Differential Detection of MSK in a Dispersive Channel and Correlated Noise

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Abstract

We consider the noise properties at the output of a differential detector under dispersive channel conditions when MSK is used as the modulation scheme. Using the autocorrelation properties of the noise term and the method of Lagrange multipliers, we present previously unreported bounds for both the noise variance and the complementary autocorrelation function of the noise term. We show that both the noise variance and the complementary autocorrelation function can increase or decrease under particular channel conditions. Based on these findings, we are able to bound the signal-to-noise ratio.

1 Introduction

The benefits of differential detection are widely acknowleged and understood. It provides an inherent robustness to phase and frequency offsets whilst enabling a low complexity solution. However, a notorious problem with differential detection is the significant increase in error rate which occurs when the channel is time dispersive [1]. This problem arises from the nonlinear behaviour of the detection process, which not only produces cross-product terms in the output signal [1, 2, 3], but also a highly correlated noise term [4, 5]. For the additive white Gaussian noise (AWGN) channel, it is known that the maximum noise variance at the output of a differential detector is twice as large as the coherent case [6], however, when the channel is time dispersive the variance is unknown.

In this paper we present analytical results for the noise term at the output of a differential detector under time dispersive channel conditions. Using the autocorrelation properties of the noise term and the method of Lagrange multipliers we establish the first upper and lower bounds (to our knowledge) for both the noise variance at the output of the differential detector and the correlation between adjacent noise terms. In addition, we also derive the form of the channel which attains these bounds. Finally, using these results, we determine an upper bound on the signal-to-noise ratio (SNR). The implications of these results in the context of detection are discussed briefly at the end of the paper.

2 Background

As a means of establishing some notation, we give a brief description of the operation of a minimum shift keying (MSK) system employing differential detection. An MSK signal has the following baseband representation

$$s(t) = \sqrt{\frac{2E_b}{T}} \exp\left[j\theta(t)\right] \tag{1}$$

where E_b is the bit energy, T is the symbol duration, $\theta(t) = \sum_i \alpha_i \pi h \int_{-\infty}^{t-iT} g(u)du$ and $j = \sqrt{-1}$. The modulation index h is equal to 0.5, the information symbols $\alpha_i \in \{-1, +1\}$ are mutually independent, and $g(t)$ is a rectangular pulse of unit area. Using the representation described in [7], the MSK signal can be represented as a linear superposition of amplitude modulated pulses, such that we can write (1) as $s(t) = \sum_{i=-\infty}^{+\infty} x_i p(t - iT)$, where

$$x_i = j\alpha_i x_{i-1}, \tag{2}$$

$x_i \in \{-1, +1, -j, +j\}$ and $p(.)$ is a suitable pulse shaping function given in [7]. Now suppose $s(t)$ is transmitted through a dispersive channel, $h_c(t)$, and is corrupted by AWGN. We write $y(t) = \sum_{i=-\infty}^{+\infty} x_i h(t - iT) + n(t)$, where the composite channel impulse response is $h(t) = \int_{-\infty}^{+\infty} p(u) h_c(t-u) du$ and $n(t)$ is complex valued AWGN with zero mean and noise power spectral density $2N_o$. If $t_k = kT + t_s$ are the sampling instants, the sampled input to the receiver is $y_k = y(kT + t_s)$, where t_s is the optimum sampling instant and k is an integer. Assuming the channel has finite duration, for convenience we define the discrete time equivalent channel as

$$y_k = b_k + n_k \tag{3}$$

where $b_k = \sum_{i=0}^{L-1} x_{k-i} h_i$. Here h_i are the complex coefficients of the channel impulse response, under the constraint of unit energy $\sum_{i=0}^{L-1} |h_i|^2$, and the variance of the noise term samples, n_k, is σ^2. We consider the complex 1-bit differential detector [8], which multiplies the current received signal sample y_k with the conjugate of the previous sample yielding

