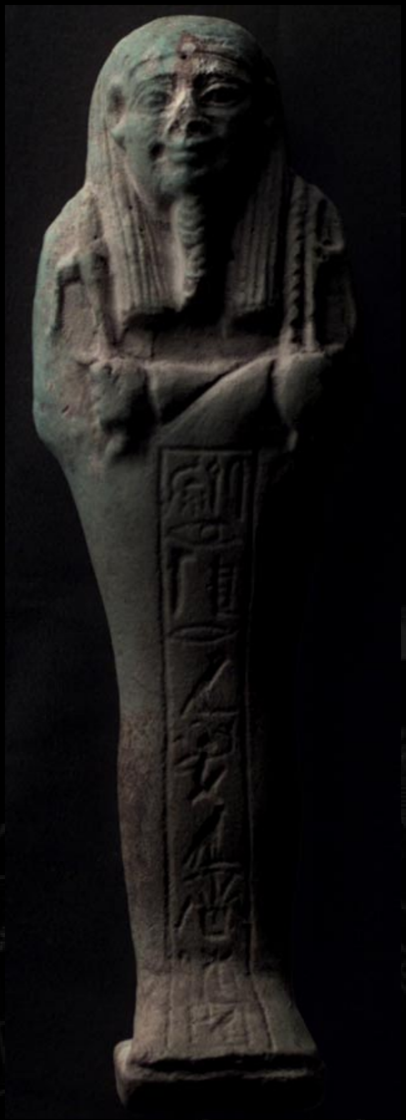

Polynomial Texture Maps

Tom Malzbender
Dan Gelb
Hans Wolters

Hewlett-Packard Laboratories

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Texture Mapping

[Catmull ‘74]

Pro:

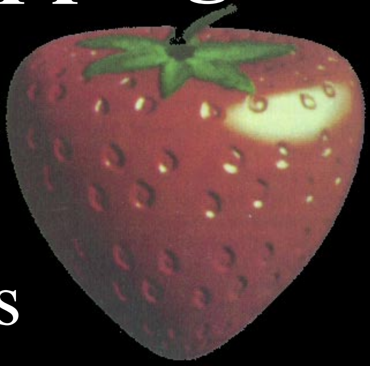
- Photographic input
- Simplicity
- Hardware Support

Con:

- Unrealistic Silhouettes
- Static Lighting

Bump Mapping

[Blinn 78]



Pro:

- Lighting Variations

Con:

- Per-pixel lighting computation
- Filtering is Problematic
- Procedural Synthesis (Not image-based.)

[Rushmeier ‘97]

PTM Demonstration:

Top: *Polynomial Texture Map*

Bottom: *Conventional Texture Map*

Advantages:

- Image based unlike Bump Mapping
- Simpler to evaluate than Bump Mapping
- Can leverage Mip Mapping

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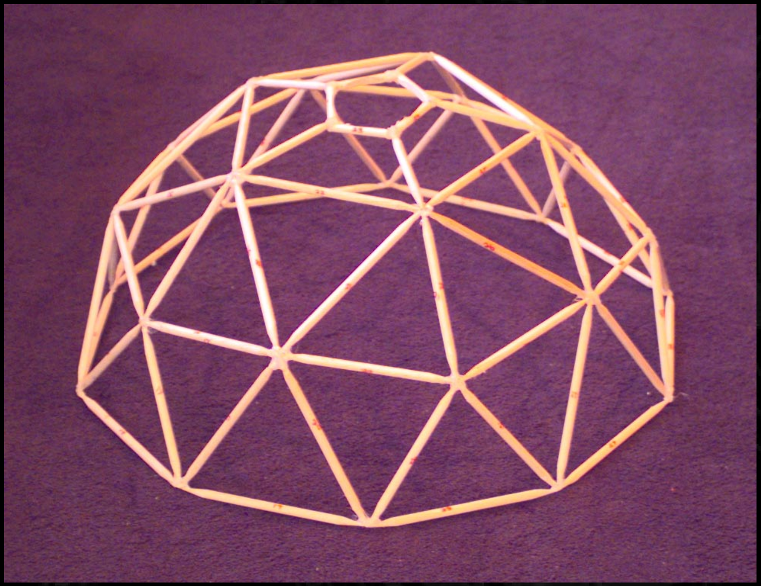
Acquiring PTM's Photographically



- Fixed object, fixed camera.
- Limited to Diffuse Objects.

