



## **Constructing Transportable Behavioural Models for Nonlinear Electronic Devices**

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# Constructing transportable behavioural models for nonlinear electronic devices

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We use radial basis functions to model the input–output response of an electronic device. A new methodology for producing models that accurately describe the response of the device over a wide range of operating points is introduced. A key to the success of the method is the ability to find a polynomial relationship between the model parameters and the operating points of the device.

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## I. INTRODUCTION

In this letter we investigate the possibility of applying methods recently developed for modeling nonlinear chaotic systems to modelling input–output dynamics of a nonlinear electronic device. The ability to describe the behaviour of modern electronic devices is becoming increasingly important. As technology advances, and the demands on current technology increase, it is becoming more and more apparent that traditional linear modelling methods fail to describe the behaviour of modern electronic devices [1]. This is particularly true when the device is being operated outside of its “normal” (linear) response regime, and is subject to complicated (nonlinear) driving signals [2,3].

It has been emphasized by researchers in nonlinear dynamics that complicated behaviour, which appears random, can sometimes arise from a low dimensional deterministic nonlinear system [4–6]. It seems apt, then, to investigate whether nonlinear modelling techniques can supplement traditional linear modelling techniques when they fail to describe a devices behaviour [7–9].

One result of our study is a new methodology for modelling electronic devices. In particular, we focus on constructing *transportable* behavioural models of such devices. In this research the dynamics of the device is represented by input–output data obtained at a few operating regimes. A behavioural model is constructed by adjusting its parameters until it accurately mimics the dynamics of the device. Within this context, a behavioural

model is transportable if it accurately mimics the dynamics of the device in operating regimes which are different from the ones used to construct the model. The ultimate goal is a single behavioural model that accurately mimics the device in all of its operating regimes.

Ideally we should compare the performance of our models to that of “standard” models using data obtained from a real device. However, testing with a real device can be expensive. Therefore, in the absence of real data, we shall regard the “standard” model for such a device as reality. If our behavioural models match the “standard” model to within a few percent then we claim that we have shown the *potential* of our methods.

To illustrate the method we will study a device with mild nonlinearities: a simplification of the “standard” Ebers–Moll model for a bi-polar junction transistor, which we embed in a self-biasing circuit [10] (see Figs. 2 and 1). That is, we are using the transistor as a simple amplifier. We will construct transportable behavioural models using time domain input–output data obtained from the Ebers–Moll model [11,7].

In Fig. 2,  $V_{bb}$  and  $V_{cc}$  are potential sources. If we apply dc-potentials at these points (so called biasing) then we are selecting an operating point for the device. A graphical summary of this part of the devices behaviour is given by the dc  $V - I$  characteristics obtained by biasing the device at different potentials. Figure 3 shows the  $V - I$  characteristics of our Ebers–Moll transistor model.

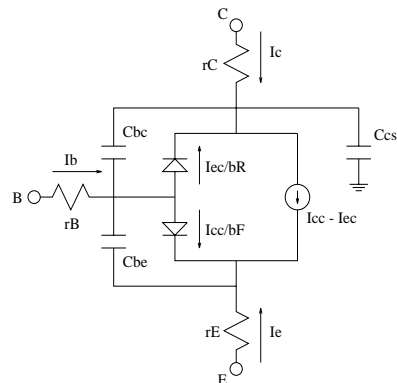


FIG. 1. Modified Ebers-Moll transistor model.

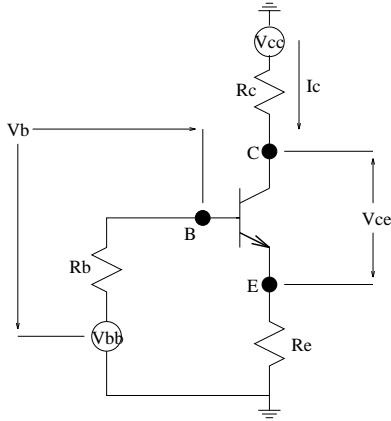


FIG. 2. Self-biasing transistor circuit.

