Analysis of Deterministic Binary Interference Channels Via a General Outer Bound

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***Abstract*—We present a new outer bound for the general two-user discrete memoryless interference channel (IFC) and use it to establish the capacity region of the binary erasure IFC, whose determination was left open in [7]. We also show that there are essentially two deterministic binary IFCs, in addition to the binary erasure IFC, whose capacity regions are not obvious from previous results. We determine the capacity region of one of these and apply the aforemenioned general outer bound to obtain the best available bound on the maximum achievable sum-rate for the other. We also show that the new general outer bound is tight for one-sided deterministic IFCs that belong to the class studied by El Gamal and Costa.**

***Index Terms*—Binary erasure interference channel, capacity, interference channel (IFC), outer bound.**

I. Introduction

The two-user interference channel (IFC) is an information-theoretic model that captures the effects of interference between two users that communicate independent information to their corresponding receivers. The capacity region of the channel gives a fundamental measure of the tradeoff between the rates that can be simultaneously achieved. Unfortunately, except for a few special cases, the capacity region is unknown [1], [2]. There are two IFC models that received particular attention in the literature: the discrete memoryless IFC (DMIFC) and the Gaussian memoryless IFC (GMIFC). Recently, the capacity region of the Gaussian IFC has been characterized with an error smaller than 1 bit/s/Hz per user through the derivation of new outer bounds [3]. However, the DMIFC lacks such a tight characterization for its capacity region. In this work we make progress in the characterization of the DMIFC capacity region by presenting a new outer bound. We use this bound to establish the capacity region of the binary erasure IFC, whose determination was left open in [6], [7].

The best known achievable region for both the DMIFC and the GMIFC is due to Han and Kobayashi [10]. In order to characterize capacity, it is of interest to obtain outer bounds to the capacity region, such as the ones proposed by Sato [11] and Carleial [12]. These bounds are obtained by allowing cooperation between the transmitters or receivers, and by enhancing the channel so that the actual channel outputs are degraded versions of the enhanced channel outputs. In the enhanced channel, each receiver is able to decode the messages sent by both transmitters. However, in many cases, the receivers in the actual channel cannot completely decode the message sent by the interfering transmitter. In these cases, the bounds of [11] and [12] are loose. In contrast, the bound that we present in this work does not require either receiver to completely decode the interfering signal. In addition to determining the capacity region of the binary erasure IFC, we present some examples that illustrate the performance of the new bound. In particular, the new bound appears to be most useful for channels with only one-sided interference.

Preliminary results of this work were presented in [14], where the bound of Theorem 1 was given in a possibly weaker form.[1] In this full version, we expand the treatment of deterministic binary IFCs and identify the remaining two-user deterministic binary IFC whose capacity region is still unknown. In addition, we show that time sharing (i.e., convexification) is required to achieve the capacity region of the binary erasure IFC, thus correcting an error in the statement and derivation of this region given in [14].

Regarding notation we use lowercase letters to denote scalars, uppercase letters to denote random variables, boldface letters to denote vectors, and calligraphic uppercase letters to denote sets. For example x is a scalar, X is a random variable, $\mathbf{x}$ is a vector, $\mathbf{X}^n = (X_1, \ldots, X_n)$ is a random vector, and $\mathcal{X}$ is a set. We use $H(\cdot)$, and $I(\cdot;\cdot)$ to denote binary entropy and mutual information. The indicator function is denoted by $1(\cdot)$, and $(x)^+$ is used to denote $\max\{x, 0\}$. The probability mass function (pmf) of the random variable X is written as p_X and, with slight abuse of notation, we use $p(x)$ to denote this function. We use $\mathcal{B}(p)$ to denote the Bernoulli distribution of parameter p. We define $h_i(p_1, \ldots, p_{i-1}) = -\sum_{j=1}^{i} p_j \log_2(p_j)$ for $p_j \geq 0, \sum_{j=1}^{i} p_j = 1$, i.e., the entropy in bits of a random variable with pmf $(p_1, \ldots, p_i)$.

II. Model

We consider a two-user discrete memoryless interference channel. In this model there are two transmitter-receiver pairs, where each transmitter wants to communicate with its corresponding receiver (cf. Fig. 1). This channel is defined by the finite input alphabets $\mathcal{X}_1$ and $\mathcal{X}_2$, the finite output alphabets $\mathcal{Y}_1$ and $\mathcal{Y}_2$, and transition probability matrices $\{p(y_1 \mid x_1, x_2)\}$ and $\{p(y_2 \mid x_1, x_2)\}$.

