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# Variations on Error Diffusion: Retrospectives and Future Trends

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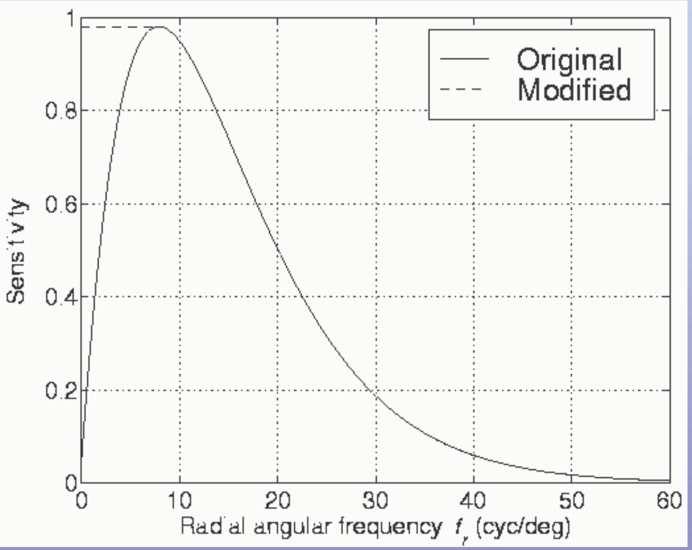
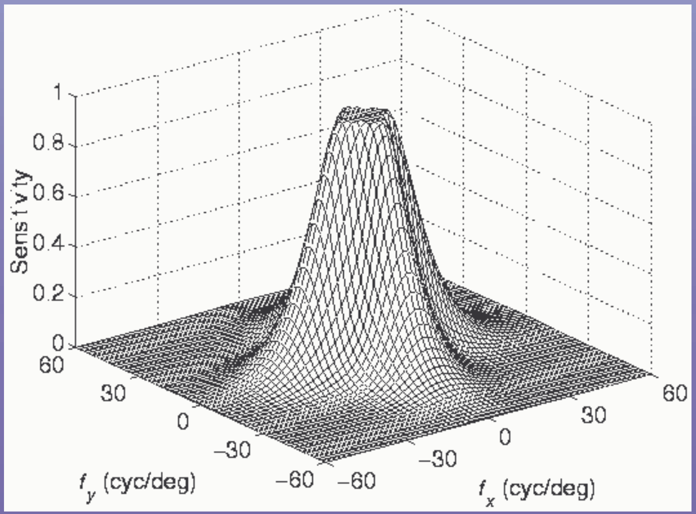
# Outline

- **Introduction**
- **Grayscale error diffusion**
  - Analysis and modeling
  - Enhancements
- **Color error diffusion halftoning**
  - Vector quantization with separable filtering
  - Matrix valued error filter methods
- **Conclusion**

# Human Visual System Modeling

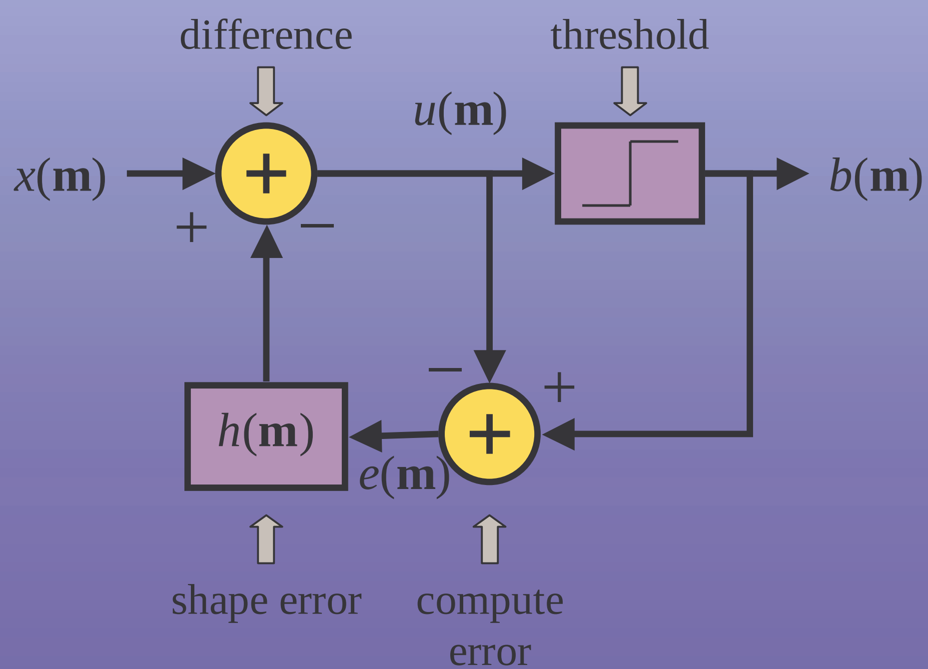
- **Contrast at particular spatial frequency for visibility**
  - Bandpass: non-dim backgrounds
    [Manos & Sakrison, 1974; 1978]
  - Lowpass: high-luminance office settings with low-contrast images
    [Georgeson & G. Sullivan, 1975]
  - Exponential decay [Näsäsen, 1984]
  - Modified lowpass version
    [e.g. J. Sullivan, Ray & Miller, 1990]
  - Angular dependence: cosine function [Sullivan, Miller & Pios, 1993]

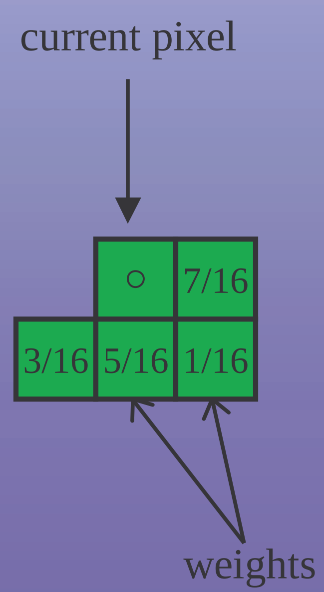


# Grayscale Error Diffusion Halftoning

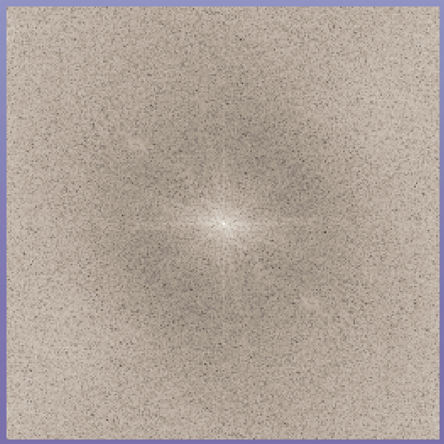
- **Nonlinear feedback system**
- **Shape quantization noise into high frequencies**
- **Design of error filter key to quality**



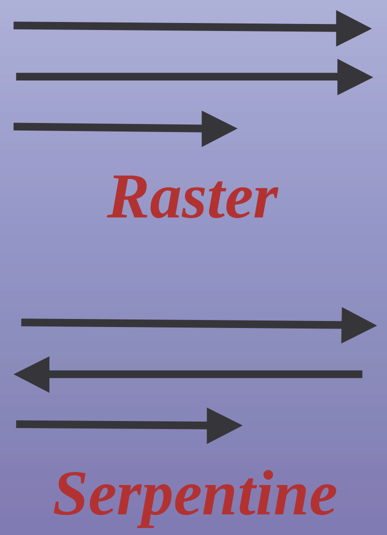


Error Diffusion


Spectrum

# Analysis of Error Diffusion I

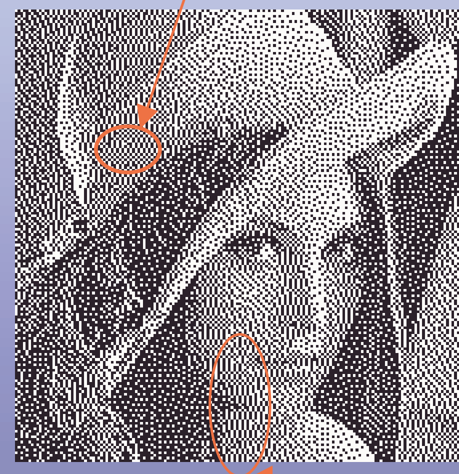
- **Error diffusion as 2-D sigma-delta modulation** [Anastassiou, 1989] [Bernard, 1991]
- **Error image** [Knox, 1992]
  - Error image correlated with input image
  - Sharpening proportional to correlation
- **Serpentine scan places more quantization error along diagonal frequencies than raster** [Knox, 1993]
- **Threshold modulation** [Knox, 1993]
  - Add signal (e.g. white noise) to quantizer input
  - Equivalent to error diffusing an input image modified by threshold modulation signal



# Example: Role of Error Image

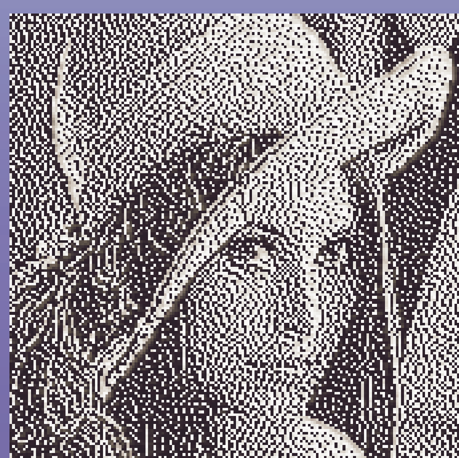
- **Sharpening proportional to correlation between error image and input image** [Knox, 1992]

