

# AM-FM Screen Design using Donut Filters

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## ABSTRACT

In this paper we introduce a class of linear filters called “donut filters” for the design of halftone screens that enable robust printing with stochastic clustered dots. The donut filter approach is a simple, yet efficient method to produce pleasing stochastic clustered-dot halftone patterns (a.k.a AM-FM halftones) suitable for systems with poor isolated dot reproduction and/or significant dot-gain. The radial profile of a donut filter resembles the radial cross section of a donut shape, with low impulse response at the center that rises to a peak and drops off rapidly as the pixel distance from the center is increased. A simple extension for the joint design of any number of colorant screens is given. This extension makes use of several optimal linear filters that may be treated as a single donut multi-filter having matrix-valued coefficients. A key contribution of this paper is the design of the parametric donut filters to be used at each graylevel. We show that given a desired spatial pair-correlation profile (a.k.a. spatial halftone statistics), optimum donut filters may be generated, such that the donut filter based screen design produces patterns possessing the desired profile in the maximum-likelihood sense. In fact, “optimal green-noise” halftone screens having the spatial statistics described by Lau, Arce and Gallagher may be produced as a special case of our design. We will also demonstrate donut filter designs that do not use an “optimum green-noise” target profile in the design and yet produce excellent stochastic clustered-dot halftone screens.

**Keywords:** image halftoning, screen design, AM-FM halftones

## 1. INTRODUCTION

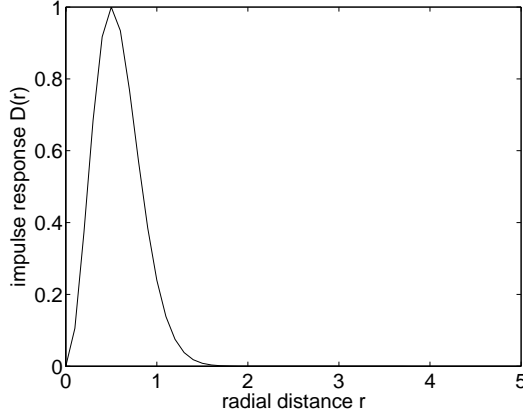
Conventional screens rely on dot size modulation (a.k.a. amplitude modulation (AM)) to reproduce tones since the dot centers are laid out on a fixed grid. The dot sizes increase to represent a darker tone and decrease to represent a lighter tone. Halftoning methods such as error diffusion use a fixed dot size (of typically one pixel) and modulate the density or frequency of the dots (frequency modulation (FM)) to reproduce grayscale. Thus the dots get closer together to represent a darker tone and further apart to represent a lighter tone. AM-FM halftones are hybrid halftones that aim to achieve a balance that takes advantage of the desirable qualities of both AM and FM halftones.

Laser printers suffer from the problem that the electro-photographic (EP) process is not able to reliably produce isolated pixel dots. This translates to dot dropouts in the highlights, plugging in the shadows and grainy appearance due to dot clumping in the mid-tones. Typically laser printers use conventional clustered-dot screens which are a periodic arrangement of pixel clusters on a grid that provide a stable dot transfer. However, since the grid frequency can interact with texture in the image or with other color planes or with a scanning grid an undesirable beat pattern called Moire could be produced. For these reasons researchers have worked on producing halftones that are formed from aperiodic spatial point processes with clusters of pixels of variable size and density.<sup>1-10</sup> Such AM-FM halftones, typically are able to provide better tone transitions and detail rendition than clustered-dot screens since their dot clusters are not confined to lie on a fixed frequency grid and offer a more stable printing process than FM halftoning for EP printing.

In this paper we introduce a method of generating AM-FM halftone screens using a class of linear filters that we call *donut filters*. The radial profile of a donut filter resembles the radial cross section of a donut shape, with low impulse response at the center that rises to a peak and drops off rapidly as the pixel distance from the

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**Figure 1.** Radial impulse response of a donut filter using difference of Gaussians:  $D(r) = 4 \left[ e^{-5.5 \frac{r^2}{2}} - e^{-5.5 r^2} \right]$ .

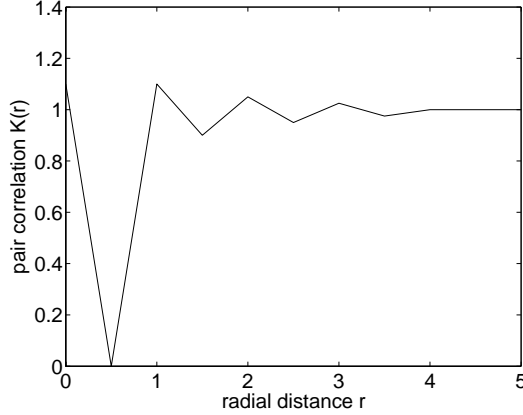
center is increased. We relate the donut filter method to the optimum maximum likelihood screen design method proposed by Lau, Arce and Gallagher<sup>5,10</sup> and establish that either method could be used to generate halftone patterns based on a probabilistic definition of the spatial statistics of the underlying point processes.