For a given block length n, user i communicates a message $m_i \in \{1, \ldots, 2^{nR_i}\}$ by choosing a codeword from a codebook

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[1]It can be shown that Theorem 1 implies the bound in [14], but we have no proof or counter-example concerning the reverse implication. We do note that the bound of [14] is also sufficient to prove Theorem 2 below, which is presently the main application of Theorem 1.

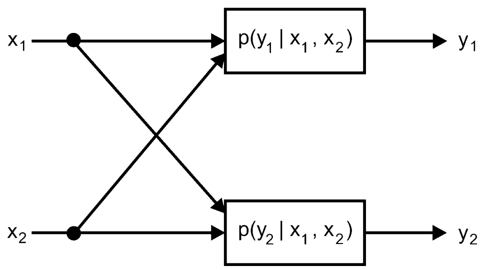

Fig. 1. Two-user discrete memoryless interference channel.

$\mathcal{C}_{i,n}$, with $|\mathcal{C}_{i,n}| = 2^{nR_i}$. The codewords $\{\mathbf{c}_i(m_i)\}$ of this codebook have symbols chosen from the input alphabet $\mathcal{X}_i$. Receiver i observes the channel outputs $\{y_{i,t} : t = 1, \dots, n\}$ and uses a decoding function $f_{i,n} : \mathcal{Y}_i^n \to \mathbb{N}$ to get the estimate $\hat{m}_i$ of the transmitted message m_i. The receiver is in error whenever $\hat{m}_i \neq m_i$. The average probability of error for user i is given by

$$\epsilon_{i,n} = E[P(\hat{m}_i \neq m_i)]$$

where the expectation is taken with respect to the random choice of the transmitted messages m_1 and m_2. Note that due to the interference among users, the probability of error of each user may depend on the codebook of the other user.

A rate pair (R_1, R_2) is achievable if there exists a family of codebook pairs $\{(\mathcal{C}_{1,n}, \mathcal{C}_{2,n})\}_n$ and decoding functions $\{f_{1,n}(\cdot), f_{2,n}(\cdot)\}_n$, such that the average decoding error probabilities $\epsilon_{1,n}, \epsilon_{2,n}$ go to zero as the block length n goes to infinity.

The capacity region $\mathcal{R}$ of the interference channel is the closure of the set of achievable rate pairs.

III. New Outer Bound

Theorem 1: Let U and Q_i, $i = 1, 2$, be auxiliary random variables with U chosen from the alphabet $\mathcal{U} = \{1, \dots, 6\}$ and Q_i chosen from the alphabets $\mathcal{Q}_i = \{1, \dots, |\mathcal{X}_i| + 1\}$, such that

$$\begin{aligned} &p_{\mathbf{X}^2,\mathbf{Y}^2,\mathbf{Q}^2,U}(\mathbf{x}^2, \mathbf{y}^2, \mathbf{q}^2, u) \\ &\quad = p_U(u) p_{Q_1\,|\,U}(q_1\,|\,u) \\ &\qquad \cdot p_{X_1\,|\,Q_1,U}(x_1\,|\,q_1,u) p_{Q_2\,|\,U}(q_2\,|\,u) \\ &\qquad \cdot p_{X_2\,|\,Q_2,U}(x_2\,|\,q_2,u) p_{Y_1\,|\,X^2}(y_1\,|\,\mathbf{x}^2) \\ &\qquad \cdot p_{Y_2\,|\,X^2}(y_2\,|\,\mathbf{x}^2) \end{aligned} \tag{1}$$

and for fixed $p_U, p_{Q_1\,|\,U}, p_{Q_2\,|\,U}, p_{X_1\,|\,Q_1,U}, p_{X_2\,|\,Q_2,U}$ let

$$\begin{aligned} r_1 = \min\{&I(X_1; Y_1\,|\,X_2, U), \\ &I(Q_2, X_1; Y_1\,|\,U), H(X_1\,|\,Q_1, U)\} \end{aligned} \tag{2}$$

$$\begin{aligned} r_2 = \min\{&I(X_2; Y_2\,|\,X_1, U), \\ &I(Q_1, X_2; Y_2\,|\,U), H(X_2\,|\,Q_2, U)\}. \end{aligned} \tag{3}$$

The closure of the region

$$\mathcal{R}_{\mathrm{OB}} = \bigcup_{\substack{p_U, p_{Q_1\,|\,U},\\ p_{Q_2\,|\,U}, p_{X_1\,|\,Q_1,U},\\ p_{X_2\,|\,Q_2,U}}} \{(R_1, R_2) : 0 \le R_1 \le r_1, 0 \le R_2 \le r_2\} \tag{4}$$

is an outer bound to the capacity region of the discrete memoryless interference channel, i.e., $\mathcal{R} \subseteq \overline{\mathcal{R}_{\mathrm{OB}}}$, where $\bar{\mathcal{A}}$ denotes the closure of the set $\mathcal{A}$.