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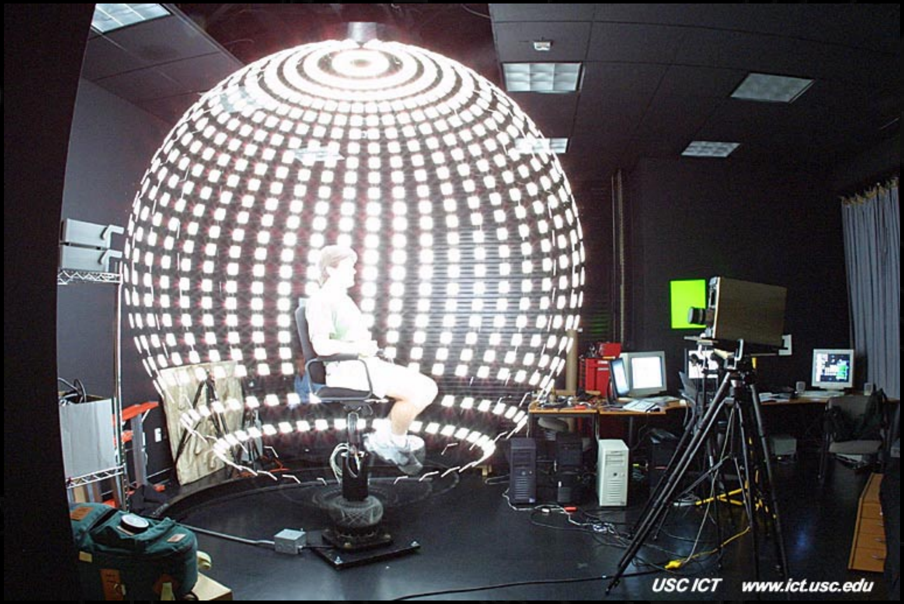
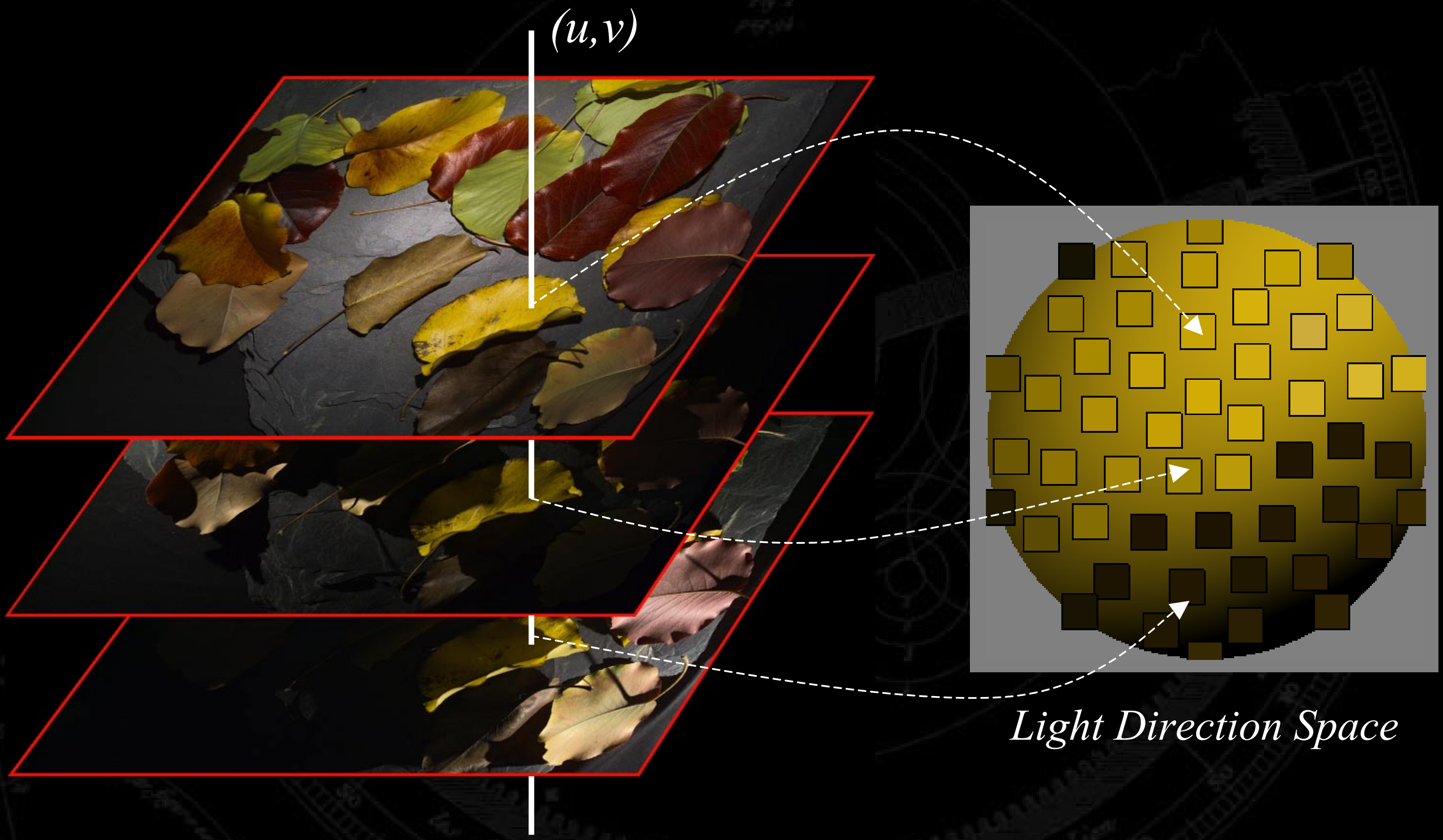
Acquiring Lighting Models

Debevec

Goerghiades '99



Modeling Pixel Color Changes Directly



$$L(u,v;l_u,l_v) = a_0 l_u^2 + a_1 l_v^2 + a_2 l_u l_v + a_3 l_u + a_4 l_v + a_5$$