$$r_k = y_k y_{k-1}^* \tag{4}$$

Substituting (3) into (4) and rearranging gives

$$r_k = \sum_{i=0}^{L-1} j\alpha_{k-i}|h_i|^2 + \sum_{\substack{n=0 \\ n\neq i}}^{L-1} \sum_{\substack{i=0 \\ i\neq n}}^{L-1} x_{k-i} x_{k-i-1}^* h_i h_n^* + z_k + n_k n_{k-1}^* \tag{5}$$

where

$$z_k = n_k b_{k-1}^* + n_{k-1}^* b_k \tag{6}$$

We note that (5) consists of four terms: a linear ISI part which is purely imaginary and contains the wanted symbol α_k, a complex nonlinear ISI part, a correlated noise term z_k which is a function of both the channel impulse response h and the symbols $(x_{k-1}^*, .., x_{k-L}^*), (x_k, .., x_{k-L+1})$, and finally a weak noise term $n_k n_{k-1}^*$. Since the n_k noise samples are i.i.d., we make the usual approximation, that is, the weak noise term $n_k n_{k-1}^*$ will be typically insignificant [6]. We neglect this term throughout the remainder of the paper.

3 Correlated Noise

Consider the autocorrelation function (ACF) of z_k, $R_z(u)$, and the complementary autocorrelation function (CACF) $\tilde{R}_z(u)$. Noting that for complex Gaussian noise we have the definitions [9], $E(n_k^2) = 0$ and $E(|n_k|^2) = \sigma^2$, we can write

$$R_z(u) = E(z_k z_{k+u}^*) = \begin{cases} \sigma^2(|b_{k-1}^*|^2 + |b_k|^2) & u = 0 \\ 0 & u \geq 1 \end{cases} \tag{7}$$

and

$$\tilde{R}_z(u) = E(z_k z_{k+u}) = \begin{cases} 0 & u = 0 \\ \sigma^2(b_{k-1}^* b_{k+1}) & u = 1 \\ 0 & u > 1 \end{cases} \tag{8}$$

Since $E(n_k^2) = 0$, the ACF will be zero for all $u > 0$, and the CACF will only be nonzero at shifts of $u = 1$. These unusual properties which exist in the structure of z_k, arise from the conjugation in the delayed complex noise term, n_{k-1}^*. If we consider the CACF for N noise terms, it can be shown that

$$E(z_k z_{k+1} z_{k+2} z_{k+3} \cdots z_{k+N}) = \begin{cases} E(z_k z_{k+1})E(z_{k+2} z_{k+3}) \cdots E(z_{k+N-2} z_{k+N-1}) & N \text{ even} \\ 0 & N \text{ odd} \end{cases} \tag{9}$$

This factoring means that an even product of noise terms breaks up naturally into the adjacent pairs $z_k z_{k+1}$, and any higher order correlations can always be calculated in terms of pairwise correlations, and as such we only ever need to consider pairs of noise terms. In addition, it reveals the fact that on a pairwise basis, the noise terms are uncorrelated. However, this does not mean that there is no correlation between z_{k+1} and z_{k+2}, since $E(z_{k+1} z_{k+2}) = \sigma^2 b_k^* b_{k+2}$ as expected, and $E(z_{k+1} z_{k+2} z_{k+3} z_{k+4}) = E(z_{k+1} z_{k+2})E(z_{k+3} z_{k+4})$. It does mean that the factoring into pairs merely depends on the noise terms under consideration (even number). Furthermore, it is obvious that when N takes on odd values we have zero correlation since $E(z_k z_{k+1} z_{k+2}) = \sigma^2 b_{k-1}^* b_{k+1} E(z_{k+2})$, where $E(z_{k+2}) = 0$.