Section 2 introduces and motivates the donut filter method of dither array generation. Section 3 reviews notions of optimum AM-FM patterns<sup>4,10</sup> and optimum green-noise halftone screen design<sup>5,10</sup> based on spatial statistics proposed by Lau, Arce and Gallagher. In section 4 we relate the donut filter method to the optimum green-noise construction method and show that the donut filter method can indeed produce optimal monochrome and color green-noise halftone patterns based on the criteria described as optimum by Lau *et. al.*. Section 5 analytically compares an empirically optimized donut filter designed without the notion of optimal statistics to the optimized donut filters of section 4. Finally section 6 concludes the paper by summarizing the contributions.

## 2. SCREEN DESIGN USING DONUT FILTERS

Donut filters, first proposed for screen design by Lin<sup>3</sup> have a spatial profile with a characteristic impulse response that is low at dc and exhibits a peak(s) away from dc, and drops off rapidly as the pixel distance from the center is increased. Fig. 1 shows an example donut filter. The filter shown in Fig. 1 may be described by a simple difference of Gaussians as  $D(r) = \gamma \left[ e^{-\lambda \frac{r^2}{2}} - e^{-\lambda r^2} \right]$  as a radial function of the average inter-minority pixel distance  $r$  for  $\lambda = 5.5$  and  $\gamma = 4$ . The parameter  $\gamma$  scales the donut filter to have a peak response of unity. The motivation for using a donut shaped impulse response is that it encourages dot clusters to form close to dot centers while inhibiting dot clusters especially strongly midway between dot clusters. The construction of an  $L$  level,  $M \times N$  screen  $S[i, j]$ , using donut filters is a simple process.

1. Set starting level  $l = l_0$
2. Set  $n = 0$ ,  $g = l/L$ . Generate donut filter  $D_g[i, j]$  for graylevel  $g$ .
3. Filter minority pixel pattern  $\phi[i, j]$  (locations where minority pixels exist are set to 1 and majority pixel locations are set to 0) for graylevel  $g$  using the donut filter  $D_g[i, j]$  to produce an output  $O_g^{(n)}[i, j]$ .
4. Find location  $[i^*, j^*]$  where  $O_g^{(n)}[i, j]$  is minimum (maximum) subject to the constraint that  $O_g^{(n)}[i, j]$  is a majority (minority) pixel when  $g \leq 0.5$  ( $g > 0.5$ ). Set  $S[i^*, j^*] = l$ .
5. If  $g \leq 0.5$  the majority pixel at  $[i^*, j^*]$  is selected and that majority pixel is converted to a minority pixel.  $\phi_g[i^*, j^*] = 1$
6. If  $g > 0.5$  the minority pixel at  $[i^*, j^*]$  is selected and that minority pixel is converted to a majority pixel.  $\phi_g[i^*, j^*] = 0$



**Figure 2.** Ideal pair correlation function for a green-noise halftone pattern. The radial distance is in units of principle wavelength  $\lambda_g$

7. If the desired concentration of minority pixels is achieved ie: if  $n = n_{desired}$ , update  $l \leftarrow l + 1$  and go to step 2. If not go to the next step. If all graylevels are processed, we are done.
8. Update filter output as

$$O_g^{(n+1)}[i, j] \leftarrow \begin{cases} O_g^{(n)}[i, j] + D_g[\text{mod}(i^* - i, M), \text{mod}(j^* - j, N)] , & g \leq 0.5 \\ O_g^{(n)}[i, j] - D_g[\text{mod}(i^* - i, M), \text{mod}(j^* - j, N)] , & g > 0.5 \end{cases}$$

9. increment n as  $n \leftarrow n + 1$
10. goto step 4.

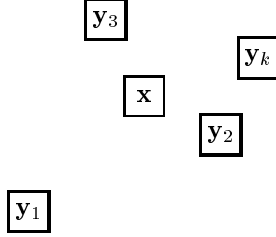
Since the filtering is linear, instead of performing step 3 every time a pixel is added, we may update the past filter output using one addition per pixel. This is encapsulated in step 8. For every new graylevel the filtering could be performed by a different donut filter using FFTs. The use of the FFTs implies that a circular convolution is used to perform the filtering, hence there designed screen tiles smoothly without boundary artifacts. Typically the pattern upto the level  $l_0$  is produced using *blue-noise* methods<sup>11, 12</sup>

### 3. HALFTONE STATISTICS AND OPTIMUM AM-FM SCREENS

Lau Arce and Gallagher<sup>4, 10</sup> analyzed the spatial patterns produced by AM-FM halftones in the spatial and frequency domains. They found that the pattern power spectra exhibited a strong mid-frequency component and hence they coined the term "green-noise" halftoning as opposed to conventional "blue-noise"<sup>13</sup> halftone patterns that had pattern spectra with strong high frequency components. Lau Arce and Gallagher<sup>4, 10</sup> also formulated criteria for "optimum green-noise" patterns by extending Ulichney's optimum homogeneous packing argument for isolated dots to dot clusters.<sup>13</sup> An optimum green-noise pattern for a graylevel  $g$  is characterized by the average distance between the dot-centers of minority pixel clusters also called the *principle wavelength*  $\lambda_g$ .