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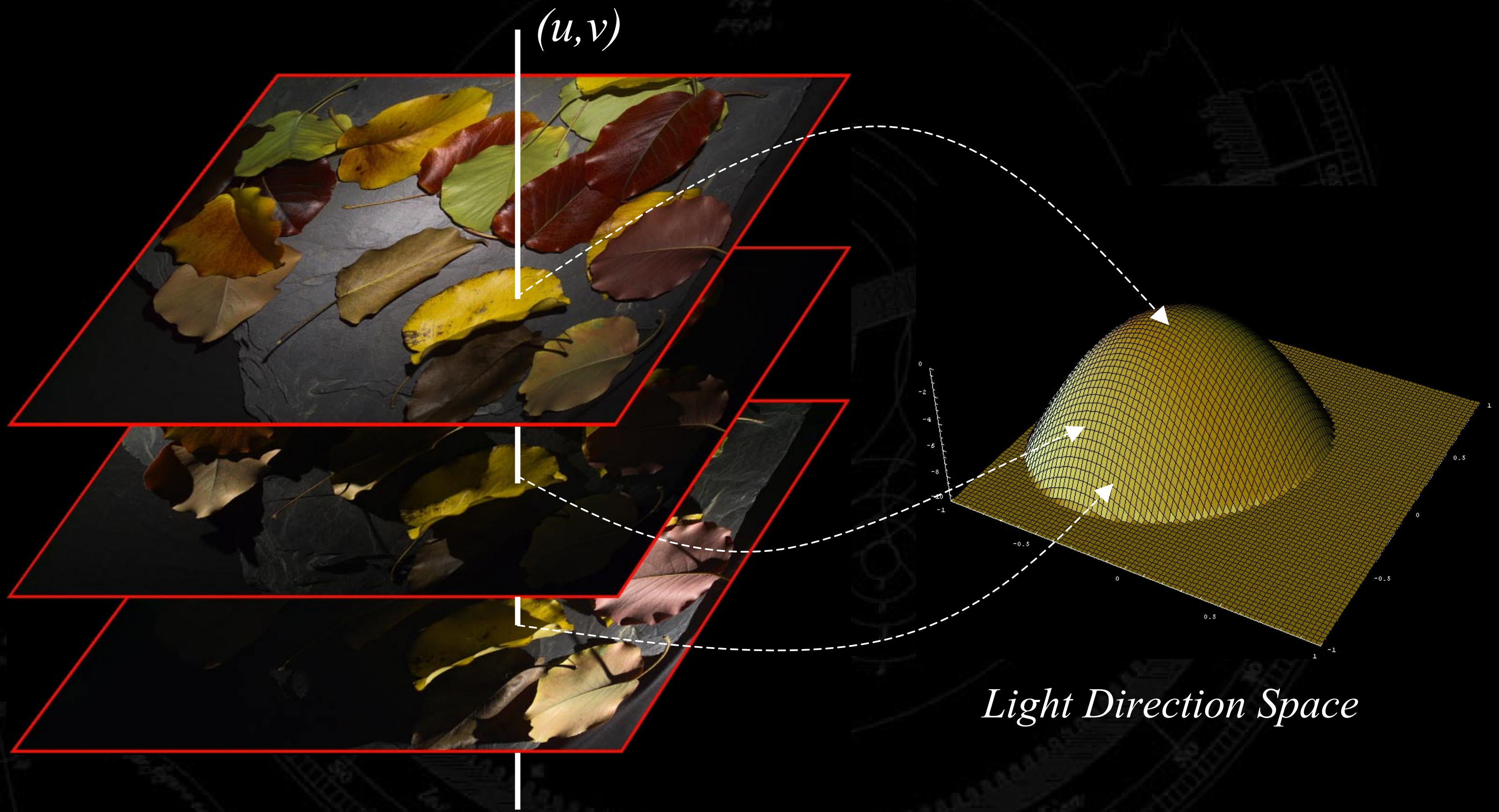
Modeling Pixel Color Changes Directly

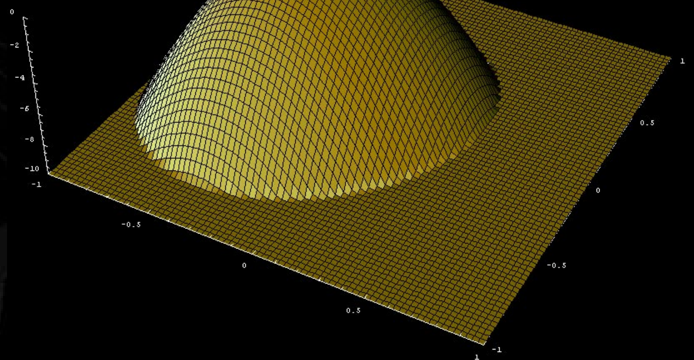


Light Direction Space

$$L(u,v;l_u,l_v) = a_0 l_u^2 + a_1 l_v^2 + a_2 l_u l_v + a_3 l_u + a_4 l_v + a_5$$

Polynomial Texture Mapp

PTM: Store RGB per pixel and
Store polynomial coefficients
(*a0*-*a5*) per textel:



$$L(u,v;l_u,l_v) = a_0 l_u^2 + a_1 l_v^2 + a_2 l_u l_v + a_3 l_u + a_4 l_v + a_5$$

$$R = L\,R'$$
$$G = L\,G'$$
$$B = L\,B'$$

Why Polynomials?

- Compact Representation
- Consist solely of multiplies and adds.
- Cheap to evaluate on both modern CPUs and VLSI

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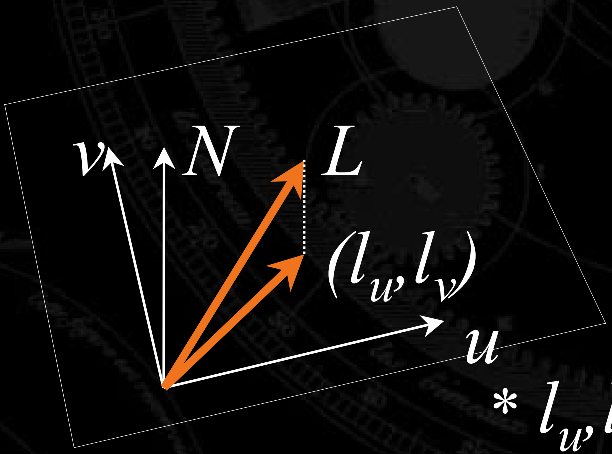
What PTMs Capture

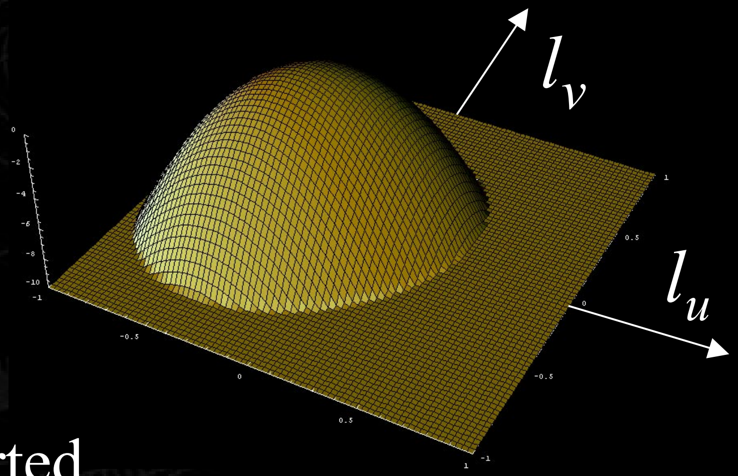
- Shading effects.
- Self shadowing.
- Interreflections.
- Sub-surface scattering.

