Tycoon: a Distributed Market-based Resource Allocation System

Kevin Lai Bernardo A. Huberman Leslie Fine {klai, bernardo.huberman, leslie.fine}@hp.com

HP Labs

September 8, 2004

Abstract

P2P clusters like the Grid and PlanetLab enable in principle the same statistical multiplexing efficiency gains for computing as the Internet provides for networking. The key unsolved problem is resource allocation. Existing solutions are not economically efficient and require high latency to acquire resources. We designed and implemented *Tycoon*, a market based distributed resource allocation system based on an *Auction Share* scheduling algorithm. Preliminary results show that Tycoon achieves low latency and high fairness while providing incentives for truth-telling on the part of strategic users.

1 Introduction

A key advantage of peer-to-peer clusters like the Grid and PlanetLab is their ability to pool together computational resources that can then be shared among peers. This allows increased throughput because of statistical multiplexing and the fact that users have a bursty utilization pattern. Sharing of nodes dispersed in the network structure allows lower delay because applications can store data close to potential users. Sharing allows greater reliability because of redundancy in hosts and network connections. Finally, sharing allows all of this at lower cost than a comparable private system.

The key problem for sharing resources in distributed systems is allocation. Allocation is an issue because demand grows to fill the available capacity. The resource demands of data mining, scientific computing, rendering, and Internet services have kept pace with hardware improvements. Problems in resource allocation are: the existence of *strategic* users who act in their own interests, a rapidly changing and unpredictable demand, and hundreds or thousands of unreliable hosts which are physically and administratively distributed. In this paper we compare resource allocation systems on the basis of: a) their *economic efficiency*, which is the percentage of *surplus* that a system generates, where the surplus for a transaction is the value of a resource to the highest-valuing recipient minus the cost to the lowestcost provider; b) utilization, which is the percentage of resources used; latency, which is the time to complete a task; c) risk, which is the lower bound on the resources a task obtains in a time interval; and d) fairness; which is the correlation between the actual distribution of utility to users to the desired distribution.

The common approach to this problem is to use a Proportional Share[11] scheduler, where users have no incentive to honestly report the value of their tasks. As a result, unimportant tasks get as much resources as critical jobs. Without further mechanisms, this causes economic efficiency to decrease as load increases (see Figure 1), eventually going to zero. To mitigate this, users engage in "horse trading" where one user agrees not to run a unimportant jobs when another user is running a critical one in exchange for the return favor in the future. This process imposes excess latency and work on users. Another approach is to use combinatorial optimization algorithms [5] to compute a schedule. However, optimal algorithms are NP-hard and share the Proportional Share problem of not eliciting users' true value for tasks.

We use an economic approach based on auctions. Economic approaches explicitly assume that users are strategic and that supply and demand vary significantly over time. We use a *strategyproof* mechanism to encourage users to reveal their true need for a resource, which allows the system to maximize economic efficiency.

However, existing economic resource allocation sys-

tems vary in how they abstract resources. The Spawn system [13] abstracts resources as reservations for specific hosts at fixed times. This is similar to the way that airline seats are allocated. Although reservations allow low risk and low latency, the utilization is also low because some tasks do not use their entire reservations. Service applications (e.g., web serving, database serving, and overlay network routing) result in particularly low utilization because they typically have bursty and unpredictable loads. An alternative is to combine Proportional Share with a market mechanism [1]. This offers higher utilization than reservations, but also higher latency and higher risk.

Our contributions are the design and implementation of Tycoon, a distributed market-based resource allocation architecture, and of Auction Share, a local resource scheduler. Tycoon distinguishes itself from other systems in that it separates the allocation mechanism (which provides incentives) from the agent strategy (which interprets preferences). This simplifies the system and allows specialization of agent strategies for different applications while providing incentives for applications to use resources efficiently and resource providers to provide valuable resources. Tycoon's distributed markets allows the system to be fault-tolerant and to allocate resources with low latency. Auction Share is the local scheduling component of Tycoon. As we show, it distinguishes itself from market-based reservations and proportional share by being high utilization, low latency, low risk, and fair.

