. In practice it will be realized by some suitably applied voltage.

[11] As θ is not an observable, it is naughty to write the commutator as $[\theta, N] = i$ (see e.g. [36]), although folk often do. $[\eta, N] = -\eta$ with $\eta = \exp i\theta$ is fine. The system is analogous to a rotor/pendulum in 2D.

[12] See, for example, chapter 6 of reference [30].

[13] Q is no longer an observable and $\pi Q/e$ is an angle.

[14] This flows in a thin ($\sim 10^{-7}$ m) surface layer of the superconductor only, set by the penetration depth [30].

due to some external source[15] of flux Φ_x, the Hamiltonian for the circuit is

$$H_L = \frac{(\Phi - \Phi_x)^2}{2L} \ . \tag{4}$$

Note that since for true integer flux states the whole of the perimeter of surface S_2 must lie within superconductor, no capacitive break is allowed in the loop. Realisitc circuits, such as those needed for SQUID magnetometers or flux qubits, have capacitive breaks. Thus in practice it is only possible to manipulate approximations to "flux eigenstates", which peak sharply at integer Φ_0 values (or thereabouts) of the continuous variable Φ.

3.3 The Josephson effect

If two pieces of superconductor are in very close proximity (for example, separated only by a thin oxide layer), it is possible for a Cooper pair of electrons to tunnel coherently from one piece to the other [37]. The operator which translates charge by $2e$ is $\exp i\theta$. Adding the reverse process with an equal amplitude[16] of $-E_J/2$ generates Josephson's famous contribution to a superconducting circuit Hamiltonian,[17]

$$H_J = E_J\,(1 - \cos\theta) = E_J\left(1 - \cos\left(\frac{2\pi\Phi}{\Phi_0}\right)\right) \ . \tag{5}$$

It is this weak link tunnelling term which enables charge or flux states to be candidate qubits.

3.4 Model charge qubit

Two pieces of superconductor close enough to exhibit Josephson tunnelling constitute a weak link, with a Hamiltonian

$$H_{cq} = H_C + H_J = \frac{(2Ne - Q_x)^2}{2C} + E_J\,(1 - \cos\theta) \ . \tag{6}$$

To a good approximation, for Q_x around $Q_d \equiv (m+\frac{1}{2})2e$ (and so for $Q'_x \equiv (Q_x - Q_d)$ around zero), the lowest two energy eigenstates just involve two adjacent charge states—those with N-eigenvalues of m and $m+1$. Denoting these respectively by $|\uparrow\rangle$ and $|\downarrow\rangle$ in the σ_z basis and ignoring the other charge states gives an effective qubit Hamiltonian

$$H_{cq} \approx \frac{e^2}{2C} + E_J + \frac{e}{C}Q'_x\sigma_z - \frac{E_J}{2}\sigma_x \ . \tag{7}$$

With the usual bit-value basis notation of $|0\rangle \equiv |\uparrow\rangle$ and $|1\rangle \equiv |\downarrow\rangle$, the Josephson term generates bit flips/rotations and the externally controllable diagonal term produces a phase difference between bit eigenstates. The system is a model *charge qubit*.

[15] This is an externally controllable parameter in the Hamiltonian, a continuous classical variable. In practice it will be realized by currents in other circuits.

[16] This is calculable, for example, from a microscopic approach to a model junction [37, 38].

[17] I choose to add in a constant E_J to fix the energy minima at zero. This is convention; it simply stops the minima disappearing off to $-\infty$ in the large E_J limit.

3.5 Model "flux" qubit

If the two pieces of superconductor forming a charge qubit are shorted by a piece of bulk superconductor (so the system resembles the flux quantization arrangement except for the Josephson tunnel barrier) the Hamiltonian is[18]

$$H_{fq} = H_C + H_L + H_J = \frac{Q^2}{2C} + \frac{(\Phi - \Phi_x)^2}{2L} + E_J \left(1 - \cos\left(\frac{2\pi\Phi}{\Phi_0}\right)\right) . \tag{8}$$

If the curvature of the cosine minima dominates that due to the inductance, expansions can be made to define an effective kinetic inductance $L_k \equiv \Phi_0^2/(4\pi^2 E_J)$ and a total $L_t^{-1} = L^{-1} + L_k^{-1}$. To a good approximation, for Φ_x around $\Phi_d \equiv (n + \frac{1}{2})\Phi_0$ (and so for $\Phi'_x \equiv (\Phi_x - \Phi_d)$ around zero), the lowest two energy eigenstates just involve two adjacent "flux" states—those with Φ expectation values[19] of around $n\Phi_0$ and $(n+1)\Phi_0$. Denoting these respectively by $|\uparrow\rangle$ and $|\downarrow\rangle$ in the σ_z basis and ignoring the other states gives an effective qubit Hamiltonian

$$H_{fq} \approx \frac{L_t}{2LL_k}\left(\frac{\Phi_0}{2}\right)^2 + \frac{1}{2}\hbar\,(L_t C)^{-\frac{1}{2}} + \frac{L_t}{LL_k}\left(\frac{\Phi_0}{2}\right)\Phi'_x\sigma_z - \frac{\Gamma}{2}\sigma_x . \tag{9}$$

The system is a model "flux" qubit, since with the usual bit-value basis notation of $|0\rangle \equiv |\uparrow\rangle$ and $|1\rangle \equiv |\downarrow\rangle$, the externally controllable diagonal term produces a phase difference between bit eigenstates and the last term generates bit flips/rotations. The coefficient $\frac{\Gamma}{2}$ of this term is the matrix element for tunnelling through the barrier between the "flux" states, generated by the Josephson potential.

Two points are particularly worth noting.

- The title *flux qubit* is not technically correct, as the states $|\uparrow\rangle$ and $|\downarrow\rangle$ are not actually flux eigenstates (although they approach these in the limit $E_J \to \infty$). An alternative term is *persistent-current qubit*, as the states exhibit oppositely circulating supercurrents.

