Using Walsh Code Selection to Reduce the Power Variance of Bandlimited Forward-link CDMA Waveforms

R. Neil Braithwaite
Electronic Systems Laboratory
HP Laboratories Palo Alto
HPL-1999-87
July, 1999

wireless communications, CDMA, multi-access communication, intermodulation distortion

This paper investigates the effects of Walsh code selection and baseband filtering on the statistical moments of the bandlimited forward-link CDMA signal. A normalized moment, the "power variance", is proposed as a measure of the signal's sensitivity to nonlinear amplification. The estimation of the power variance for the case of a bandlimited QPSK-modulated random waveform is presented as a baseline for illustrating the effect of filtering on a sampled signal. The bandlimited CDMA signal case illustrates the interaction between the Walsh codes and the wave-shaping filter, and quantifies the influence of "missing Hadamard basis functions". Results verify the accuracy of the statistical models and illustrate the importance of Walsh code selection in the reduction of the power variance of a bandlimited CDMA waveform.

Using Walsh Code Selection to Reduce the Power Variance of Bandlimited Forward-link CDMA Waveforms

R. Neil Braithwaite
Hewlett-Packard Company, HP Labs, Palo Alto CA 94304

Abstract

This paper investigates the effects of Walsh code selection and baseband filtering on the statistical moments of the bandlimited forward-link CDMA signal. A normalized moment, the "power variance", is proposed as a measure of the signal's sensitivity to nonlinear amplification. The estimation of the power variance for the case of a bandlimited QPSK-modulated random waveform is presented as a baseline for illustrating the effect of filtering on a sampled signal. The bandlimited CDMA signal case illustrates the interaction between the Walsh codes and the wave-shaping filter, and quantifies the influence of "missing Hadamard basis functions". Results verify the accuracy of the statistical models and illustrate the importance of Walsh code selection in the reduction of the power variance of a bandlimited CDMA waveform.

IEEE keywords: multi-access communication, intermodulation distortion.

1.0 Introduction

CDMA, or Code Division Multiple Access, is a modulation format that uses spread spectrum to transmit multiple channels over a common bandwidth [14], [16]. It is generally accepted that the capacity of a CDMA system is limited by the interference from other users [5], [6]. However, if the transmit portion of a base-station contains nonlinear components, such as power amplifiers (PAs), intermodulation distortion becomes an additional source of interference [4]. In general, it is the AM component of the input signal that is converted by nonlinearities into intermodulation distortion.

"Walsh code selection" is the task of choosing which Walsh codes are to be used for data transmission when only a subset of the available codes are required. The combination of Walsh code selection and filtering affect the AM component of the CDMA waveform. For the purpose of

comparison, the AM component of the input signal is quantified using a statistical measure referred to as the “power variance”. A lower power variance, in general, will result in less intermodulation distortion. Another influence on the power variance of the CDMA signal---correlation between data sequences---is beyond the scope of this paper; it is discussed in [2].

The remainder of the introduction is as follows. Section 1.1 provides an overview of the transmission architecture used to generate a bandlimited forward-link CDMA waveform, including a power amplifier exhibiting mild nonlinearities. Section 1.2 introduces the power variance as a measure of the signal’s sensitivity to nonlinearities. Walsh code selection is proposed in Section 1.3 as a means of reducing the power variance of a bandlimited CDMA waveform. The outline of the paper is found in Section 1.4.

1.1 Overview of Forward-link CDMA Waveform

The simplified block diagram of the forward-link (base-station to mobile) CDMA transmit architecture is shown in Figure 1. For the purpose of analyzing the CDMA waveform, there are five key signals: sampled Walsh-coded signal $x_{AM}(nT)$; sampled (digital) CDMA signal $x(nT)$; bandlimited (analog) baseband signal $x_h(t)$; RF input signal $x_{RF}(t)$; and RF output signal $z_{RF}(t)$. The Walsh-coded signal, $x_{AM}(nT)$, is a bipolar sequence which contains the amplitude information. It is QPSK modulated to form the sampled CDMA signal, $x(nT)$. The sampled CDMA signal is lowpass filtered, up-converted, then amplified to form signals $x_h(t)$, $x_{RF}(t)$, and $z_{RF}(t)$, respectively.

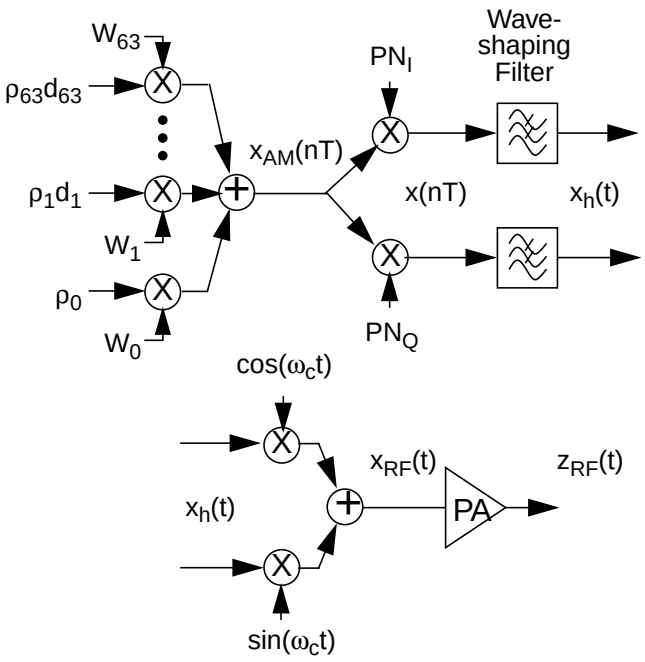

Fig. 1. Forward-link CDMA Transmit chain.

The equations for the baseband signals shown in Figure 1 are as follows. The Walsh-coded signal, $x_{AM}(nT)$, is

$$x_{AM}(nT) = \sum_{i=0}^{63} \rho_i d_i(nT) \cdot W_i(nT) \tag{1}$$

where T is the sample (chip) interval and $d_0 = 1$. Each data symbol, d_i, is held constant at +1 or -1 over an interval of 64T. Each Walsh code, W_i, comprises 64 chips that are repeated for each data symbol:

$$W_i = \left[W_i(0)\ W_i(T)\ \ldots\ W_i(63T) \right]. \tag{2}$$

The scale term, ρ_i, is used to adjust the transmitted power of the individual channel associated with Walsh code W_i, and it is assumed to constant over the data symbol interval [0,63T].