3.1 Non-dispersive AWGN Channel

We investigate the correlation properties for the non-dispersive AWGN channel. If $L = h_0 = 1$ then $b_k = x_k$, and $R_z(u)$ and $\tilde{R}_z(u)$ are both real-valued functions $\forall u$. If σ_z^2 is the variance of the noise term z_k, then $\sigma_z^2 = R_z(0) = 2\sigma^2$, which is invariant to a shift in k. However, $\tilde{R}_z(1)$ is dependent on the symbols $(x_{k-1}^* x_{k+1})$. If we use the relationship of (2), we can write

$$\tilde{R}_z(1) = -\sigma^2 \alpha_k \alpha_{k+1} \tag{10}$$

where $\tilde{R}_z(1) \in \pm\sigma^2$. In (10), an intimate relationship is revealed between pairs of information symbols, which gives an underlying connection with all previously transmitted symbols. Clearly, $\tilde{R}_z(u)$ is not invariant to a shift in k, and therefore, we can conclude that z_k is not strictly stationary [9].

3.2 Dispersive Channel

We now consider the noise at the output of the differential detector when the channel is dispersive. Unlike the AWGN channel, σ_z^2, is not invariant to a shift in k, and is dependent on both the channel impulse response and the symbols x_k within the range $\{k, k-1, ..., k-L\}$. We now present Theorem 1.

Theorem 1 *Under the constraint* $\sum_{i=0}^{L-1} |h_i|^2 = 1$ *and* $L > 1$, *the variance,* σ_z^2, *is upper bounded by* $2L\sigma^2$.

Proof: Let $\sigma_z^2 = \sigma^2(|b_{k-1}^*|^2 + |b_k|^2)$ be the variance at the output of the differential detector, let $x_{k-i}h_i = \gamma_i + j\beta_i$, and consider $|b_k|^2$, in the form

$$F(\gamma_i, \beta_i) = \left[\sum_{i=0}^{L-1} \gamma_i\right]^2 + \left[\sum_{i=0}^{L-1} \beta_i\right]^2 \tag{11}$$

We wish to maximise $F(\gamma_i, \beta_i)$ over possible γ_i, β_i under the constraint $\sum_{i=0}^{L-1} |h_i|^2 = 1$. We write

$$\sum_{i=0}^{L-1} \gamma_i^2 + \sum_{i=0}^{L-1} \beta_i^2 = 1 \tag{12}$$

$$G = \sum_{i=0}^{L-1} \gamma_i^2 + \sum_{i=0}^{L-1} \beta_i^2 - 1 \tag{13}$$

and use the method of Lagrange multipliers, with $H(\gamma_i, \beta_i) = F(\gamma_i, \beta_i) - \lambda G$ and

$$\frac{\partial H(\gamma_i, \beta_i)}{\partial \gamma_i} = 2\sum_{i=0}^{L-1} \gamma_i - 2\lambda\gamma_i = 0 \tag{14}$$

$$\frac{\partial H(\gamma_i, \beta_i)}{\partial \beta_i} = 2\sum_{i=0}^{L-1} \beta_i - 2\lambda\beta_i = 0 \tag{15}$$

¿From (14) and (15), $\sum_{i=0}^{L-1} \gamma_i = \lambda\gamma_i$, and $\sum_{i=0}^{L-1} \beta_i = \lambda\beta_i$. This implies that γ_i and β_i are independent of i. Letting $\gamma = \gamma_i$ and $\beta = \beta_i$ $\forall i$, then from (12)

$$L(\gamma^2 + \beta^2) = 1 \tag{16}$$

which from (11) gives

$$F(\gamma, \beta) = L^2(\gamma^2 + \beta^2) = L \tag{17}$$

Clearly then $\sigma_z^2 = \sigma^2(|b_{k-1}^*|^2 + |b_k|^2) \leq 2L\sigma^2$ and the upper bound for σ_z^2 can only be attained in the case that $|b_{k-1}^*|^2$ and $|b_k|^2$ are equal and maximise at L.