- The Josephson effect plays a different role in the two cases. For a charge qubit eq. (7), the amplitude of σ_x is directly proportional to the Josephson tunnelling, so turning this off prevents the action of bit flip/rotation. In the flux case eq. (9), the amplitude of σ_x is given by the tunnelling through the barrier[20] generated by the Josephson potential. Thus *increasing* the Josephson tunnelling (so it dominates the other terms in eq. (8)) is the way to turn off bit flip/rotation for a flux qubit. Clearly, in both cases it is desirable to have the ability of external control over the Josephson amplitude E_J.

[18] With the short in place an external voltage source cannot be connected across the junction; however, an external magnetic flux source (which can be varied with time, as appropriate) can be coupled inductively to the system.

[19] The potential minima lie at $\Phi' \equiv \Phi - \Phi_d \approx -\frac{L_t}{L_k}\left(\frac{\Phi_0}{2}\right) + \frac{L_t}{L}\Phi'_x$ and $\Phi' \approx \frac{L_t}{L_k}\left(\frac{\Phi_0}{2}\right) + \frac{L_t}{L}\Phi'_x$.

[20] This is exponentially damped according to the barrier area.

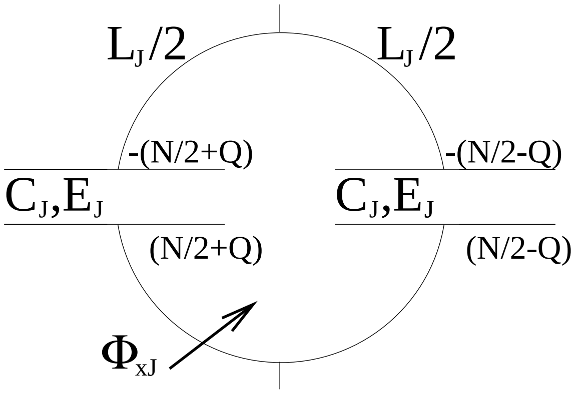

Figure 2: The circuit for a tunable E_J, which combines features of the model charge and flux qubits. Two weak links (with capacitance C_J and Josephson energy E_J) are placed in a ring of total inductance L_J which is subject to an external flux Φ_{xJ}.

3.6 Tunable E_J

As noted by the various superconducting qubit proposers [16, 19], it is indeed possible to construct a subcircuit which behaves as an externally controllable E_J in a qubit circuit. This is illustrated in figure 2. The system has two pairs of conjugate variables, the discrete charge $2Ne$ with the phase difference γ across the whole system and the electric flux Q with the total magnetic flux Φ threading the ring. The Hamiltonian is

$$H_{tot} = \frac{(2Ne)^2}{2(2C_J)} + \frac{Q^2}{2(C_J/2)} + \frac{(\Phi - \Phi_{xJ})^2}{2L_J} + E_J\left(1 - \cos\left(\gamma - \frac{\pi\Phi}{\Phi_0}\right)\right) + E_J\left(1 - \cos\left(\gamma + \frac{\pi\Phi}{\Phi_0}\right)\right). \tag{10}$$

Provided that (i) L_J is small (so $L_J\dot{Q} \ll \Phi_{xJ}$ and $\Phi \approx \Phi_{xJ}$), (ii) $\Phi_0^2/L_J \gg e^2/C_J$, $(2\pi)^2 E_J$ (so the Φ wavefunction peaks sharply around $\Phi \approx \Phi_{xJ}$) and (iii) $\hbar\omega_J \equiv \hbar(L_J C_J/2)^{-1/2} \gg E_J$, e^2/C_J , $k_B T$, then the Φ,Q system exhibits ground state behaviour with $\Phi \approx \Phi_{xJ}$. The effective Hamiltonian for the N,γ system is therefore

$$H_{varE} = \frac{(2Ne)^2}{2(2C_J)} + 2E_J\left(1 - \cos\left(\frac{\pi\Phi_{xJ}}{\Phi_0}\right)\cos\gamma\right) . \tag{11}$$

There is an effective tunnelling energy $E_{Je} \equiv 2E_J \cos\left(\frac{\pi\Phi_{xJ}}{\Phi_0}\right)\cos\gamma$, tunable through the applied flux. The capacitances add in parallel, $C_{Je} = 2C_J$.

4 Proposed superconducting qubits

The proposals for superconducting qubits are based on the model systems described, but with practical modifications.

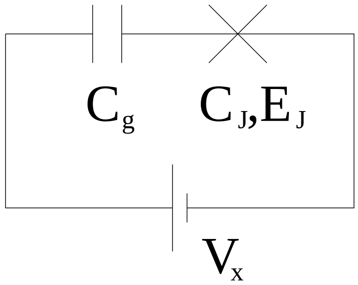

Figure 3: The circuit for a charge qubit. The voltage source V_x is connected through a gate capacitance C_g. This is shown as a single capacitor but in practice it may be composite (e.g. $C_g/2$ either side of the weak link, in series.) The weak link (capacitance C_J and tunnelling energy E_J) is simply denoted by X. In practice this can be a composite controllable weak link as described in subsection 3.6.

4.1 Charge qubit

For charge qubits [14, 15, 16] an external bias voltage is connected through a gate capacitor to provide a controllable polarization charge, so the actual circuit is shown in figure 3. The charge degree of freedom $2Ne$ for such a system is in fact the *excess* number of charges on the "island" of superconductor between the two capacitors, isolating it from external leads. The relevant (N-dependent) piece of the charging energy is [39][21]

$$H_{C_gJ} = \frac{(2Ne - C_g V_x)^2}{2(C_g + C_J)} \ . \tag{12}$$

This is similar in form to the first term of eq. (6). With the tunnelling term as before, the charge island qubit is therefore clearly describable by the Hamiltonian eq. (7) with a suitable redefinition of the parameters.