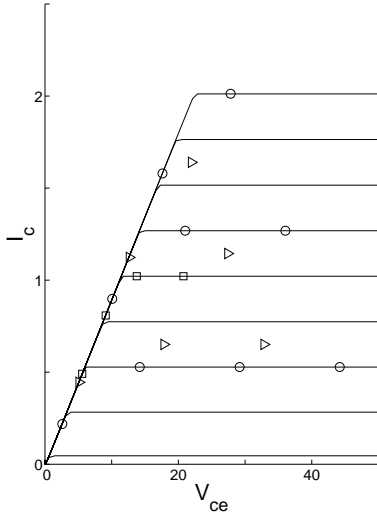


FIG. 3. dc-characteristics of Ebers-Moll transistor model. Each characteristic curve corresponds to  $V_{bb}$  ranging from 1 to 9 volts. Points along a particular curve corresponds to  $V_{ce}$  ranging from 0 to 60 volts.

The work reported in this letter does not attempt to describe a method for constructing completely transportable behavioural models. A complete study of transportability would include an investigation of the models ability to describe a device operating at *all bias points* and subjected to *all possible classes* of drive signals. Because, the drive signal a device “sees” is very much application dependent, and in lieu of knowledge of a specific application, we will investigate a restricted notion of transportability. We demonstrate our methods for the device operating at many different dc-bias potentials, but subjected to the same class of amplitude modulated (AM) drive signals. Thus, in this letter we investigate

transportability across dc-bias potentials and avoid the issue of transportability across signal class.

The class of AM drive signal we investigate is [10]

$$V = [(1 + A \sin(\omega t))V_c] \sin(\Omega t), \quad (1)$$

with  $V_c = 5V$ ,  $\omega = 5\text{MHz}$ ,  $\Omega = 500\text{MHz}$ , and  $A = 4/5$ . This signal is added to the fixed dc-offset at  $V_{bb}$  (see Fig. 2) with fixed dc-offset at  $V_{ce}$  (see Fig. 2). The input-output data we work with is obtained by integrating the circuit equations for the Ebers-Moll model of Fig. 1 embedded in the amplifier shown in Fig. 2. Approximately  $N = 3000$  input-output data points are generated from these equations by integrating them from 0 to  $3\mu s$ , sampling every  $1ns$ . The inputs for the behavioural models are potential differences, ( $V_b$ ,  $V_{ce}$ ), the output data is the current,  $I_c$ . In Fig. 4 we show examples of input-output data generated at the four different offset bias points indicated by the squares in Fig. 3. (We will refer to these data sets as pancakes.)

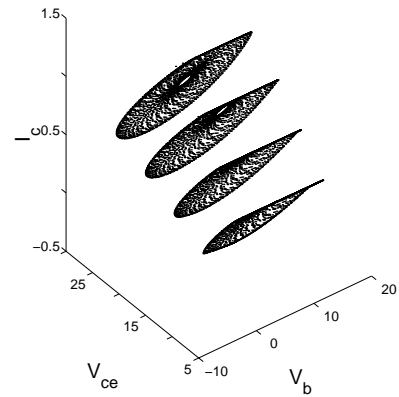


FIG. 4. Four input-output data sets (pancakes) along the characteristic curve corresponding to  $V_{bb} = 5V$ , indicated by the squares in Fig. 3.

## II. NONLINEAR MODELLING

The nonlinear behavioural models we reconstruct are radial basis function models. They attempt to predict the measured current,  $I_c$  (the output) from the measured potential differences,  $V_b$  and  $V_{ce}$  (the inputs). In passing, we remark that the inputs to the behavioural models are not the same as the AM signal used to drive the device.