The sampled CDMA signal is generated by QPSK modulation using two PN sequences [10], [11] (denoted by PN_I and PN_Q):

$$x(nT) = x_{AM}(nT) \cdot 2^{-0.5} \cdot [PN_I(nT) + j \cdot PN_Q(nT)] \tag{3}$$

The phase modulation has the effect of spreading the spectrum of the signal, as well as ensuring that each baseband sample is uncorrelated with its neighbor (that is, E[x(nT) x(mT)] = 0 when n is not equal to m).

The wave-shaping filter, h(t), is used to bandlimit the sampled CDMA signal:

$$x_h(t) = \sum_n x(nT) \cdot h(t - nT)\,. \tag{4}$$

For illustrative purposes, the wave-shaping filter, h(t), is assumed to perform perfect bandlimiting: that is,

$$h(t - nT) = \frac{\sin[0.5\omega_s(t-nT)]}{0.5\omega_s(t-nT)}\,, \tag{5}$$

where ω_s is the sample (chip) rate ($\omega_s = 2\pi/T$). Equation (5) is a deviation from the IS-95 specification [8]. The RF input signal is given by

$$x_{RF}(t) = I \cdot \cos(\omega_c t) + Q \cdot \sin(\omega_c t) \tag{6}$$

where ω_c is the carrier frequency; $I = \mathrm{Re}\{x_h(t)\}$ and $Q = \mathrm{Im}\{x_h(t)\}$.

The RF output signal is described by

$$z_{RF}(t) = G_{PA}(|x_{RF}(t)|) \cdot x_{RF}(t)\,, \tag{7}$$

where G_{PA} is the nonlinear gain of the power amplifier and $|x_{RF}(t)|$ denotes the envelope of the RF input signal (equal to the modulus of the baseband signal $x_h(t)$). It is assumed that G_{PA} can be represented, adequately, by the following series [13]:

$$G_{PA}(|x_{RF}|) = G_o \cdot \sum_i a_i \cdot |x_{RF}|^i \tag{8}$$

where a_i are complex coefficients, $a_0 = 1$, and G_o is the nominal gain of the power amplifier. Since the PA model is defined in terms of the envelope of $x_{RF}(t)$, any measure of the effect of nonlinearities on the transmitted waveform must include statistics of the RF input signal. One such nonlin-

ear measure, the power variance, is presented in Section 1.2. An input signal with a reduced power variance has the desirable property of allowing the linearity requirements of the amplifier to be relaxed.

1.2 Power Variance and Sensitivity to Nonlinearities

A perfectly linear amplifier has a constant gain, $G_{PA} = G_o$. Thus, the variation in G_{PA} is a good measure of the nonlinearity of the amplifier for a given input signal format and power level. Using a PA model with a second-order gain variation (that is, $a_0 = 1$ and $|a_2| > 0$), we assign the following cost function:

$$J = E\left[\frac{\left|G_{PA}\{|x_{RF}(t)|\} - G_o\right|^2}{|G_o|^2}\right] = 2 \cdot |a_2|^2 \cdot E[|x_{RF}|^4] \tag{9}$$

where E[] denotes expected value. With respect to distortion in the output signal, this simple model captures third-order intermodulation products [13], and depends on the fourth-order moment of the input signal, $E[|x_{RF}|^4]$.

In this paper, we are interested in comparing the nonlinear sensitivity of various input signals created within the CDMA modulation format. Rather than use the fourth-order moment directly, it is useful to normalize the cost function to remove dependence on the average power:

$$\nu_e^2 = \frac{E[|x_{RF}|^4]}{\left\{E[|x_{RF}|^2]\right\}^2}. \tag{10}$$

The measure in (10), denoted by ν_e^2, is the normalized power variance; however, for convenience, it will be referred to as the “power variance”.

An alternative measure of the sensitivity of modulation formats to nonlinearities can be found in [3], which is based on distortion power rather than gain error.

1.3 Reducing Power Variance

IS-95 CDMA forward-link transmissions do not use all 64 Walsh codes at a given time. As a result, the basestation can select which set of Walsh codes will be in-use (or “active”). Walsh code selection can be used to reduce the power variance of the transmitted signal.

Consider as an example a nine-channel forward-link CDMA transmission comprising a pilot channel, sync channel, paging channel, and six traffic channels. The pilot and sync channels are always assigned to W_0 and W_{32}, respectively. The paging channel is one of W_1 to W_7. The traffic channels can be assigned to any of the remaining Walsh codes. There are 288,868,427 possible Walsh code sets that fulfil the above-mentioned restrictions for a nine-channel transmission using a 64 Walsh code space.

The goal of this paper is to identify which Walsh code sets can transmit the given data sequences with the lowest power variance. To achieve this goal, the relationship between the selected Walsh codes and the wave-shaping filter needs to be established. Once established, the relationships can be exploited to reduce the power variance of the bandlimited CDMA signal.

1.4 Outline

The remainder of the paper is as follows. The power variance measure is applied to the band-limited random waveform in Section 2.0, and to the CDMA waveform in Section 3.0. The results from the former are used as baselines for comparison with the latter. Section 4.0 describes an approach based on Walsh code selection that reduces the power variance of a bandlimited CDMA signal. Section 5.0 provides results of simulations, verifying the statistical models derived in Section 2.0 and Section 3.0, as well as demonstrating the reduction in power variance provided by the approach proposed in Section 4.0. Section 6.0 contains the concluding remarks.

2.0 Estimation of Power Variance for a Bandlimited Random Waveform

In the following, the second- and fourth-order moments of bandlimited random waveforms are derived. Two baseline waveforms are discussed: a signal generated from a sampled random

sequence, with zero mean, that is strictly bandlimited using the filter defined by (5); and a sampled random sequence that is QPSK-modulated, as shown in (3), then bandlimited. The effect of bandlimiting on the power variance in each case is of interest. For the case of the first baseline waveform, bandlimiting makes the “power variation” of the waveform more Gaussian in nature compared to the sampled signal. For the second baseline waveform, the statistics are moved towards a Rayleigh distribution [9].