Light Direction Parametrization

$$L(u,v;l_u,l_v) = a_0 l_u^2 + a_1 l_v^2 + a_2 l_u l_v + a_3 l_u + a_4 l_v + a_5$$

u,v - texture coordinates
a_0-a_5 - fitted coefficients stored in texture map
l_u,l_v - projection of light direction into texture plane





* l_u,l_v can be
scan-converted
without normalization.

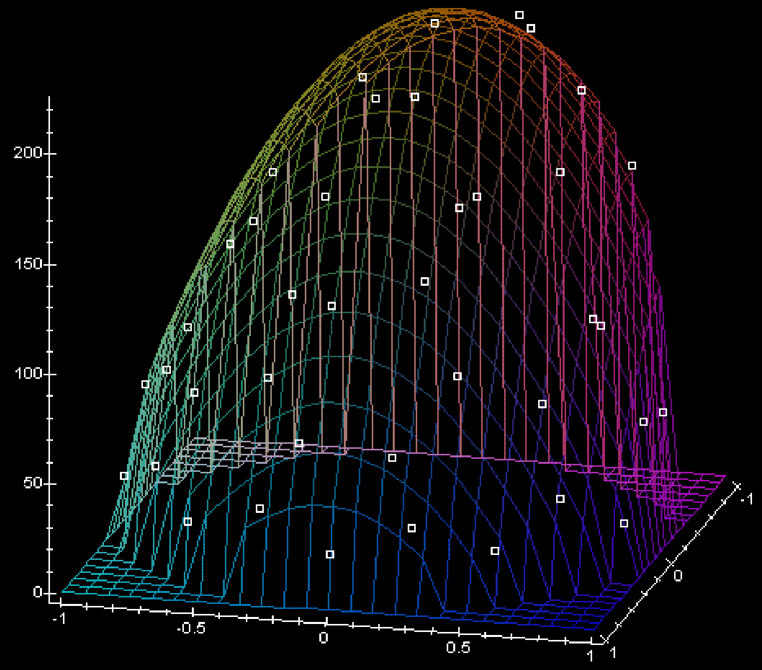
Fitting PTMs to Image Data

- Given *N* light sources we compute the best fit for (a_0-a_6) in the L_2 norm using S.V.D.

- SVD computed once for a given lighting arrangement.

$$\begin{bmatrix} l_{u0}^2 & l_{v0}^2 & l_{u0}l_{v0} & l_{u0} & l_{v0} & 1 \\ l_{u1}^2 & l_{v1}^2 & l_{u1}l_{v1} & l_{u1} & l_{v1} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ l_{uN-1}^2 & l_{vN-1}^2 & l_{uN-1}l_{vN-1} & l_{uN-1} & l_{vN-1} & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_5 \end{bmatrix} = \begin{bmatrix} L_0 \\ L_1 \\ \vdots \\ L_{N-1} \end{bmatrix}$$

PTM Fitting Errors



- Smoothing is not spatial, it occurs in light space.
- High spatial frequencies are well preserved.
- Hard shadows become softer.
- Point lights become area lights.

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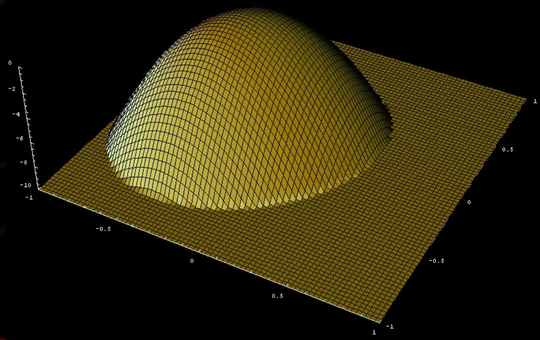
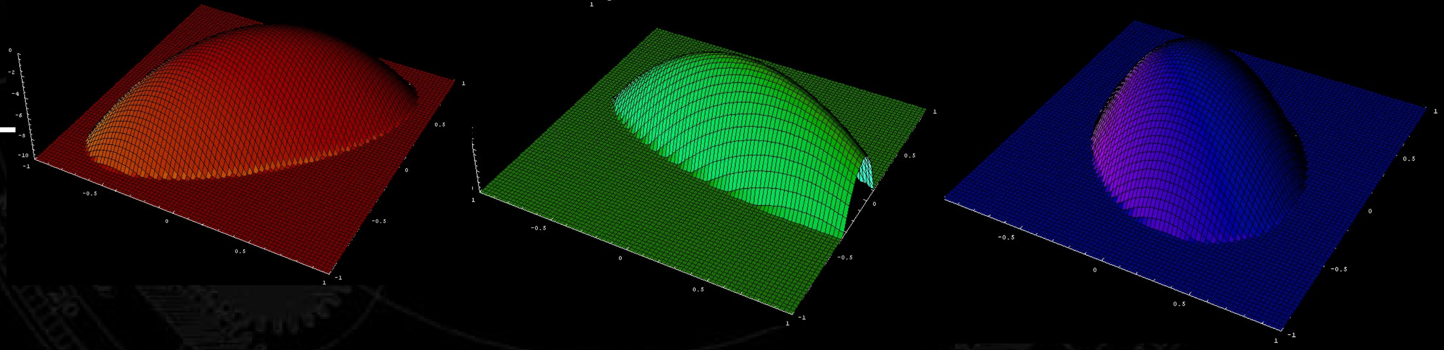
PTM Formats

Format — Per Pixel Storage

- LRGB -  + R,G,B
- RGB - 
- ENC - Index to L.U.T. storing Polynomial Coefficients

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Scale and Bias

$$a_i = \lambda_i (a_i' - \Omega_i)$$

- Allows polynomial coefficients to be stored as 8 bit values.