4.2 Flux qubit I

One proposal [18] for a flux qubit is to use a standard radio frequency (rf) SQUID circuit shown in figure 4. This system is decribed by the Hamiltonian eq. (8) and so forms an effective qubit describable by eq. (9). The use of a quantum flux parametron (QFP) has also been proposed [18] as a flux qubit. In the rf SQUID case, for the external flux Φ_x close to a degenerate point $\Phi_d \equiv (n + \frac{1}{2})\Phi_0$, the bit value states are distinguished through the introduction of an extra flux quantum in the SQUID loop. In the QFP case (which is effectively two rf SQUIDs linked by a common inductor) the bit value states correspond to

[21] Excess charge on the island is due to charge tunnelling across the weak link and is given by the difference between the capacitor charges, $2Ne = Q_J - Q_g$. Kirchoff's law gives $V_x = Q_g/C_g + Q_J/C_J$ so the capacitor charges are determined in terms of the quantum variable N and the external source V_x. For a given N the total charging energy is given by the sum of the stored energies on the capacitors plus the work done by the voltage source in re-establishing the equilibrium values after each of N tunnelling events, $E_{tot} = Q_g^2/2C_g + Q_J^2/2C_J - 2NeC_GV_x/(C_g + C_J)$.

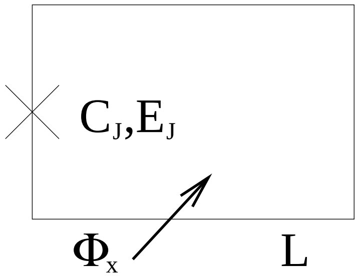

Figure 4: The rf SQUID circuit for a flux qubit. The weak link (capacitance C_J and tunnelling energy E_J) is simply denoted by X. In practice this can be a composite controllable weak link as described in subsection 3.6.

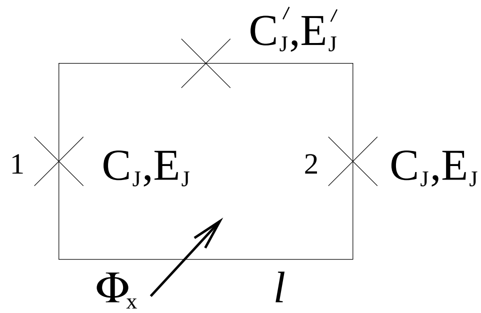

Figure 5: The multi-link circuit for a flux qubit. In practice the weak link with capacitance C'_J and tunnelling energy E'_J can be composite and controllable as described in subsection 3.6.

an extra flux quantum in one SQUID or the other. It is suggested [18] that this symmetric arrangement for a flux qubit may have fabrication advantages over a single rf SQUID.

4.3 Flux qubit II

One potential problem with rf SQUID-based flux qubits is that, in order to achieve a usable double-well potential, the inductance L has to be relatively large. (See subsection 3.5; the tunnelling potential curvature has to dominate that of the inductive potential, $L > L_k$.) Such a loop may well be susceptible to pick-up from stray external fields and control lines/coils associated with other qubits in a many-qubit system. This would result in noise and errors in the applied flux Φ_x and form a source of decoherence. Consequently, a proposal for a flux qubit with a smaller loop has been made [19]. This system, shown in figure 5, incorporates three weak links in a loop and in practice one of these can be a composite and controllable. If the composite weak link contains two links with equal tunnelling energy of $E'_J \equiv \alpha E_J$, the phase difference across link i (see figure 5) is γ_i and the inductors l and L_J are small enough so their contributions to the fluxes in the loops are negligible, the (two-dimensional) potential for the system is

$$U_{fq}/E_J = 2(1+\alpha) - \cos\gamma_1 - \cos\gamma_2 - 2\alpha\cos(f_2\pi)\cos(2f_1\pi + f_2\pi + \gamma_1 - \gamma_2) \ . \qquad (13)$$

The applied fluxes have been defined as $\Phi_x \equiv f_1\Phi_0$ for the loop shown in figure 5 and $\Phi_{xJ} \equiv f_2\Phi_0$ for the loop in the composite weak link. This potential is periodic in both γ_1 and γ_2. However, it is possible to identify a cell which contains a double-well potential along a chosen direction[22] in the γ_1-γ_2 plane [19]. Inter-cell tunnelling is suppressed by a much larger barrier, preventing the development of momentum in γ-space (i.e. voltages across junctions) and thus decoupling the system from external charge [19]. With suitably chosen parameters [19, 40] it is possible to manipulate this flux system in the double-well potential analogous to the description given in subsection 3.5. It is therefore describable by an effective Hamiltonian of the form eq. (9), with the coefficients of σ_z and σ_x controllable through the independent external fluxes f_1 and f_2 [19, 40].

5 Superconducting qubits versus the "DiVincenzo" checklist

To discuss the superconducting qubit proposals in more detail, it is very helpful to refer to David DiVincenzo's checklist Five criteria which must be satisfied by candidate quantum computing hardware have been elucidated [41]: (i) Clearly identifiable qubits (an enumerable Hilbert space) and the ability to scale up in number; (ii) "Cold" starting states (e.g. the ability to prepare the thermal ground state of whole system); (iii) Quantum gates (the ability to realise a universal set of gates through control of the system Hamiltonian); (iv) Low decoherence (so that error correction techniques [23, 24, 25] may be used in a fault-tolerant manner [26, 27])—an approximate benchmark is a fidelity loss of $\sim 10^{-4}$ per elementary quantum gate operation; (v) Measurement (the ability to perform quantum measurements on the qubits to obtain the result of the computation). This list provides a basis for consideration of superconducting qubits.

5.1 Identifiable qubits and scalability

There is no problem regarding the identifiable nature of superconducting qubits. For both charge and flux systems the long term goal is clearly a fabricated structure of enough qubits to do something interesting and useful. By its very nature the structure will contain qubits identifiable (and addressable via external control lines and circuits) by their spatial location.