The models have the following form

$$I_c(t) = F[V_b(t), V_{ce}(t)], \quad (2)$$

where  $F[u(t)]$  is a radial basis function with affine terms,

$$F[u(t)] = \beta + \alpha u(t) + \sum_{i=1}^K \omega_i \phi(\|c_i - u(t)\|). \quad (3)$$

Here,  $u(t) = [V_b(t), V_{ce}(t)]$ , while  $\beta$ ,  $\alpha$ , and  $\omega_i$  are parameters to be estimated. The  $c_i$ 's are called centres and also need to be estimated. The function  $\phi$  is a Gaussian

$$\phi(r) = \exp\left(-\frac{r^2}{2\sigma^2}\right),$$

where  $\sigma$  is another parameter which is kept fixed at  $\sigma = 4$  (corresponding to half the standard deviation of the  $V_{ce}$  data) throughout our investigations.

For many cases it may be necessary to embed the voltages,  $V_b$  and  $V_{ce}$ , using their past as well as present values in the function approximation of Eq. (2). This will certainly be the case for complicated drive signals. However, for the class of AM driving signals we examined, the device produces signals which are simple enough that embedding can be avoided.

When using radial basis functions an important issue is how many centres should be used, and where should they be located. An effective method of choosing appropriate centres from a large set of candidates is the subset selection method of Judd and Mees [12]. This method attempts to find the *best* selection of centres from a set of candidates. It selects the best set by evaluating a description length criteria. We will use this method (subject to the Schwarz Information Criterion) to determine (in some sense) an optimal model structure using data collected at one bias point.

### A. Constructing Simple Models

To illustrate the effectiveness of using radial basis functions to model the input-output relationship between  $(V_b, V_{ce})$  and  $I_c$ , consider the data from the bias point at  $V_{bb} = 5V$  and  $V_{cc} = 18V$ . In Fig. 5 we show the input space for this data. (The full input-output space corresponding to this data is the second pancake from the bottom of Fig. 4.)

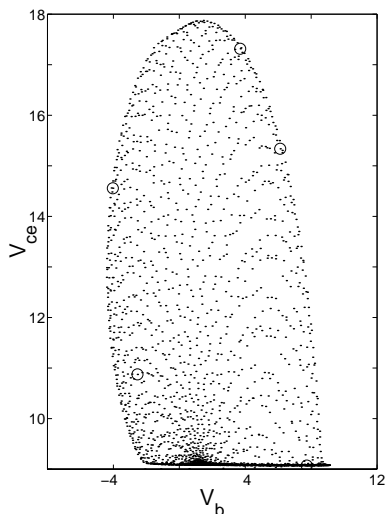


FIG. 5. The input space of the data generated at  $(V_{bb}, V_{cc}) = (5, 18)V$ . The five circles are the locations of data points chosen as centres by the modelling method.

The modeling method was given every fifth data point as a candidate centre.

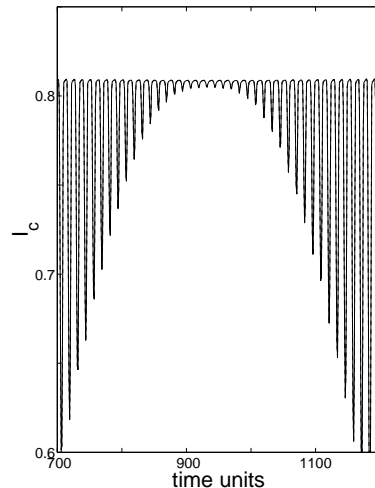


FIG. 6. A section of the response signal and the predicted signal of the nonlinear model reconstructed at  $(V_{bb}, V_{cc}) = (5, 18)V$ . We see that the fit is exceptional with the errors too difficult to see.

The modeling method kept the linear terms in Eq. (3) plus five radial basis terms. The data points selected as centres are indicated by the circles in Fig. 5. Figure 6 shows *both* the true  $I_c$  output and the output predicted by the behavioural model. At this scale the two curves are indistinguishable, with  $RMS = 6.13 \times 10^{-4}$  or  $[RMS/STD(I_c)] = 0.003$ . Clearly, for this data set a very good fit is possible with a small number of centres. We argue that because the modeling method chose nonlinear terms the data is better described by a nonlinear model than a purely linear one.

### B. Constructing Transportable Models

Given the effectiveness of our method at constructing nonlinear models at *one* operating point, our goal now is to deliver a single nonlinear model which “works” over a wide range of operating points.