Proof: Given a sequence of block-length n codes of rates R_1 and R_2 with vanishing error probability, using Fano's inequality, we can write

$$\begin{aligned} R_1 &\le \frac{1}{n} I(\mathbf{X}_1^n; \mathbf{Y}_1^n) + \epsilon_n \overset{(a)}{\le} \frac{1}{n} I(\mathbf{X}_1^n; \mathbf{Y}_1^n, \mathbf{X}_2^n) + \epsilon_n \\ &\overset{(b)}{=} \frac{1}{n} I(\mathbf{X}_1^n; \mathbf{Y}_1^n\,|\,\mathbf{X}_2^n) + \epsilon_n \\ &= \frac{1}{n} [H(\mathbf{Y}_1^n\,|\,\mathbf{X}_2^n) - H(\mathbf{Y}_1^n\,|\,\mathbf{X}_1^n, \mathbf{X}_2^n)] + \epsilon_n \\ &= \frac{1}{n} \sum_{i=1}^n \big[H(Y_{1,i}\,|\,\mathbf{Y}_1^{i-1}, \mathbf{X}_2^n) \\ &\qquad - H(Y_{1,i}\,|\,\mathbf{Y}_1^{i-1}, \mathbf{X}_1^n, \mathbf{X}_2^n)\big] + \epsilon_n \\ &\overset{(c)}{=} \frac{1}{n} \sum_{i=1}^n \big[H(Y_{1,i}\,|\,\mathbf{Y}_1^{i-1}, \mathbf{X}_2^n) \\ &\qquad - H(Y_{1,i}\,|\,X_{1,i}, X_{2,i})\big] + \epsilon_n \\ &\overset{(d)}{\le} \frac{1}{n} \sum_{i=1}^n [H(Y_{1,i}\,|\,X_{2,i}) - H(Y_{1,i}\,|\,X_{1,i}, X_{2,i})] + \epsilon_n \\ &= \frac{1}{n} \sum_{i=1}^n I(X_{1,i}; Y_{1,i}\,|\,X_{2,i}) + \epsilon_n \end{aligned} \tag{5}$$

where (a) follows from the non-negativity of mutual information, (b) follows from the independence between $\mathbf{X}_1^n$ and $\mathbf{X}_2^n$, (c) follows from the memoryless property of the channel, (d) follows from the fact that conditioning reduces entropy.

Next, we will need the following inequality:

$$\begin{aligned} H(\mathbf{Y}_1^n\,|\,\mathbf{X}_1^n) &= \sum_{i=1}^n H(Y_{1,i}\,|\,\mathbf{Y}_1^{i-1}, \mathbf{X}_1^n) \\ &\overset{(a)}{\ge} \sum_{i=1}^n H(Y_{1,i}\,|\,\mathbf{X}_2^{i-1}, \mathbf{Y}_1^{i-1}, \mathbf{X}_1^n) \\ &\overset{(b)}{=} \sum_{i=1}^n H(Y_{1,i}\,|\,\mathbf{X}_2^{i-1}, X_{1,i}) \end{aligned} \tag{6}$$

where (a) follows from the fact that extra conditioning reduces conditional entropy, and (b) follows from the equality

$$p(Y_{1,i}\,|\,\mathbf{X}_2^{i-1}, \mathbf{Y}_1^{i-1}, \mathbf{X}_1^n) = p(Y_{1,i}\,|\,\mathbf{X}_2^{i-1}, X_{1,i})$$

which is a consequence of the memoryless property of the channel.

Resorting again to Fano's inequality

$$\begin{aligned} R_1 &\le \frac{1}{n} I(\mathbf{X}_1^n; \mathbf{Y}_1^n) + \epsilon_n \\ &= \frac{1}{n} [H(\mathbf{Y}_1^n) - H(\mathbf{Y}_1^n\,|\,\mathbf{X}_1^n)] + \epsilon_n \\ &\overset{(a)}{\le} \frac{1}{n} \sum_{i=1}^n \big[H(Y_{1,i}) - H\big(Y_{1,i}\,|\,\mathbf{X}_2^{i-1}, X_{1,i}\big)\big] + \epsilon_n \\ &\overset{(b)}{=} \frac{1}{n} \sum_{i=1}^n I(Q_{2,i}, X_{1,i}; Y_{1,i}) + \epsilon_n \end{aligned} \tag{7}$$

where (a) follows from (6) and the fact that removing conditioning increases entropy. In step (b), we defined the random variables $Q_{2,i}$ with alphabets $\mathcal{Q}_{2,i} = \{1, \ldots, |\mathcal{Q}_{2,i}|\}$, $|\mathcal{Q}_{2,i}| = |\mathcal{X}_2|^{i-1}$, to represent the subsequences $\mathbf{X}_2^{i-1}$.