In § 2, we review related work in cluster resource allocation and scheduling. In § 3, we describe the *Ty*-coon architecture. In § 4, we describe the *Auction Share* scheduler. We conclude in § 5.

2 Related Work

In this section, we describe related work in resource allocation. There are two main groups: those that consider the problem of strategic users (the economic approach) and those that do not (the computer science approach).

Examples of systems that use the economic approach are Spawn [13], the Millennium resource allocator [1], and work by Stoica, et al. [9]. These systems are *strategyproof.* A strategyproof mechanism forces truthtelling to be the dominant strategy for each entity regardless of the behavior of other entities. This ensures that rational entities will tell the truth about their preferences (e.g., how important particular resources are to them) and allow the overall system to be economically efficient. As a result, these systems mitigate the effect of strategic users. These systems differ in how they abstract resources. Spawn uses a reservation abstraction that results in low latency, low risk and low utilization (as described in § 1. The Millennium resource allocator uses Proportional Share, described in more detail below. Stoica, et al. use a centralized priority queue of tasks that is not suitable for a P2P system like Planet-Lab.

Proportional Share (PS) [11] is one of the noneconomic resource allocators. Each PS process i has a weight w_i . The share of a resource that process i receives over some interval t where n processes are running is

$$\frac{w_i}{\sum_{j=0}^{n-1} w_j}.$$
(1)

PS maximizes utilization because it always provides resources to needy processes. Most recent work [14] [8] on PS has focused on computationally efficient and fair implementations, where fair is defined as having a minimal difference between the actual allocation and the ideal one. We show in § 4.2 that the Auction Share (AS) scheduler is fair and computationally efficient. One problem with PS is its high risk, where risk is defined as the lower bound on the resources a process can obtain in a time interval. A process's share goes to zero as the sum of weights increases. Stoica, et al. [7] show that a PS scheduler can fix the shares of processes that need controlled risk while varying the shares of other processes. This is the approach we use in the AS scheduler to control risk.

Another issue is latency. Any one process must wait for all the others run. This delay makes no difference to a batch application like a renderer, but could significantly affect a service application like a web server. As described in § 4.2, PS scheduling can increase latency by a factor of 10 even for four processes. Borrowed Virtual Time (BVT) [2] schedulers are a form of PS scheduler that addresses this problem. Our Auction Share (AS) scheduler is similar to BVT in that it considers scheduling latency and uses admission control to reduce risk. However, AS is a simpler abstraction than BVT. BVT processes must specify three variables: a warp value, a warp time limit, and an unwarp time requirement. In contrast, AS processes only need to specify one: an expected number of processor-seconds needed during an interval (the interval is the same for all the processes if none are part of a distributed application, as BVT assumes). Also, if an application incorrectly sets its BVT parameters, it can significantly affect the performance of other applications. In § 4.2, we show that if an AS process sets its parameter incorrectly, it only affects its own performance.

Lottery scheduling [14] is a PS-based abstraction that is similar to the economic approach in that processes are issued tickets that represent their allocations. Sullivan and Seltzer [10] extend this to allow processes to barter these tickets. Although this work provides the software infrastructure for an economic mechanism¹, it does not provide the mechanism itself.

Similarly, SHARP [3] provides the distributed infrastructure to manage tickets, but not the mechanism or bartering agent strategies. In addition, SHARP and work by Urgaonkar, et al. [12] use an overbooking resource abstraction instead of PS. An overbooking system promises probabilistic resources to applications. Overbooking has essentially the same risk as PS because in the worst case, unexpected demand can deprive an application of its resources.

Another class of non-economic algorithms are those based on combinatorial optimization [4] [5]. This approach assumes that the load is deterministic and uses a centralized NP-hard algorithm to compute the optimal algorithm. As a result, it would perform poorly with the rapidly changing and unpredictable loads typical on the Grid and PlanetLab. In addition, the centralized optimizer would impose bottlenecks and decrease the reliability of an otherwise decentralized system.