There are two approaches which can be followed. The first is to develop “training” data which covers as wide a range of the devices input-output space as possible, and then construct a model using this data. The second approach is to generate “identical” data sets at many bias points, reconstruct models at each bias point, and form a global model by interpolating between the reconstructed models. We have attempted both approaches and have, so far, found the latter approach to be more fruitful. Therefore, it will be described below.

In general, the relationship between the modeling parameters and the bias points is

$$\begin{aligned}\beta &= \beta(V_{bb}, V_{cc}) \\ \alpha &= \alpha(V_{bb}, V_{cc}) \\ \omega_i &= \omega_i(V_{bb}, V_{cc}) \\ c_i &= c_i(V_{bb}, V_{cc}).\end{aligned}$$

We will determine these functional relationships by constructing models from input–output data corresponding to a few different bias points. Once this step is completed, parameter values at bias points not explicitly in the “training” data can be estimated via interpolation or extrapolation. In this fashion, we can deliver a nonlinear model that is transportable over a wide regime of bias points. The information needed for simulating the dynamics of a device is the operating bias point (from which we determine the parameters of the model) and the input signal sequence.

Referring once more to Fig. 4 we notice that the pancakes appear to change smoothly with changes in the operating bias point. In Fig. 7 we show that the location of the centres shown in Fig. 5 slowly vary as we move along the characteristic curve corresponding to  $V_{bb} = 5V$ . We have found that this behaviour is true for all of the bias curves in Fig. 3. Also, the data shown in Fig. 4 was generated in an “identical” fashion at each bias point. That is, the input–output data point with index 50 (say) in one pancake corresponds to the data point with index 50 in any other pancake. Thus, if the number of centres is fixed, and if the centres in each pancake have the same indices, then the nonlinear models constructed with these centres will have the same structure at every bias point.

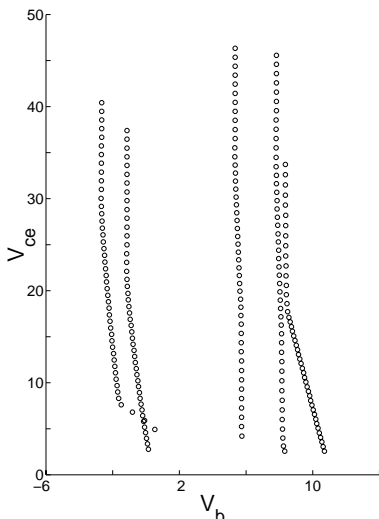


FIG. 7. The evolution of the five centres chosen by description length along the characteristic curve. We observe that their locations smoothly change with changes in the bias points.

When constructing a transportable model we keep the number of centres, and their indices, fixed. The exact location of a centre is given by curves like those in Fig. 7. Therefore, only a least squares estimate of the weight parameters is necessary. Implicit in our approach is the assumption that a smooth change in the bias points will result in a smooth change in the response signal, which will manifest itself as a smooth change in the estimated model parameters.

This is indeed the case as can be seen in Figs. 8. Figures 8(a) and 8(b) show two ways of obtaining an average estimate of the evolution of the constant weight  $\beta$  along the characteristic curve corresponding to  $V_{bb} = 5V$  ( $V_{cc} = 5 - 50V$ ). Figures 8(c) and 8(d) are the results for one of the linear weights  $\alpha_1$ , and Figs. 8(e) and 8(f) show the same for a nonlinear weight,  $\omega_3$ .

To understand how these estimates were obtained, recall that at each sample bias point we generated a  $N = 3000$  point data set. At each bias point we used two methods to divide this large set into five different 600 point data sets. In the first method, the first 600 points of the large set forms the first small set, the next 600 points forms the second small set, and so on. A (five centre) radial basis mode was constructed for each of the smaller data sets. The five estimates for the radial basis parameters are given by the dotted lines in Figs. 8(a), (c) and (e). The solid lines in the figures are the average values of the parameter estimates, while the starred lines are quadratic polynomial fits to the average values.