Consider a bandlimited random waveform, $y_{RF}(t)$, that is generated using random digital samples y(nT), interpolated using h(t), up-converted, then amplified; that is,

$$y_{RF}(t) = y_h(t) \cdot G_o \cdot \exp(j\omega_c t) \tag{11}$$

where

$$y_h(t) = \sum_n y(nT) \cdot h(t - nT) \tag{12}$$

and G_o is chosen to be equal to unity, without loss of generality. The even-order statistics for both the RF envelope and baseband signal, $|y_{RF}(t)|$ and $y_h(t)$, respectively, are equal. Thus, for the purpose of measuring the power variance, we can use the even-order statistics of $y_h(t)$ instead of $y_{RF}(t)$, allowing us to consider only the baseband signal, as described by (12). In addition, the average power (denoted by σ^2) is equal for both the sampled and bandlimited signals:

$$\sigma^2 = E[|y_{RF}(t)|^2] = E[y_h{}^2(t)] = E[y^2(nT)]\,. \tag{13}$$

Let us look at the auto-correlation of the sampled signals, y(nT) and y^2(nT). Since the baseband samples, y(nT), are random with zero mean, we get

$$E[y(nT) \cdot y(mT)] = \begin{cases} \sigma^2 & \text{for} \quad n = m \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

and

$$E[y^2(nT)\cdot y^2(mT)] = \begin{cases} E[y^4(nT)] & \text{for } n = m \\ \sigma^4 & \text{otherwise} \end{cases} . \quad (15)$$

From (14) and (15), it is apparent that all of the second- and fourth-order statistics of the sampled signal are derived from σ^2 and $E[y^4(nT)]$.

Now let us look at $E[y^4(t)]$, where t can be any instant of time, at or between the sample instants, nT. Using (12), (14), and (15), and noting that

$$\sum_n h^2(t-nT) = 1 , \quad (16)$$

and

$$\sum_n h^4(t-nT) = f(t) = \frac{2}{3} + \frac{1}{3}\cdot\cos\left(\frac{2\pi t}{T}\right) \quad (17)$$

we get

$$E[{y_h}^4(t)] = 3\sigma^4\cdot(1-f(t)) + E[y^4(nT)]\cdot f(t) . \quad (18)$$

It is apparent from (18) that the fourth-order moment is, in general, cyclostationary; that is, it is dependent on the fractional portion of t/T (see the cosine term within (17)).

To remove the dependence on t, $E[{y_h}^4(t)]$ is averaged over one sample interval [0, T] (see cyclostationary processes in [9]): as a result,

$$E[y^4] = \frac{1}{T}\cdot\int_0^T E[{y_h}^4(t)]dt \quad (19)$$

or

$$E[y^4] = 3\sigma^4\cdot(1-\bar{f}) + E[y^4(nT)]\cdot\bar{f} \quad (20)$$

where

$$\bar{f} = \frac{1}{T} \cdot \int_0^T f(t)dt = \frac{2}{3}. \tag{21}$$

It is apparent from (20) that, in general, $E[y^4]$ is not equal to $E[y^4(nT)]$; instead, the statistical distribution of the bandlimited waveform ($E[y^4]$) is a "blending" of two-thirds of the sampled signal statistics ($E[y^4(nT)]$) and one-third of a Gaussian distribution ($3\sigma^4$). Thus, bandlimiting makes the distribution of the waveform, $y_h(t)$, more Gaussian than the sampled signal, y(nT).

Figure 2 shows a bandlimited waveform $y_{iq,h}(t)$ generated from a sampled random signal that is QPSK-modulated using two PN sequences, PN_I and PN_Q, then filtered along the in-phase (I) and quadrature (Q) paths using h(t). Assuming that the PN sequences are independent, the Gaussian "blendings" within the I and Q bandlimited waveforms are also independent. As a result, the one-third blending component within () is replaced by a Rayleigh distribution ($2\sigma^4$). On the other-hand, the two-thirds blending component is unaffected because the sampled random signal, y(nT), is common to both the in-phase and quadrature signals. Thus, for bandlimited QPSK-modulated signals, (20) is replaced by

$$E[|y_{iq}|^4] = 2\sigma^4 \cdot (1 - \bar{f}) + E[y^4(nT)] \cdot \bar{f}, \tag{22}$$

where y_{iq} is the time-average of $y_{iq,h}(t)$ over the interval [0,T].

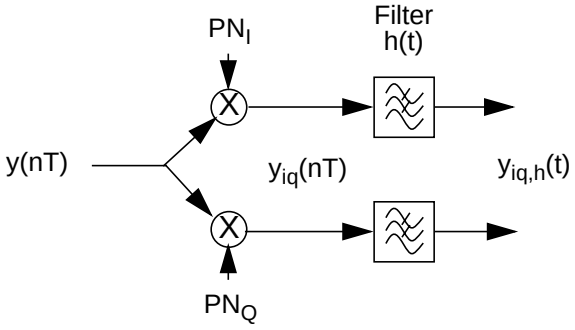

Fig. 2. Sampled random amplitude y(nT), QPSK-modulated random signal $y_{iq}(nT)$, and bandlimited QPSK-modulated random waveform $y_{iq,h}(t)$.

We now have enough information to compute the power variance. From (10), (), and (21), the power variance of a bandlimited QPSK-modulated signal is

$$v_e^2 = \frac{E[|y_{iq}|^4]}{\sigma^4}. \tag{23}$$

Baseline power variances derived from (23) appear in Section 5.0.

The key observation from this section is that QPSK-modulation and bandlimiting alter the power variance of the original sampled signal, making it more like that of a signal whose envelope has a Rayleigh distribution.

3.0 Estimation of Power Variance for the Forward-Link CDMA Waveform

This section develops statistical models for estimating the power variance of the forward-link CDMA signal. Two even-order moments are derived, $E[|x|^2]$ and $E[|x|^4]$, which are used in (10) to obtain the power variance. Section 3.1 discusses the intermodulation of Walsh codes, which is present in calculation of higher-order moments. In Section 3.2 the even-order statistics of the sampled CDMA signal are derived. The effect of bandlimiting is investigated in Section 3.3. Of particular interest is the interaction between the Walsh codes and wave-shaping filter that leads to different power variances for different Walsh code sets.