A point about superconducting circuits (and, for that matter, some other candidate qubits) which shouldn't be forgotten is that they aren't actually qubits. It is only an approximation that the superconducting circuits discussed here possess some sort of electromagnetic degree of freedom which lives in a two dimensional Hilbert space. This approximation is pretty good for the restrictions quoted on the system parameters and the external sources. However, in practice, small amplitudes of states outside the truncated Hilbert space can arise and contribute in quantum circuit evolution. Sitting in the qubit space, this can be viewed (at least loosely) as a contribution to the decoherence [42].

[22] The maximum is a minimum in the orthogonal direction, forming a saddle point constraining the system to motion in the double well.

One potential advantage of condensed matter qubits in general, in comparison to those made from more fundamental systems (photons, electrons, atoms/ions, nuclei...) is seen to be their potential for scalability. We know how to fabricate stuff at pretty small lengthscales (sub-micron) and there is the future potential for so-called self assembly at even smaller scales. It is certainly true that lithographic and related techniques offer the possibility of producing many qubits, once the specific techniques involved have been mastered and tuned for the fabrication of a few. However, there are a number of facets to the fabrication approach worthy of note (e.g. [40]), which apply to the superconducting approaches in general. Condensed matter systems have high densities of states—many microscopic degrees of freedom. This environment also has a tendency for coupling to the qubit degrees of freedom. The use of superconductivity helps combat this decoherence, but it does not eliminate it completely. (Decoherence is discussed in more detail below.) Fabrication is also imperfect—fabricated qubits will never be identical in the same way that fundamental systems can be (like two identical calcium ions). This means that fabricated systems either have to be insensitive to manufacturing variations (such as the qubit being dependent on some topological quantity) or have to be measured and tweaked to chosen values (e.g. of E_{Je}) prior to use. On the other hand, fabrication does offer great flexibility and the potential for combatting the disadvantages. Design freedom means the circuits can be constructed to minimize certain environment couplings and to couple qubits as desired. It has also been stressed [40] that complicated devices which perform a number of functions require at least as many tunable design parameters, a constraint which is met by flexible fabrication.

5.2 State preparation

For a superconducting qubit with an effective Hamiltonian of eq. (7) or eq. (9), state preparation should be relatively straightforward, compared to other items on the checklist. The coefficient of σ_z should be made large (so the level separation is greater than that when the qubit is in use) and negative and the coefficient of σ_x should be turned off. If the qubit attains thermal equilibrium it will then be in state $|0\rangle$ to a good approximation.[23] The corrections to the unit fidelity are clearly calculable and small (by an appropriate Boltzmann factor). Superconducting qubits clearly require operation at temperatures low compared to the typical level separations of qubits in use, so this will guarantee the smallness of the impurity of prepared states. The importance of control over the σ_x tunnelling term is apparent here. Note that if this is non-zero during state preparation, an amplitude of the other qubit state will be introduced. This is given by the ratio of the tunnelling to the σ_z energy and is not exponentially small.

5.3 Quantum gates

It is known that arbitrary single qubit gates and a two-qubit gate capable of creating maximal entanglement from a product state of two qubits form a universal set for quantum computation [43, 44]. Single qubit gates can discussed generally for superconducting qubits

[23]Likewise, if the coefficient of σ_z is made large and positive, $|1\rangle$ will be prepared.

as these are based on the Hamiltonians already discussed. Two-qubit gates require coupling and separate discussion for the charge and flux approaches.

Arbitrary single qubit gates can be constructed using the Hamiltonians eq. (7) or eq. (9). Control over qubit rotations around two orthognal axes (x and z), by turning on the appropriate term in the Hamiltonian for a set length of time, is sufficient to enable an arbitrary qubit rotation.[24] Single superconducting qubit gates are thus old hat theoretically. Whilst this is not the case experimentally yet, this clearly has to become so if superconducting quantum computing is to be taken seriously.

There are various proposed methods for coupling superconducting qubits, so it is appropriate to discuss these separately.

5.3.1 Charge qubits

The first proposal for coupling charge qubits [14] involves placing the qubits (each with their accompanying external voltage source) in parallel and connecting a parallel superconducting inductor across the qubit system. This generates a coupling between qubits i and j proportional to $\sigma_y^i \sigma_y^j$.[25] Such an interaction term (acting for a specific time) can be used with single qubit gates to form a universal set. In the original discussion [14] this interaction term is permanently on, so two-qubit gates have to be realised by operating qubits at different voltages and bringing them into resonance, either suddenly for a fixed (short) time period, or adiabatically and using an oscillatory external voltage difference pulse. However, having the qubit interactions turned on permanently is something of a nuisance, although clearly it can be handled with external applied pulses, as is also done in NMR systems.

An alternative adiabatic manipulation of charge qubits has also been discussed [15]. Here, in effect, the interaction is turned on and off through spatial movement of the charges. By using a 1D array of superconducting islands to represent each qubit, it is possible to turn on a capacitive coupling between qubits by adiabatic manipulation of the charges spatially. Two-qubit gates are realised here through a $\sigma_z^i \sigma_z^j$ term. This can generate a conditional phase shift, which combined with a series of single qubit rotations, can be used to achieve a controlled-NOT (or XOR) gate, a universal two-qubit gate.

The inductive qubit coupling can be controlled [16, 45] if the charge qubits have variable E_{Je}, as in subsection 3.6. For m qubits (each of capactitance $C_t = (C_g^{-1} + C_{Je}^{-1})^{-1}$) in parallel with a shunt inductor L_s, the oscillation frequency $\omega = (L_s m C_t)^{-1/2}$ must be large compared to any qubit frequencies, so this degree of freedom remains adiabatically in its ground state.[26]

[24]For example, using an oscillatory piece of the flux f_1 for a type II flux qubit, it is possible to induce familiar Rabi oscillations. Driving at a resonant frequency of 20π GHz and with $E_J = 200$ GHz, it is possible to arrange a Rabi frequency of around 90 MHz, so a π-pulse applied to the qubit would take about 20 ns [40].