In the second method we decimate the data by taking every 5<sup>th</sup> point to obtain the 600 point data sets. The rest of the procedure for obtaining the model parameters shown in Figs. 8(b), (d) and (f) is the same as that just outlined.

The point of the above is to emphasize that the parameter estimates themselves have “error bars” and although the quadratic fits to the average are bad in places, they nevertheless describe the parameter evolution quite well as our final results show.

### C. New methodology

Our investigation leads us to propose the following methodology for producing a transportable model for the electronic device.

1. Sample the space of bias voltages, and generate input–output data sets, in an “identical” manner, at each bias point.
2. Choose one of the data sets to reconstruct a model using the subset selection and description length methodology proposed by Judd and Mees [12].
3. For each input–output data set, reconstruct a nonlinear model (using centres whose indices were determined in step 2) and store the estimated parameters together with their corresponding bias points.

- Fit a polynomial relationship between the estimated parameters and the bias points.

Models constructed using this methodology should be transportable over a range of operating points.

To use the behavioural model to predict the input–output dynamics of the device at one particular bias point, one estimates the values of the parameters in the model by interpolating the relationships between the parameters and the bias point. Predicted responses to drive signals are generated by inserting the drive signal into the radial basis model.

We have used the above methodology to produce a transportable radial basis model of the circuit discussed above. Behavioural models were obtained from data generated at twelve bias points ( $V_{bb} = 3, 6$  and  $9$ ,  $V_{cc} = 5, 20, 35$ , and  $50$ ) corresponding to four points on each of three characteristic curves. The locations of these points are shown by circles in Fig. 3. Because Fig. 3 is a two dimensional projection of a three dimensional input–output space there are only nine circles indicating twelve bias points. Bias points (3, 5), (6, 5) and (9, 5) are actually indicated by the same circle, and similiary for (6, 20) and (9, 20).

At each bias point an AM driving signal was added as described above. The indices for the centres shown in Fig. 5 were used for all of the models, and a quadratic polynomial was used to describe the relationship between the model parameters and the bias points.

The transportability of the models was tested using input–output data generated at nine bias points. These points are indicated by triangles in Fig. 3. Once again and for the same reasons as above, bias points (3.5, 10), (5.5, 10) and (7.5, 10) are represented by the same triangle. (Similiary for (5.5, 25) and (7.5, 25).) We emphasize that this test data represents bias points that are completely different from those used to “train” the model. The results of these out of sample tests are shown in Table I. We see that good performance is achieved and transportability of the model is clearly exhibited over this wide range of operating points.

Test set ( $V_{bb}, V_{cc}$ )	RMS	RMS/std( $I_c$ )
(3.5, 10)	0.0053	0.0324
(3.5, 25)	0.0047	0.0143
(3.5, 40)	0.0099	0.0278
(5.5, 10)	0.0052	0.0412
(5.5, 25)	0.0017	0.0061
(5.5, 40)	0.0129	0.0361
(7.5, 10)	0.0046	0.0850
(7.5, 25)	0.0050	0.0296
(7.5, 40)	0.0070	0.0231

TABLE I. The errors produced by the transportable model for twelve out of sample test sets.

### III. CONCLUSION

We have introduced a new methodology for modelling the input–output dynamics of nonlinear electronic devices. Our study was particularly focused on producing a nonlinear model which was transportable over a wide range of operating points. We used radial basis functions to model the device. Transportability was achieved by finding a relationship between the models parameters and the operating points of the device. A key to the success was the fixing of model structure (i.e., fixing appropriate centres) so that interpolation of the parameters with respect to the bias points was possible. A topic for future work is to apply our methods to additional electronic devices. A second is to investigate the “other half” of the transportability question, namely, can the response of the device subject to other classes of drive signals be predicted accurately by our models.