3.1 Walsh Code Intermodulation

High-order Walsh code products arise when the CDMA signal passes through a nonlinearity. For example, the PA gain model in (8) comprises a weighted sum of nonlinear operators applied to the input signal, where the i-th order operator is $|x_{RF}|^i$. Walsh code products also arise in the calculation of the second- and fourth-order moments, presented in Section 3.2.

Consider the response of a CDMA signal to a second-order operator. At sample points, t = nT, (5) has no ISI, therefore

$$\left|x_{RF}(nT)\right|^2 = |x(nT)|^2 = x_{AM}^2(nT) \tag{24}$$

and

$$x_{AM}^2(nT) = \sum_{i,\,j\,=\,0}^{63} \rho_i \rho_j \cdot d_i(nT) d_j(nT) \cdot W_i(nT) W_j(nT)\,. \tag{25}$$

Equation (25) contains second-order intermodulation terms of the data sequences (d_i^2 and $d_i d_j$) and of the Walsh codes (W_i^2 and $W_i W_j$). The intermodulation of Walsh codes is discussed below. Data intermodulation is not discussed because it is assumed later that the data sequences are uncorrelated, making $E[d_i^2] = 1$ and $E[d_i d_j] = 0$ when i is not equal to j.

Each of the 64 Walsh codes can be represented as the product of 6 (or less) Hadamard basis functions, which are shown in Figure 3 (see also Rademacher functions in [12]). A Walsh code W_i is defined as

$$W_i = B_1 \cdot B_2 \cdot B_4 \cdot B_8 \cdot B_{16} \cdot B_{32} \tag{26}$$

where

$$i = c_1 + 2c_2 + 4c_4 + 8c_8 + 16c_{16} + 32c_{32} \tag{27}$$

and

$$B_k = \begin{cases} W_k & \text{if the basis is active } (c_k = 1) \\ 1 & \text{if the basis is inactive } (c_k = 0) \end{cases}\,. \tag{28}$$

A Hadamard basis function B_k that is used in the definition of a Walsh code W_i is referred to as “active”, and this active state is indicated by $c_k = 1$. An inactive state is indicated by $c_k = 0$. Two examples of Walsh codes decomposed into the product of Hadamard basis functions are $W_{14} = W_8\, W_4\, W_2$ and $W_{40} = W_{32}\, W_8$.

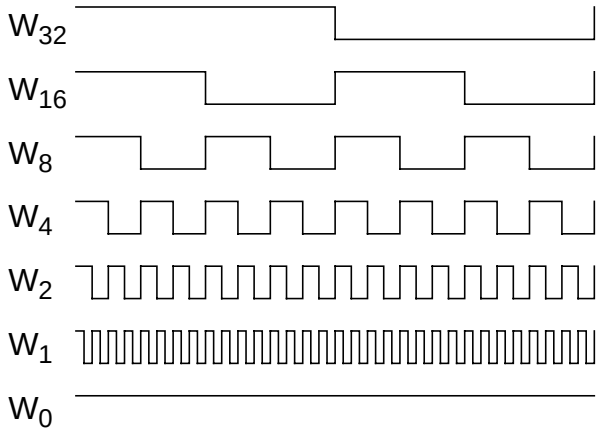

Fig. 3. Hadamard basis functions used to form Walsh codes.

The product of two Walsh codes, W_i and W_j, can be understood by noting an important property: $W_i\ W_i = W_0 = 1$. An application of this property is $W_{14}\ W_{40} = W_{32}\ W_4\ W_2 = W_{38}$. If the indices 14 and 40 are rewritten in binary form (001110 and 101100), it can be seen that the resultant index 38 (100110) is obtained using an exclusive-or operation. In general, the product of two Walsh codes, W_i and W_j, produces a new Walsh code, which for convenience, is written as $W_i\ W_j = W_{i\oplus j}$. This Walsh code intermodulation can be extended to higher-order products using the exclusive-or of the corresponding set of Walsh codes: $W_i\ W_j ... W_z = W_{i\oplus j\oplus ...\oplus z}$.

The active basis functions for an intermodulation code $W_{i\oplus j}$ are determined by the basis functions active within W_i and W_j. Let the states of the basis function B_k for Walsh codes W_i and W_j be denoted by $c_k(i)$ and $c_k(j)$, respectively. For the case of $W_{i\oplus j}$, (27) becomes

$$i \oplus j = [c_1(i) \oplus c_1(j)] + \ldots + 32 \cdot [c_{32}(i) \oplus c_{32}(j)] \tag{29}$$

where $\oplus$ is the exclusive-or operator.

Returning to the $W_{14}W_{40}$ example, we see that the basis functions B_1 and B_{16} are inactive in both W_{14} and W_{40}, as well as in the resulting product W_{38}. In general, if both $c_k(i)$ and $c_k(j)$ are zero, the basis B_k for $W_{i\oplus j}$ is inactive ($c_k(i\oplus j) = 0$). This means that Hadamard basis functions that are “missing” from the initial Walsh code set (B_1 and B_{16} in the $W_{14}W_{40}$ example) are not activated by intermodulation. Thus, intermodulation does not increase the dimension of the Walsh

code set. It should be noted that this “constant dimension” property holds for higher-order inter-modulation products ($W_iW_jW_k$...), in addition to second-order products. The effect of missing Hadamard basis functions on the power variance of the CDMA waveform is discussed in Section 4.0.

The intermodulation properties of Walsh codes, mentioned above, are used in Section 3.2 to derive fourth-order statistical properties of the CDMA waveform.