3 Tycoon

In this section, we describe the *Tycoon* design principles and architecture and provide some preliminary simulation results demonstrating its properties.

3.1 Design Principles

We use two design principles in the design of the *Ty*coon architecture: separation of mechanism and strategy and distribution of allocation. Separation of mechanism and strategy is important because they have different requirements and consequences for complexity, security, and efficiency.

A strategy interprets a user's and an application's high level preferences for how an application should be run into valuations of resources. For example, web server may be more concerned with latency than throughput and is therefore willing to consume a few resources on many hosts in the hope one of its hosts will be close to a new client. A database server or a rendering application is willing to make a different tradeoff. Such preferences may not even be be technical: an application distributing sensitive information may wish to avoid hosts in certain countries. As a result of this diversity of preferences, strategies that are specialized to particular users and applications are more efficient than those that are not. However, if a resource allocation system were to incorporate strategies as part of its mechanism, it would either have to limit the preferences of applications or increase the complexity of its mechanism. Examples of the former approach are the system by Urgaonkar, et al. [12], which optimizes for throughout and shortest-job-first allocation, which optimizes for latency.

A mechanism provides incentives for users to truthfully reveal their values for resources and for providers to provide desirable resources. The mechanism also needs to provide primitives for expressing preferences. *Tycoon* allows applications to specify on which hosts they wish to run and the *Auction Share* scheduler allows them to specify how they wish to tradeoff throughput, latency, and risk. The mechanism is critical to the security and efficiency of the system, so it must be simple to understand and implement.

By separating strategy and mechanism, we allow the mechanism to be simple while not limiting the preferences expressed by users and applications. Instead, *Tycoon* provides incentives for users and application writers to specialize and optimize strategies. This principle is similar to the original conception of how functionality should be split between an operating system kernel and applications (and was applied again with microkernels) and the end-to-end argument [6] for how functionality should be split in computer networks.

The other *Tycoon* design principle is distribution of allocation. Since our motivation is to allocate resources for very large systems like the Grid or PlanetLab, we

¹By *mechanism* we mean the system that provides an incentive for users to reveal the truth (e.g., an auction)

distribute the allocation of resources as much as possible (the bank is still centralized, as described below). This increases reliability because the failure of one host will not prevent allocating resources on another. In addition, distribution mitigates accidental or malicious misbehavior by one host (e.g., charging credits without providing resources). Users or parent agents (see below) will eventually notice that some hosts have poor price/performance and run on other hosts. Finally, distributed allocation reduces the latency to change allocations because all allocation decisions are made local to a host.

3.2 Architecture

Using the principles described in the previous section, we split *Tycoon* into the following components: Parent Agent, Child Agent, Auctioneer, Service Location Service, and Bank. The Parent Agent and Child Agent implement the strategy, while the Auctioneer implements the mechanism. The Service Location Service and the Bank are infrastructure.

• **Parent Agent:** The parent agent does all highlevel distributed resource management on behalf of a user. Its two main tasks are budgeting and managing child agents. Budgeting is important because it removes the burden of managing budgets from the user (at the cost of some flexibility). Parent agents should be specialized for specific applications, but our current implementation includes a sample parent agent for batch application. The user specifies a number of credits, a deadline, and number of hosts to run on. If the user specifies to spend \$700 for 100 minutes on seven hosts, then the batch parent agent budgets \$1 for each host per minute.

Managing the child agents is important because some hosts may be more cost-effective than others. This may be because heterogeneity in the host platform or because one host is more lightly loaded than another. The batch parent agent monitors progress and costs associated with candidate hosts by querying the child agents. If a child agent has a low performance to cost ratio, it kills the child agent and associated application process running on that host. It replaces that child agent with a randomly selected host in the hopes that it will perform better. The key concern with this algorithm is the overhead associated with copying code to a new host, especially if it is not significantly better than the old candidates. We are still evaluating the effectiveness of this algorithm.