Now

$$b_k = \sum_{i=0}^{L-1} x_{k-i}h_i \equiv \sum_{i=0}^{L-1} (\gamma_i + j\beta_i) \tag{18}$$

and since from (2), $x_{k-1} = -j\alpha_k x_k$, we can write

$$b_{k-1} = \sum_{i=0}^{L-1} x_{k-i-1}h_i = \sum_{i=0}^{L-1} \alpha_{k-i}(\beta_i - j\gamma_i) \tag{19}$$

Inserting (18) and (19) into (7) yields

$$\sigma_z^2 = \sigma^2 \left[\left(\sum_{i=0}^{L-1} \gamma_i \right)^2 + \left(\sum_{i=0}^{L-1} \beta_i \right)^2 + \left(\sum_{i=0}^{L-1} \alpha_{k-i} \beta_i \right)^2 + \left(\sum_{i=0}^{L-1} \alpha_{k-i} \gamma_i \right)^2 \right] \tag{20}$$

Under the conditions $\gamma_i = \gamma$ and $\beta_i = \beta \ \forall i$ we get

$$\sigma_z^2 = \sigma^2 \left[L + \frac{1}{L} \left(\sum_{i=0}^{L-1} \alpha_{k-i} \right)^2 \right] \tag{21}$$

So we see that σ_z^2 attains the upper bound of $2\sigma^2 L$ exactly when $|\sum_{i=0}^{L-1} \alpha_{k-i}| = L$. This requires all α_{k-i} in the summation to have the same sign. □

We next consider the CACF for the dispersive channel. It is obvious that, like the AWGN channel, $\tilde{R}_z(u)$ is not invariant to a shift in k, and so z_k is not strictly stationary. We now present our second theorem.

Theorem 2 *Under the constraint* $\sum_{i=0}^{L-1} |h_i|^2 = 1$ *and* $L > 1$, $|\tilde{R}_z(1)|$ *is upper bounded by* $L\sigma^2$.

Proof It is clear that $|\tilde{R}_z(1)|^2 = (\sigma^2)^2 |b_{k-1}^*|^2 |b_{k+1}|^2$ has a maximum of $\sigma^4 L^2$ as $|b_{k-1}^*|^2$ and $|b_{k+1}|^2$ each have a maximum of L. However, $|\tilde{R}_z(1)|^2$ can only attain its maximum in cases where $|b_{k-1}^*|^2$ and $|b_{k+1}|^2$ are equal and maximise at L.

We write $x_{k-1-i} h_i = \delta_i + j\epsilon_i$. Clearly, $|b_{k-1}^*|^2$ maximises under the conditions $\delta_i = \delta$, $\epsilon_i = \epsilon$ and $\delta^2 + \epsilon^2 = \frac{1}{L}$ (as in Theorem 1).

Now

$$|\tilde{R}_z(1)|^2 = \sigma^4 \left[\left(\sum_{i=0}^{L-1} \alpha_{k+1-i} \alpha_{k-i} \delta_i \right)^2 + \left(\sum_{i=0}^{L-1} \alpha_{k+1-i} \alpha_{k-i} \epsilon_i \right)^2 \right] \left[\left(\sum_{i=0}^{L-1} \delta_i \right)^2 + \left(\sum_{i=0}^{L-1} \epsilon_i \right)^2 \right] \tag{22}$$

for the case $\delta_i = \delta$, $\epsilon_i = \epsilon$ and $\delta^2 + \epsilon^2 = \frac{1}{L}$ we get

$$|\tilde{R}_z(1)|^2 = \sigma^4 \left(\sum_{i=0}^{L-1} \alpha_{k+1-i} \alpha_{k-i} \right)^2 \tag{23}$$

It is clear that $|\tilde{R}_z(1)|^2$ can only attain its maximum by satisfying $|\sum_{i=0}^{L-1} \alpha_{k+1-i} \alpha_{k-i}| = L$. This requires all α_j in the summation to have the same sign, or for the signs to alternate. □

4 Worst Case Channel

We now present the channel that achieves the upper bound on σ_z^2 and $|\tilde{R}_z(1)|$. This channel is important, since it can be considered as the worst case channel as it always exhibits the highest noise variance. This will become clearer in the next section when we consider the SNR at the output of the differential detector.