$$\lambda_g = \begin{cases} 1/\sqrt{g/\overline{M}} & , \quad 0 < g \leq 1/2 \\ 1/\sqrt{1 - g/\overline{M}} & , \quad 1/2 < g \leq 1 \end{cases} \quad (1)$$

where  $\overline{M}$  is the average number of minority pixels per cluster. Following up on their work, Lau Arce and Gallagher presented a method to construct green-noise masks having this property.<sup>5</sup> Lau *et. al.* used spatial statistics such as the *pair correlation function* commonly employed in *stochastic geometry*<sup>14</sup> to characterize green-noise halftones. The pair correlation function  $K(r)$  is defined as the ratio of the expected number of minority pixels



**Figure 3.** Evaluating majority pixel locations at which to add a minority pixel for  $0 < g \leq 0.5$ . The pixels drawn with thick lines indicate current minority pixels. The pixel drawn with thin lines represents a candidate majority pixel location for adding the next minority pixel.

at distance  $r$  given that the distance is measured from a minority pixel to the expected number of minority pixels at a distance  $r$  from an arbitrary pixel. Fig. 2 shows the pair correlation function for an optimum green-noise pattern. The pair correlation function for an optimum green-noise pattern exhibits a peak near the origin and has multiple peaks at positive integer multiples of  $\lambda_g$  with valleys in between. As the distance from a dot-cluster increases the pair correlation function asymptotically equals 1. Lau *et. al.*<sup>5</sup> use the pair correlation function shown in Fig. 2 to construct optimum green noise screens using their binary pair correlation construction algorithm (BIPCCA). The algorithm is essentially a maximum likelihood algorithm that initially assigns probabilities to pixels in an uncorrelated manner. As each new minority pixel is added, the probabilities of all neighboring majority pixels is adjusted according to the desired pair correlation of the desired pattern. The probability of a majority pixel to become a minority pixel is increased if the majority pixel is at a distance  $r$  from the newest minority pixel and  $K(r) > 1$ . The probability of a majority pixel to become a minority pixel is decreased if the majority pixel is at a distance  $r$  from the newest minority pixel and  $K(r) < 1$ . Lau *et. al.* also use a multiplicative *concentration matrix* derived from the normalized output of a Gaussian filter to promote dot growth in areas where large voids exist, by increasing the probability of majority pixels in such regions to become minority pixels. Imposing the stacking constraint, BIPCCA was extended to construct a screen.<sup>5</sup>

#### 4. OPTIMUM DONUT FILTERS

In this section we show that given a spatial halftone statistics we may use appropriately designed donut filters to construct halftone patterns possessing that statistic in the maximum likelihood sense. This unifies the theory of filter based screen design methods (ex. void and cluster) and the optimal maximum likelihood method proposed by Lau *et. al.* The optimum screen design problem reduces to the design of the optimum donut filter to be used at each graylevel. In section 4.1 we consider the case of monochrome screen design using optimum donut filters. Section 4.2 generalizes this result to joint color screen design using optimized donut filters.

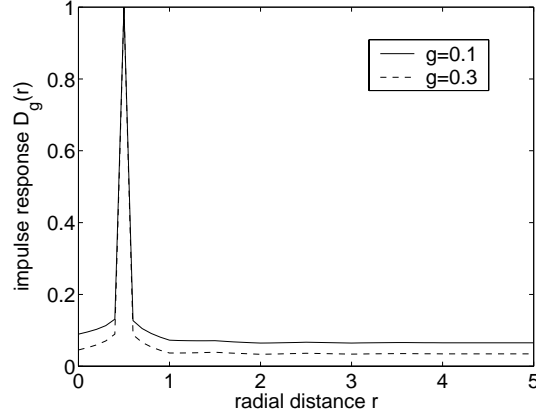
##### 4.1. Optimum AM-FM monochrome screen design

We use a modification of the pair correlation function to specify the desired statistics. We define a related function called the *spatial probability profile* that encodes the probability of seeing a minority pixel at a radial distance  $r$  from the center of any dot-cluster of minority pixels. It is essentially a scaled version of the pair correlation function defined by

$$Z_g(r) = \begin{cases} g K(r) & , 0 < g \leq 1/2 \\ (1 - g) K(r) & , 1/2 < g \leq 1 \end{cases} \quad (2)$$

According to the spatial probability profile, the probability of seeing a minority pixel at given distance  $r$  from a minority pixel becomes equal to the unconditional probability of seeing a minority pixel as  $r$  gets large.