- Child Agent: Child agents bid for resources on hosts and monitor application progress. Although we describe a child agent as "bidding", a child agent actually transfers a lump sum to the auctioneer which then does the fine-grained bidding itself (described in more detail in § 4. This is more efficient than communication between the child agent and the auctioneer and removes the need to communicate frequently with the bank. Child agents monitor application progress by maintaining application specific statistics, e.g., the latency and throughput of transactions on a web server or the rate of frames rendered for a rendering application.
- Auctioneer: Auctioneers schedule local resources in a way that approximates proportional share, but allows flexibility for latency-sensitive and risk-averse applications. Auctioneers do efficient first or second price sealed bid auctions for finegrained resources, e.g., 10 ms CPU timeslices. This allows for high utilization and the agility to adapt very quickly to changes in demand and/or supply. We describe this mechanism in more detail in § 4.
- **Bank:** The bank maintains account balances for all users and providers. The two key issues with the bank are security and funding policy. The security problem is counterfeiting of currency. We deal with this problem by only allowing transfers between accounts. Users pay providers by directly transferring funds from one account to another. This prevents counterfeiting, but involves the bank in all transactions, which could limit scalability. We intend to examine this, but we do not believe it will be a problem in practice because (as described above) transfers only occur when a child agent 1) initially funds its application, 2) refreshes those funds when they are exhausted, and 3) the parent agent's budget changes.

Funding policy determines how users obtain funds. We define *open loop* and *closed loop* funding policies. In an open loop funding policy, users receive an allotment of funds when they join and at set intervals. The system administrators set their income rate based on exogenously determined priorities. Providers accumulate funds and return them to the system administrators. In a closed loop (or *peer-to-peer*) funding policy, users themselves bring resources to the system when they join. They receive an initial allotment of funds, but they do not receive funding grants after joining. Instead, they must earn funds by enticing other users to pay for their resources. A closed loop funding policy is preferable because it encourages service providers to provide desirable resources and therefore should result in higher economic efficiency.

• Service Location Service (SLS): Parent agents use the SLS to locate particular kinds of resources and auctioneers use it to advertise resources. Although we currently use a simple centralized softstate server, we can use any of the distributed SLSs described in the literature. The key point is that *Tycoon* does not require strong consistency. Parent agents monitor and optimize the end-to-end performance of their applications, so stale information in the SLS will simply delay the parent agent's from converging on an efficient set of resources.

3.3 Results



Figure 1: The utility of different user behaviors and mechanism as a function of system load.

We have preliminary simulation results. These results show that a market for computational resources effectively maintains a high utility despite strategic users. For the simulations in this section, we use a marketbased proportional share instead of the *Auction Share* described in § 4. We use a separate simulator for the *Auction Share* scheduler. This is our initial approach because the *Tycoon* simulator simulates events on many hosts on large timescales while the *Auction Share* simulator simulates events on one host on small time scales. We are working on merging the simulators.

The simulation results are of 100 users submitting tasks with a Poisson inter-arrival time. The simulation runs for 1000 seconds. There are ten hosts available for running tasks. We assume in the simulation that there is no overhead for distributing and starting tasks. The size and deadline of the tasks are also from a Poisson distribution. Each task has a value selected uniformly at random from (0, 1]. If a task completes by the deadline, then the user receives value * size utility, otherwise nothing. We simulate three different user behaviors: obedient, strategic without a market, and strategic with a market. Obedient users assign a Proportional Share weight to their tasks equal to the task's value. Non-market strategic users assign the maximum possible weight to all their tasks. Market strategic users have a limited budget for assigning weights. These users have an income of \$1 per time unit. They can save this income or spend it by assigning some of it as the weight of one of their tasks.