### ACKNOWLEDGMENT

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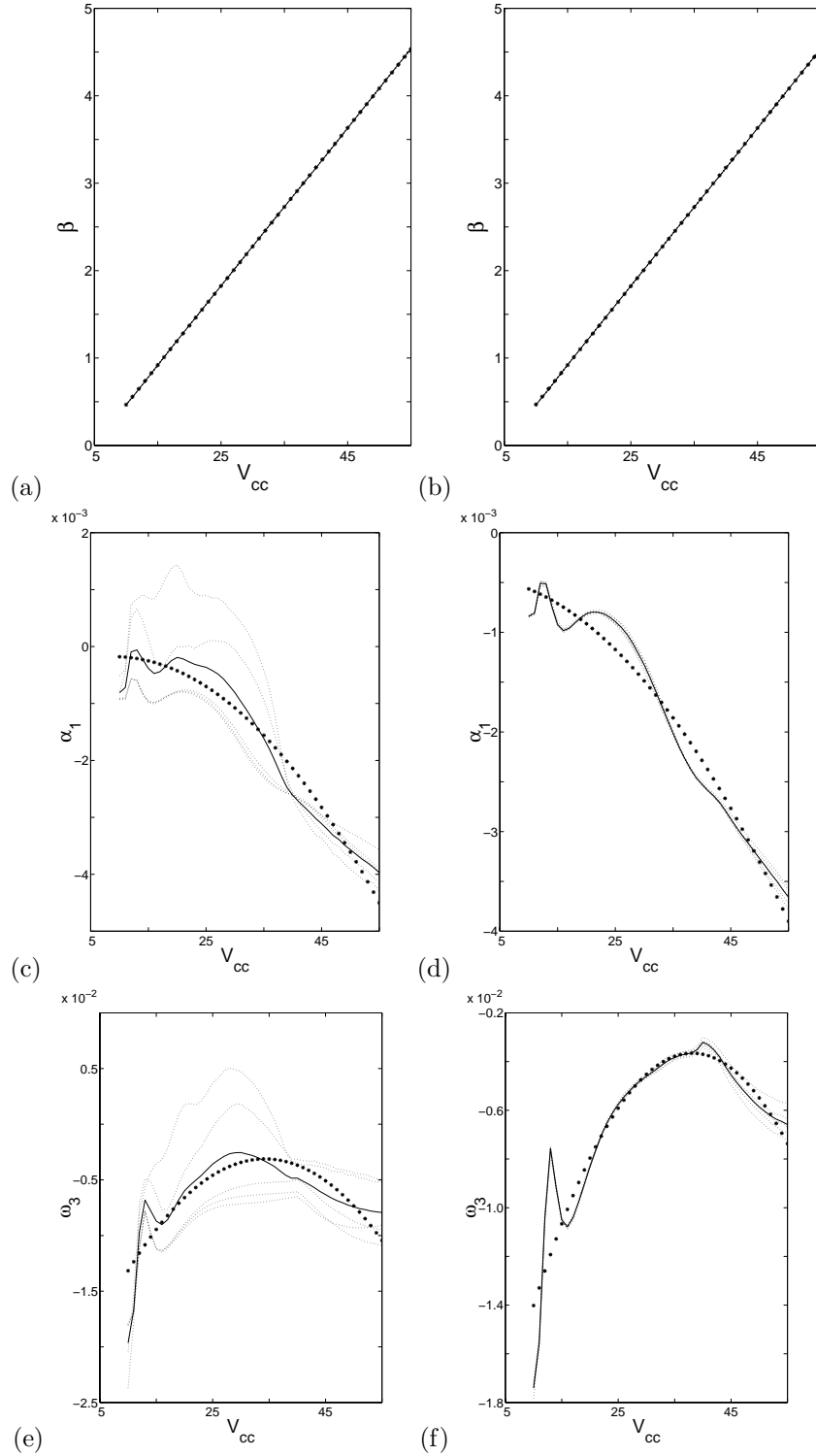


FIG. 8. The evolution of the estimated parameters along the characteristic curve corresponding to  $V_{bb} = 5V$ . The dotted lines show the estimates obtained from the chopped (a), (c) and (e), and the decimated (b), (d) and (f) data sets. The solid line is the average of the dotted lines. Also shown (starred) line) is the quadratic fit to the parameters using the bias points.