[25]On the assumption that the inductor-qubits oscillator is high frequency (and so in its ground state), the important interaction term is proportional to I^2 where I is the sum of the currents through the qubits. In the spin picture a qubit tunnelling energy (proportional to $\cos\theta$) is represented by σ_x (see subsection 3.4) and so the tunnelling current (proportional to $\sin\theta$ [37]) is represented by σ_y.

[26]Note that the frequency decreases as $m^{-1/2}$, so the argument cannot hold for arbitrarily large m. This is not an issue for short term superconducting quantum computing with a few qubits, but is a long term

The interaction term is then $-\frac{1}{2}L_s I^2$ where I is the sum of the qubit currents [16, 45]. The cross terms give qubit interactions, e.g.

$$H^{ij}_{cint} = -\frac{\pi^2 C_t^2 L_s}{C_{Je}^2 \Phi_0^2} E^i_{Je} E^j_{Je} \sigma^i_y \sigma^j_y \ . \tag{14}$$

A nice feature of this coupling [16, 45] is that single qubit manipulations only require the turning on of one E^i_{Je} (using the external flux Φ^i_{xJ}), in which case the interaction remains off. However, this assumes that there is no attempt to perform independent single qubit gates simultaneously—this would result in the interaction being turned on whether you like it or not. Nevertheless, all this is still probably preferable to having to live with the interactions full time. It is certainly good to have the ability to perform single and two-qubit gates using the same external sources, Φ^i_{xJ} and V^i_x for each charge qubit.

5.3.2 Flux qubits

One method for coupling flux qubits (type II) is the use of closed superconducing inductive loops, or flux transformers [19, 40]. For the case where one of the three weak links in the flux qubit (type II) is a composite controllable link, there are three usable positions for a flux transformer link two qubits (see subsection 4.3 for the flux numbering): (i) Coupling the current in loop$^i_1$ to that in loop$^j_1$; (ii) Coupling the current in loop$^i_1$ to that in loop$^j_2$; (iii) Coupling the current in loop$^i_2$ to that in loop$^j_1$. The effect is that the applied flux seen by one of the loops of qubit^i contains a contribution from the superconducting current flowing in a loop of qubit^j, and vice versa. Position (i) couples the σ_z terms, position (ii) couples σ^i_z to σ^j_x and position (iii) the reverse of (ii). The interaction Hamiltonian is therefore [19, 40][27]

$$H^{ij}_{fint} = \kappa_{zz}\sigma^i_z\sigma^j_z + \kappa_{zx}\sigma^i_z\sigma^j_x + \kappa_{xz}\sigma^i_x\sigma^j_z \ . \tag{15}$$

Which terms are used depends on the choice of which flux transformers are put in place. The coupling coefficients will be small compared to the qubit energies, as the inductances (self and mutual) are deliberately kept small in this approach II to avoid pick-up. An estimate is $\kappa \sim 0.01E_J$ [40]. There is clearly ample freedom in the interaction eq. (15) to realise a universal two-qubit gate. For example, the first term can generate a controlled phase shift which, in combination with a series of single qubit rotations, can be used to achieve a controlled-NOT (or XOR) gate. It is clearly important that the external flux sources applied to qubits do not couple directly to the flux transformers or else a single qubit gate applied to a chosen site will introduce effects at others. Clearly if the flux transformers are left in place permanently, the interaction eq. (15) will act all the time, a nuisance although it can be handled. It may be possible to activate the flux transformers only when required by using Josephson switches [40]. However, this switching may introduce additional unwanted noise.

An alternative approach for coupling flux qubits (type I and type II) exists [45] which enables control using the same external sources applied to achieve single qubit gates. This

issue. This coupling mechanism will work for registers of qubits up to some maximum m set by the system parameters. An estimate is $m_{max} \sim 70$ [16].

[27] Such coupling should also be usable for type I flux qubits, although it was not discussed for the spin picture in [18].

involves coupling the main flux loop (as shown in figures 4 and 5) of each of a 1D array of qubits (via a mutual inductance M) to the superconducting inductor L_s of a separate L_s–C_s oscillator. This oscillator has to be high frequency compared to any qubit frequencies, so it remains adiabatically in its ground state. The interaction term is then $-\frac{1}{2}C_sV^2$ where V is the sum of the qubit voltages induced in the oscillator circuit due to the flux tunnelling in the qubit circuits [45].[28] The cross terms give qubit interactions, e.g.

$$H^{ij}_{fint} = -\frac{\pi^2 C_s M^2 \delta\Phi^i \delta\Phi^j}{4L_s^2 e^2 \Phi_0^2}\Gamma^i\Gamma^j\sigma_y^i\sigma_y^j\ . \tag{16}$$

As in the charge case, a nice feature of this coupling [45] is that single qubit manipulations only require the manipulation of one Γ^i (through variation of the barrier height E^i_{Je} using the external flux Φ^i_{xJ}), in which case the interaction term can remain exponentially suppressed through the other Γ^j staying "off". Again, this does assume that there is no attempt to perform independent single qubit gates simultaneously—this would result in the interaction being turned on whether you like it or not. As in the charge case of eq. (14), this approach enables the performance of single and two-qubit gates using the same external sources, Φ^i_{xJ} and Φ^i_x for each charge qubit.

5.3.3 Linear chain addressing

The addressing of a linear chain of qubits through global pulses [46, 47] may be applicable to superconducting qubits. This requires a repeated sequence of qubits such as $ABABAB$... where all the A qubits respond to one type (frequency) of external pulse and all the B qubits to another. Ideally the A systems should all be identical (which is clearly possible if they are fundamental—built by Nature—) and the same for B. The practical problem for application of this approach to any form of fabricated system is therefore to tune all the qubits to form a good approximation to such an ideal array. If this can be done, one great advantage is that qubit-specific pulses are not needed. The qubits only have to be addressed individually for tuning (effectively at dc). Computation is achieved by blasting the whole array with appropriate pulses [46, 47]. In the superconducting case this could be realised by placing the whole qubit array in an appropriate microwave cavity. This has been suggested in the flux case [40] but is equally feasible for the charge case.