Fig. 3 shows the situation in which a minority pixel is to be added to an existing pattern of minority pixels for a graylevel  $g \leq 0.5$ . If the positions of all existing minority pixels is given by the set  $\mathcal{Y} = \{\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_t\}$ ,



**Figure 4.** Impulse responses of optimal donut filters for different graylevels. The radial distance is in units of principle wavelength  $\lambda_g$

then the optimum majority pixel location  $\mathbf{x}^* \in \mathcal{X}$  at which to add the next minority pixel is the location that maximizes the probability of observing minority pixels at  $\mathcal{Y}$  given that a minority pixel is added at  $\mathbf{x}$ .

$$\mathbf{x}^* = \operatorname{argmax}_{\mathbf{x} \in \mathcal{X}} P(\mathcal{Y}|\mathbf{x}) = \operatorname{argmax}_{\mathbf{x} \in \mathcal{X}} \prod_{k=1}^t P(\mathbf{y}_k|\mathbf{x}) \quad (3)$$

where we have assumed that, given a minority pixel at  $\mathbf{x}$ , seeing a minority pixel at a location  $\mathbf{y}_i \in \mathcal{Y}$  is independent of seeing a minority pixel at a location  $\mathbf{y}_j \in \mathcal{Y}$  for  $i \neq j$ . This assumption is in fact implied by the optimal spatial probability profile which assigns a probability to a minority pixel  $\mathbf{y}_k \in \mathcal{Y}$  that only depends on its distance to  $\mathbf{x}$ . Taking the negative logarithm converts equation (3) to a minimization problem.

$$\mathbf{x}^* = \operatorname{argmin}_{\mathbf{x} \in \mathcal{X}} \sum_{k=1}^t -\log(P(\mathbf{y}_k|\mathbf{x})) = \operatorname{argmin}_{\mathbf{x} \in \mathcal{X}} \sum_{k=1}^t -\log(Z_g(\|\mathbf{y}_k - \mathbf{x}\|)) \quad (4)$$

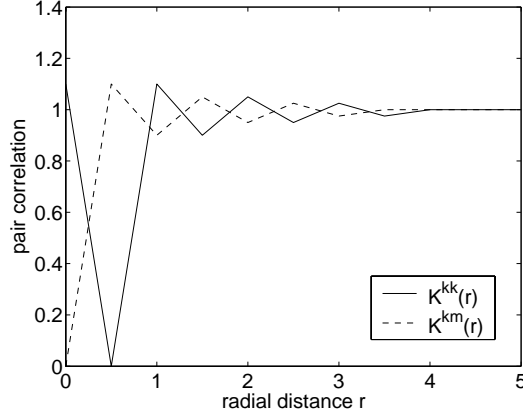
Since the minority pixel pattern consists of ones and zeros the above summation may be regarded as a linear filtering operation. Thus the maximum likelihood solution to the minority pixel placement problem is obtained by filtering the existing minority pixel pattern using a radial linear filter with a radial impulse response  $-\log(Z_g(r))$  and adding a minority pixel to the majority pixel location where the filter output is minimum. When  $g > 0.5$  we need to convert minority pixels to majority pixels in order to satisfy the stacking constraint. In this case we need to find the minority pixel with the lowest likelihood of being a minority pixel and convert it to a majority pixel. In this case the optimal minority pixel location is given by

$$\mathbf{y}^* = \operatorname{argmax}_{\mathbf{y} \in \mathcal{Y}} \sum_{k=1}^t -\log(P(\mathbf{y}_k|\mathbf{y})) = \operatorname{argmax}_{\mathbf{y} \in \mathcal{Y}} \sum_{k=1}^t -\log(Z_g(\|\mathbf{y}_k - \mathbf{y}\|)) \quad (5)$$

Using the maximum likelihood solution as described above does not constrain the dot growth to be homogeneous. This solution does not necessarily encourage pixels to form in regions where there are large voids. The optimal donut filter may be constructed according to the following formula

$$D_g(r) = (1 - \alpha) \frac{\log(\delta + Z_g(r))}{\log(\delta + Z_g(0))} + \alpha e^{-r^2} \quad (6)$$

The parameter  $\delta$  is a small constant used to avoid the  $\log(0)$  situation (we use  $\delta = 10^{-15}$ ). The parameter  $\alpha \in [0, 1]$  provides a compromise between satisfying the optimal spatial statistics and achieving homogeneous dot growth, both of which are important. At locations where the minority pixel density is large the additional



**Figure 5.** Ideal pair correlation functions  $K^{kk}(r)$  and  $K^{km}(r)$ ,  $k \neq m$  for a color green-noise halftone pattern. The radial distance is in units of principle wavelength  $\lambda_g$

term provides a large response while it provides a low response when a void is encountered. It must be noted that the optimum donut filter is not unique, but defines an equivalence class of allowed linear filters. Indeed once  $D_g(r)$  is designed any filter of the form  $D'_g(r) = K_1 D_g(r) + K_2$  where  $K_1 > 0$  would also produce the same results. The optimal spatial probability profile may be derived from the optimal pair correlation function of Fig. 2. Fig. 4 shows the radial impulse response of the optimal donut filters for  $g = 0.1$  and  $g = 0.3$ . They have the characteristic donut filter shape. A complete optimal monochrome screen design may be carried out using the procedure described in section 2. A void and cluster approach was used to design blue-noise patterns upto graylevel 5%. Fig. 7 shows the results of halftoning constant graylevels of 10%, 30% and 50% respectively using a screen designed using the optimum donut filters along with their respective spatial probability profiles. Note the strongest peaks occur at the principle wavelength  $\lambda_g$ . The corresponding patterns for the darker tones are similar.