3.2 Sampled CDMA signal

Consider a CDMA waveform formed by 64 Walsh codes with various scale terms, ρ_i. Because of the PN sequence, the auto-correlation of the sampled CDMA signal is

$$E[x(nT)\cdot x^*(mT)] = \begin{cases} \sigma^2 & \text{for } n = m \\ 0 & \text{otherwise} \end{cases}, \tag{30}$$

where the average power is

$$\sigma^2 = \sum_{i=0}^{63} \rho_i^2 . \tag{31}$$

The important fourth-order statistics of the sampled CDMA signal, x(nT), are as follows:

$$E[|x(nT)|^4] = 3\sigma^4 - 2\cdot\left[\sum_{i=0}^{63}\rho_i^4\right] + Q_C(n,n) \tag{32}$$

$$E[|x(nT)|^2\cdot|x(mT)|^2] = \sigma^4 + Q_R(n,m) + Q_C(n,m) \tag{33}$$

where

$$Q_R(n,m) = 2\cdot\sum_{i=0}^{63}\left[\sum_{\substack{j=0 \\ i\neq j}}^{63}\rho_i^2\rho_j^2\cdot R_{i\oplus j}(n,m)\right] \tag{34}$$

$$Q_C(n, m) = \sum_{S(i, j, k, l)} \rho_i \rho_j \rho_k \rho_l \cdot C_{ijkl}(n, m) \tag{35}$$

$$R_i(n, m) = E[W_i(nT) W_i(mT) \cdot d_i(nT) d_i(mT)] \tag{36}$$

$$C_{ijkl}(n, m) = E[W_{i \oplus j}(nT) W_{k \oplus l}(mT) d_{ij}(nT) d_{kl}(mT)] . \tag{37}$$

The notation S(i, j, k, l) within (35) indicates that the summation includes all C_{ijkl} except for the following terms: (i = j = k = l); (i = j and k = l); and (i = l and j = k). The term d_{ij} represents the product d_i d_j.

If the data symbols (d_i) for a given Walsh code, W_i, are uncorrelated in time, we get

$$R_i(n, m) = \begin{cases} W_i(nT) \cdot W_i(mT) & \text{for } 0 \le n, m \le 63 \\ 0 & \text{otherwise} \end{cases} . \tag{38}$$

If the data sequences, d_1... d_{63}, are uncorrelated in time and across Walsh code channels, then $Q_c(n,m) = 0$. In the following, it is assumed that the data sequences are uncorrelated so that $Q_c = 0$.

The effect of data correlation, which is beyond the scope of this paper, is described in [2].

3.3 Bandlimited CDMA Waveform

In the following, the fourth-order moment of the bandlimited CDMA waveform is derived relative to that of the bandlimited QPSK-modulated random waveform.

Let us assume that the second- and fourth-order moments are the same for both the sampled Walsh-coded and sampled random signals: that is,

$$E[x_{AM}{}^2(nT)] = E[y^2(nT)] \tag{39}$$

$$E[x_{AM}{}^4(nT)] = E[y^4(nT)] . \tag{40}$$

Despite having the same statistics at the sample instants (nT), the bandlimited CDMA and random waveforms will differ at times between samples. In the following, the difference will be investigated.

The bandlimited CDMA waveform is cyclostationary over a data symbol interval [0,64T], which implies that the fourth-order moment is dependent on the fractional portion of t/64T. To remove the dependence on t, averaging over the data symbol interval [0, 64T] is used (see cyclostationary processes in [9]):

$$E[|x|^4] = \frac{1}{64T} \cdot \int_0^{64T} E[|x_h(t)|^4] dt \,. \tag{41}$$

where it is assumed that t is within the interval [0, 64T].

Let us assume that the data sequences are uncorrelated (so that $Q_C = 0$). We can write (41) with respect to the bandlimited QPSK-modulated random waveform:

$$E[|x|^4] = E[|y_{iq}|^4] + \frac{2}{3} \cdot \left[\frac{1}{64T} \cdot \int_0^{64T} q_0(t) dt \right] \tag{42}$$

where

$$q_0(t) = 3 \cdot \sum_n h^2(t - nT) \sum_{\substack{m \\ m \neq n}} Q_R(n, m) \cdot h^2(t - mT) \,. \tag{43}$$

The term $q_o(t)$ in (43) accounts for the effects of bandlimiting, but not QPSK-modulation. To compensate for the effects of QPSK-modulation, a 2/3 scale factor is included in (42) (recall Section 2.0 and (22)).

Let us introduce a new measure, referred to as the “relative power variance”. It is the difference in the power variances for the bandlimited CDMA and random signals, under the assumption that (39) and (40) are valid, and is denoted by $\Delta(\nu_e^2)$:

$$\Delta(\nu_e^2) = \frac{E[|x|^4] - E[|y_{iq}|^4]}{\sigma^4} = \frac{\sum_i \alpha_i \gamma_i}{\sigma^4} \,. \tag{44}$$

where γ_i is the contribution of W_i (see below to (45)) and α_i is the corresponding weight which is dependent on the channel power and the intermodulation (see equations (34) and (35)).

Since it is assumed that the data ensembles are uncorrelated, so that $Q_C = 0$, we have

$$\gamma_i = \sum_n \left[\sum_m W_i(nT) \cdot W_i(mT) \cdot \bar{\eta}_o(nT, mT) \right] \tag{45}$$

where

$$\bar{\eta}_o(nT, mT) = \begin{cases} \bar{\eta}(nT, mT) & \text{for } n \neq m \\ 0 & \text{for } n = m \end{cases}, \tag{46}$$

and

$$\bar{\eta}(nT, mT) = 3 \cdot \int_0^{64T} \left[\frac{h^2(t - nT) \cdot h^2(t - mT)}{64T} \right] dt \,. \tag{47}$$

Note that $\bar{\eta}$ is a function of the wave-shaping filter h(t), and is represented by a matrix with a band-like structure where the significant values are found near the diagonal. Also note that the diagonal elements of the matrix $\bar{\eta}_o$ are zero.