Again we assume the conditions $\gamma_i = \gamma$ and $\beta_i = \beta \ \forall i$, this gives $x_{k-i} h_i = x_{k+1-i} h_{i-1}$ for $i = 1 \ldots L-1$. However: $x_{k+1-i} = j\alpha_{k+1-i} x_{k-i}$, which implies that $h_i = j\alpha_{k+1-i} h_{i-1}$. We have established already that $|\sum_{i=0}^{L-1} \alpha_{k-i}| = L$ is necessary to maximise σ_z^2, which suggests that there are two channels:

$$h_i = \begin{cases} jh_{i-1} & \text{for } i = 1 \ldots L-1 \quad (a) \\ -jh_{i-1} & \text{for } i = 1 \ldots L-1 \quad (b) \end{cases} \tag{24}$$

In each case h_0 satisfies $|h_0|^2 = \frac{1}{L}$, which implies that

$$h_0 = \frac{e^{j\phi}}{\sqrt{L}} \tag{25}$$

where ϕ is an arbitrary phase shift. We now show that σ_z^2 can be zero for the same channel impulse response that achieves the upper bound.

Consider the channel that attains the maximum of σ_z^2, but examine the conditions when σ_z^2 vanishes. Now $b_k = \sum_{i=0}^{L-1} x_{k-i}h_i$, so if b_k is to vanish by terms cancelling, then L has to be even. Consider odd values of i, then using channel (a) with $h_i = jh_{i-1}$ and $x_{k-i} = -j\alpha_{k-i+1}x_{k-i+1}$ we can write

$$h_i x_{k-i} = \alpha_{k-(i-1)} h_{i-1} x_{k-(i-1)}$$

So, if $h_i x_{k-i}$ is to cancel the term $h_{i-1}x_{k-(i-1)}$ then $\alpha_{k-(i-1)}$ has to be negative. Similarly, in b_{k-1}, if this is to vanish by cancellation then for odd i, $\alpha_{k-1-(i-1)} = -1$. Hence, for channel (a), the relevant string of $\alpha_k \ldots \alpha_{k-L+1}$ (dependent on the choice of k) all have to equal -1. For channel (b), when $h_i = -jh_{i-1}$, there is a sign reversal, so the relevant $\alpha_k \ldots \alpha_{k-L+1}$ have to equal +1. This will now be demonstrated using a specific example.

Example 1: If $L = 2$, the upper and lower bound for σ_z^2 and $|\tilde{R}_z(1)|$ is attained by the channel impulse response $h = \frac{1}{\sqrt{2}}(1, j)$. Using (7) and (8), we tabulate σ_z^2 and $|\tilde{R}_z(1)|$ normalised to σ^2, and we include their associated sequences for $k = 0$, see Table I. For convenience, we let $\phi = 0$. In comparison, for the non-dispersive AWGN channel, it is simple to show that $\sigma_z^2/\sigma^2 = 2$ and $|\tilde{R}_z(1)|/\sigma^2 = 1$ holds for all sequences. In this example, we note the maximum of σ_z^2/σ^2 for the dispersive channel is attained when $\alpha_0 = \alpha_{-1} = 1$ and the minimum is attained for $\alpha_0 = \alpha_{-1} = -1$. Similarly, the maximum of $|\tilde{R}_z(1)|/\sigma^2$ is attained when $\alpha_1 = \alpha_0 = \alpha_{-1} = 1$ or $\alpha_1 = -\alpha_0 = \alpha_{-1} = 1$. We note also, these sequences will invert for channel (b) $h_i = -jh_{i-1}$. Here we have used the simplest channel model to illustrate the bounds proved earlier. Obviously, under realistic channel conditions, for example, a Rayleigh fading exponentially decaying power delay profile, these values will vary (depending on the rms delay spread of the channel) within the upper and lower bounds.