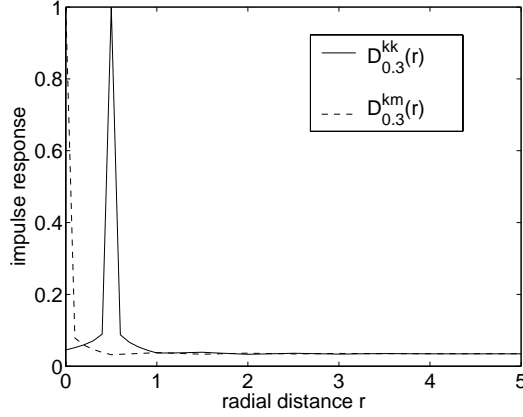
## 4.2. Optimum AM-FM color screen design

The approach of section 4.1 may be extended to the joint design of color screens. Joint color statistics may be defined by joint pair correlation functions.<sup>5</sup> Fig.5 shows the joint pair correlation function used by Lau *et al.* to generate joint colorant screens where overlap between different colorants is discouraged. Spatial probability profiles may be derived from these pair correlation functions using equation (2). The spatial probability profile functions  $Z_g^{kk}(r)$  represent the probability of seeing a minority pixel in the  $k^{\text{th}}$  colorant plane at a distance  $r$  from another minority pixel in the  $k^{\text{th}}$  colorant plane. The spatial probability profile functions  $Z_g^{km}(r)$ ,  $k \neq m$  represent the probability of seeing a minority pixel in the  $m^{\text{th}}$  colorant plane at a distance  $r$  from another minority pixel in the  $k^{\text{th}}$  colorant plane. The optimal donut filters in this case are given by the equation:

$$D_g^{km}(r) = \begin{cases} \gamma_{kk} \left[ (1 - \alpha) \frac{\log(\delta + Z_g^{kk}(r))}{\log(\delta + Z_g^{kk}(0))} + \alpha e^{-r^2} \right] & , \quad k = m \\ \gamma_{km} [\log(\delta + Z_g^{km}(r))] & , \quad k \neq m \end{cases} \quad (7)$$

where the homogeneity term is omitted when  $k \neq m$  since it is already taken into account while designing the individual colorant planes. The constants  $\gamma_{kk}$  and  $\gamma_{km}$  scale the filter responses to achieve a peak value of unity. Fig. 6 shows the radial impulse responses of the donut filters for graylevels  $g = 0.1$  and  $g = 0.3$ . Let us consider the joint design of an  $L$  level,  $M \times N$  screen  $\mathbf{S}[i, j]$  with  $C$  colorants. The filtering operations may be expressed as linear filtering using  $1 \times C$  multi-filters (ie: filters with matrix-valued coefficients).

$$\tilde{\mathbf{D}}_g^k[i, j] = [ \beta_g^{k1} D_g^{k1}[i, j], \quad \beta_g^{k2} D_g^{k2}[i, j], \quad \dots, \quad \beta_g^{kC} D_g^{kC}[i, j] ] \quad (8)$$



**Figure 6.** Radial impulse responses  $D_g^{kk}(r)$  and  $D_g^{km}(r)$ ,  $k \neq m$  of color donut filters at graylevel  $g = 0.3$ .

where  $\beta_g^{km}$  is a graylevel dependent relative weighting factor that weights the influence of the  $m^{\text{th}}$  colorant plane on the statistics of the  $k^{\text{th}}$  colorant plane. The weighting constants satisfy the equations

$$\sum_{m=1}^C \beta_{km} = 1, \forall k \quad (9)$$

$$\beta_{km} \geq 0, \forall k, m \quad (10)$$

A filtering of the minority pixel color patterns  $\Phi[i, j] = [\phi_g^1[i, j], \phi_g^2[i, j], \dots, \phi_g^C[i, j]]^T$  using this multi-filter is performed according to

$$O_g^k[i, j] = \left( \tilde{\mathbf{D}}_g^k \star \Phi \right) [i, j] = \sum_{m=1}^C \beta_g^{km} (D_g^{km} \star \phi_g^m) [i, j] \quad (11)$$

where the matrix-vector convolution operator  $\star$  is represented using the scalar convolution operator  $*$ . As with monochrome screen design, the color screens are designed one graylevel at a time.

1. Set starting level  $l = l_0$
2. Set  $k = 1$ ,  $g = l/L$ ,  $n = 1$ . Generate donut multi-filters  $\tilde{\mathbf{D}}_g^k[i, j]$  for graylevel  $g$ ,  $\forall k$ .
3. Filter minority pixel pattern  $\Phi[i, j]$  for graylevel  $g$  using the donut multi-filter  $\tilde{\mathbf{D}}_g^k[i, j]$  to produce an output  $O_g^{k(n)}[i, j]$ .
4. Find location  $[i^*, j^*]$  in colorant plane  $k$  where  $O_g^{k(n)}[i, j]$  is minimum (maximum) subject to the constraint that  $O_g^{k(n)}[i, j]$  is a majority (minority) pixel when  $g \leq 0.5$  ( $g > 0.5$ ). Set  $S_k[i^*, j^*] = l$ .
5. If  $g \leq 0.5$  the majority pixel in colorant plane  $k$  at  $[i^*, j^*]$  is selected and that majority pixel is converted to a minority pixel.  $\phi_g^k[i^*, j^*] = 1$ .
6. If  $g > 0.5$  the minority pixel at  $[i^*, j^*]$  in colorant plane  $k$  is selected and that minority pixel is converted to a majority pixel.  $\phi_g^k[i^*, j^*] = 0$ .
7. Update filter output for colorant plane  $k$  as