**Floyd-Steinberg (1976)**



**Limit Cycles**

**Jarvis (1976)**



***Error images***

***Halftones***

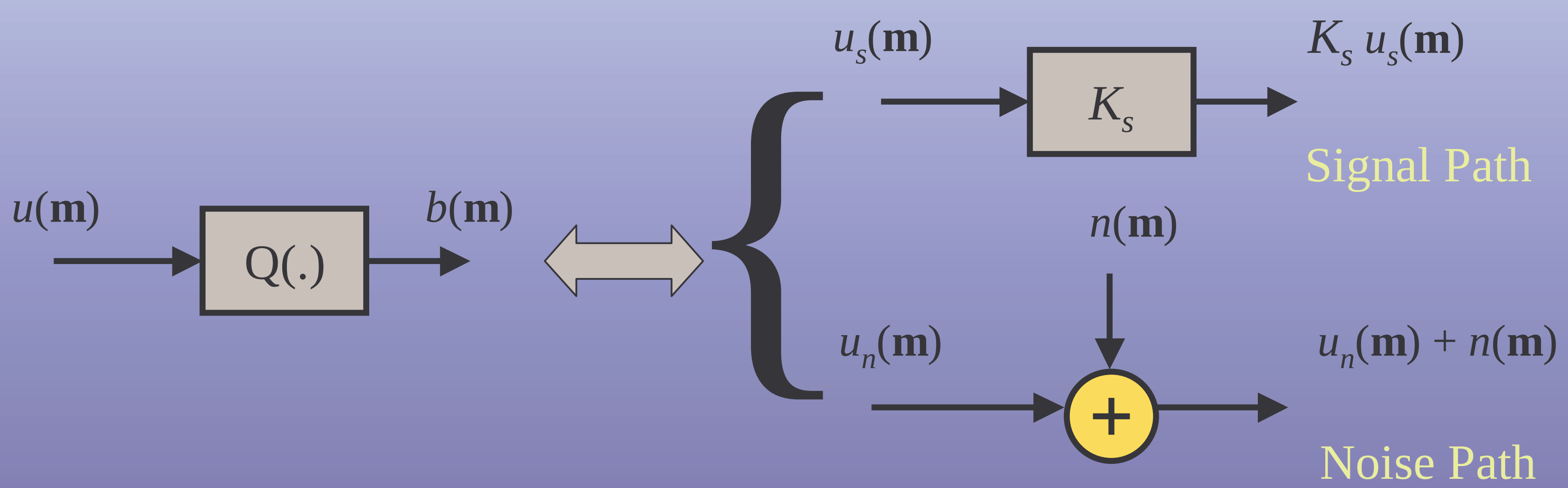
# Analysis of Error Diffusion II

- **Limit cycle behavior** [Fan & Eschbach, 1993]
  - For a limit cycle pattern, quantified likelihood of occurrence for given constant input as function of filter weights
  - Reduced likelihood of limit cycle patterns by changing filter weights
- **Stability of error diffusion** [Fan, 1993]
  - Sufficient conditions for bounded-input bounded-error stability: sum of absolute values of filter coefficients is one
- **Green noise error diffusion** [Levien, 1993] [Lau, Arce & Gallagher, 1998]
  - Promotes minority dot clustering


- **Linear gain model for quantizer** [Kite, Evans & Bovik, 2000]
  - Models sharpening and noise shaping effects

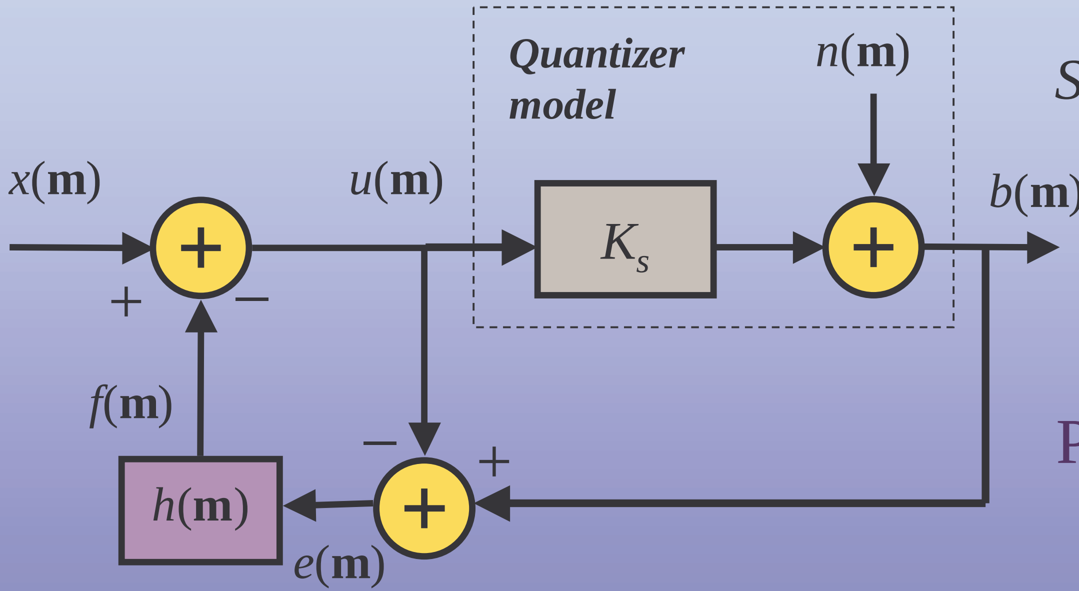
# Linear Gain Model for Quantizer

- **Extend sigma-delta modulation analysis to 2-D**
  - Linear gain model for quantizer in 1-D [Ardalan and Paulos, 1988]
  - Linear gain model for grayscale image [Kite, Evans, Bovik, 1997]




- **Error diffusion is modeled as linear, shift-invariant**
  - Signal transfer function (STF): quantizer acts as scalar gain
  - Noise transfer function (NTF): quantizer acts as additive noise

# Linear Gain Model for Quantizer



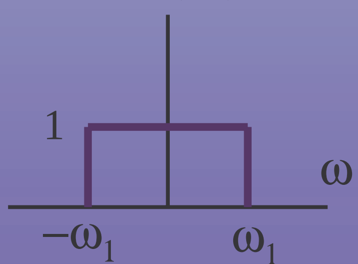

$$STF = \frac{B_s(\mathbf{z})}{X(\mathbf{z})} = \frac{K_s}{1 + (K_s - 1)H(\mathbf{z})}$$

$$NTF = \frac{B_n(\mathbf{z})}{N(\mathbf{z})} = 1 - H(\mathbf{z})$$

Put noise in high frequencies

$H(\mathbf{z})$ must be lowpass

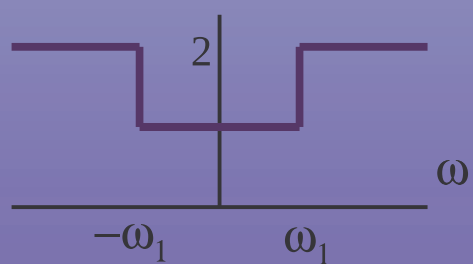
$H(\omega)$




Also, let $K_s = 2$

(Floyd-Steinberg)

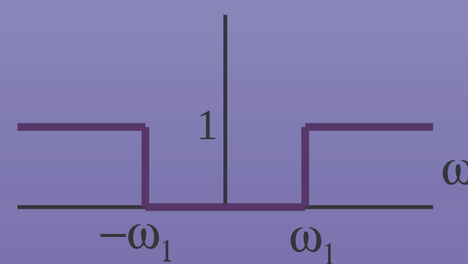
*STF*




Pass low frequencies

Enhance high frequencies

*NTF*




Highpass response

(independent of $K_s$ )

# Linear Gain Model for Quantizer

- **Best linear fit for $K_s$ between quantizer input $u$(m) and halftone $b$(m)**

$$K_s = \arg\min_{\alpha} \sum_{\mathbf{m}} \left(\alpha\, u(\mathbf{m}) - b(\mathbf{m})\right)^2$$

$$K_s = \frac{1}{2}\frac{\sum_{\mathbf{m}} |u(\mathbf{m})|}{\sum_{\mathbf{m}} u^2(\mathbf{m})} = \frac{1}{2}\frac{E\{|u(\mathbf{m})|\}}{E\{u^2(\mathbf{m}\}}$$

| *Image* | *Floyd* | *Stucki* | *Jarvis* |
|---|---|---|---|
| *barbara* | 2.01 | 3.62 | 3.76 |
| *boats* | 1.98 | 4.28 | 4.93 |
| *lena* | 2.09 | 4.49 | 5.32 |
| *mandrill* | 2.03 | 3.38 | 3.45 |
| *Average* | 2.03 | 3.94 | 4.37 |

  - Does not vary much for Floyd-Steinberg
  - Can use average value to estimate $K_s$ from only error filter

# Visual Quality Measures [Kite, Evans & Bovik, 2000]

- **Sharpening: proportional to $K_s$**

  Value of $K_s$: Floyd Steinberg < Stucki < Jarvis

- **Impact of noise on human visual system**

  Signal-to-noise (SNR) measures appropriate when noise is additive and signal independent

  Create unsharpened halftone $y[m_1,m_2]$ with flat signal transfer function using threshold modulation

  Weight signal/noise by contrast sensitivity function $C[k_1,k_2]$

$$\text{WSNR (dB)} = 10\log_{10} \frac{\sum_{k_1,k_2} \left| X[k_1,k_2]\, C[k_1,k_2] \right|^2}{\sum_{k_1,k_2} \left| \left( X[k_1,k_2] - Y[k_1,k_2] \right) C[k_1,k_2] \right|^2}$$

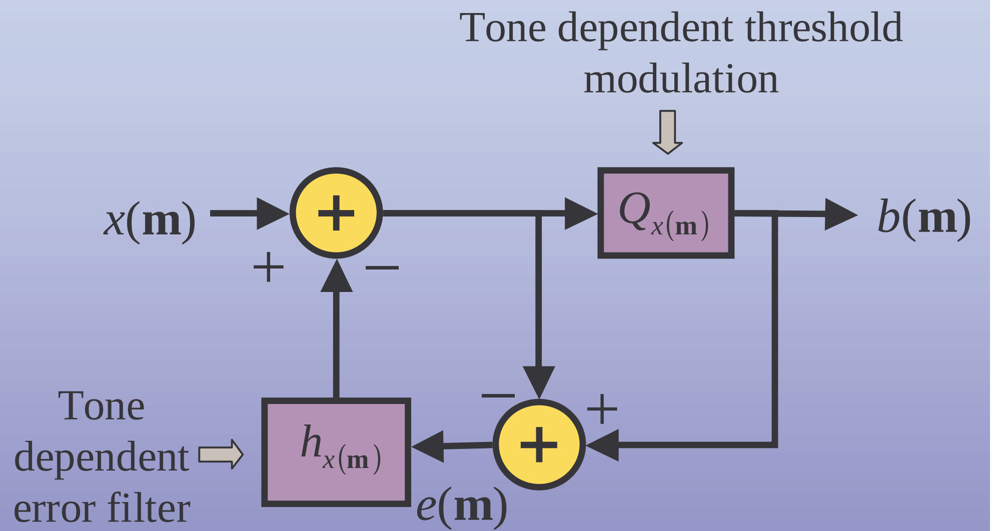
  Floyd-Steinberg > Stucki > Jarvis at all viewing distances

# Enhancements I: Error Filter Design

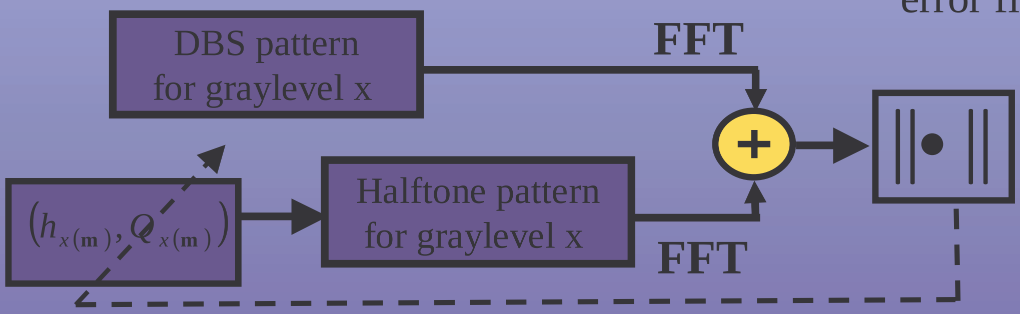
- **Longer error filters reduce directional artifacts**
  [Jarvis, Judice & Ninke, 1976] [Stucki, 1981] [Shiau & Fan, 1996]
- **Fixed error filter design: minimize mean-squared error weighted by a contrast sensitivity function**
  - Assume error image is white noise [Kolpatzik & Bouman, 1992]
  - Off-line training on images [Wong & Allebach, 1998]
- **Adaptive least squares error filter** [Wong, 1996]
- **Tone dependent filter weights for each gray level**
  [Eschbach, 1993] [Shu, 1995] [Ostromoukhov, 1998] [Li & Allebach, 2002]