5.4 Decoherence

One important contribution to decoherence which applies to both charge and flux systems is the presence of fluctuations/errors in the external sources coupled to qubits. There will always be some level of imperfection in these sources. At minimum they will have finite coupling back to room temperature apparatus and so will contain some level of thermal fluctuations; however, in practice they will clearly also be subject to experimental error in

[28] This again gives $\sigma_y^i\sigma_y^j$ coupling. The voltage induced in the oscillator circuit is $V^i = \frac{M}{L_s}\dot{\Phi}^i = \frac{M\delta\Phi^i\Gamma^i\sigma_y^i}{2\hbar L_s}$, where $\delta\Phi^i$ is the separation between the minima in the double-well flux potential and Γ^i is the tunnelling amplitude through the barrier (see eq. (9)).

their definition.[29] For example, both charge and flux systems are susceptible to variations in the external magnetic flux applied to control the effective Josephson tunnelling energy $E_{Je}(\Phi_{xJ})$, although in different ways. In the charge case (where E_{Je} is the actual tunnelling amplitude), when the tunnelling is turned "off" this will suffer linear fluctuations in Φ_{xJ} . On the other hand, with the tunnelling "on", if the system can be operated at a maximum of E_{Je} then only second order fluctuations impact. In the flux case (where E_{Je} forms the barrier through which the tunnelling occurs), then when the tunnelling is "off" there is exponential suppression of the sensitivity to fluctuations in Φ_{xJ}. On the other hand, when the tunnelling is on, the system sees linear fluctuations.

Another important issue which applies to both charge and flux qubits is the fact that the Hilbert space is truncated by hand to make the system look like a two-state qubit. It is clearly a reasonable approximation to use just the lowest two energy levels of a system when their separation is much smaller than that to higher levels. However, it is not perfect. Sudden (irreversible) projective resolution into different spin bases will carry errors. Evolution under time-varying sources can excite amplitudes of higher energy states outside the qubit basis.[30] These effects will look like decoherence as far as the qubits are concerned. It is certainly true that in principle this decoherence should be negligible if "dc" sources are only ever varied adiabatically and resonant ac source pulses are employed. However, detailed calculations for model qubits are needed to estimate the impact of these effects during realistic quantum gates. A related concern with the Hilbert space truncation is that of unitary *quantum leakage* to states outside the qubit space. This has been considered in detail in [42] for the case of charge qubits. For single qubit gates it is estimated that with a Josephson to charging energy ratio of about 0.02 (which is physically reasonable), about 4000 gates would result in a fidelity loss due to leakage of order unity. For two-qubit gates (for a given Josephson to charging energy ratio), there is an optimum coupling energy (the coefficient in eq. (14)) to reduce the damage due to leakage.[31] Similar analysis is clearly possible for flux qubits.

5.4.1 Charge qubits

Some detailed analysis has been made of decoherence effects which are specific to charge qubits [14, 16, 45]. Basically, a charge qubit will be susceptible to the effects of external charge/voltage noise; these couple to σ_z (in the spin notation) and cause decoherence. This noise could be due to charged impurities in the fabricated circuit (e.g. in the substrate material) or voltage fluctuations in circuits coupled to the qubit. (Quasiparticle effects in the superconducting circuits themselves, whilst not zero as the temperature is finite for any

[29] It is interesting to note that the decoherence effects seen in the charge qubit experiments [17] may well be dominated by such source fluctuations/errors [48]. In a sense this is very encouraging, because it is more likely that these can be improved in future experiments, compared to reducing the intrinsic decoherence (for example, due to microscopic environment degrees of freedom coupled to the charge or flux quantum variable).

[30] This has been studied using the time-dependent Schrödinger equation for a SQUID device in [49]. Unexpected (with respect to standard perturbation theory) transition probabilities can arise due to the non-linear dependence of the device energy levels on externally applied flux, so simple perturbative estimates for transition rates out of the qubit space should be treated with caution.

[31] For example, with a Josephson to charging energy ratio of 0.025, the minimum fidelity *loss* for a two-qubit gate is about 0.007 for a coupling to charging energy ratio of about 0.003. Correction of such leakage "errors" through measurement which only occurs if the system steps outside the qubit space is a potential remedy [42].

real system, are thought to contribute less to the overall charge qubit decoherence [14].) The external noise can be modelled as a dissipative circuit (with an effective resistance R_V) coupled to the qubit. Using standard techniques, the decoherence time due to this dissipation can be calculated as [16][32]

$$\tau_V = \frac{\hbar R_Q}{4\pi E_{Je} R_V} \left(\frac{C_{Je}}{C_t}\right)^2 \tanh\left(\frac{E_{Je}}{2kT}\right) , \tag{17}$$

where the so-called quantum resistance is given by $R_Q = h/e^2$. With capacitances down at the $10^{-16} \sim 10^{-17}$ F level, $T \sim 50$ mK and E_{Je} at a similar value in temperature units (all physically reasonable choices), a typical (single qubit) gate time is about $\tau_{op} \sim 7\ 10^{-11}$ s [16]. An estimate of the effective resistance as $R_V \sim 50\ \Omega$ gives a ratio $\tau_V/\tau_{op} \sim 4000$ [16]. This is certainly reasonable enough for demonstrations of superconducting quantum computing with charge qubits, and it is around the estimated benchmark required for fault-tolerant methods (assuming that two-qubit gates run with a similar τ_{op}).

