Nonlinear Modelling of a Bi-Polar Junction Transistor

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Abstract

We reconstruct nonlinear models of an Bi–Polar Juntion transistor from time domain input–output data. We show that for the drive signals considered here — amplitude modulated signals — the models are transportable.

I. INTRODUCTION

We attempt to reconstruct transportable nonlinear models of electronic device components. The nonlinear models are reconstructed using time domain input–output time series data. The goal is to produce a model which given the values of present and past voltages (and perhaps past currents) accurately predicts the values of the present currents.

We are interested in developing such models because it is becoming apparant that today's circuit simulators are failing to provide accurate simulations of electronic devices which are operated in regimes where memory, heating and nonlinear effects play an important rôle. A reason for this is because the models which underpin the simulators are typically based on linear theory, and those models that are nonlinear are subject to artificially imposed cut–offs to avoid divergence.

We believe that there is scope for the use of nonlinear behavioural models reconstructed from actual measurement data in circuit simulators. Such models may be better able to capture the behaviour of a device when subject to signals which cause nonlinear responses due to any factors.

In order to convince the "circuit simulator community" of the potential of such models we must first show that behavioural models can match the performance of standard models in places where standard models do well. We will therefore restrict our study here to one device component namely the Ebers–Moll model of a Bi–Polar Junction Transistor (BJT) [11].

A circuit diagram of the Ebers–Moll models is shown in Figure 1. The model is a nonlinear model due to the capacitors and current element being described by nonlinear functions. The behaviour of the BJT model is studied using Kirchoff's Laws and by integrating the resulting differential equations. We are interested in reconstructing nonlinear models from the time series data obtained from the BJT model. We require our models to predict the output current I_c given present and past values of V_{be} — the base–emitter voltage — and V_{ce} — the collector–emitter voltage.

FIGURES

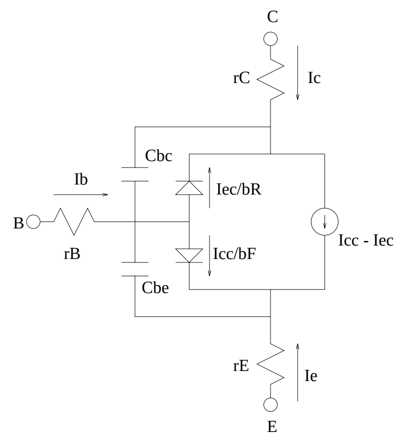

FIG. 1. Ebers-Moll BJT model.

A good summary of the device behaviour is given by the dc v–i characteristics of the device. The characteristics of the Ebers–Moll model are shown in Figure 2. This figure is obtained by providing fixed dc voltages for V_{be} and V_{ce} — so–called biasing. These voltages are referred to as bias points or operating points of the device.

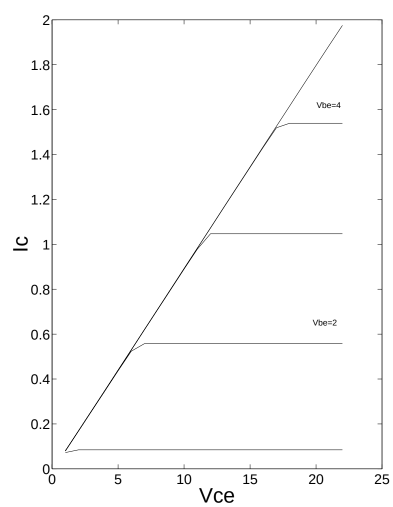

FIG. 2. The v–i characteristics of the BJT model.

An important property required of our black–box models is what engineers refer to as *transportability*. That is, can a (nonlinear) model reconstructed using data at *one* bias point adequately describe the behaviour of the device at a *different* bias point. This second bias point may lie on the same characteristic curve as the first, or lie on another characteristic curve.

A second transportability question is how do models reconstructed using data generated (measured) at *one* bias point subject to *one* drive signal generalise to data produced using *other* drive signals.

In order to test transportability an important problem is the design of a drive signal which can excite the device to fully explore its response behaviour. The design of such a drive signal goes hand–in–hand with the question of transportability. The design of such a "universal" drive signal is an open question. Indeed it may not even be possible since a device can be used in all manner of ways. Perhaps the best one can hope for is to design a "best" drive signal for a given application.

In earlier work we developed a method to reconstruct transportable behavioural models across bias points subject to *one class* of drive signal. In this report we develop transportable models over different classes of drive signal and across bias points.

We will consider amplitude–modulated (AM) drive and test signals. We will also only study transportability along one characteristic curve to show the *potential* ot our methods. The training data will therefore consist of applying an AM signal at V_{be}. The AM signal is

$$V = (1 + m\sin(\omega_m t))V_c \sin(\omega_c t). \tag{1}$$

We will apply a one–tone signal at V_{ce}. This has the effect of moving the AM signal back and forth along a characteristic curve.