Market strategic users use a *budgeting* strategy. They assign weights at each host at each time unit to be

$$\frac{balance * value}{num_hosts * (deadline - now)}$$

where *balance* is the user's current credit balance, *value* is the value of the user's currently most valuable task, *num_hosts* is the number of hosts to run on, *deadline* is the deadline of the currently most valuable task, and *now* is the current time.

Figure 1 shows the simulation results. The y-axis is the mean utility per host per time unit. This cannot exceed 1.0, but the only way for that to be the maximum is if there is always a value 1.0 task in the system, which is not true in most cases. The y-axis shows the mean interarrival of tasks in the system and is a measure of overall system load. Each point in the graph is a run of the simulator. As the load increases to the left, the obedient users without a market are able to maintain a high level of utility. In contrast, the non-market strategic users are able to maintain a high level of utility when the system is moderately loaded (from 120 to 100), but when the load saturates the system, utility drops to zero. This is because the system wastes resources running tasks that never meet their deadlines and therefore provide no utility. As the number of tasks increases, this becomes more likely. In a system without a mechanism or significant social pressure, this is inevitable. With the market mechanism, the strategic users are forced to truthfully reveal the value of their tasks and the system can maintain a high utility.

4 Auction Share Scheduling

In this section, we describe the *Auction Share* scheduling component of *Tycoon* and use simulations to compare it with Proportional Share scheduling.

4.1 Algorithm

The *Auction Share* scheduler achieves the high utilization of a proportional share scheduler, the low latency of a Borrowed Virtual Time scheduler, the low risk of reservations, and the strategyproofness of a market scheduler. In addition, it is fair and computationally efficient.

While we only describe the use of the auction scheduling algorithm for CPU scheduling, it has a straightforward extension to other resources like network bandwidth and disk storage. For CPU scheduling, the resources are 10ms timeslices of the processor. The algorithm consists of child agents that bid for resources for an application process and an auctioneer that resolves the bids, allocates resources, and collects credits. In a typical operating system like Linux, part of the auctioneer resides in the kernel's processor scheduler.

Each child agent *i* has a balance of b_i credits, an expected funding interval of $E(t_i)$, and an expected number of processor-seconds needed during $E(t_i)$ of q_i . A parent agent funds its child agents periodically in proportion to their importance to it (§ 3 describes this budgeting process in more detail). $E(t_i)$ is the average amount of time between such fundings. We assume that $E(t_i)$ is on the order of seconds and therefore large relative to the timeslice size.

The child agent of a batch application sets q_i to be $E(t_i)$ in processor-seconds because batch application want to run as much as possible. The child agent of a delay-sensitive application sets q_i to be less than $E(t_i)$ because the application is willing to sacrifice some processor-seconds for lower delay. For example, a web

server is willing to sleep sometimes in return for having priority when a request comes in. More willing an application is to trade throughput for delay, the smaller its q_i is relative to its $E(t_i)$.

To allocate a timeslice, the auctioneer computes the bid of each thread *i* as b_i/q_i . The auctioneer allocates the timeslice to the thread with the highest bid. After *elapsed* elapsed seconds, the running thread is context-switched either because its allocation finished or because another thread with a higher bid becomes runnable. At this point, the thread pays its bid to the auctioneer in proportion to the amount of elapsed time:

$$\frac{elapsed}{timeslice} * \frac{b_i}{q_i}$$

The auctioneer then deducts this amount from the winning process's balance. Alternatively, the auctioneer can charge the winning process the second highest bidder's bid. We are still investigating the tradeoffs of using the first or second price.

This algorithm is strategyproof because it corresponds to a series of first or second price sealed bid auctions. The only difference is that the *Auction Share* auctioneer automatically computes bids for the clients instead of having them do it. If they wish, clients can micromanage the bidding by changing q_i , but only clients that wish to change their latency-throughput tradeoff gain anything from doing so.

Auction Share is computationally efficient because the only work the auctioneer needs to do each timeslice is update the previous winning processes balance and select the highest (and possibly second highest) current bid. The scheduler implementations of current operating systems already do similar calculations at low overhead. A typical implementation keeps process priorities in a heap which allows the selection of the highest value in O(1) time, and updating one of the values in O(logn) time, where n is the number of values. Changing q_i and funding (which changes b_i) will also require O(logn), but these happen infrequently.