5.4.2 Flux qubits

Some detailed analysis has been made of decoherence effects which are specific to flux qubits [40, 45, 50]. Basically, a flux qubit will be susceptible to the effects of external flux/current noise; these couple to σ_z (in the spin notation) and cause decoherence. Such noise could be due to magnetic impurities in the fabricated circuit (e.g. in the substrate material) or current fluctuations in circuits coupled to the qubit. This external noise can all be modelled as a dissipative circuit (with an effective resistance R_I) coupled to the qubit. The decoherence time due to this dissipation is [45][33] (see eq. (9))

$$\tau_V = \frac{a\hbar R_I}{\Gamma R_Q} \left(\frac{\Phi_0^2}{4\pi^2 M E_{Je}}\right)^2 \tanh\left(\frac{\Gamma}{2kT}\right) , \tag{18}$$

where a is a prefactor of order unity for both designs I and II of flux qubit. M is the mutual inductance coupling and, as this will be rather smaller for type II flux qubits [19] compared to type I qubits [18], demonstrates why decoherence due to external flux/current noise should be rather smaller for the former, with correspondingly longer decoherence times.

Other decoherence effects relevant for flux qubits are [50]: (i) quasiparticle effects in the actual superconducting circuits; (ii) magnetic fluctuations due to nuclear spins in the

[32] This is a simplification for the degenerate point $Q'_x = 0$, (see eq. (7)). In general there are two timescales, that for dephasing τ_d and that for energy relaxation τ_r, given by [45]

$$\tau_d = \nu \frac{\hbar}{\Delta} \left(\frac{1}{2} \coth \frac{\Delta}{2kT} \sin^2 \eta + \frac{2kT}{\Delta} \cos^2 \eta\right)^{-1}$$

and

$$\tau_r = \nu \frac{\hbar}{\Delta} \left(\coth \frac{\Delta}{2kT} \sin^2 \eta\right)^{-1} .$$

For the charge qubit case the level separation is $\Delta = \left(e^2 Q'^2_x/C_t^2 + E_{Je}^2\right)^{1/2}$, $\tan\eta \equiv C_t E_{Je}/eQ'_x$ and $\nu_V = R_Q C_{Je}^2/4\pi R_V C_t^2$. Similar analysis can be employed to study the effects of the E_{Je} fluctuations (due to noise in Φ_{xJ}) which couple to σ_x [45].

[33] Again, comments and more general expressions analogous to those in footnote 32 apply.

solid; (iii) electromagnetic radiation by the qubit (from the oscillatory currents, for example during Rabi oscillations); (iv) unwanted but unavoidable coupling to other flux qubits. In the analysis of [50], the last effect is seen to dominate the decoherence time for type II flux qubits. Given that the controlled coupling between qubits is achieved either through flux transformers or an oscillator link, the direct magnetic dipole interaction between qubits (assuming that this cannot be screened)

$$H_{dip} = \sum_{i,j} \hbar \lambda_{i,j} \sigma_z^i \sigma_z^j \tag{19}$$

is taken as a source of decoherence. The coupling $\hbar\lambda_{i,j} \approx \frac{\mu_i \mu_j}{|\boldsymbol{r}_i - \boldsymbol{r}_j|^3}$ between the dipoles μ is clearly largest for nearest neighbours. With a separation of about 10 μm, this gives $\lambda \sim 6$ kHz and a corresponding decoherence time of $\tau \sim 0.2$ ms. If the operation time is estimated from the Rabi frequency for type II flux qubits of about 100 MHz, this gives an estimate for $\tau/\tau_{op} \sim 10^5$ [50]. Once again, this compares favourably with the estimated benchmark required for fault-tolerant methods.

5.5 Measurement

5.5.1 Charge qubits

In principle, the charge on a charge qubit can be measured by capacitively coupling it to a conventional single-electron transistor (SET) [16, 45, 51].[34] This forms a decent approximation to a projective quantum measurement of the qubit charge; a detailed analysis has been given in [51]. Clearly measurements should be at the whim of a quantum computer operator, and ideally the measurement apparatus should have no effect on the quantum system prior to this. If the channel voltage applied to the SET is kept at zero, then no dissipative current flows and it behaves as a reactive circuit element. The only effect of the SET is to renormalize the capacitance of the qubit to which it is coupled. (This should not be a problem and it is certainly preferable to attempting to turn off the capacitive coupling.) To make a measurement the SET voltage is turned on; it is assumed that the qubit evolution will have been first "frozen", through manipulation of the sources applied to it.[35] Study of the evolution of the density operator of the coupled system shows that there is a fast dephasing of the qubit state (in the charge basis) and a slower development of correlation between the SET current probability distribution and the charge probabilities of the qubit state [51]. Measurement of the SET current (a relatively standard procedure) at this point thus effects a measurement of the qubit charge.

[34] To be more precise, the superconducting island of the qubit is capacitively coupled to the central charge island of the SET. This coupling means that there is a conditional modification to the tunnelling rate in the SET, dependent upon the charge on the qubit island. When a current flows and this entanglement involves states of the SET which are then distinguished by a conventional current measurement, the charge measurement is effected.

[35] E_{Je} at least needs to be turned off, so there is no further change in the magnitudes of the state amplitudes in the bit value (σ_z) basis.

5.5.2 Flux qubits

In principle, a flux qubit can be measured in its bit value (σ_z) basis by coupling it to a SQUID magnetometer [19, 40]. I have not seen a detailed analysis of this coupled system, analogous to that in [51] for the charge measurement, but the procedure will have to be similar as it will need to form a decent approximation to a projective measurement of flux [40]. Although it is potentially easier to turn off inductive coupling (compared to capacitive) by use of a flux transformer containing a Josephson switch, this may introduce noise. It is probably therefore better to couple the measurement SQUID permanently and operate it in a manner analogous to the SET for a charge measurement. Until the measurement is to be made, the SQUID should be biased so that it simply behaves like a linear inductive element. Its only effect is then a renormalization of the flux qubit inductance. To make a measurement, the SQUID needs to be switched to a voltage carrying state (a non-linear region of its response) in which it is capable of measurement.[36] It is again assumed that the qubit evolution will have been first "frozen", through manipulation of the sources applied to it.[37] It is likely that there will be a fast dephasing of the qubit state (in the flux basis) and a slower development of correlation between the SQUID voltage probability distribution and the flux probabilities of the qubit state, but a detailed study of a model coupled system is needed to demonstrate this. Measurement of the SQUID voltage (to effect a measurement of the qubit flux) is routine as SQUID magnetometers are widely used instruments.