II. NONLINEAR MODELLING

The nonlinear models we will reconstruct have the following form

$$I_c(t) = F[V_{be}(t-(k-1)\tau), \ldots, V_{be}(t), V_{ce}(t-(k-1)\tau), \ldots, V_{ce}(t)] \tag{2}$$

where τ is a time–delay lag and k is the number of past voltages used. The classes of nonlinear models we reconstruct will be polynomial models and radial basis function models. For example, with $k = 1$ a second order polynomial will have the following structure

$$\begin{aligned} I_c(t) &= F[V_{be}(t), V_{ce}(t)] \\ &= a_1 + a_2 V_{be}(t) + a_3 V_{ce}(t) + \\ &\quad a_4 V_{be}^2(t) + a_5 V_{be}(t) V_{ce}(t) + a_6 V_{ce}^2(t) \end{aligned} \tag{3}$$

The parameters a_i are determined from the training data. A radial basis model will have the form

$$I_c(t) = \beta + \alpha_1 V_{be}(t) + \alpha_2 V_{ce}(t) + \sum_{i=1}^{N} \omega_i \phi(\| c_i - (V_{be}(t), V_{ce}(t)) \|) \tag{4}$$

where the parameters to be estimated are β, α and ω_i. The c_i are called centres and they can also be estimated. We will explain the method we use later. The function ϕ is typically a Gaussian and this is what we will use. So,

$$\phi(r) = \exp\big(-\frac{1}{2\sigma^2} r' r\big).$$

σ is also another parameter to be specified (estimated) and we will explain our choice later.

We remark that (2) is a *static* approximation since there is no feedback of the currents $I_c(t)$. If we include past currents as well as voltages in (2) then the model will be a dynamic model. We will come back to feedback models later.

An important consideration of nonlinear modelling arises because of the "curse of dimensionality". That is, how to prevent an explosion of the number of parameters to be estimated as dimension increases and model complexity increases, e.g., in high dimensions higher order polynomials have many parameters. For radial basis models the "curse of dimensionality" is less severe but the problem of the number of centres to use exists, and furthermore the problem of where the centres should be is an issue.

An extremely effective method recently developed by Judd and Mees [12] uses description length ideas in an attempt to reconstruct radial basis models while controlling model size — number of centres — and choosing the "best" location for the chosen centres. Description length is their preferred criterion for comparing different models but the models can equally be compared by performance on a test data set. The best model is the one with minimum description length or best test set prediction performance.

Judd and Mees' method is not restricted to radial basis function approximations. It can be readily adapted to polynomial function approximation. The usefulness can be explained as follows. Suppose that the current I_c in (3) can be predicted accurately by using only the voltage $V_{ce}(t)$. That is, the voltage $V_{be}(t)$ is unnecessary. The parameters a_i can be estimated by finding the least–squares solution to

$$I_c = \Psi A$$

where the columns of Ψ are written as $[1\ V_{be}\ V_{ce}\ V_{be}^2\ V_{be}V_{ce}\ V_{ce}^2]$. If terms involving V_{be} are unnecessary for prediction of I_c then the method of Judd and Mees would (probably) select columns 1, 3 and 6 from Ψ to perform the least–squares estimation of the parameters a_i. Clearly, in addition to finding the correct relationship we have also reduced the number of parameters to be fitted and circumvented the "curse of dimensionality" somewhat.

The method of Judd and Mees is a subset selection algorithm based on sensitivity analysis from Lagrangian theory. The columns of Ψ are the elements of a candidate set and the algorithm picks the best subset of columns to describe the data. The size of the subset can be controlled by a description length criteria or a test error criteria.

In radial basis function approximation the columns of Ψ correspond to different centres and even different basis functions with different scales. When the algorithm selects the best subset in this case it is also selecting the centres in the best location.

III. RESULTS

A good test to study the transportability of a model is to see how well the model matches the dc–characteristics of the actual device. This test will give a guide as to what bias points the model is applicable. It is unlikely that the dc–characteristics will be well approximated for bias values outside of the region of the training data. This motivates the search for a drive signal which will excite the device to explore as much of its response space as possible.

Initially we will demonstrate that nonlinear models of the BJT can be reconstructed using the above methods from time series data. We show that such models can then be used to predict the behaviour of the BJT subject to simple drive signals at other operating points thus illustrating transportability.