		$h = \frac{1}{\sqrt{2}}(1, j)$	
$\alpha_1, \alpha_0, \alpha_{-1}$	x_1, x_0, x_{-1}, x_{-2}	σ_z^2/σ^2	$\lvert\tilde{R}_z(1)\rvert/\sigma^2$
$-1-1-1$	$+j-1-j+1$	0	0
$-1-1+1$	$-j+1+j+1$	2	0
$-1+1-1$	$-j+1-j+1$	2	0
$-1+1+1$	$+j-1+j+1$	4	0
$+1-1-1$	$-j-1-j+1$	0	0
$+1-1+1$	$+j+1+j+1$	2	2
$+1+1-1$	$+j+1-j+1$	2	0
$+1+1+1$	$-j-1+j+1$	4	2

TABLE I

Normalised σ_z^2 and $|\tilde{R}_z(1)|$ for the time dipsersive channel

5 Signal-to-Noise Ratio

Using our results from Theorem 1 and Theorem 2, we now consider the SNR at the output of the differential detector. Suppose $n_k = 0$, then from (4), the signal power is

$$s_k = r_k r_k^* = |b_k|^2 |b_{k-1}|^2 \tag{26}$$

Now the noise power is $E(|z_k|^2) = \sigma_z^2$, and so we write the SNR as

$$\text{SNR} = \frac{|b_k|^2 |b_{k-1}|^2}{\sigma^2 \left(|b_k|^2 + |b_{k-1}|^2\right)} \tag{27}$$

As shown in the proof of Theorem 2, it is clear that s_k maximises at L^2 as $|b_k|^2$ and $|b_{k-1}|^2$ each maximise at L. Futhermore, it is clear that s_k vanishes when σ_z^2 vanishes. As shown previously, for σ_z^2 to vanish requires $|b_k|^2$ and $|b_{k-1}|^2$ to both vanish. For s_k to vanish there is only a requirement that $|b_k|^2$ or $|b_{k-1}|^2$ vanish. For channel (a), this means that either $\alpha_{k-(i-1)} = -1$ or $\alpha_{k-i} = -1$ for odd i, and every other $\alpha_{k-i} = +1$. For channel (b), there is a sign reversal. We now present our third theorem

Theorem 3 *Under the constraint* $\sum_{i=0}^{L-1} |h_i|^2 = 1$, *the* SNR *is upper bounded by* $L/2\sigma^2$

Proof For the channel that attains the maximum of σ_z^2 there are sequences for which both $|b_k|^2$ and $|b_{k-1}|^2$ are not equal to their maximum value, however, both are non-zero. Let $|b_k|^2 = L - c_k$ and $|b_{k-1}|^2 = L - c_{k-1}$ where $0 < c_k, c_{k-1} < L$. From (27), we can write the SNR as

$$\text{SNR} = \frac{1}{\sigma^2} \frac{(L - c_k)(L - c_{k-1})}{2L - c_k - c_{k-1}} \tag{28}$$

which can be rewritten as

$$\text{SNR} = \frac{1}{\sigma^2} \left(\frac{L}{2} - \frac{1}{2} \left[\frac{c_k \left(L - c_{k-1}\right) + c_{k-1} \left(L - c_k\right)}{2L - c_k - c_{k-1}} \right] \right) \tag{29}$$

Since $0 < c_k, c_{k-1} < L$, it is clear that the term in square brackets in (29) is positive. As this is subtracted from $L/2$, this means that $L/2\sigma^2$ is the maximum SNR $\square$.

We now present a second example to illustrate this point.