- Handles large dynamic range among coefficients.

- 12 Global values stored per texture map (LRGB).

Mip Mapping PTM's vs Bump Maps

- Mip-mapping bump maps effectively smooths geometry.
  [Schilling 97]

- PTMs are linear in polynomial coefficients
  so mip-mapping PTMs is accurate.

$$L(u,v;l_u,l_v) = a_0 l_u^2 + a_1 l_v^2 + a_2 l_u l_v + a_3 l_u + a_4 l_v + a_5$$

$$\frac{1}{n}\sum_{i,j\in\Omega} L_{r,g,b}(a_{0-5}(u_i,v_j)) = L_{r,g,b}\left(\frac{1}{n}\sum_{i,j\in\Omega} a_{0-5}(u_i,v_j)\right)$$

Evaluation – MMX / SSD Implementation

Parallel computation

- Fixed point arithmetic
- Pack 4 PTM coefficients in 64 bit integer MMX register
- Parallel multiply/adds

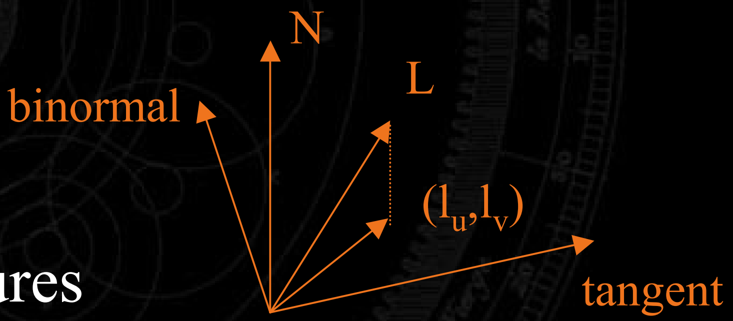
Yields 6.5M pixels/sec on a 1 Ghz CPU (software only).

Evaluation – Programmable Hardware

Vertex Processing

- Store precomputed tangent and binormal per vertex
- Vertex code projects light vector onto tangent and binormal

Pixel Processing

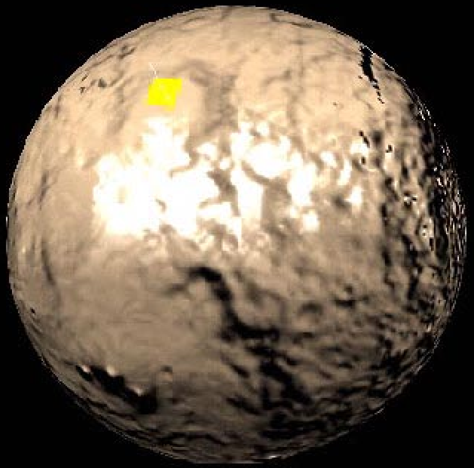


- l_u,l_v passed from vertex stage
- PTM coefficients stored in 2 textures
- Calculate using dot products / multiplies / adds
- Single pass on current hardware

Bump Maps as PTM’s

- If PTM rendering methods are implemented, they can be used for rendering bump maps.
- Provides specular and diffuse effects.

$$I = I_a k_a + I_d k_d (N \cdot L) + I_s k_s (N \cdot H)^n$$



- Precompute $N \cdot V$ PTM L.U.T.
- Convert Normals to PTM using L.U.T.
- Render PTM
  - Diffuse - evaluate $N \cdot L$
  - Specular - evaluate $(N \cdot H)^n$

Complex Shading Effects

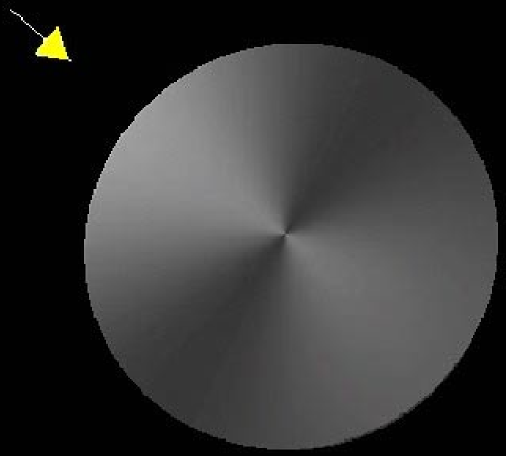
Combine with hardware lighting

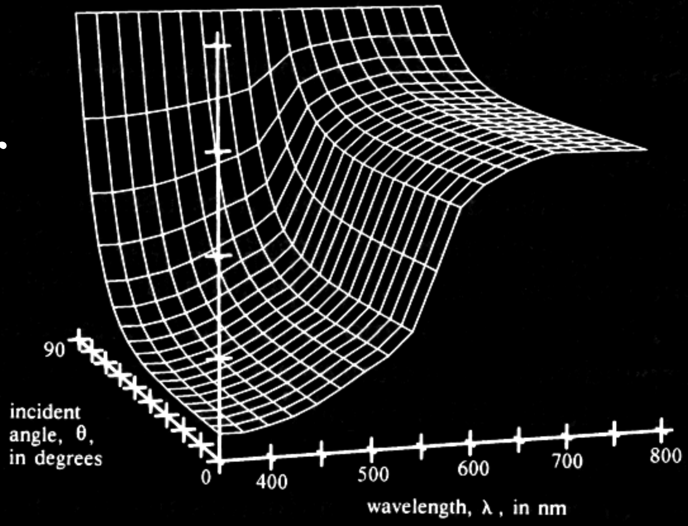
- Use with existing Phong lighting
- PTM models more complex reflectance effects
  - Examples:

Anisotropic

Fresnel

Off-specular



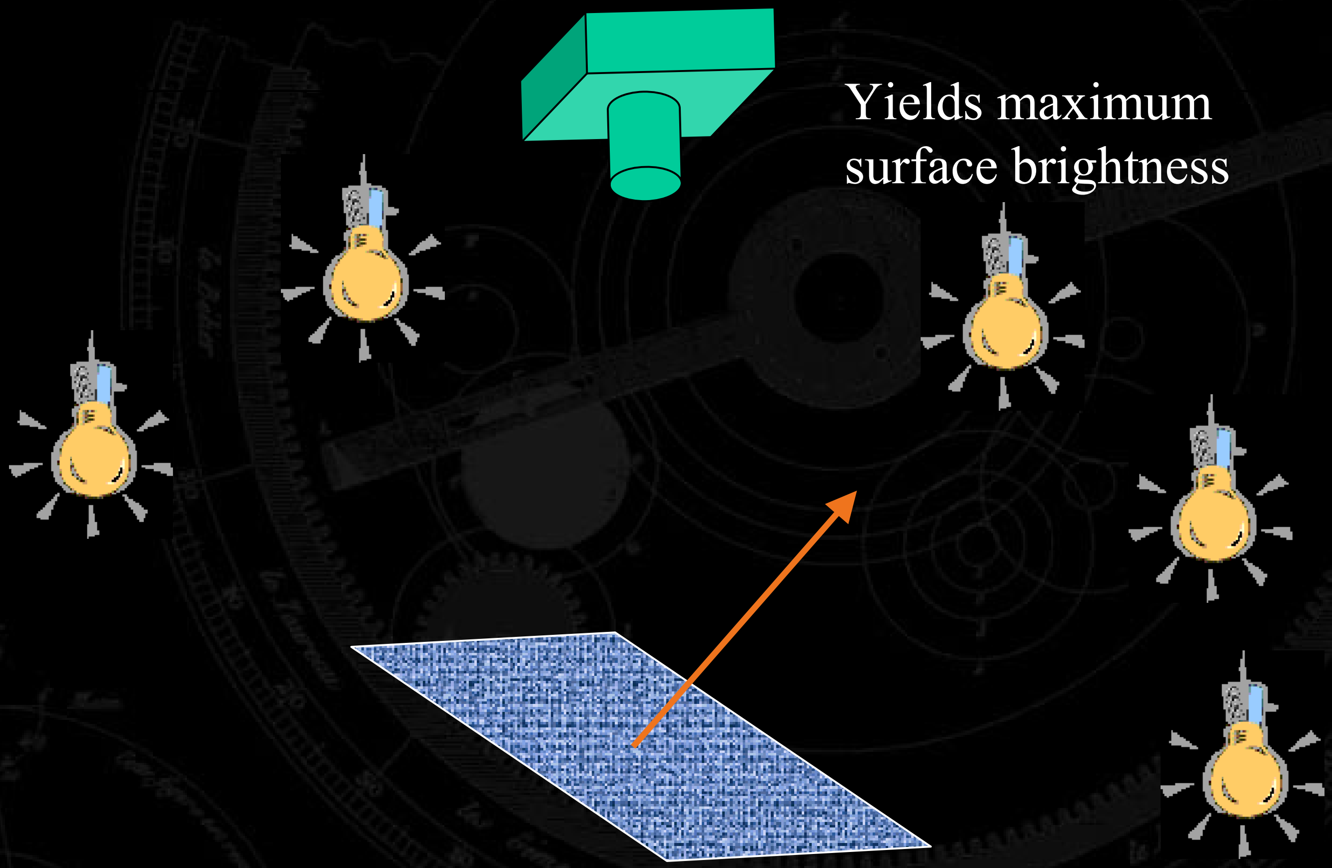


approximate Fresnel reflectance for copper using the measured properties: $n = 0.617$ $k = 2.630$

2D Applications

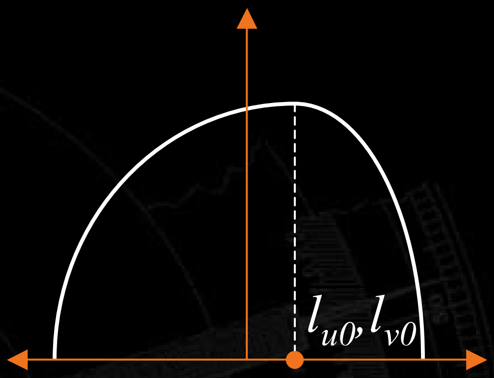
- Enhancement of Cuneiform Tablets w/ Zuckerman USC
- PTMs for Short Image Sequences
- PTMs for Depth of Focus Effects.

Surface Normal Extraction



Yields maximum surface brightness

Surface Normal Extraction



For a diffuse object, coordinates of (l_u, l_v) that maximize luminance yield local surface normals.

Setting $\frac{\partial L}{\partial u} = \frac{\partial L}{\partial v} = 0$ yields:

$$l_{u0} = \frac{a_2 a_4 - 2a_1 a_3}{4a_0 a_1 - {a_2}^2}$$

$$l_{v0} = \frac{a_2 a_3 - 2a_0 a_4}{4a_0 a_1 - {a_2}^2}$$

Providing a surface normal per textel:

$$\vec{N} = (l_{u0}, l_{v0}, \sqrt{1 - {l_{u0}}^2 - {l_{v0}}^2})$$

Specular Enhancement

Cuneiform tablet courtesy of
Dr. Bruce Zuckerman at U.S.C.