# Example: Tone Dependent Error Diffusion

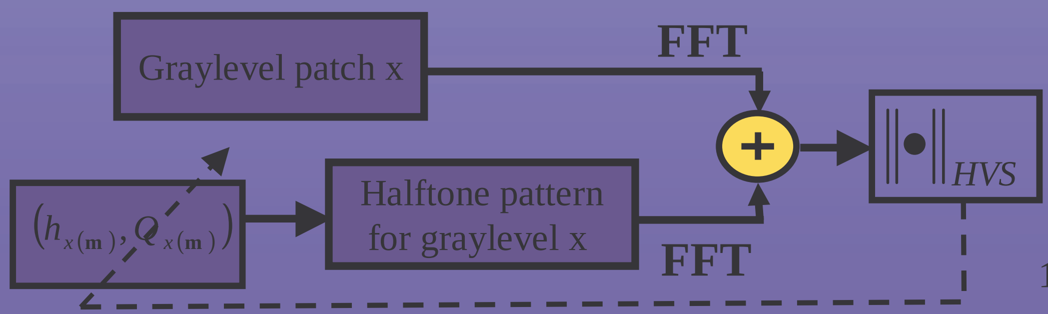
- **Train error diffusion weights and threshold modulation**
  [Li & Allebach, 2002]




**Midtone regions**

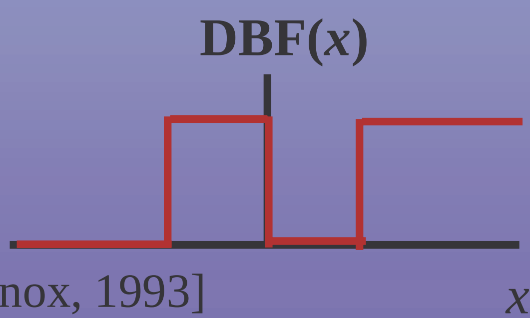



**Highlights and shadows**



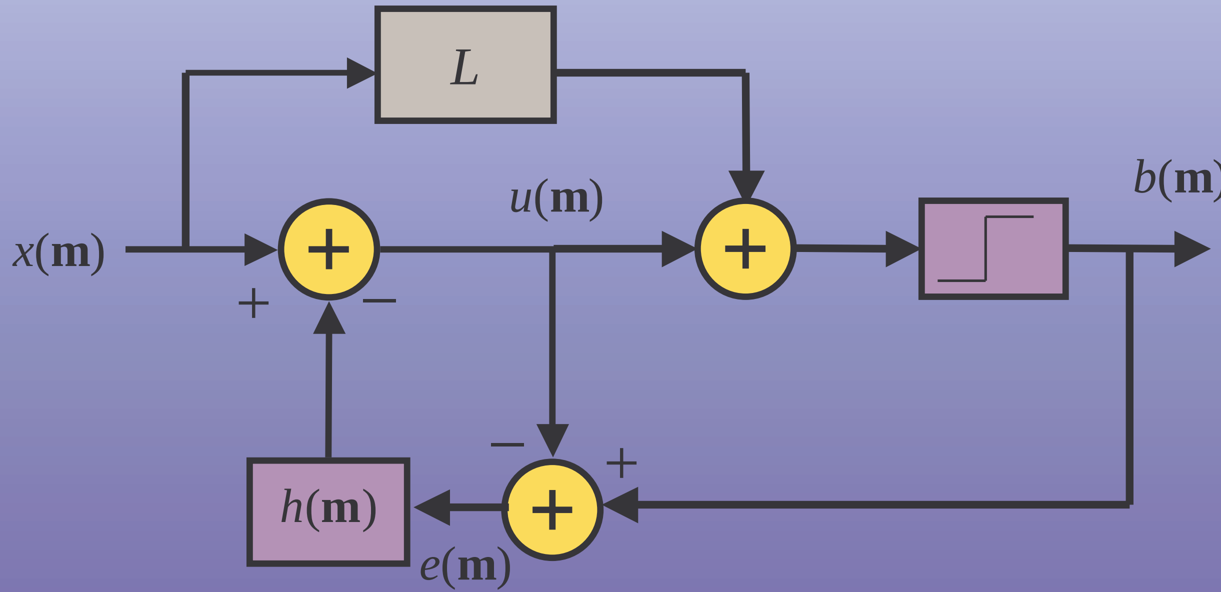
# Enhancements II: Controlling Artifacts

- **Sharpness control**
  - Edge enhancement error diffusion [Eschbach & Knox, 1991]
  - Linear frequency distortion removal [Kite, Evans & Bovik 1991]
  - Adaptive linear frequency distortion removal [Damera-Venkata & Evans, 2001]
- **Reducing worms in highlights & shadows** [Eschbach, 1993] [Shu, 1993] [Levien, 1993] [Eschbach, 1996] [Marcu, 1998]
- **Reducing mid-tone artifacts**
  - Filter weight perturbation [Ulichney, 1988]
  - Threshold modulation with noise array [Knox, 1993]
  - Deterministic bit flipping quant. [Damera-Venkata & Evans, 2001]
  - Tone dependent modification [Li & Allebach, 2002]



# Example: Sharpness Control in Error Diffusion

- **Adjust by threshold modulation** [Eschbach & Knox, 1991]
  - Scale image by gain $L$ and add it to quantizer input
  - Low complexity: one multiplication, one addition per pixel




- **Flatten signal transfer function** [Kite, Evans & Bovik, 2000]

$$L = \frac{1}{K_s} - 1 = \frac{1 - K_s}{K_s} \qquad L \in (-1, 0] \text{ since } K_s \geq 1$$

# Results

**Original**

**Floyd-Steinberg**

**Edge enhanced**

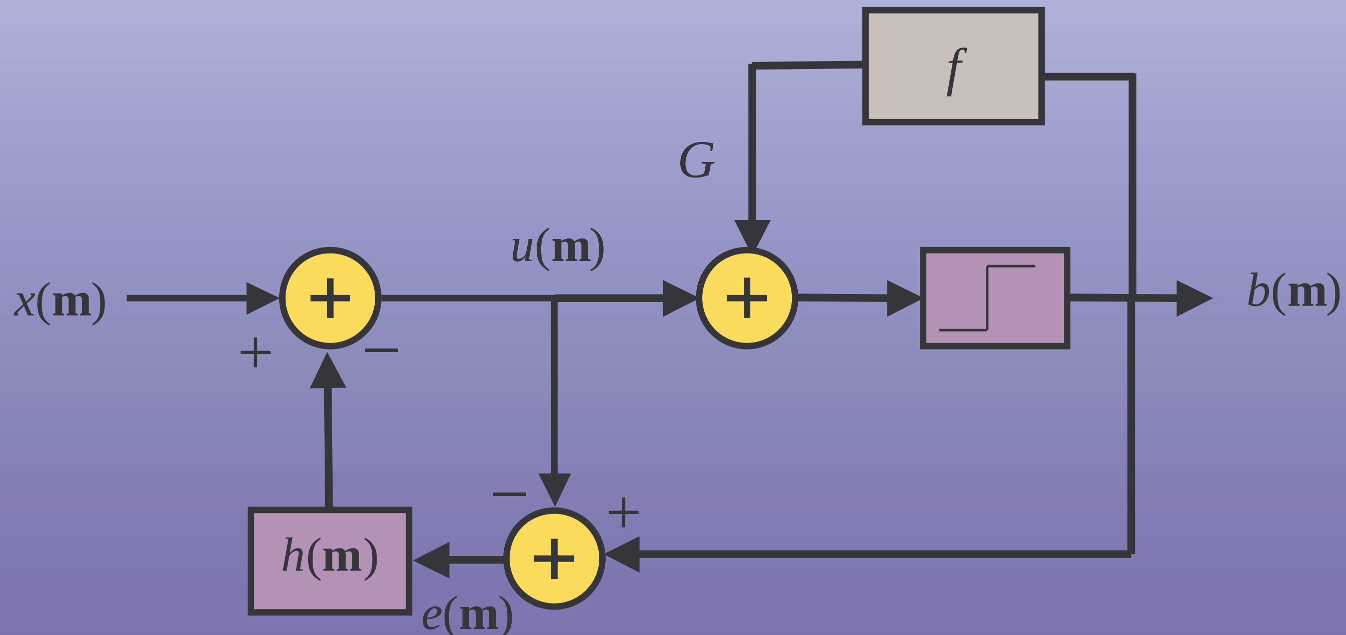
**Unsharpened**

# Enhancements III: Clustered Dot Error Diffusion

- **Feedback output to quantizer input** [Levien, 1993]
- **Dot to dot error diffusion** [Fan, 1993]
  - Apply clustered dot screen on block and diffuse error
  - Reduces contouring
- **Clustered minority pixel diffusion** [Li & Allebach, 2000]
- **Block error diffusion** [Damera-Venkata & Evans, 2001]
- **Clustered dot error diffusion using laser pulse width modulation** [He & Bouman, 2002]
  - Simultaneous optimization of dot density and dot size
  - Minimize distortion based on tone reproduction curve

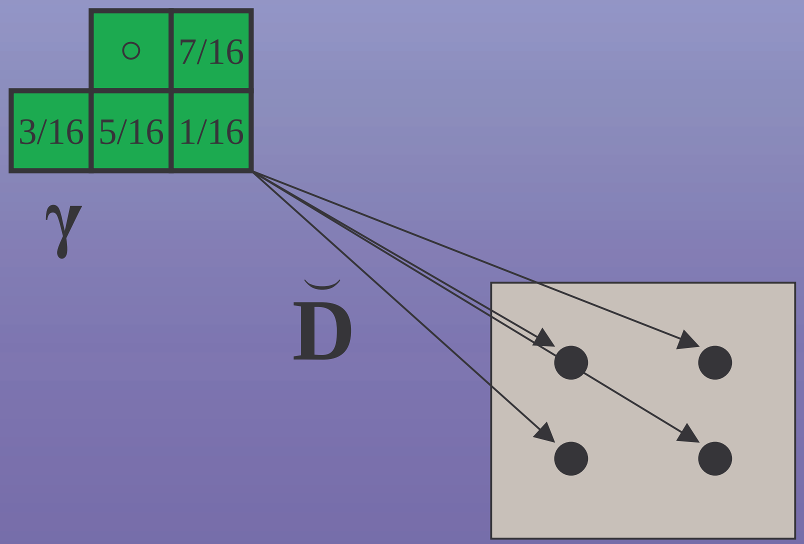
# Example #1: Green Noise Error Diffusion