We will present the results of our study at three test bias points along various characteristic curves. The training data along each curve is obtained by applying an AM signal with (see (1)) $m = 4/5$, $V_c = 5V$, $\omega_c = 5GHz$ and $\omega_m = 50MHz$ at V_{be}. The signal applied at V_{ce} is a one–tone drive given by

$$V_{ce} = V_{max} \sin(\frac{N\pi}{T} t), \tag{5}$$

where $V_{max} = 20V$, $T = 1e - 6s$ and $N = 100$. We integrate from $0s$ to T sampling every $0.1ns$ so obtaining $10,000$ data points.

An operating point of the device is selected by adding a constant bias value to V_{be} and V_{ce}. The bias at V_{be} has the effect of picking a charateristic curve in Figure 2, and the bias at V_{ce} picks a point along this curve. The addition of a sinusoidal driving term at V_{ce} has the effect of moving backward and forward along the chosen characteristic curve. The AM signal at V_{be} has the effect of circulating the chosen characteristic curve. So, using such a drive signal we are exciting the device across many bias points. We therefore expect that a nonlinear model reconstructed using the above data will test well on data sets subject to similar one and two tone driving at a variety of bias points thus illustrating transportability.

In Figure 3 we show the input–output time series obtained by setting $V_{be} = 3V$ and

$V_{ce} = 2V$. The test sets we will use are generated by applying a fixed dc voltage at V_{ce} and applying the AM signal above at V_{be}.

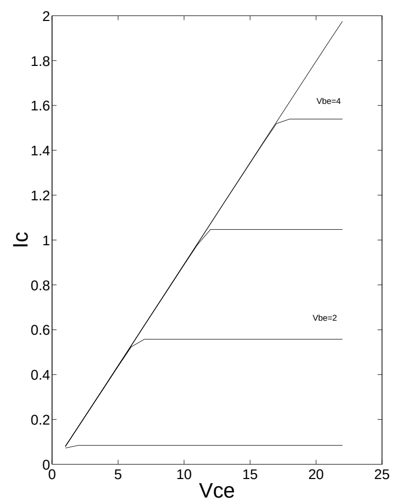

FIG. 3. Input–Output time series data.

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We will reconstruct radial basis models and polynomial models from the training data. We embed each measured voltage in 2 dimensions with a lag of 10. (We adapted the methods of Rhodes and Morari, and Cao et. al) to determine the embedding dimension. The lag was chosen by examining the average mutual information of the drive signals as well as the autocorrelation function.)

The candidate centres for the radial basis models are chosen from the data, and we make 10 candidate centres available for selection. The maximum order allowed for the polynomial models is four, and so there are 40 parameters which can be selected.

In Table I we show the results of reconstructing and testing radial basis and polynomial models at three bias points on the characteristic curve corresponding to $V_{be} = 3V$.

TABLES

Model (bias)	Model size	Error percent	(5,19) 250	(5,19) 500	(5,19) 750	(5,23) 250	(5,23) 500	(5,23) 750	(5,31) 250	(5,31) 500	(5,31) 750
R (5,19)	16	0.13	2.69	0.14	3.73	115	178	251	254	384	553
P (5,19)	18	0.09	2.5	0.11	2.7	117	178	250	257	385	555
R (5,23)	27	0.08	142	226	333	1.17	0.11	1.23	167	253	364
P (5,23)	18	0.33	111	228	252	58	0.33	91	280	320	527
R (5,31)	30	0.06	427	678	997	230	354	498	0.71	0.05	0.92
P (5,31)	15	0.3	448	595	901	262	318	469	35	0.35	33

TABLE I. This table shows the results of modelling the transistor at three bias points with radial basis function and polynomial aproximations. The training data is generated using an AM drive signal and the test sets are data generated using simpler sinusoidal drives. The modelling and test errors are given as percentages and are RMSE/STD(DATA) $\times$ 100 % .

There are several things to be read from Table I. The first is to notice that nonlinear models with a small number of parameters are reconstructed. We also notice that the accuracy of the training fit is excellent with errors less that 1% and there seems no advantage in reconstructing a radial basis model over a polynomial model.

The results of the test error calculations are also interesting. We notice that both the radial basis models and the polynomial models generalise very well to test set generated at the *same* bias point as the training data. We obseve that where test error performance is good radial basis models appear to outperform the polynomial models we have recosntructed. We do acknowledge, however, that we have not attempted to reconstruct a very complicated polynomial model.

The generalisation to other bias points, however, is abysmal. This drop-off in performance is easily explained. The reason for such terrible results at different bias points is that this new data lies in a different part of "phase space" than the training data. The reconstructed models thus have no knowledge of this space beacause "they have not seen it".

The above results suggest that to be able to reconstruct a nonlinear behavioural model from data which generalises well, the data must be rich enough to explore as much of the devices "phase space" as possible.