Example 2: Let $L = 4$, so the channel (a) impulse response is $h = \frac{1}{\sqrt{4}}(1, j, -1, -j)$. Using (27), we tabulate σ^2.SNR, and we include the associated sequences for $k = 0$, see Table II. Again for convenience, we let $\phi = 0$, and for the non-dispersive AWGN channel, it is simple to show that $\sigma^2.\text{SNR} = 1/2$ holds for all sequences. In this example, we note the maximum of σ^2.SNR for the dispersive channel is attained when $\alpha_0 = \alpha_{-1} = \alpha_{-2} = \alpha_{-3} = 1$, the same sequence that maximises σ_z^2 for this channel. We note also, these sequences will invert for channel (b). Again we have used the simplest channel model to illustrate the variation in SNR.

We have used the symbol $0\dagger$ in Table II to indicate the situation when both s_k and σ_z^2 vanish. In these cases where $s_k = \sigma_z^2 = 0$, the ratio has been set to zero by virtue of the $n_k n_{k-1}^*$ term in (5) being small but finite. From (5), let $\tilde{z}_k = z_k + n_k n_{k-1}^*$, then $E(\tilde{z}_k \tilde{z}_k^*) = \sigma_z^2 + \sigma^4$, so the denominator of the SNR is in fact still finite even when $\sigma_z^2 = 0$.

We remark, the results of Table II suggest appropriate coding of signals can be used to increase the SNR.

		$h = \frac{1}{\sqrt{4}}(1, j, -1, -j)$
$\alpha_0, \alpha_{-1}, \alpha_{-2}, \alpha_{-3}$	$x_0, x_{-1}, x_{-2}, x_{-3}, x_{-4}$	σ^2.SNR
$-1-1-1-1$	$+1+j-1-j+1$	0†
$-1-1-1+1$	$-1+j-1-j+1$	0
$-1-1+1-1$	$-1-j-1-j+1$	0
$-1-1+1+1$	$+1-j-1-j+1$	0.5
$-1+1-1-1$	$-1-j+1-j+1$	0
$-1+1-1+1$	$+1-j+1-j+1$	0†
$-1+1+1-1$	$+1+j+1-j+1$	0.5
$-1+1+1+1$	$-1+j+1-j+1$	0.8
$+1-1-1-1$	$-1-j+1+j+1$	0
$+1-1-1+1$	$+1-j+1+j+1$	0.5
$+1-1+1-1$	$+1+j+1+j+1$	0†
$+1-1+1+1$	$-1+j+1+j+1$	0
$+1+1-1-1$	$+1+j-1+j+1$	0.5
$+1+1-1+1$	$-1+j-1+j+1$	0
$+1+1+1-1$	$-1-j-1+j+1$	0.8
$+1+1+1+1$	$+1-j-1+j+1$	2

TABLE II

σ^2.SNR for the time dispersive channel

6 Conclusions

We have presented some insight into the behaviour of the noise term z_k defined in (6). In particular, we have obtained bounds for σ_z^2 and $|\tilde{R}_z(1)|$ under time dispersive channel conditions. We have presented the channel that attains these bounds and the associated data sequences. Using these results we have determined the upper bound on the SNR for differential detection of MSK under dispersive channel conditions.

Finally, we mention the technique of multiple symbol differential detection (MSDD) of MPSK described in [10], which can also be generalized to GMSK modulation [11]. This technique uses $|\tilde{R}_z(1)|$ to achieve the same performance as coherent detection. This is done by allowing the observation interval over which symbol decisions are made to be longer than two symbol intervals, whilst making a joint decision on several symbols simultaneously. However, MSDD in its present form does not reflect the dependency of $|\tilde{R}_z(1)|$ and σ_z^2 on x and h, relying on the assumption that $|\tilde{R}_z(1)|$ and σ_z^2 are constant. As shown, for the time dispersive channel this is not the case. It is anticipated that a form of MSDD can also be used for the time dispersive channel. The development of this technique is the subject of continuing investigation.

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