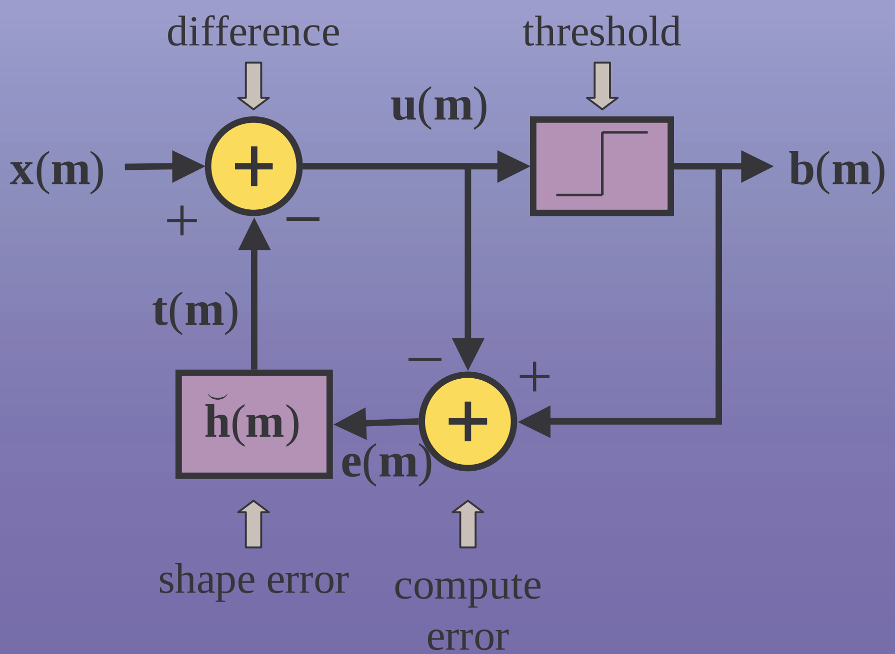
- **Output fed back to quantizer input** [Levien, 1993]
  - Gain $G$ controls coarseness of dot clusters
  - Hysteresis filter $f$ affects dot cluster shape



# Example #2: Block Error Diffusion

- **Process a pixel-block using a multifilter**
  [Damera-Venkata & Evans, 2001]
  - FM nature controlled by scalar filter prototype
  - Diffusion matrix decides distribution of error in block
  - In-block diffusions constant for all blocks to preserve isotropy



# Results

**Block error diffusion**

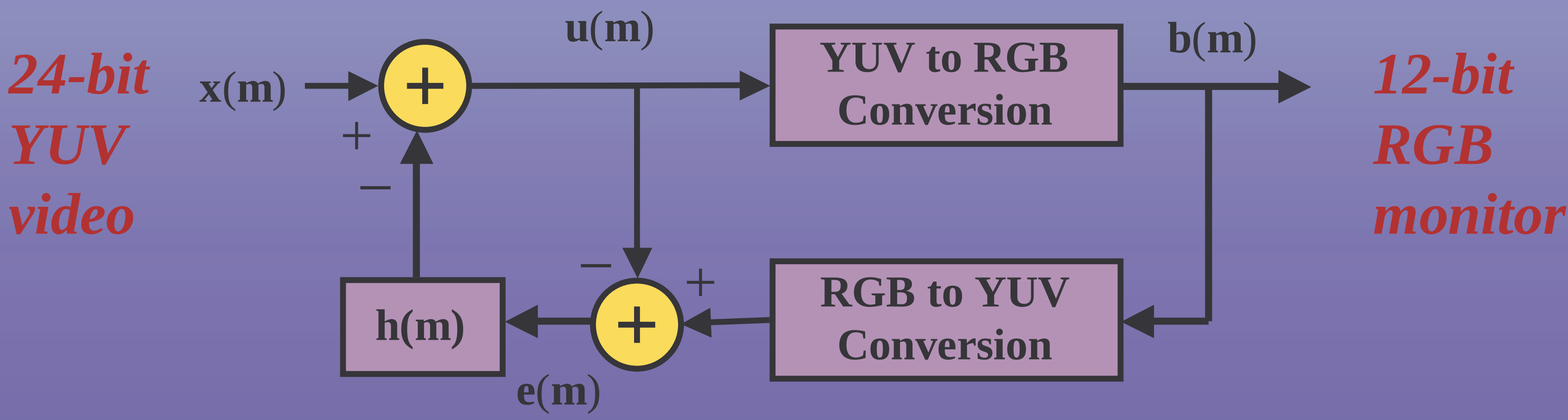
**Green-noise**

**DBF quantizer**

**Tone dependent**

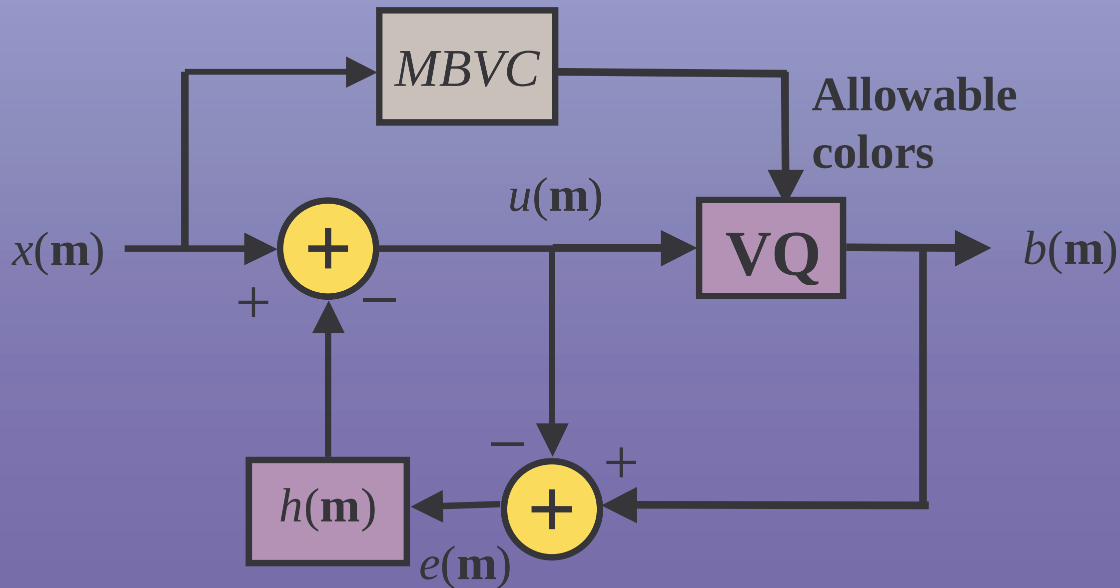
# Color Monitor Display Example (Palettization)

- **YUV color space**
  - Luminance (Y) and chrominance (U,V) channels
  - Widely used in video compression standards
  - Contrast sensitivity functions available for Y, U, and V
- **Display YUV on lower-resolution RGB monitor: use error diffusion on Y, U, V channels separably**

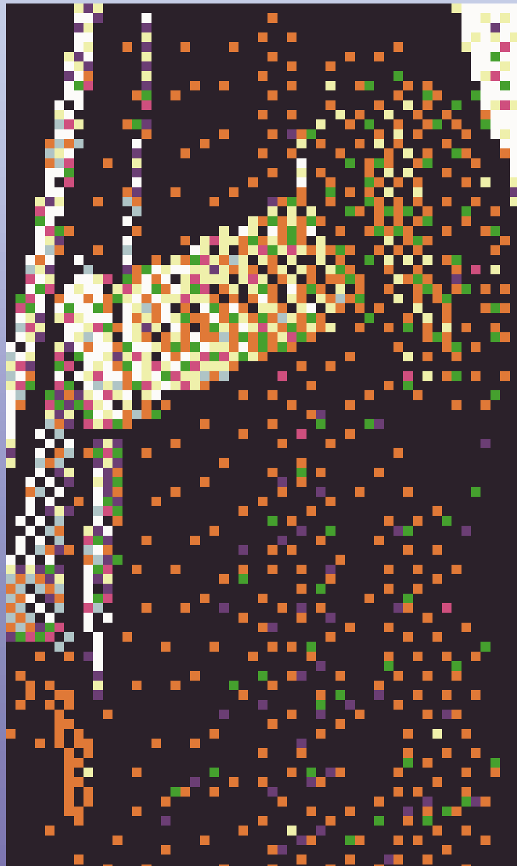


# Vector Quantization but Separable Filtering

- **Minimum Brightness Variation Criterion (MBVC)**
  [Shaked, Arad, Fitzhugh & Sobel, 1996]
  - Limit number of output colors to reduce luminance variation
  - Efficient tree-based quantization to render best color among allowable colors
  - Diffuse errors separably

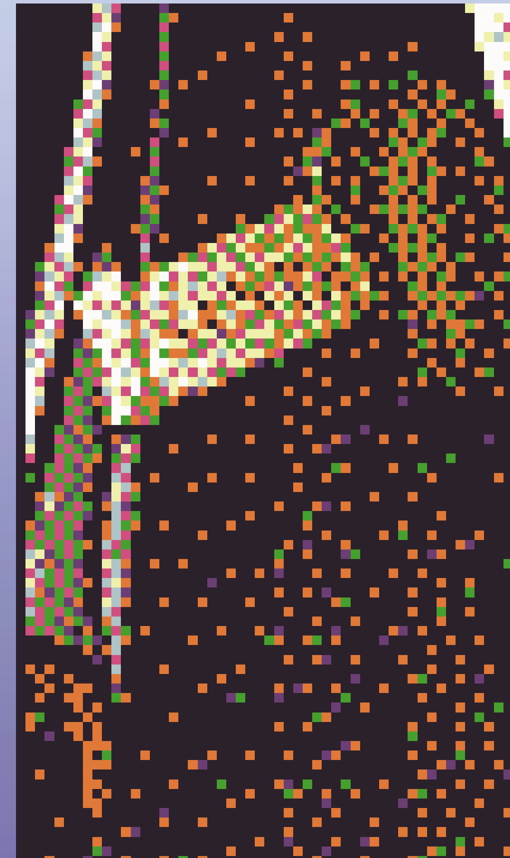


# Results



**Separable Floyd-Steinberg**

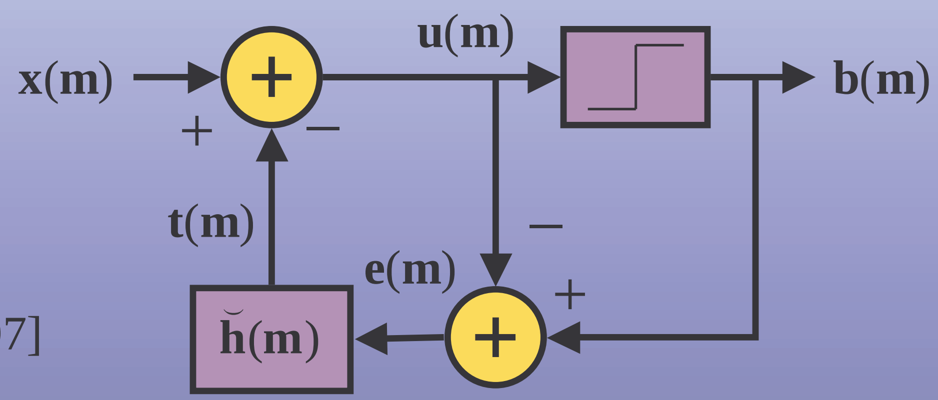
**Original**



**MBVC halftone**

# Non-Separable Color Halftoning for Display

- **Input image has a vector of values at each pixel (e.g. vector of red, green, and blue components)**
  - Error filter has matrix-valued coefficients
  - Algorithm for adapting matrix coefficients based on mean-squared error in RGB space [Akarun, Yardimci & Cetin, 1997]




$$\mathbf{t}(\mathbf{m}) = \sum_{\mathbf{k} \in \wp} \underbrace{\breve{\mathbf{h}}(\mathbf{k})}_{\text{matrix}} \underbrace{\mathbf{e}(\mathbf{m} - \mathbf{k})}_{\text{vector}}$$