Model (bias,RR)	Model size	Error percent	(5,19) 250	(5,19) 500	(5,19) 750	(5,23) 250	(5,23) 500	(5,23) 750	(5,31) 250	(5,31) 500	(5,31) 750
R (3,5,5)	177	3.0	103	204	322	103	184	277	103	165	209
P (3,5,5)	27	10	103	233	334	96	211	324	96	183	284
R (5,11,4)	142	2.5	131	2.9	181	112	3.2	139	84	2.4	81
P (5,11,4)	26	3.1	482	3.3	607	386	2.1	476	294	1.2	307
R (8,10,5)	107	2.5	149	132	205	160	131	201	162	142	240
P (8,10,5)	27	2.2	1093	1033	2118	884	829	1622	635	602	1144

TABLE II. This table shows the results of modelling the transistor along three characteristic curver with radial basis function and polynomial aproximations. The training data is generated using an AM drive signal and the test sets are data generated using simpler sinusoidal drives. The modelling and test errors are given as percentages and are RMSE/STD(DATA) $\times$ 100 % .

In Table II we show the results of modelling the transistor using data generated as above along three different characteristic curves. We notice that the size of the reconstructed are much larger that those shown in Table I. This can be attributed to the fact that modelling the dynamics along an entire characteristic curve is more complicated than modelling the dynamics at *one* bias point.

The ability of the reconstructed models to reproduce the dc-characteristics of the device is important. This could be considered a “first” test to perform in the sense that if the dc-characteristics are poorly approximated at certain bias points then it is unlikely that the ac-tests will be good at these points.

In Figure (4) we show the ability of the radial basis models of Table II to reproduce the characteristic curve on which they were built. We see that the characteristics are adequately reproduced in the areas where there was training data. Outside of these areas, however, performance is severely compromised.

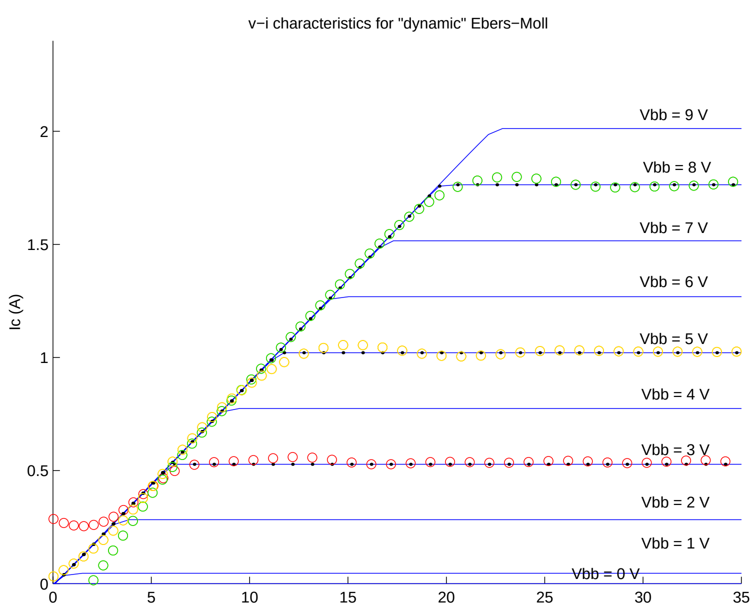

FIG. 4. The test fit of radial basis models to the dc-characteristics of Ebers-Moll transistor model.

IV. CONCLUSION

We have presented preliminary results of nonlinear modelling of an electronic device. We have seen that if the operating (bias) point of the device is known *a priori* then an appropriate drive signal can be designed to reconstruct a nonlinear model which can be used to predict the behaviour of the device at that operating point.

If a more global model is required, valid over a wide range of operating points and test signals then it is crucial to design a drive signal which produces a training set which explores as much of the devices response space as possible. If such a drive is forthcoming then our results suggest that it is possible to reconstruct an accurate nonlinear model.

In our investiagtions we have considered AM signals and we have seen that although such signals are adequate for some purposes, reconstructing at one bias piont, characteristic curve testing, they are lacking the complexity to deliver a "global" model. The consideration of such signals, however, has highlighted the potential of nonlinear modelling methods to the modelling of electronic devices but has also illuminated areas for further development.

There are two key questions which have arisen from this work. The first is the question of what is a "good" drive signal to use which probes the rich response behaviour of the device over a wide range of operating conditions. The design of such a signal is crucial if a "global" model is to be produced from one data set. The second question which we have considered briefly is the problem of interpolating between many models reconstructed at many bias points.

The two questions are intimately related since if the answer to the second question suggests that a "global" can be reconstructed by interpolating models between operating points then the design of an appropriate drive signal may be made easier.

It is also clear from this study that knowledge of the intended use of the device under study in terms of operating points and likely drive signals is a crucial piece of information best not dropped.

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