$$O_g^{k(n+1)}[i, j] \leftarrow \begin{cases} O_g^{k(n)}[i, j] + \sum_{m=1}^C \beta_g^{km} D_g^{km}[\text{mod}(i^* - i, M), \text{mod}(j^* - j, N)], & g \leq 0.5 \\ O_g^{k(n)}[i, j] - \sum_{m=1}^C \beta_g^{km} D_g^{km}[\text{mod}(i^* - i, M), \text{mod}(j^* - j, N)], & g > 0.5 \end{cases}$$

8. If  $k = C$  goto the next step, else increment  $k$  as  $k \leftarrow k + 1$  and goto step 3 if  $n = 1$  or step 4 if  $n > 1$ .
9. If the desired concentration of minority pixels in all the colorant planes is achieved ie: if  $n = n_{desired}$ , update  $l \leftarrow l + 1$  and go to step 2. If not go to the next step. If all graylevels are processed, we are done.
10. increment  $n$  as  $n \leftarrow n + 1$
11. goto step 4

Fig. 8 shows the results of halftoning constant Cyan-Magenta graylevels of 10%, 30% and 50% respectively using a screen designed using the optimum donut multi-filters along with their respective spatial probability profiles. We used  $\beta_g^{kk} = 0.7$  and  $\beta_g^{km} = 0.3, k \neq m$ . Note that the strongest peak of  $Z_g^{kk}(r)$  and the corresponding strongest valley of  $Z_g^{km}(r)$  occurs at the principle wavelength  $\lambda_g$ . The corresponding patterns for the darker tones are similar.

## 5. SUB-OPTIMAL DONUT FILTERS

It is interesting to compare the performance of donut filters not designed using the optimal method described in section 4 with optimal donut filters. We will refer to the empirically designed donut filters as sub-optimal donut filters. Consider the parametric donut filter profile  $D(r) = \gamma \left[ e^{-\lambda \frac{r^2}{2}} - e^{-\lambda r^2} \right]$ . The constant  $\gamma$  is chosen to normalize the peak of the donut response to unity. This design generates a one parameter family of donut filters which may be empirically optimized by varying  $\lambda$ . We obtained the best results for  $\lambda \approx 5.5, \gamma \approx 4$ . Fig. 9 shows the results of halftoning constant graylevels of 10%, 30% and 50% respectively using a screen designed using the sub-optimal donut filter along with their respective spatial probability profiles. Note that the strongest peaks in the probability profiles occur away from the principle wavelength  $\lambda_g$ .

## 6. CONCLUSIONS

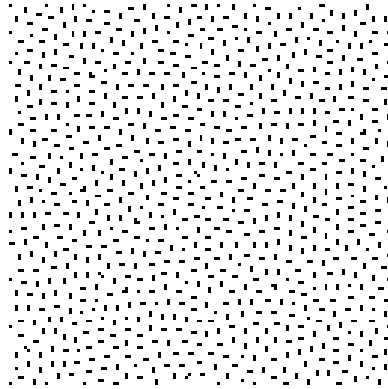
In this paper we have presented a method of generating AM-FM halftone screens using a class of linear filters that we call *donut filters*. We have shown how the donut filters may be designed to produce optimum green-noise screens based on a given pair-correlation function. We have also shown how optimum AM-FM color screens may be produced using novel donut multi-filters. This work relates the donut filter method to the optimum maximum likelihood screen design method proposed by Lau, Arce and Gallagher and establishes that either method could be used to generate halftone patterns based on a given pair correlation function. Further, we demonstrated sub-optimal donut filters that were not designed with the 'optimum' green-noise pair-correlation function but which also produce excellent AM-FM patterns in practice.