- **Optimization problem**
  - Given a human visual system model, find color error filter that minimizes average visible noise power subject to diffusion constraints [Damera-Venkata & Evans, 2001]
  - Linearize color vector error diffusion, and use linear vision model in which Euclidean distance has perceptual meaning

# Matrix Gain Model for the Quantizer

- **Replace scalar gain w/ matrix** [Damera-Venkata & Evans, 2001]

$$\breve{\mathbf{K}}_s = \arg\min_{\breve{\mathbf{A}}} E\left(\left\|\mathbf{b}(\mathbf{m}) - \breve{\mathbf{A}}\,\mathbf{u}(\mathbf{m})\right\|^2\right) = \breve{\mathbf{C}}_{\mathbf{bu}}\breve{\mathbf{C}}_{\mathbf{uu}}^{-1}$$

$$\breve{\mathbf{K}}_n = \breve{\mathbf{I}}$$

$\mathbf{u}(\mathbf{m})$ quantizer input
$\mathbf{b}(\mathbf{m})$ quantizer output

  - Noise uncorrelated with signal component of quantizer input
  - Convolution becomes matrix–vector multiplication in frequency domain

***Noise component of output***

$$\mathbf{B}_n(\mathbf{z}) = \left(\breve{\mathbf{I}} - \breve{\mathbf{H}}(\mathbf{z})\right)\mathbf{N}(\mathbf{z})$$

***Signal component of output***

$$\mathbf{B}_s(\mathbf{z}) = \breve{\mathbf{K}}\left(\breve{\mathbf{I}} + \breve{\mathbf{H}}(\mathbf{z})\left(\breve{\mathbf{K}} - \breve{\mathbf{I}}\right)\right)^{-1}\mathbf{X}(\mathbf{z})$$

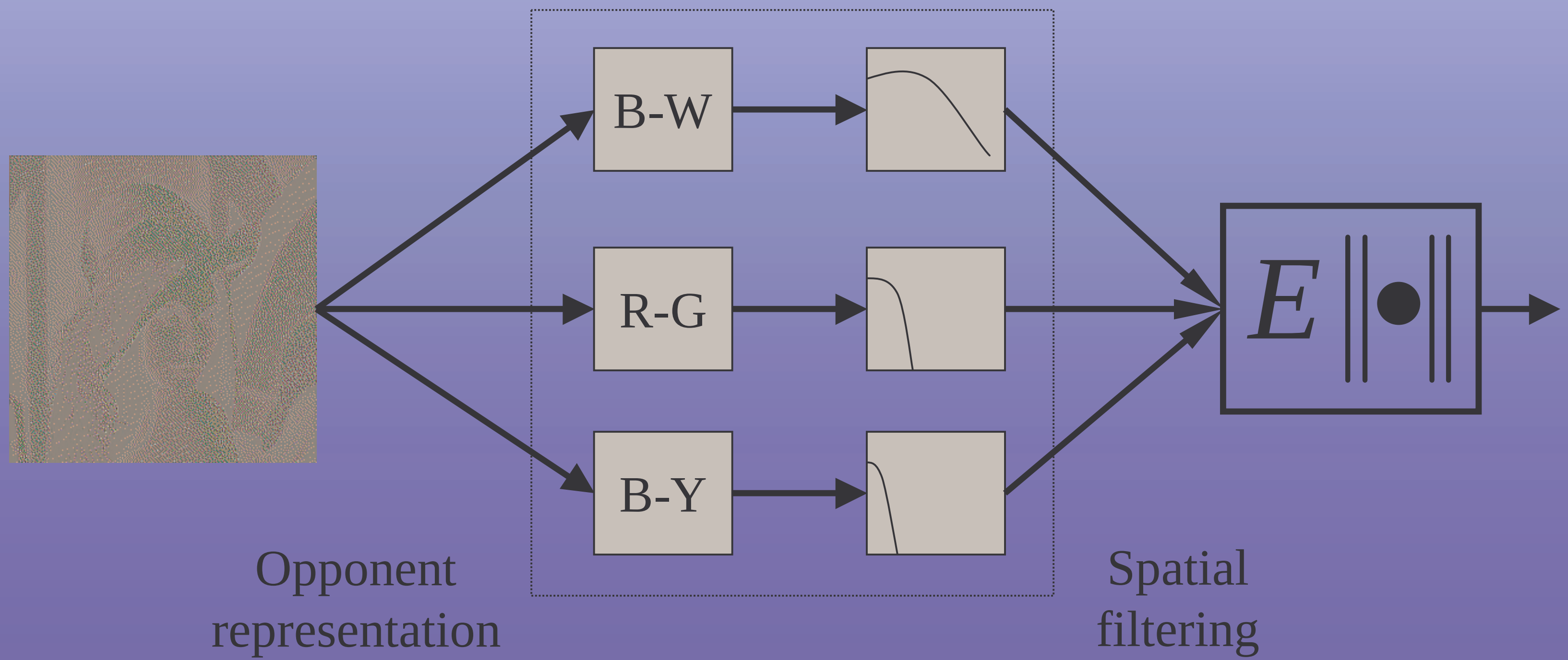
***Grayscale results***

$$\left(1 - H(\mathbf{z})\right)N(\mathbf{z})$$

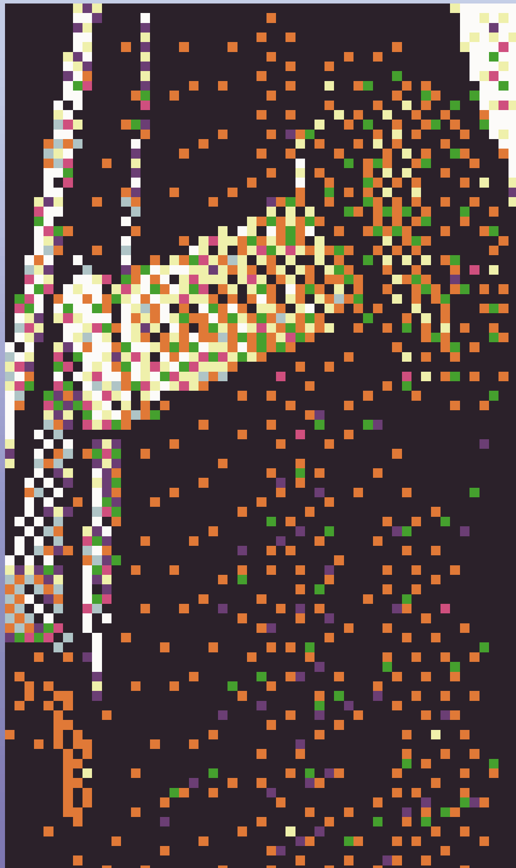
$$\frac{K_s\,X(\mathbf{z})}{1 + \left(K_s - 1\right)H(\mathbf{z})}$$

# Linear Color Vision Model

- **Undo gamma correction to map to sRGB**
- **Pattern-color separable model** [Poirson & Wandell, 1993]
  - Forms the basis for Spatial CIELab [Zhang & Wandell, 1996]
  - Pixel-based color transformation

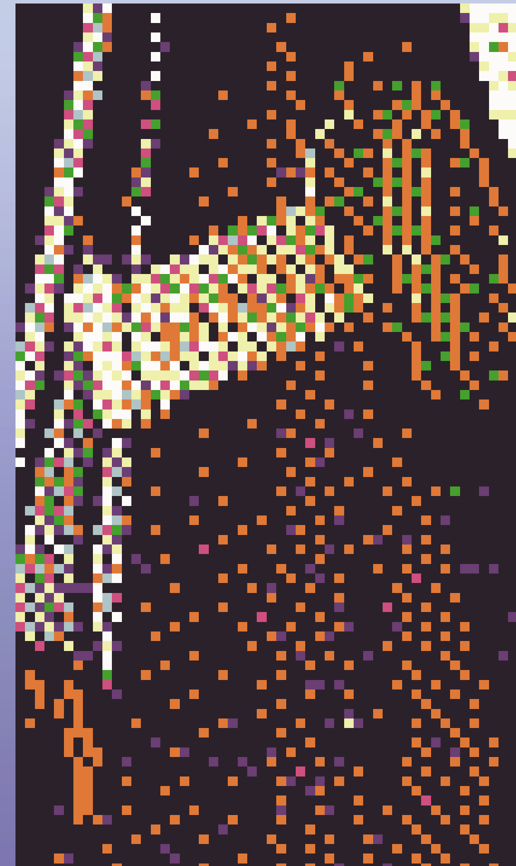


# Example



**Separable Floyd-Steinberg**

**Original**



**Optimum vector error filter**

# Evaluating Linear Vision Models

[Monga, Geisler & Evans, 2003]

- **An objective measure is the improvement in noise shaping over separable Floyd-Steinberg**
- **Subjective testing based on *paired comparison task***
  - Observer chooses halftone that looks closer to original
  - Online at www.ece.utexas.edu/~vishal/cgi-bin/test.html