Figure 4 shows γ_i as a function of the number of zero-crossings in the Walsh code W_i, denoted by N_{zero}. It can be seen that there is a monotonic decrease in γ_i as the number of zero crossings increases. A list of the number of zero crossings and the corresponding Walsh code appears in Table 1. The corresponding value of α_i depends on the power levels of the Walsh codes involved in creating the intermodulation code W_i:

$$\alpha_i = 2 \cdot \left[\sum_{j=0}^{63} \rho_j^2 \cdot (\rho_{i \oplus j})^2 \right]. \tag{48}$$

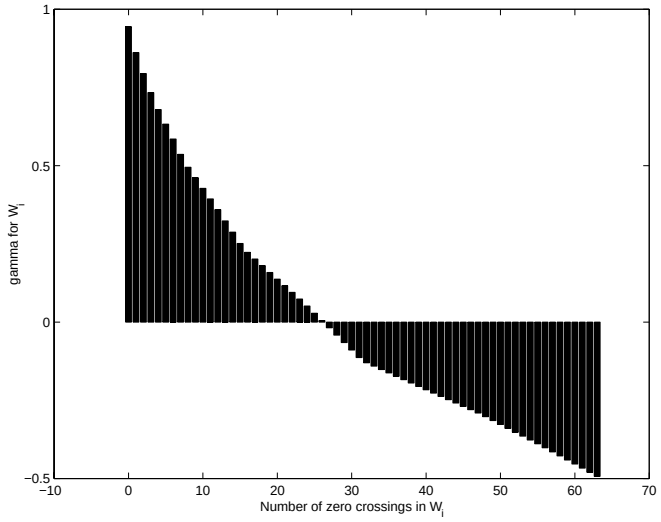

Fig. 4. The contribution of W_i to the relative power variance, γ_i of (44), as a function of the number of zero-crossings (N_{zero}) in W_i. The transition from positive to negative γ_i occurs between N_{zero} = 26 and 27. This graph shows that intermodulation power falling on Walsh codes with few (many) zero crossings increases (decreases) the power variance of the bandlimited CDMA signal. The pairings of N_{zero} with W_i are listed in Table 1.

Table 1. The number of zero crossings, N_{zero}, over the interval [0,64T] and the corresponding Walsh code (W_i).

N_{zero}	W_i	N_{zero}	W_i	N_{zero}	W_i	N_{zero}	W_i
0	W_0	16	W_6	32	W_3	48	W_5
1	W_{32}	17	W_{38}	33	W_{35}	49	W_{37}
2	W_{48}	18	W_{54}	34	W_{51}	50	W_{53}
3	W_{16}	19	W_{22}	35	W_{19}	51	W_{21}
4	W_{24}	20	W_{30}	36	W_{27}	52	W_{29}
5	W_{56}	21	W_{62}	37	W_{59}	53	W_{61}
6	W_{40}	22	W_{46}	38	W_{43}	54	W_{45}
7	W_8	23	W_{14}	39	W_{11}	55	W_{13}
8	W_{12}	24	W_{10}	40	W_{15}	56	W_9
9	W_{44}	25	W_{42}	41	W_{47}	57	W_{41}
10	W_{60}	26	W_{58}	42	W_{63}	58	W_{57}
11	W_{28}	27	W_{26}	43	W_{31}	59	W_{25}
12	W_{20}	28	W_{18}	44	W_{23}	60	W_{17}
13	W_{52}	29	W_{50}	45	W_{55}	61	W_{49}
14	W_{36}	30	W_{34}	46	W_{39}	62	W_{33}
15	W_4	31	W_2	47	W_7	63	W_1

From the viewpoint of limiting the power variance, one should select the set of active Walsh codes that minimizes (44). That is, a desirable situation would be to have large (small) values for any α_i associated with Walsh codes W_i comprising many (few) zero crossings, N_{zero}. However, it is important to remember that the W_i associated with α_i is an intermodulation Walsh code ($W_i = W_j W_k$). For example, if the active Walsh codes were chosen to be W_0, W_1, and W_{32}, the intermodulation Walsh codes to be included within (44) would be W_0, W_1, W_{32}, and W_{33}.

The key observation is that it is not possible to select only Walsh codes with high N_{zero} because intermodulation will produce new codes with low N_{zero}. In the above-mentioned example, W_1 and W_{33} have the maximum number of zero crossings (N_{zero} = 63 and 62, respectively), whereas W_0 and W_{32} have the minimum (N_{zero} = 0 and 1, respectively). The selection of Walsh codes for the purpose of reducing the power variance is discussed in Section 4.0 and Section 5.0.

4.0 Reducing the Power Variance of a CDMA waveform

In the following, the effect of missing basis functions (see Section 3.1) on the power variance is presented. In particular, it is shown that the presence of Hadamard basis function B_1 is essential in reducing the power variance below that of a bandlimited random waveform. Note that within this section, the estimates of the relative power variance are based on assumptions that the data sequences are not correlated (so that $Q_c = 0$, see (35) and (44)) and that h(t) is used as the wave-shaping filter.

To illustrate the effect of a missing basis function B_k on the power variance, consider a set of 32 active Walsh codes with equal power. If the set comprises the even-numbered Walsh codes (W_0, W_2, W_4,..., W_{62}), the basis function B_1 is missing. The value of the relative power variance, in this case, is +0.37. In contrast, if the set is missing B_{16} (that is, the set contains W_0,..., W_{15}; W_{32},..., W_{47}), then the value of the relative power variance is -0.04. Recall from (44), that a positive (negative) value indicates that the power variance is greater (less) than that of a bandlimited QPSK-modulated random waveform.

To better understand why there is such a large disparity in the power variances for the cases where B_1 and B_{16} are missing, let us refer back to Table 1 and Figure 4. Consider first the case when B_1 is missing. Recalling the "constant dimension" property from Section 3.1, all higher-order intermodulations (W_i W_j...) are restricted to Walsh codes that are also missing B_1. It can be seen from Table 1 that this set includes all intermodulation Walsh codes with $N_{zero} < 32$ (codes W_0, W_2, W_4,..., W_{62}) and, from Figure 4, that the intermodulation power will be concentrated in the 32 highest γ_i's. As a result, the power variance will be large. In contrast, the set that is missing B_{16} will find it's intermodulation power on 32 Walsh codes whose respective γ_i's are distributed over the full range shown in Figure 4. As a result, the relative power variance is close to zero.

As a further illustration, consider a set of 16 equal power Walsh codes that is missing two basis functions. If B_1 and B_2 are missing, the relative power variance is +0.65. In contrast, if B_8 and B_{16} are missing, the relative power variance is -0.07. Returning again to Table 1 and Figure 4, we see that when both B_1 and B_2 are missing, the intermodulation power is restricted to the Walsh codes where $N_{zero} < 16$. This corresponds to the 16 highest γ_i's, which leads to an even higher power variance. In contrast, for the set that is missing both B_8 and B_{16}, the available γ_i's are still distributed over the full range, which leads to a relative power variance that is near zero.