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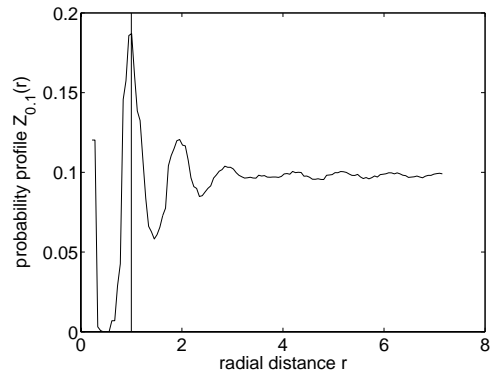
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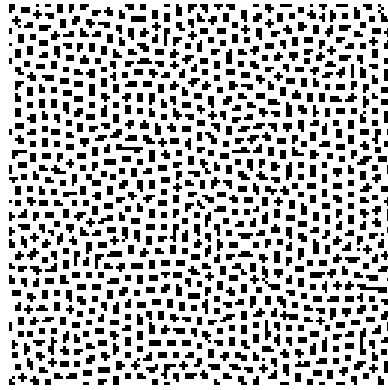
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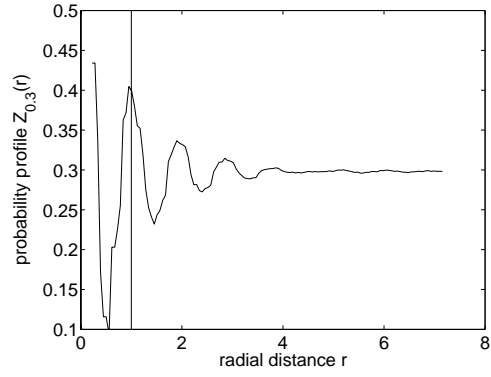
(a)  $g = 10\%$



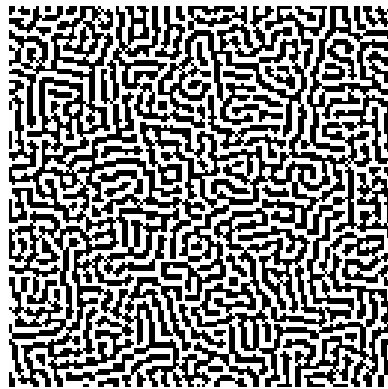
(d)  $g = 10\%$



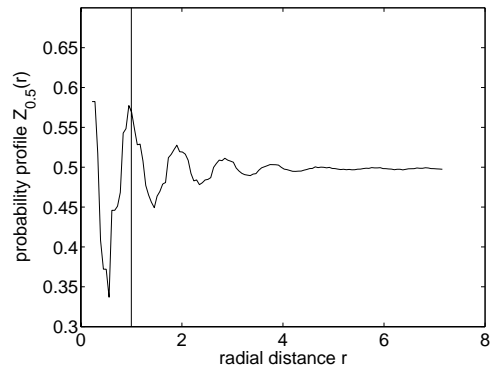
(b)  $g = 30\%$



(e)  $g = 30\%$

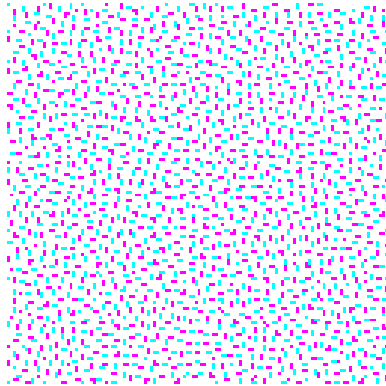


(c)  $g = 50\%$

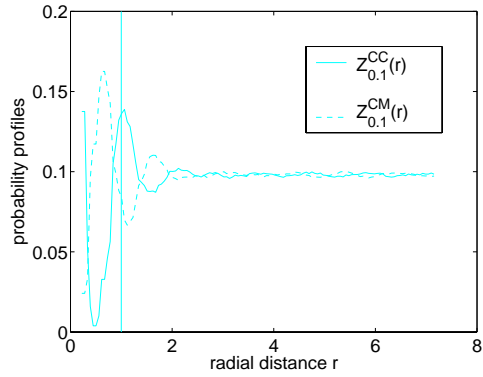


(f)  $g = 50\%$

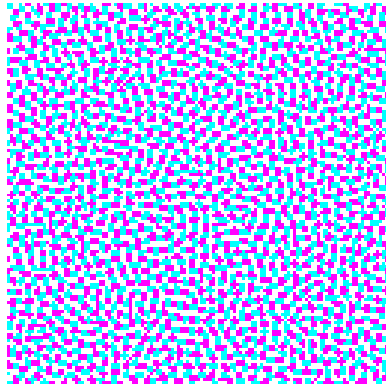
**Figure 7.** Performance of optimum monochrome donut filters. (a)-(c) show halftones at various graylevels. (d)-(f) show the computed spatial probability profile. The vertical line indicates the location of  $\lambda_g$ .