**halftone A**

**original**

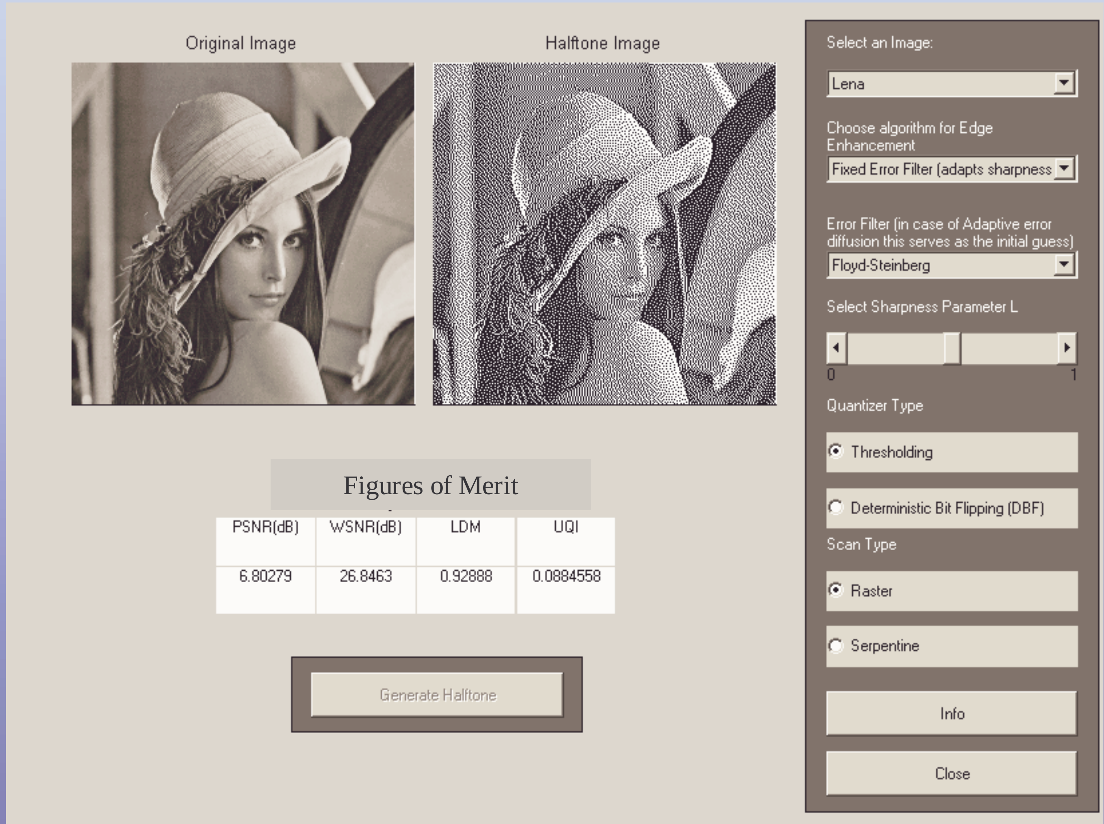
**halftone B**

# Subjective Testing

- **Binomial parameter estimation model**
  - Halftone generated by particular HVS model considered better if picked over another 60% or more of the time
  - Need 960 paired comparison of each model to determine results within tolerance of 0.03 with 95% confidence
  - Four models would correspond to 6 comparison pairs, total 6 x 960 = 5760 comparisons needed
  - Observation data collected from over 60 subjects each of whom judged 96 comparisons
- **In decreasing subjective (and objective) quality**

  Linearized CIELab >> Opponent > YUV ≥ YIQ

# *UT Austin Halftoning Toolbox 1.1 for MATLAB*




**Grayscale & color halftoning methods**

1. Classical and user-defined screens
2. Classical error diffusion methods
3. Edge enhancement error diffusion
4. Green noise error diffusion
5. Block error diffusion

**Additional color halftoning methods**

1. Minimum brightness variation quadruple error diffusion
2. Vector error diffusion

**Figures of merit for halftone evaluation**

1. Peak signal-to-noise ratio (PSNR)
2. Weighted signal-to-noise ratio (WSNR)
3. Linear distortion measure (LDM)
4. Universal quality index (UQI)

***Freely distributable software available at***
**http://ww.ece.utexas.edu/~bevans/projects/halftoning/toolbox**

UT Austin Center for Perceptual Systems, www.cps.utexas.edu

# Selected Open Problems

- **Analysis and modeling**
  - Find less restrictive sufficient conditions for stability of color vector error filters
  - Find link between spectral characteristics of the halftone pattern and linear gain model at a given graylevel
  - Model statistical properties of quantization noise
- **Enhancements**
  - Find vector error filters and threshold modulation for optimal tone-dependent vector color error diffusion
  - Incorporate printer models into optimal framework for vector color error filter design

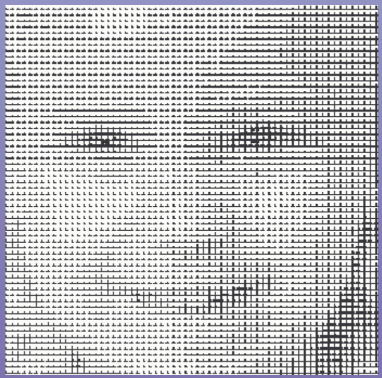
# Backup Slides

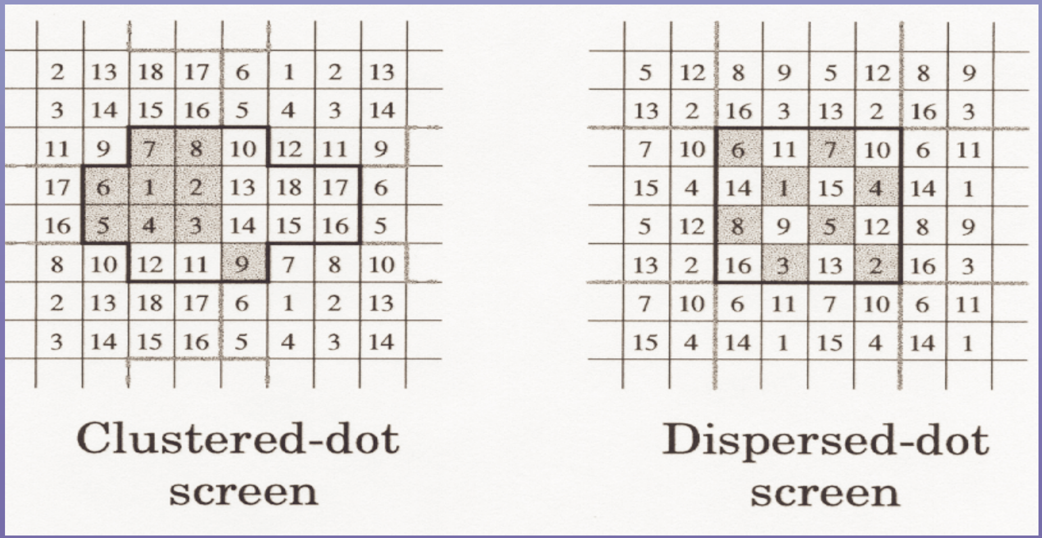
# Need for Digital Image Halftoning

- **Examples of reduced grayscale/color resolution**
  - Laser and inkjet printers
  - Facsimile machines
  - Low-cost liquid crystal displays
- **Halftoning is wordlength reduction for images**
  - Grayscale: 8-bit to 1-bit (binary)
  - Color displays: 24-bit RGB to 8-bit RGB
  - Color printers: 24-bit RGB to CMY (each color binarized)
- **Halftoning tries to reproduce full range of gray/ color while preserving quality & spatial resolution**
  - Screening methods are pixel-parallel, fast, and simple
  - Error diffusion gives better results on some media

# Screening (Masking) Methods

- **Periodic array of thresholds smaller than image**
  - Spatial resampling leads to aliasing (gridding effect)
  - Clustered dot screening produces a coarse image that is more resistant to printer defects such as ink spread
  - Dispersed dot screening has higher spatial resolution
  - Blue noise masking uses large array of thresholds



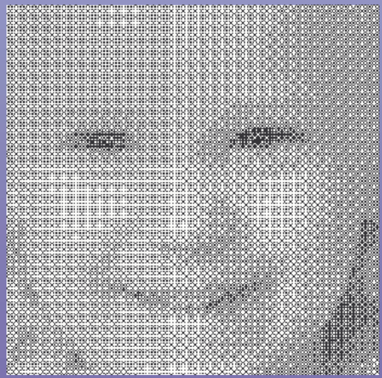


| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 | 13 | 18 | 17 | 6 | 1 | 2 | 13 |
| 3 | 14 | 15 | 16 | 5 | 4 | 3 | 14 |
| 11 | 9 | 7 | 8 | 10 | 12 | 11 | 9 |
| 17 | 6 | 1 | 2 | 13 | 18 | 17 | 6 |
| 16 | 5 | 4 | 3 | 14 | 15 | 16 | 5 |
| 8 | 10 | 12 | 11 | 9 | 7 | 8 | 10 |
| 2 | 13 | 18 | 17 | 6 | 1 | 2 | 13 |
| 3 | 14 | 15 | 16 | 5 | 4 | 3 | 14 |

Clustered-dot screen

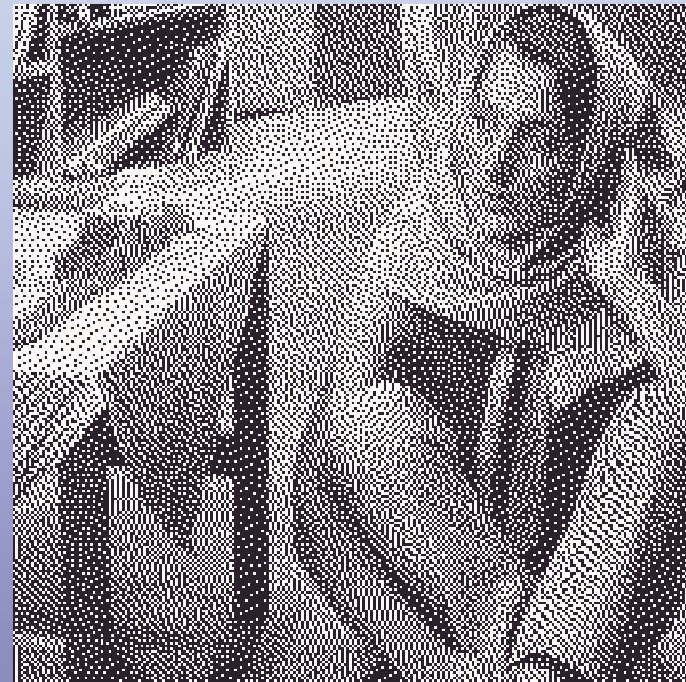
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 5 | 12 | 8 | 9 | 5 | 12 | 8 | 9 |
| 13 | 2 | 16 | 3 | 13 | 2 | 16 | 3 |
| 7 | 10 | 6 | 11 | 7 | 10 | 6 | 11 |
| 15 | 4 | 14 | 1 | 15 | 4 | 14 | 1 |
| 5 | 12 | 8 | 9 | 5 | 12 | 8 | 9 |
| 13 | 2 | 16 | 3 | 13 | 2 | 16 | 3 |
| 7 | 10 | 6 | 11 | 7 | 10 | 6 | 11 |
| 15 | 4 | 14 | 1 | 15 | 4 | 14 | 1 |