Now consider code sets with nine channels, which will be used in Section 5.0. Nine-channel code sets can be formed that are missing either zero, one, or two basis functions. The process of intermodulation will tend to distribute power amongst Walsh codes that span the active basis functions. As long as B_1 and, to a lesser extent, B_2 are active basis functions, a wide range of γ_i's will be available, and the relative power variance will be small (assuming $Q_c = 0$). Thus the specific Walsh codes in-use are less important than which basis functions are active.

In summary, minimizing the power variance requires that the basis function B_1 be active within the Walsh code set. Assigning W_1 as the paging channel is a recommended means of making B_1 active.

5.0 Results

In this section, examples are provided to illustrate the effects of Walsh code selection on the power variance of the CDMA signal. Section 5.1 compares measured and baseline values for the pilot-only waveform, illustrating the effect of bandlimiting on the power variance. In Section 5.2, the measured power variances for six different nine-channel CDMA signals are compared to illustrate the dependence of the power variance on Walsh code selection. Measured and predicted power variances are also compared to verify the accuracy of the statistical models presented in Section 2.0 and Section 3.0.

For the examples shown in this section, the measured power variances are obtained from computer simulations where the bandlimited signals, $x_h(t)$ and $y_h(t)$, are over-sampled by a factor of four compared to the chip rate. The IS-95 baseband filter [8] is used for the wave-shaping instead of h(t) described by (5). Note that the IS-95 baseband filter has a blending factor (to be used in (22)) of

$$\bar{f}_{is95} = \frac{4 \cdot \sum_n h_{is95}{}^4(t - nT)}{\left[\sum_n h_{is95}{}^2(t - nT)\right]^2} = 0.733 \tag{49}$$

instead of $\bar{f} = 0.667$ for h(t) (note that the factor of 4 in the numerator of (49) is due to the four times over-sampling). The phase equalizing filter [8] specified in IS-95 is not included.

The nine-channel forward-link CDMA signal is considered by many to be a “standard” test waveform [7]. The Walsh code set contains pilot, paging, sync, and six traffic channels. Within this paper, the pilot, paging, and sync channels are assigned relative scale factors (ρ_i/ρ_0) of 1.0, 0.9, and 0.45, respectively. The traffic channels each have a relative scale factor of 0.8. The power variance of the sampled Walsh-coded signal ($E[x_{AM}{}^4(nT)]/\sigma^4$) is 2.76. However, the power variance of the CDMA waveform, after QPSK-modulation and bandlimiting, varies depending on the active Walsh codes.

Uncorrelated data vectors, to be used in the nine-channel examples, are formed as follows. For the case of nine active channels, the data vector has 256 distinct values (2^8 because only 8 of the

data bits can change signs; the pilot data is constant, $d_0 = 1$). Since there are 512 symbol intervals within a PN sequence period, each value is used twice. The time positions of the 512 data vectors over the PN sequence period are selected using a random permutation.

Two baseline power variances, based on the bandlimited random waveforms described in Section 2.0, appear in Table 2. They are generated from random sequences having sampled power variances ($E[y^4(nT)]/\sigma^4$) of 1.0 and 2.76, which then are QPSK-modulated and bandlimited by the IS-95 filter. Using the prediction model described by (22), and $\bar{f}_{is95} = 0.733$, the predicted (baseline) power variances are 1.27 and 2.56, respectively. These predictions are used as baselines for the pilot-only signal and the nine-channel CDMA signal, respectively.

Table 2. Sampled and baseline power variances. Baseline waveform is QPSK-modulated and bandlimited using the IS-95 filter.

Baseline Signal	Sampled Power Variance	Baseline Power Variance
Pilot-only Baseline: $E[y^4(nT)] = \sigma^4$	1.0	1.27
9-Channel Baseline: $E[y^4(nT)\} = 2.76\ \sigma^4$	2.76	2.56

5.1 CDMA Signal: Pilot-only

The measured power variance of the pilot-only waveform is 1.28. This confirms that bandlimiting increases the power variance from unity towards the Rayleigh distribution's power variance of 2. The measured value compares well with the baseline prediction of 1.27: an error of only 0.7 percent. This level of accuracy should be viewed as a validation of the prediction model described by (22) and (49).

5.2 CDMA Signals: Nine Channels

Six Walsh code sets forming nine-channel CDMA signals are presented in Table 3. Within each set, the pilot and sync channels are W_0 and W_{32}, respectively. The paging channel is W_1 for

all sets except for Set 1, where it is W_4. The remaining Walsh codes are traffic channels. Within Set 1 and Set 4, the paging and traffic channels are selected such that two Hadamard basis functions are missing: B_1 and B_2 for Set 1; and B_8 and B_{16} for Set 4. Set 5 is missing B_{16}. The remaining sets have six active basis functions. Note that Set 5 is the default nine-channel forward-link code assignment in [7].

Table 3. Measured and predicted power variances for various nine-channel Walsh code sets.

Set	Active Walsh codes ($\rho_i > 0$)	Measured Power Variance	Predicted Power Variance
Set 1	W_0, W_4, W_{32}, W_8, W_{16}, W_{24}, W_{40}, W_{48}, W_{56}	3.21	3.20
Set 2	W_0, W_1, W_{32}, W_{58}, W_{59}, W_{60}, W_{61}, W_{62}, W_{63}	2.46	2.45
Set 3	W_0, W_1, W_{32}, W_2, W_4, W_8, W_{15}, W_{16}, W_{51}	2.56	2.55
Set 4	W_0, W_1, W_{32}, W_2, W_3, W_4, W_5, W_6, W_7	2.44	2.44
Set 5	W_0, W_1, W_{32}, W_8, W_9, W_{10}, W_{11}, W_{12}, W_{13}	2.49	2.48
Set 6	W_0, W_1, W_{32}, W_2, W_4, W_8, W_{13}, W_{16}, W_{18}	2.60	2.59

The range of measured power variances in Table 3 indicates that a CDMA signal formed from uncorrelated data is not the same, in general, as the baseline example of a random sequence that is QPSK-modulated and bandlimited. The power variance for the latter is 2.56, whereas the former has measured values ranging from 2.44 to 3.21. The dependence of the power variance on the code set indicates the importance of Walsh code selection. Note that the power variance for Set 1 is significantly higher than any of the other five code sets. This confirms that it is beneficial to specify Walsh code W_1 as the paging channel.