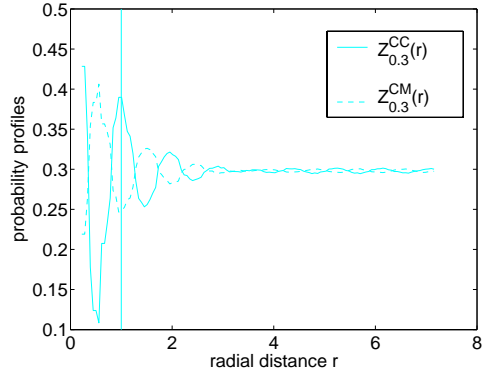
(a)  $g = 10\%$



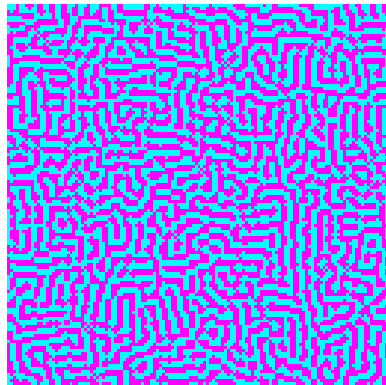
(d)  $g = 10\%$



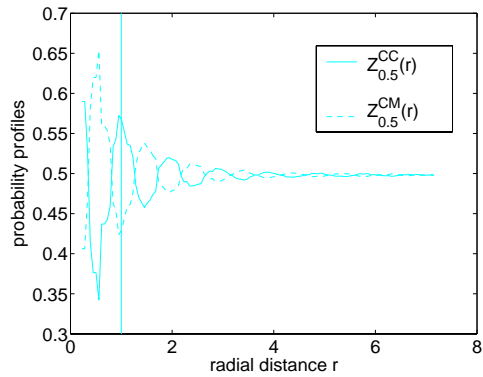
(b)  $g = 30\%$



(e)  $g = 30\%$

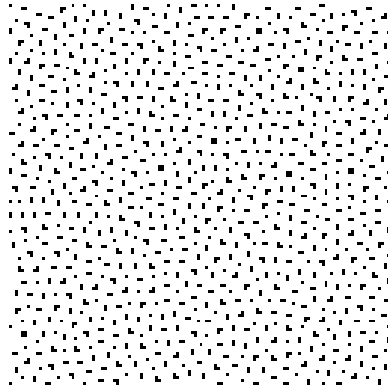


(c)  $g = 50\%$

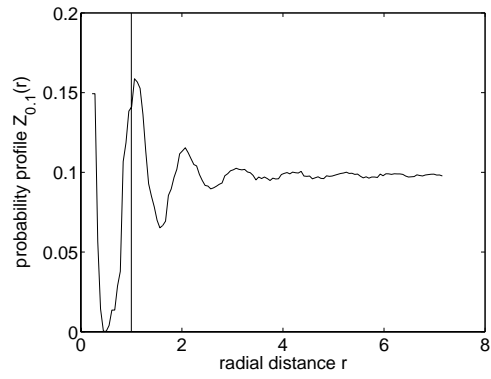


(f)  $g = 50\%$

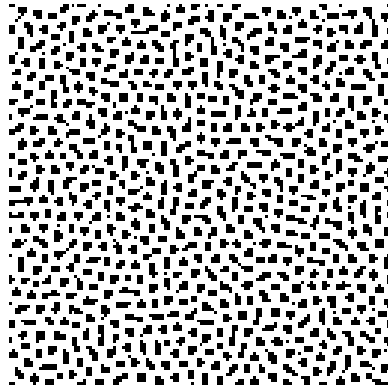
**Figure 8.** Performance of optimum color donut multi-filters. (a)-(c) show Cyan-Magenta halftones at various graylevels. (d)-(f) show the computed spatial probability profiles for the Cyan color plane. The vertical line indicates the location of  $\lambda_g$ .



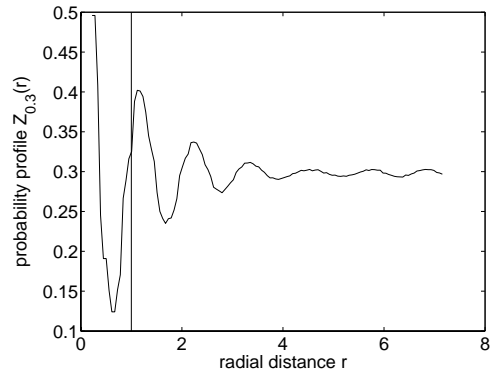
(a)  $g = 10\%$



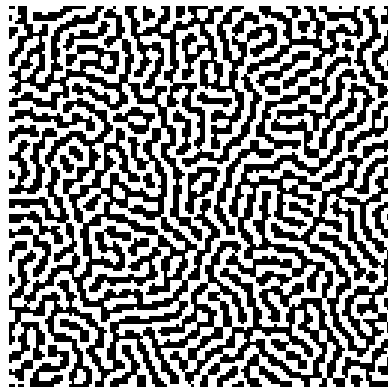
(d)  $g = 10\%$



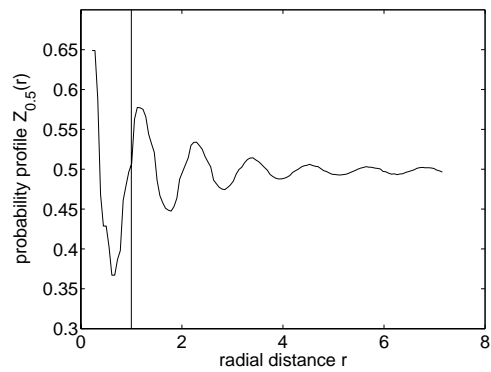
(b)  $g = 30\%$



(e)  $g = 30\%$



(c)  $g = 50\%$



(f)  $g = 50\%$

**Figure 9.** Performance of empirically optimized sub-optimal donut filter. (a)-(c) show halftones at various graylevels. (d)-(f) show the computed spatial probability profiles. The vertical line indicates the location of  $\lambda_g$ .