This basic algorithm has high utilization, low latency, strategyproofness, fairness, and low overhead, but it still has significant risk. The arrival of new child agents will reduce the resources allocated to all the other child agents using the processor. Some risk-averse users would prefer having a higher lower bound on the resources they receive in an interval instead of having more total resources in that interval. An example is a

Algorithm	Weight	Yields CPU	Scheduling Error	Mean Latency
Proportional	1/10	yes	0.09	81 ms
Share				
Proportional	7/10	yes	0.01	4.4 ms
Share				
Proportional	7/10	no	1.16	4.7 ms
Share				
Auction Share	1/10	yes	0.01	3.6 ms
Auction Share	1/10	no	0.02	96 ms

Table 1: The scheduling error and latency for different scheduling mechanisms with different application behaviors.

real-time process like a game server that would benefit more from processing all its requests by their deadlines rather than finishing some very quickly and some very slowly.

To satisfy these processes, Auction Share offers a form of reservation. The idea is to use recent history as a guide to calculate a price for the reservation. A process can request a percentage of the process r for a time period of p timeslices. In some cases, the auctioneer must reject the reservation immediate because it has already sold its limit of reservations. If this is not the case, the auctioneer calculates the price for this reservation as

$$(\mu + \sigma) * r * p$$

where μ is the average price per timeslice, and σ is the standard deviation of the price. The process can either reject this price, or pay it, in which case, p begins immediately. During the reservation, the auctioneer enters a proxy bid in its own auction such that the reserving process always receives r of the processor.

This assumes the price in the recent past is indicative of the price in the near future and that price is normally distributed. We are still investigating methods for pricing reservations when these assumptions do not hold. Another issue is that the auctioneer must limit how much of the resource is reserved and the length of reservations to maintain liquidity in its market. We have not determined how these limits should be set.

4.2 Results

Our simulation results demonstrate that Auction Share achieves high utilization, low latency, and high fairness while providing an incentive for truth-telling to rational users. A proportional share scheduler can achieve high utilization and either low latency or fairness, but not both, and it does not provide incentives for truthtelling. We simulate a latency sensitive application like a web server running with 3 batch applications on a single processor. The desired long term processor share for the web-serving application is 1/10. During each timeslice, the web server has a 10% probability to receive a request, which takes 10ms of CPU cycles to service. Otherwise, the web server sleeps. The batch applications are always ready to run. For the proportional share scheduler, we initially set the weight of the web server and batch applications to be 1, 2, 3, and 4, respectively. For the auction scheduler, we set the income rates to be 1, 2, 3, and 4. For the auction scheduler, the processes are not funded at precise intervals. Instead, the income rates specify the mean interarrival times of funding. We run 1,000 timeslices of 10ms.

Table 1 shows the latency and fairness for different mechanisms and different application behaviors. "Weight" is the weight (for proportional share) or income rate (for auction scheduling) given the web server. "Yields CPU" is whether the web server correctly yields the CPU after servicing a request. "Scheduling Error" measures by how much the actual CPU time used by applications deviates from the amount intended. This is computed as the sum of the relative errors for each of the applications. For example, 0.09 indicates that the sum of the relative errors is 9%. Fairness is inversely proportional to the scheduling error. "Mean Latency" is the mean latency for the latency-sensitive application to service requests.

The second row of Table 1 shows that proportional share scheduling provides low error, but high latency. Note that this latency is proportional to the total number of runnable processes in the system, which is only four in our simulations. We can reduce the latency by increasing the weight of the web server, as shown in the third row. This assumes that the web server yields the processor after finishing a request. However, a rational user will exploit the extra weight granted to his application to do other computations to his benefit. Unfortunately, as shown in the fourth row, this is at the expense of the overall fairness of the system.