The high power variance for Set 1 is a consequence of an unfavorable concentration of intermodulation power (α_i's of (48), excluding α_0) amongst codes with low N_{zero}'s. Set 1 has 72.9 and 100 percent of the total intermodulation power ($\Sigma_{i=1,63}\ \alpha_i$) on codes with $0 < N_{zero} < 8$ and $0 <$

$N_{zero} < 16$, respectively. The lowest power variances are Set 2 and Set 4; for each code set, only 1.4 and 15.4 percent of the total intermodulation power appears on codes with $0 < N_{zero} < 8$ and $0 < N_{zero} < 16$, respectively. The remaining three code sets (Set 3, Set 5, and Set 6) have balanced distributions of α_i as a function of N_{zero} (27.5, 23.9, and 31 percent of $\Sigma\alpha_i$, respectively, for $0 < N_{zero} < 16$). As a result, the power variances are close to the baseline value. The unfavorable distribution of α_i's for Set 1 is consistent with the predictions discussed in Section 4.0 (Set 1 is missing basis functions B_1 and B_2).

The predicted power variances in Table 3 are obtained as follows. The relative power variances are predicted based on (42) and (44), and the filter h(t). Since the IS-95 filter has a higher blending factor than h(t), the predicted relative power variances are decreased by a factor of 0.667/0.733 (for the six sets, the relative power variances are reduced from [0.71, -0.12, -0.01, -0.13, -0.09, +0.03] to [0.64, -0.11, -0.01, -0.12, -0.08, 0.03]). These adjusted values are then added to the baseline power variance of 2.56. The predicted values are accurate; the typical error is -0.01, or 0.4 percent of the baseline power variance. Note that even if the incorrect blending factor of 2/3 is used, the relative power variance measure provides an accurate ranking for the purpose of selecting the Walsh code set that minimizes the power variance.

One final observation is that multiplying a given code set by a Walsh code W_k has no effect on the power variance (see (48)). For example, if Set 2 is multiplied by W_{63}, we obtain a new set (W_{63}, W_{62}, W_{31}, W_5, W_4, W_3, W_2, W_1, W_0). Since Set 4 and the new set (W_{63} * Set 2) share six of nine Walsh codes, it should not be surprising that Set 4 and Set 2 have similar power variances. In addition. this observation allows missing basis functions properties to be extended to include code sets with "common factor" basis functions. Although the requirement for a pilot channel W_0 prevents the use of sets for which all codes contain a common factor W_k (where k not equal to zero), this observation allows one to predict the influence of a code subset possessing common basis functions.

6.0 Conclusion

The power variance of a bandlimited forward-link CDMA waveform is defined in terms of selected Walsh codes. It is shown that the power variance is largest when the Hadamard basis functions B_1 and B_2 are missing from the Walsh code set.

Statistical models used to predict the power variance for bandlimited CDMA waveforms and QPSK-modulated random waveforms are shown to be accurate. This information allows potential code sets to be ranked in terms of their power variance, without the need for simulation or testing.

7.0 References

[1] R. A. Birgenheier, "Overview of code-domain power, timing, and phase measurements," *HP Journal*, pp. 73-93, Feb. 1996.

[2] R. N. Braithwaite, "Nonlinear amplification of CDMA waveforms: an analysis of power amplifier gain errors and spectral regrowth," in *Proc. IEEE Veh. Tech. Conf.*, Ottawa, Canada, May 1998, pp. 2160-2166.

[3] J. K. Cavers, "The effect of data modulation format on intermodulation power in nonlinear amplifiers," in *Proc. IEEE Veh. Tech. Conf.*, Stockholm, Sweden, June 1994, pp. 489-492.

[4] S.-W. Chen. W. Panton, and R. Gilmore, "Effects of nonlinear distortion on CDMA communication systems," *IEEE Trans. Microwave Theory and Techniques*, vol. 44, no. 12, pt. 2, pp. 2743-2750, Dec. 1996.

[5] A. O. Fapojuwo, "Radio capacity of direct sequence code division multiple access mobile radio systems," *IEE Proceedings-I*, vol. 140, no. 5, pp. 402-408, Oct. 1993.

[6] K. S. Gilhousen, I. Jacobs, R. Padovani, A. Viterbi, L. Weaver Jr., and C. Wheatley III, "On the capacity of a cellular CDMA system," *IEEE Trans. Vehicular Technology*, vol. 40, no. 2, 303-312, May 1991.

[7] “HP ESG-D series signal generators, dual arbitrary waveform generator and multi-channel CDMA personality,” User’s and Programming Guide, HP Part No. E4400-90148, March 1998.

[8] “Mobile station-base station compatibility standard for dual-mode wideband spread spectrum cellular system,” TIA/EIA IS-95-A, Arlington VA, May 1995.

[9] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*. McGraw Hill, 1984.

[10] R. L. Pickholtz, L. B. Milstein, and D. L. Schilling, “Spread spectrum for mobile communications,” *IEEE Trans. Vehicular Technology*, vol. 40, no. 2, pp. 313-321, May 1991.

[11] R. L. Pickholtz, D. L. Schilling, and L. B. Milstein, “Theory of spread-spectrum communications---A tutorial,” *IEEE Trans. Communications*, vol. COM-30, no. 5, pp. 855-884, May 1982.

[12] M. Schetzen, *The Volterra and Wiener Theories of Nonlinear Systems*. Krieger Publishing, 1989.

[13] S. P. Stapleton, G. S. Kandola, and J. K. Cavers, “Simulation and analysis of an adaptive predistorter utilizing a complex spectral convolution,” *IEEE Trans. Vehicular Technology,* vol. 41, no. 4, pp. 387-394, Nov. 1992.

[14] A. J. Viterbi, *CDMA: Principles of Spread Spectrum Communication*. Addison-Wesley, 1995.

[15] T. Wada, T. Yamazato, M. Katayama, and A. Ogawa, “A constant amplitude coding for orthogonal multi-code CDMA systems,” *IEICE Trans. Fundamentals*, vol. E80-A, no. 12, pp. 2477-2484, Dec. 1997.

[16] D. P. Whipple, “North American cellular CDMA,” *HP Journal*, vol. 44, no. 6, pp. 90-97, Dec. 1993.