Dispersed-dot screen

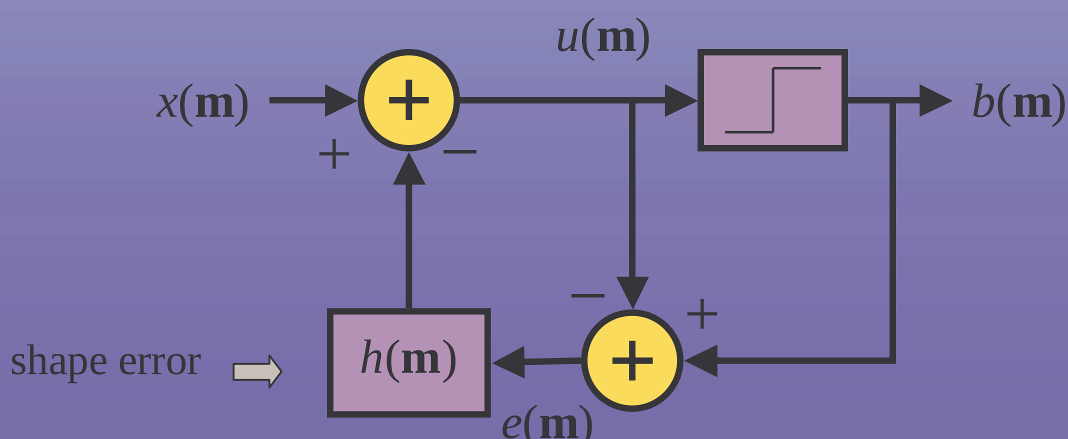


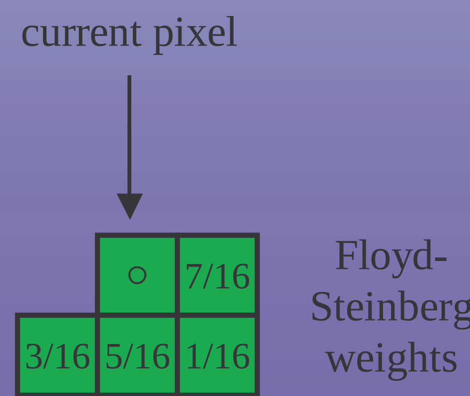
# Basic Grayscale Error Diffusion

**Original**



**Halftone**



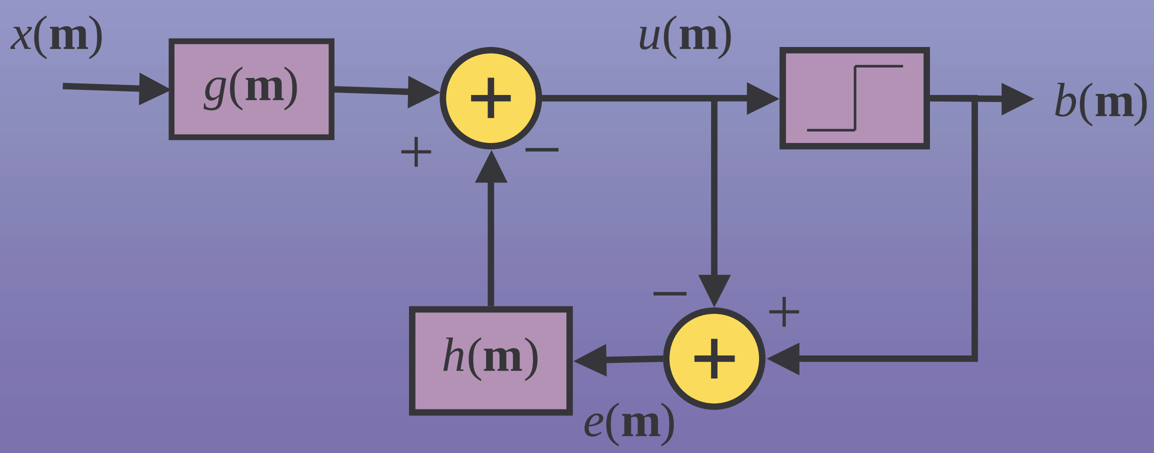
# Compensation for Frequency Distortion

- **Flatten signal transfer function** [Kite, Evans, Bovik, 2000]

$$L = \frac{1 - K_s}{K_s} \quad ( \quad L \in (-1, 0] \text{ since } K_s \geq 1$$

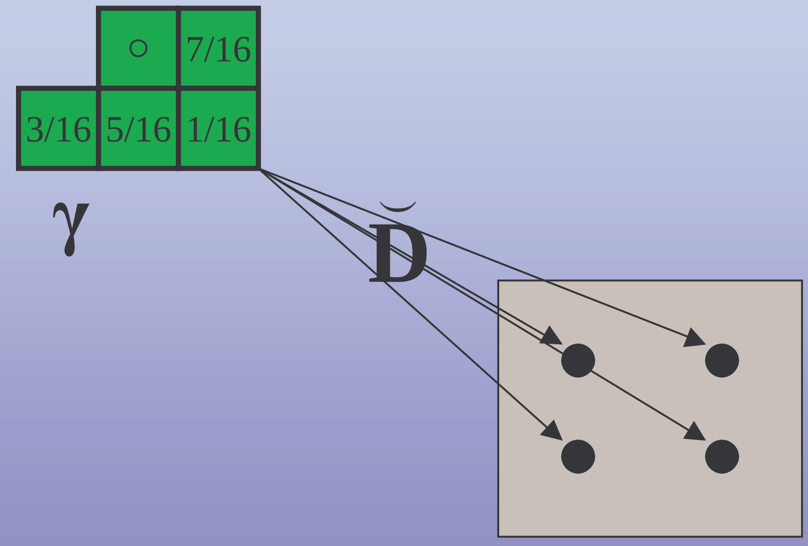
- **Pre-filtering equivalent to threshold modulation**

$$G(z) = 1 + L(1 - H(z)) \quad ( \quad \text{FIR filter}$$



# Block FM Halftoning Error Filter Design

- **FM nature of algorithm controlled by scalar filter prototype**
- **Diffusion matrix decides distribution of error within a block**
- **In-block diffusions are constant for all blocks to preserve isotropy**




$$\breve{\Gamma} = \boldsymbol{\gamma} \otimes \breve{\mathbf{D}}$$

$\breve{\mathbf{D}}$ diffusion matrix

$$\breve{\mathbf{D}} = \frac{1}{N^2}\left[\breve{\mathbf{1}}\right]$$

$N$ is the block size

# Linear Color Vision Model

- **Undo gamma correction on RGB image**
- **Color separation** [Damera-Venkata & Evans, 2001]
  - Measure power spectral distribution of RGB phosphor excitations
  - Measure absorption rates of long, medium, short (LMS) cones
  - Device dependent transformation **C** from RGB to LMS space
  - Transform LMS to opponent representation using **O**
  - Color separation may be expressed as **T** = **OC**
- **Spatial filtering included using matrix filter** $\breve{\mathbf{d}}(\mathbf{m})$
- **Linear color vision model**

$\breve{\mathbf{v}}(\mathbf{m}) = \breve{\mathbf{d}}(\mathbf{m})\breve{\mathbf{T}}$ where $\breve{\mathbf{d}}(\mathbf{m})$ is a diagonal matrix

# Designing the Error Filter

- **Eliminate linear distortion filtering before error diffusion**
- **Optimize error filter h(m) for noise shaping**

$$\min E\left[\left\|\mathbf{b}_n(\mathbf{m})\right\|^2\right] = E\left\{\left\|\breve{\mathbf{v}}(\mathbf{m}) * \left(\breve{\mathbf{I}} - \breve{\mathbf{h}}(\mathbf{m})\right) * \mathbf{n}(\mathbf{m})\right\|^2\right\}$$

Subject to diffusion constraints

$$\left(\sum_{\mathbf{m}} \breve{\mathbf{h}}(\mathbf{m})\right)\mathbf{1} = \mathbf{1}$$

where $\breve{\mathbf{v}}(\mathbf{m})$ linear model of human visual system

$*$ matrix-valued convolution

# Generalized Optimum Solution

- **Differentiate scalar objective function for visual noise shaping w/r to matrix-valued coefficients**

$$\frac{d\left\{E\left[\left\|\mathbf{b}_n(\mathbf{m})\right\|^2\right]\right\}}{d\mathbf{h}(\mathbf{i})} = \mathbf{0} \quad \forall\, \mathbf{i} \in \wp \qquad \left\|\mathbf{x}\right\| = Tr\left(\mathbf{x}\mathbf{x}'\right)$$

- **Write norm as trace and differentiate trace using identities from linear algebra**

$$\frac{d\left\{Tr\left(\breve{\mathbf{A}}\breve{\mathbf{X}}\right)\right\}}{d\breve{\mathbf{X}}} = \breve{\mathbf{A}}' \qquad \frac{d\left\{Tr\left(\breve{\mathbf{X}}'\breve{\mathbf{A}}\breve{\mathbf{X}}\breve{\mathbf{B}}\right)\right\}}{d\breve{\mathbf{X}}} = \breve{\mathbf{A}}\breve{\mathbf{X}}\breve{\mathbf{B}} + \breve{\mathbf{A}}'\breve{\mathbf{X}}\breve{\mathbf{B}}'$$

$$\frac{d\left\{Tr\left(\breve{\mathbf{A}}\breve{\mathbf{X}}\breve{\mathbf{B}}\right)\right\}}{d\breve{\mathbf{X}}} = \breve{\mathbf{A}}'\breve{\mathbf{B}}' \qquad Tr\left(\breve{\mathbf{A}}\breve{\mathbf{B}}\right) = Tr\left(\breve{\mathbf{B}}\breve{\mathbf{A}}\right)$$

# Generalized Optimum Solution (cont.)

- **Differentiating and using linearity of expectation operator give a generalization of the Yule-Walker equations**

$$\sum_{\mathbf{k}} \breve{\mathbf{v}}'(\mathbf{k})\breve{\mathbf{r}}_{\mathbf{an}}(-\mathbf{i}-\mathbf{k}) = \sum_{\mathbf{p}}\sum_{\mathbf{q}}\sum_{\mathbf{s}} \breve{\mathbf{v}}'(\mathbf{s})\breve{\mathbf{v}}(\mathbf{q})\breve{\mathbf{h}}(\mathbf{p})\breve{\mathbf{r}}_{\mathbf{nn}}(-\mathbf{i}-\mathbf{s}+\mathbf{p}+\mathbf{q})$$

**where**

$$\mathbf{a}(\mathbf{m}) = \breve{\mathbf{v}}(\mathbf{m}) * \mathbf{n}(\mathbf{m})$$

- **Assuming white noise injection**

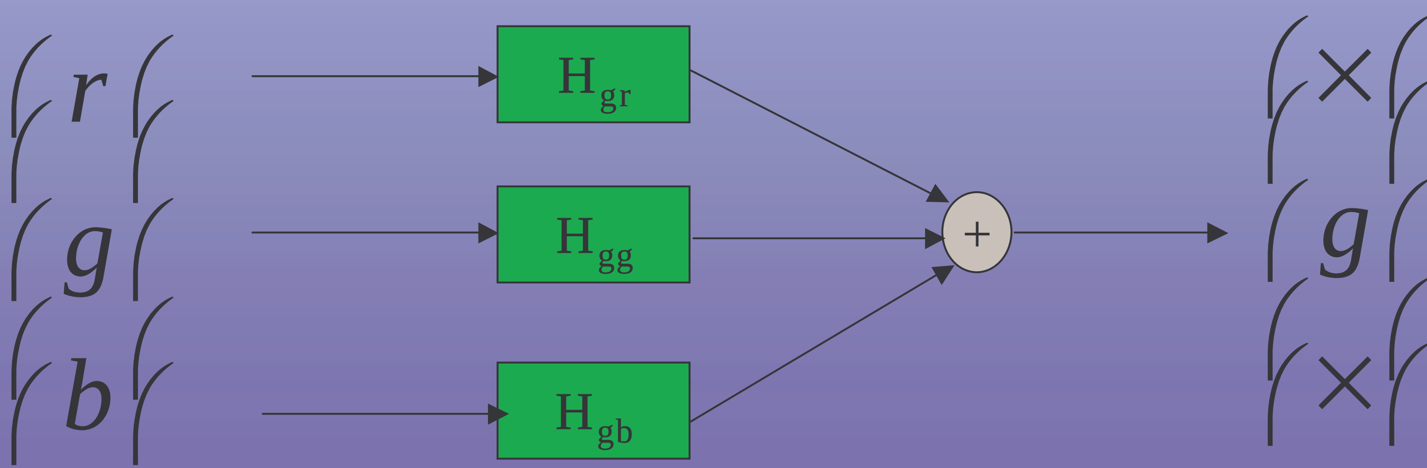
$$\mathbf{r}_{\mathbf{nn}}(\mathbf{k}) = E[\mathbf{n}(\mathbf{m})\,\mathbf{n}'(\mathbf{m}+\mathbf{k})] \approx \delta(\mathbf{k})$$

$$\mathbf{r}_{\mathbf{an}}(\mathbf{k}) = E[\mathbf{a}(\mathbf{m})\,\mathbf{n}'(\mathbf{m}+\mathbf{k})] \approx \breve{\mathbf{v}}(-\mathbf{k})$$