With auction share scheduling, the weight of the web server does not need to be boosted to achieve low latency (as shown in the fifth row). More importantly, if the web server mis-estimates the resources it requires (accidentally or deliberately), as shown in the last row, it only penalizes its own latency. The overall fairness of the system remains high. This provides the incentive for child agents to truthfully reveal their requirements for resources and therefore allows the system to achieve high economic efficiency. In addition, *Auction Share* has the same utilization as Proportional Share because the processor is always utilized.

5 Conclusion

The contributions of this paper are the *Tycoon* distributed market-based resource allocation architecture and the *Auction Share* local resource scheduler. Through simulation, we show that a market-based system has greater utility than a non-market-based proportional share system. In addition, we show through simulation that *Auction Share* is high utilization, low latency, and fair.

We are planning to implement *Auction Share* in the Linux kernel. Using this, we hope to deploy our implementation of *Tycoon* on a large cluster and take measurements of realistic workloads.

References

- CHUN, B. N., AND CULLER, D. E. Market-based Proportional Resource Sharing for Clusters. Technical Report CSD-1092, University of California at Berkeley, Computer Science Division, January 2000.
- [2] DUDA, K. J., AND CHERITON, D. R. Borrowed-Virtual-Time (BVT) scheduling: supporting latencysensitive threads in a general-purpose schedular. In *Symposium on Operating Systems Principles* (1999), pp. 261–276.
- [3] FU, Y., CHASE, J., CHUN, B., SCHWAB, S., AND VAHDAT, A. SHARP: An Architecture for Secure Resource Peering. In ACM Symposium on Operating Systems Principles (SOSP) (October 2003).
- [4] PAPADIMITRIOU, C. H., AND STEIGLITZ, K. Combinatorial Optimization. Dover Publications, Inc., 1982.
- [5] PINEDO, M. Scheduling. Prentice Hall, 2002.
- [6] SALTZER, J. H., REED, D. P., AND CLARK, D. D. End-to-end arguments in system design. ACM Transactions on Computer Systems 2, 4 (1984), 277–288.

- [7] STOICA, I., ABDEL-WAHAB, H., AND JEFFAY, K. On the Duality between Resource Reservation and Proportional Share Resource Allocation. In *Multimedia Computing and Networking* (February 1997), vol. 3020 of *SPIE Proceedings Series*, pp. 207–214.
- [8] STOICA, I., ABDEL-WAHAB, H., JEFFAY, K., BARUAH, S., GEHRKE, J., AND PLAXTON, G. C. A Proportional Share Resource Allocation Algorithm for Real-Time, Time-Shared Systems. In *IEEE Real-Time Systems Symposium* (December 1996).
- [9] STOICA, I., ABDEL-WAHAB, H., AND POTHEN, A. A Microeconomic Scheduler for Parallel Computers. In Proceedings of the Workshop on Job Scheduling Strategies for Parallel Processing (April 1995), pp. 122–135.
- [10] SULLIVAN, D. G., AND SELTZER, M. I. Isolation with Flexibility: a Resource Management Framework for Central Servers. In *Proceedings of the USENIX Annual Technical Conference* (2000), pp. 337–350.
- [11] TIJDEMAN, R. The Chairman Assignment Problem. Discrete Mathematica 32 (1980).
- [12] URGAONKAR, B., SHENOY, P., AND ROSCOE, T. Resource Overbooking and Application Profiling in Shared Hosting Platforms. In *Proceedings of Operating Systems Design and Implementation* (December 2002).
- [13] WALDSPURGER, C. A., HOGG, T., HUBERMAN, B. A., KEPHART, J. O., AND STORNETTA, W. S. Spawn: A Distributed Computational Economy. *Software Engineering* 18, 2 (1992), 103–117.
- [14] WALDSPURGER, C. A., AND WEIHL, W. E. Lottery Scheduling: Flexible Proportional-Share Resource Management. In *Operating Systems Design and Implementation* (1994), pp. 1–11.