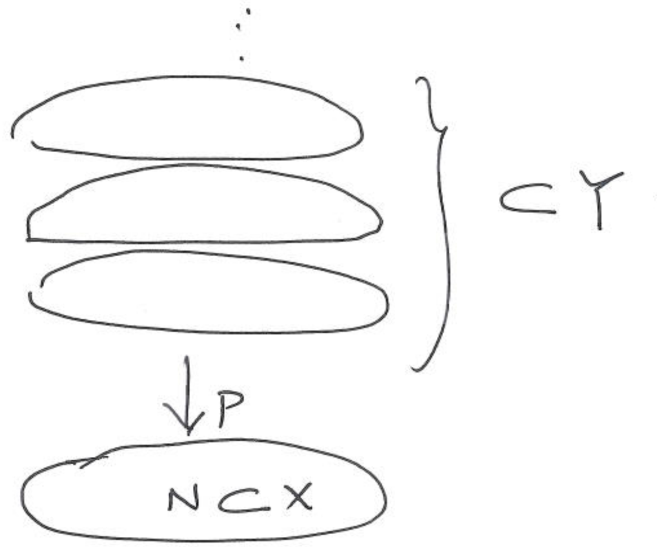
SOME PRELIMINARY FACTS ABOUT COVERING SPACES

Definition: X & Y are topological spaces. A covering map from Y to X is a continuous map $p: Y \to X$ s.t. each point in X has a nbd N $p^{-1}(N)$ is a disjoint union of open sets, ~~for~~ each of which is mapped homeomorphically onto N. We say that Y covers X. Thus "locally", a covering looks like



Basically, a covering is a locally trivial map.

Automorphisms of coverings: Let $p: Y \to X$ be a covering map. A map $\pi: Y \to Y$ is an automorphism of this covering if ~~for~~

$$p = \pi \circ p.$$

Clearly the set of automorphisms of a covering forms a group.

"Galois Theory": Let $\pi_1(X)$ be the <u>fundamental group</u> of X in what follows:

<u>FACT</u>: Every topological space X has a unique simply connected cover $\tilde{X}$ called it's <u>Universal cover</u>.

<u>FACT</u>: $\mathrm{Aut}(\tilde{X}/X) \cong \pi_1(X)$.

Galois Theory: Every subgroup of $\pi_1(X)$ has as it's "fixed cover" a subcover of $\tilde{X}$.

$$\begin{array}{ccc} \tilde{X} & \qquad\qquad & \{1\} \\ | & & | \\ H \backslash \tilde{X} & & H \\ | & & | \\ X & & \pi_1(X) \end{array}$$

Riemann Surfaces:

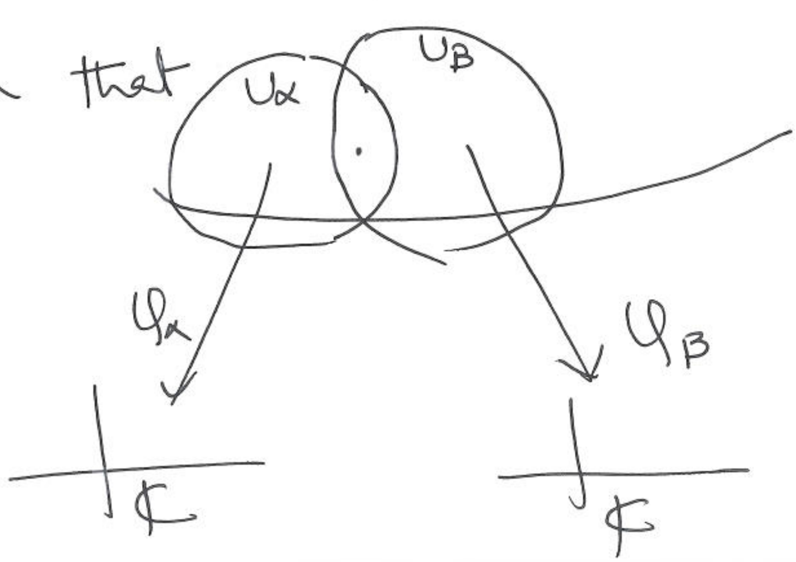
A complex "manifold" is a topological space that is locally like $\mathbb{C}^n$ everywhere for some n. n is called the dimension of a the manifold.

(ex: the circle is a real manifold of dimension 1; the figure ∞ is not a manifold).

A <u>Riemann Surface</u> S is a 1 dimensional complex manifold together with an "analytic structure". By this we mean that S can be covered with a collection of coordinate charts

$$\varphi_\alpha : U_\alpha \longrightarrow \mathbb{C}$$

Such that



$\varphi_\alpha^{-1} \cdot \varphi_\beta$ is an analytic function from $\mathbb{C}$ to $\mathbb{C}$, for all α & β.

THEOREM: Every compact Riemann Surface is the Riemann Surface of an algebraic Curve.

THEOREM (Riemann Mapping). The universal cover of every Riemann Surface is either

(1) $\mathbb{C}$ (covers only genus 1 Riemann surfaces)

(2) $\mathbb{H}$ (covers everything else!!)

(3) S^2 (covers only S^2)