6 Experiments

At the experimental level, charge qubits currently lead flux qubits. However, the game has only just started and the score is only 1 – 0. Coupling between superconducting qubits has yet to be demonstrated. It is to be hoped that this section of the paper dates rather rapidly...

Following on from evidence for quantum behaviour of charges and discrete states [52, 53], coherent quantum oscillations[38] of a single charge qubit have been reported [17]. These experiments show clearly that state preparation, single qubit state manipulation (effectively, control over the two important terms in the Hamiltonian eq. (7)) and (a simple form of) charge measurement are possible. The decoherence was small enough for quantum coherence to be seen, but will need to be significantly smaller for a usable superconducting qubit.[39]

[36] To be more precise, the current in the flux qubit is inductively coupled to the loop of the SQUID. This coupling means that there is a conditional modification to the flux tunnelling rate in the SQUID (which gives rise to the voltage), dependent upon the current of the flux qubit. This entanglement involves states of the SQUID which are then distinguished by a conventional voltage measurement, and the flux/current measurement is effected.

[37] E_{Je} at least needs to be turned to a large value, so there is no further change in the magnitudes of the state amplitudes in the bit value (σ_z) basis.

[38] The preparation of an energy eigenstate superposition, which produces oscillatory behaviour in the expectation value of some other observable—in this case charge.

[39] Oscillations with a period of $\sim$ 80 ps were seen for durations up to $\sim$ 2 ns. No demonstration of qubit-qubit interactions exists yet and, of course, the decoherence will probably be worse when more stuff is placed near qubits. However, the measurement process in [17] was rather robust; refinement of this to be more like the theoretical proposal will, on the other hand, reduce decoherence.

Nevertheless, it is a promising start.

No analogous demonstration of coherent quantum oscillations for a flux system exists to date (at least to my knowledge), although folk have tried [54, 55, 56, 57]. However, there is some evidence for the energy level structure and tunnelling in superconducting weak link rings [58, 59, 60, 61, 62] and, more recently, the spectroscopic mapping out of anticrossings between excited levels [63].[40] Actually, all this is not entirely a fair comparison with the charge case because such flux experiments have really concentrated on trying to achieve macroscopic quantum coherence—a Schrödinger cat—in order to test out fundamental aspects of quantum mechanics [64]. For the purposes of quantum computing it does not matter whether the qubits are macroscopic, mesoscopic or whatever; they simply need to stand up to the checklist and do the job. Certainly from the flux perspective they will probably suffer less decoherence if they are smaller, hence the proposal for type II flux qubits.

7 Comments for the future

The superconducting approach to qubits is certainly a promising one. For future progress and with the potential development of superconducting quantum computing in mind, I think the following points are worth noting.

- More experimental progress is needed! At present we only have one charge qubit. I am optimistic that this will change fairly soon, with the most likely breakthrough on the flux qubit front coming from the type II approach. The next hurdle will be a two-qubit gate, or the demonstration of entanglement between two superconducting qubits. This is a reasonable goal for the next couple of years; if we have to wait rather longer then the validity of the superconducting route to quantum computing will become questionable.
- (Something of an aside...) If any developments of superconducting qubits have any passing resemblance to cats (in the sense of being "macroscopic"), this will be extremely interesting from the perspective of fundamental quantum physics and the classical-quantum interface. Never mind one cat, how about entangled cats?! From the technology and fabrication point of view, it's probably preferable if the qubits are rather smaller.
- Decoherence is the most important factor in the DiVincenzo checklist. Theoretical proposals exist for implementing a universal set of gates (for both charge and flux qubits) and measuring appropriate charge or flux states of qubits, based on physically reasonable system parameters. However, further investigations are needed to determine if these can be implemented experimentally with sufficiently low system decoherence to enable quantum computing. Modelling and estimates certainly suggest that a decent number (10^3 to 10^5) of quantum gate operations should be achievable within a decoherence time. However, as is stressed in [19], the real test is experiment.

[40] Such anticrossings are highly suggestive of superposition states, but coherent oscillation of flux through a superposition of energy eigenstates remains to be demonstrated.

- In the longer term and if few qubit superconducting experiments are successful, the methods proposed for implementing gates will probably need further development, to enable parallel operations on different qubits.

- From the theoretical perspective there seems to be no clear winner at present between charge and flux (at least type II) qubits. Given there is so far to go, the initial experimental lead of the charge approach should not be taken to heart. At present both routes are worthy of further investigation. If there is to be a winner, this will probably be decided in the laboratory rather than on paper.

- There may well be no winner, and hybrid systems, using both superconducting charge and flux states (maybe combined with other non-superconducting qubits) may prove to be the most useful for actual technology. After all, bits are realized in different ways (for processing, RAM, longer term memory, communication, etc.) in conventional classical information technology.

If the superconducting qubit experiments progress over the next few years, then at some point the larger scale fabrication issues will need some serious research and development. This will be on a totally different investment scale from the work up to that point. Once all the fundamental investigations are complete and some principles for scaling up in qubit number have been demonstrated experimentally,[41] then someone will have to bite the bullet and invest big bucks if we are to have a quantum information technology industry in the future. Getting over this hurdle may well require careful collaboration between the research funding agencies and industry, as neither may be prepared to jump alone.[42]

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[41] More than likely, all this will have been done with "custom built" circuits.

[42] Of course, these last remarks apply to all the potential routes for scalable quantum computing, and not just the superconducting ones.

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