Diffuse Gain - a reflection transformation that:

- Keeps the surface normal fixed.
- Increases the curvature (second derivative) of the reflectance function by *g*.

$$a_0' = g a_0$$

$$a_1' = g a_1$$

$$a_2' = g a_2$$

$$a_3' = (1 - g)(2a_0 l_{u0} + a_2 l_{v0}) + a_3$$

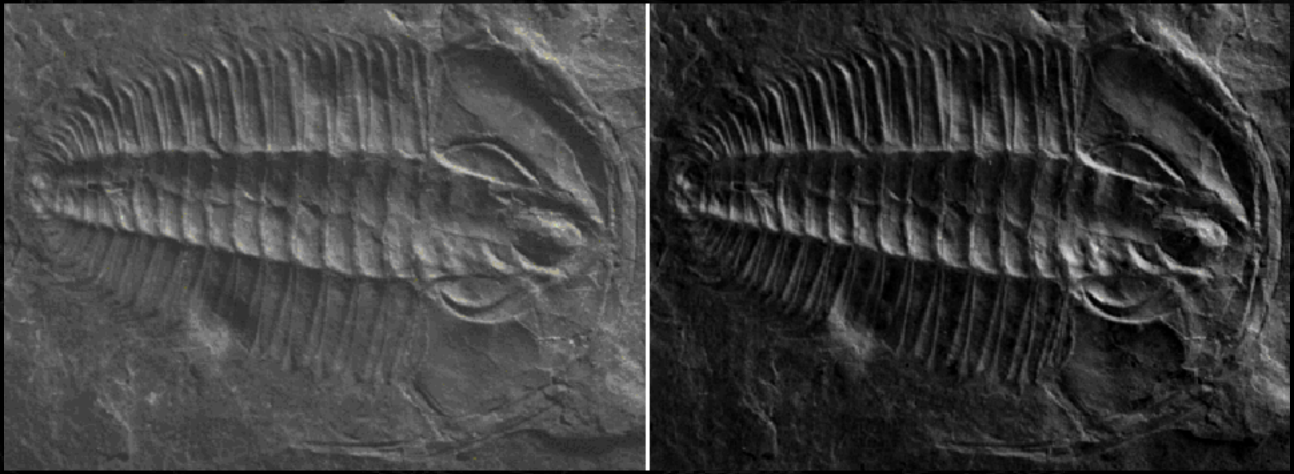
$$a_4' = (1 - g)(2a_1 l_{v0} + a_2 l_{u0}) + a_4$$

$$a_5' = (1 - g)(a_0 l_{u0}^{\,2} + a_1 l_{v0}^{\,2} + a_2 l_{u0} l_{v0}) +$$
$$(a_3 - a_3')l_{u0} + (a_4 - a_4')l_{v0} + a_5$$

Light Direction Extrapolation

- Input images are collected across a hemisphere of light directions, i.e. $-1 <= l_u, l_v <= 1$

- PTM's can be evaluated outside of the hemisphere, ($l_u, l_v < -1$ or $l_u, l_v > 1$)

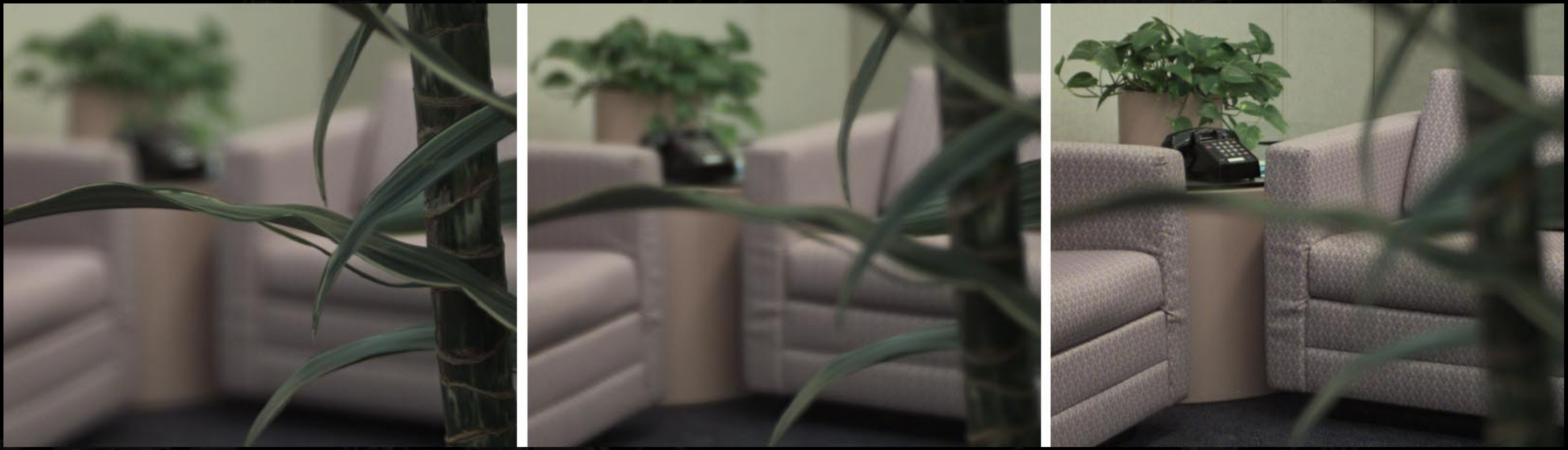


PTMs as Parametric Images

For each *(u,v)* we have:

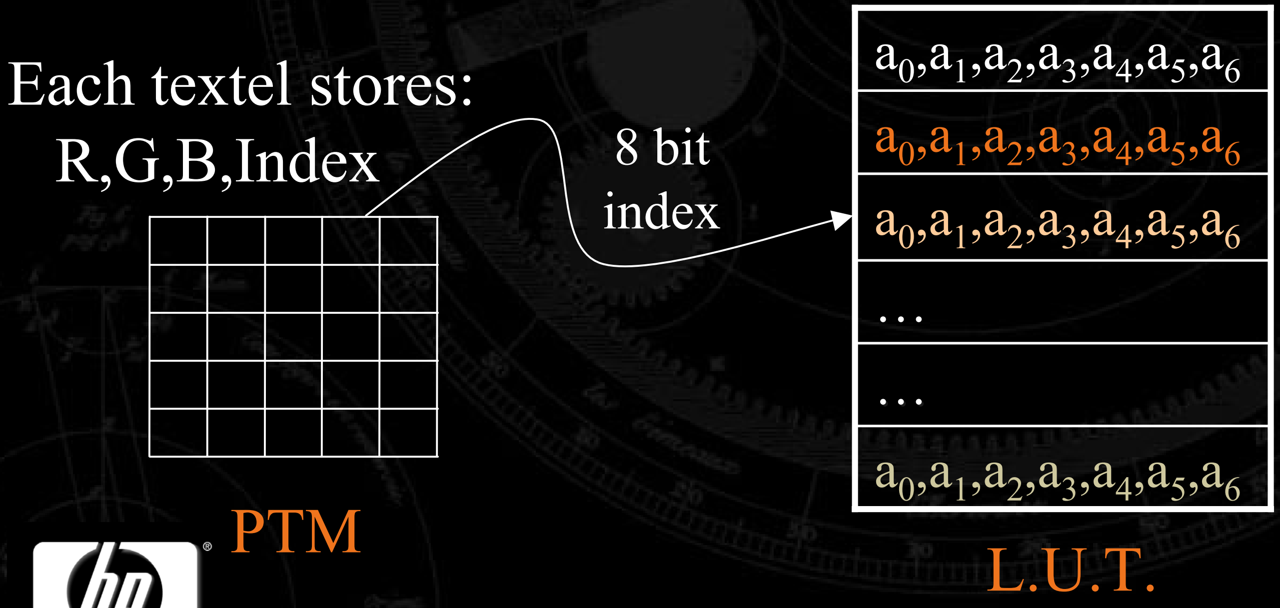
$$a_0 l_u^2 + a_1 l_v^2 + a_2 l_u l_v + a_3 l_u + a_4 l_v + a_5$$

Depth of Focus



Palletization

- Random Access – Each pixel treated independently.
- Light Space Lookup table – contains polynomials.
- LUT constructed by K-means clustering.
- RGB values can be stored in L.U.T. or image space.



Each textel stores:
R,G,B,Index

8 bit index

$a_0,a_1,a_2,a_3,a_4,a_5,a_6$

$a_0,a_1,a_2,a_3,a_4,a_5,a_6$

$a_0,a_1,a_2,a_3,a_4,a_5,a_6$

…

…

$a_0,a_1,a_2,a_3,a_4,a_5,a_6$

PTM

L.U.T.

Palletization

- Random Access – Each pixel treated independently.
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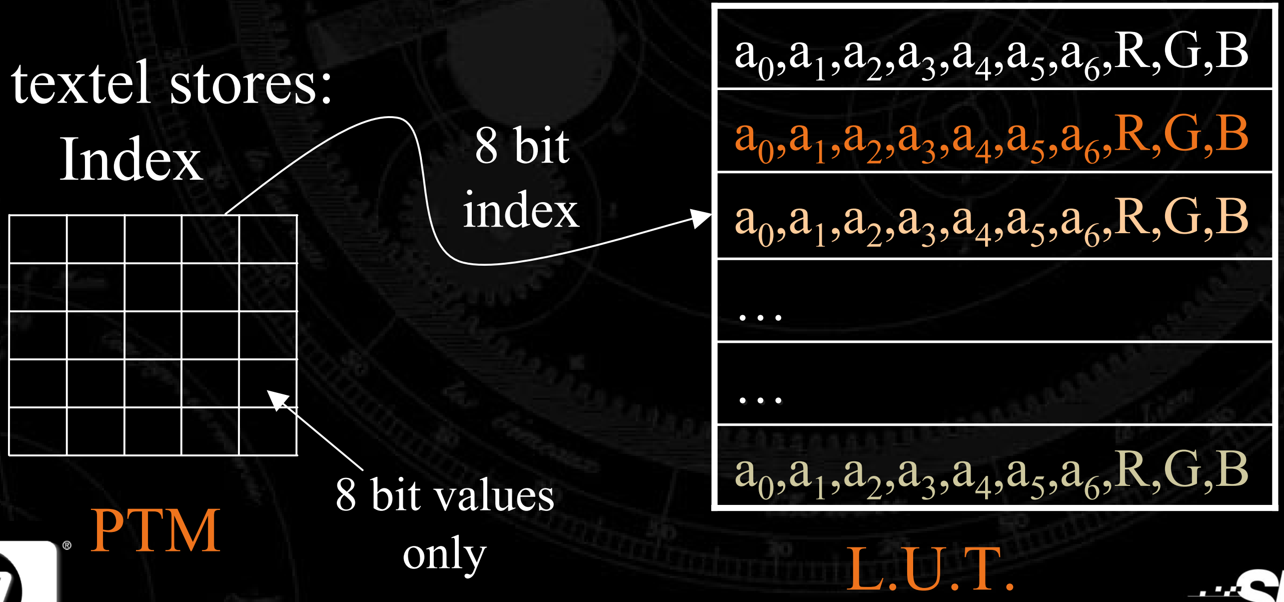

Each textel stores:

Index

8 bit index

8 bit values only

PTM

$a_0,a_1,a_2,a_3,a_4,a_5,a_6$,R,G,B

$a_0,a_1,a_2,a_3,a_4,a_5,a_6$,R,G,B

$a_0,a_1,a_2,a_3,a_4,a_5,a_6$,R,G,B

…

…

$a_0,a_1,a_2,a_3,a_4,a_5,a_6$,R,G,B

L.U.T.

Compression

- Allows better rate/distortion tradeoff than palletization.
- Similar to compression of multispectral images.
- Removes correlations within and between byte planes.
- Sacrifices pixel independence.
- Visible artifacts don't appear until ~10 bits/pixel.

Perceptually lossless results:

Original Size	Lossless	Loss = 1 grey level	Loss = 2 grey levels	Loss = 4 grey levels
72 - 144 bits	27.4 bits	17.1 bits	13.5 bits	10.1 bits

Future Work, Conclusions + Web Tools

- PTM's are fast, compact, effective representations.
- PTM's encoding opacity channels?
- Full BRDF's can be modelled using PTMs by trading off spatial variation with viewing angle.
- Applications in Medicine, Forensics, Paleontology

Tools available at hpl.hp.com/ptm

- sample PTMs
- PTM viewer
- Polynomial Fitter
- PTM format document

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The End

hpl.hp.com/ptm

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