- **Solve using gradient descent with projection onto constraint set**

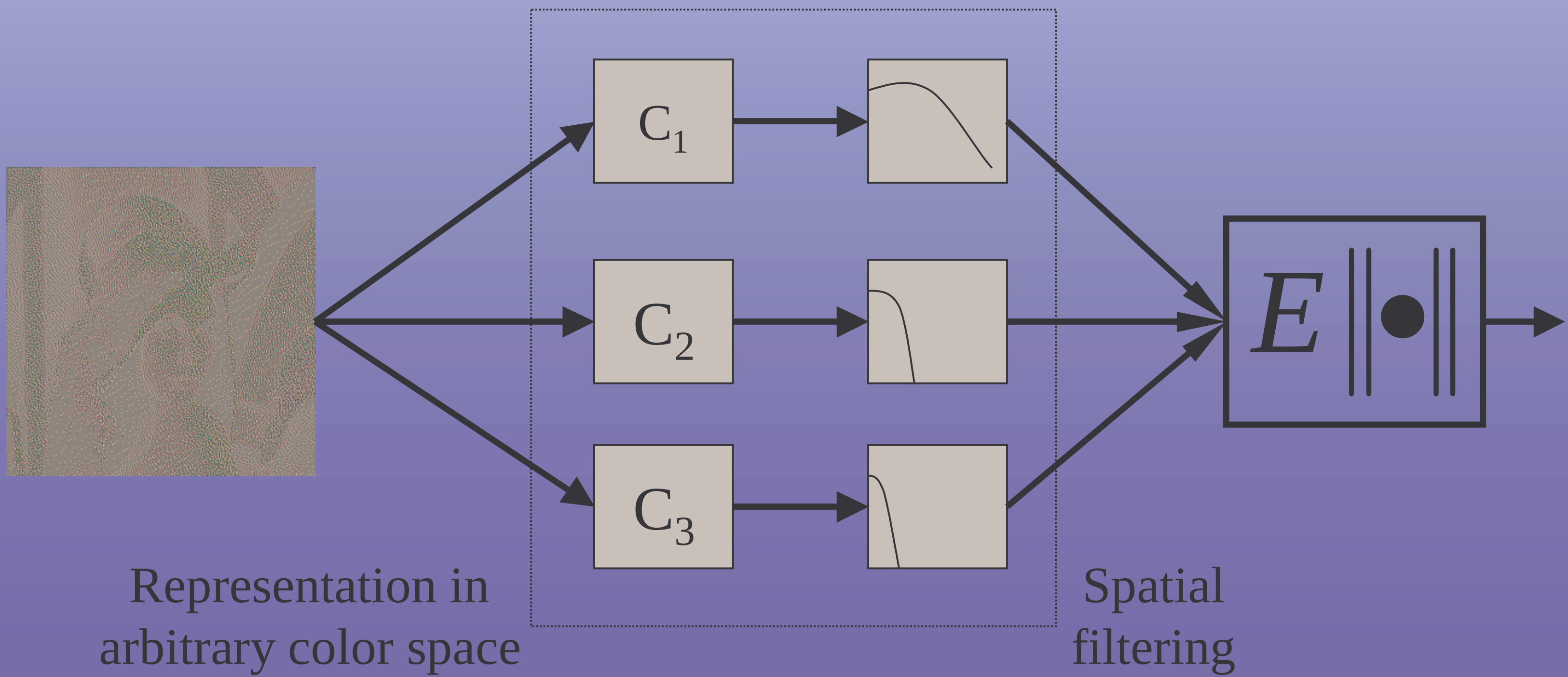
# Implementation of Vector Color Error Diffusion

$$\breve{\mathbf{H}}(\mathbf{z}) = \begin{pmatrix} H_{rr}(\mathbf{z}) & H_{rg}(\mathbf{z}) & H_{rb}(\mathbf{z}) \\ H_{gr}(\mathbf{z}) & H_{gg}(\mathbf{z}) & H_{gb}(\mathbf{z}) \\ H_{br}(\mathbf{z}) & H_{bg}(\mathbf{z}) & H_{bb}(\mathbf{z}) \end{pmatrix}$$



# Generalized Linear Color Vision Model

- **Separate image into channels/visual pathways**
  - Pixel based linear transformation of RGB into color space
  - Spatial filtering based on HVS characteristics & color space
  - Best color space/HVS model for vector error diffusion?
    [Monga, Geisler & Evans 2002]



# Linear CIELab Space Transformation

[Flohr, Kolpatzik, R.Balasubramanian, Carrara, Bouman, Allebach, 1993]

- **Linearized CIELab using HVS Model by**

  Yy = 116 Y/Yn – 116 L = 116 f (Y/Yn) – 116

  Cx = 200[X/Xn – Y/Yn] a = 200[ f(X/Xn ) – f(Y/Yn ) ]

  Cz = 500 [Y/Yn – Z/Zn] b = 500 [ f(Y/Yn ) – f(Z/Zn ) ]

  where

  f(x) = 7.787x + 16/116 0<= x <= 0.008856

  f(x) = (x)1/3 0.008856 <= x <= 1

- **Linearize the CIELab Color Space about D65 white point**

  Decouples incremental changes in Yy, Cx, Cz at white point on (L,a,b) values

$$\nabla_{(Y_y, C_x, C_z)}(L, a, b) = (1/3)\mathbf{I}$$

*T* is sRGB → CIEXYZ →Linearized CIELab

# Spatial Filtering

- **Opponent** [Wandell, Zhang 1997]
  - Data in each plane filtered by 2-D separable spatial kernels

$$f = k \sum_i w_i E_i \qquad E_i = k_i \exp[-(x^2 + y^2)/\sigma_i^2].$$

  - Parameters $(w_i, \sigma_i)$ for the three color planes are

| Plane | Weights $w_i$ | Spreads $\sigma_i$ |
|---|---|---|
| Luminance | 0.921 | 0.0283 |
| | 0.105 | 0.133 |
| | -0.108 | 4.336 |
| Red-green | 0.531 | 0.0392 |
| | 0.330 | 0.494 |
| Blue-yellow | 0.488 | 0.0536 |
| | 0.371 | 0.386 |

# Spatial Filtering

- **Spatial Filters for Linearized CIELab and YUV,YIQ based on:**

  **Luminance frequency Response [ Nasanen and Sullivan – 1984]**

$$W_{(Y_y)}(\tilde{p}) = K(L)\exp[-\alpha(L)\tilde{p}]$$

L – average luminance of display, $\tilde{p}$ the radial spatial frequency and

$$\alpha(L) = \frac{1}{c\ln(L)+d} \qquad K(L) = aL^b \qquad \tilde{p} = \frac{p}{s(\phi)}$$

where $p = (u^2+v^2)^{1/2}$ *and* $$s(\phi) = \frac{1-w}{2}\cos(4\phi) + \frac{1+w}{2}$$

w – symmetry parameter = 0.7 and $\phi = \arctan(\frac{v}{u})$

$s(\phi)$ effectively reduces contrast sensitivity at odd multiples of 45 degrees which is equivalent to dumping the luminance error across the diagonals where the eye is least sensitive.

# Spatial Filtering

**Chrominance Frequency Response [Kolpatzik and Bouman – 1992]**

$$W_{(C_x, C_z)}(p) = A\exp[-\alpha p]$$

**Using this chrominance response as opposed to same for both luminance and chrominance allows more low frequency chromatic error not perceived by the human viewer.**

- The problem hence is of designing 2D-FIR filters which most closely match the desired Luminance and Chrominance frequency responses.
- In addition we need zero phase as well.

***The filters ( 5 x 5 and 15 x 15 were designed using the frequency sampling approach and were real and circularly symmetric).***

Filter coefficients at: ***http://www.ece.utexas.edu/~vishal/halftoning.html***

- Matrix valued Vector Error Filters for each of the Color Spaces at

***http://www.ece.utexas.edu/~vishal/mat_filter.html***

# Color Spaces

- **Desired characteristics**
  - Independent of display device
  - Score well in perceptual uniformity [Poynton color FAQ http://comuphase.cmetric.com]
  - Approximately pattern color separable [Wandell *et al.*, 1993]
- **Candidate linear color spaces**
  - Opponent color space [Poirson and Wandell, 1993]
  - YIQ: NTSC video
  - YUV: PAL video
  - Linearized CIELab [Flohr, Bouman, Kolpatzik, Balasubramanian, Carrara, Allebach, 1993]

← Eye more sensitive to luminance; reduce chrominance bandwidth

# Monitor Calibration

- **How to calibrate monitor?**

  sRGB standard default RGB space by HP and Microsoft

  Transformation based on an sRGB monitor (which is linear)

- **Include sRGB monitor transformation**

  *T*: sRGB → CIEXYZ →Opponent Representation
  [Wandell & Zhang, 1996]

  Transformations sRGB → YUV, YIQ from S-CIELab Code at http://white.stanford.edu/~brian/scielab/scielab1-1-1/

- **Including sRGB monitor into model enables Web-based subjective testing**

  http://www.ece.utexas.edu/~vishal/cgi-bin/test.html

# Spatial Filtering

- **Opponent** [Wandell, Zhang 1997]

  Data in each plane filtered by 2-D separable spatial kernels

  $$f = k \sum_i w_i E_i \qquad E_i = k_i \exp[-(x^2 + y^2)/\sigma_i^2].$$

- **Linearized CIELab, YUV, and YIQ**

  Luminance frequency response [Näsänen and Sullivan, 1984]

  $$W_{(Y_y)}(\rho) = K(L)\, e^{-\alpha(L)\,\rho}$$

  L average luminance of display

  ρ radial spatial frequency

  Chrominance frequency response [Kolpatzik and Bouman, 1992]

  $$W_{(C_x, C_z)}(\rho) = A e^{-\alpha\,\rho}$$

  Chrominance response allows more low frequency chromatic error not to be perceived vs. luminance response