Finally, it follows trivially that

$$\begin{aligned} R_1 &\leq \frac{1}{n} H\left(\mathbf{X}_1^n\right) + \epsilon_n \\ &= \frac{1}{n}\sum_{i=1}^{n} H\left(X_{1,i} \,|\, \mathbf{X}_1^{i-1}\right) + \epsilon_n && (8) \\ &= \frac{1}{n}\sum_{i=1}^{n} H(X_{1,i} \,|\, Q_{1,i}) + \epsilon_n && (9) \end{aligned}$$

where we use $Q_{1,i}$ to represent the subsequence $\mathbf{X}_1^{i-1}$.

Let U be uniformly distributed on $\{1, \ldots, n\}$ and let $Q_1, Q_2, X_1, X_2, Y_1, Y_2$ conditioned on $U = i$ have the same joint distribution as $Q_{1,i}, Q_{2,i}, X_{1,i}, X_{2,i}, Y_{1,i}, Y_{2,i}$ in (5), (7), and (9). These three inequalities then imply that

$$\begin{aligned} R_1 \leq \min\{&I(X_1; Y_1 \,|\, X_2, U), I(Q_2, X_1; Y_1 \,|\, U), \\ &H(X_1 \,|\, Q_1, U)\} + \epsilon_n. \quad (10) \end{aligned}$$

Analogous inequalities for R_2 can be derived similarly, yielding

$$\begin{aligned} R_2 \leq \min\{&I(X_2; Y_2 \,|\, X_1, U), I(Q_1, X_2; Y_2 \,|\, U), \\ &H(X_2 \,|\, Q_2, U)\} + \epsilon_n. \quad (11) \end{aligned}$$

It is easy to see that the joint distribution of the resulting $U, Q_1, Q_2, X_1, X_2, Y_1, Y_2$ is of the form specified by (1). Moreover, using Carathéodory's theorem [5] and the procedure of [13], one can find an alternative set of random variables $U, Q_1, Q_2, X_1, X_2, Y_1, Y_2$, again with a joint distribution of the form of (1), that induce the same values for the conditional mutual information and entropy expressions in (10) and (11), but for which the alphabet sizes of U, Q_1, and Q_2 are as in the statement of the theorem. Since ϵ_n can be made arbitrarily small, this together with (10) and (11) prove that the achievable rate pair (R_1, R_2) is in the closure of $\mathcal{R}_{\mathrm{OB}}$. ■

Remark 1: One of the components of our bound, namely $I(Q_2, X_1; Y_1 \,|\, U)$, is equivalent to a bound in [15, Theorem 3] which applies to the "Z" channel and one sided interference channel satisfying certain Markovity condition. This result, which was simultaneous with the conference version of the present work [14], uses essentially the same auxiliary random variable that we introduced in (7).

The first conditional mutual informations appearing in the rate bounds (2) and (3) can be interpreted as the maximum single user rate that results when a genie provides signal x_i to receiver j $(i \neq j)$. The remaining information quantities involve an auxiliary random variable Q_i whose alphabet size can be restricted to $|\mathcal{X}_i| + 1$. The role of Q_i is to capture the memory in the input codeword $\mathbf{x}_i$.

The bound of Theorem 1 can be strengthened by incorporating the bounds of Sato [11] and Carleial [12]. While in the examples that we consider in this paper adding these bounds does not lead to tighter results, the additional bounds may prove useful for more general channels. In Section V-A2 we discuss how to incorporate the bounds of [11], [12] in the numerical evaluation of our bound.

IV. Capacity Region of the Binary Erasure Interference Channel

The binary erasure interference channel has input and output alphabets $\mathcal{X}_1 = \mathcal{X}_2 = \mathcal{Y}_1 = \mathcal{Y}_2 = \{0, 1\}$ and is defined by the equations: $y_1 = x_1 \cdot x_2$, $y_2 = x_2$. We see that receiver 2 observes a clean version of x_2, but whenever $x_2 = 0$ the symbol x_1 is "erased" at y_1.

This channel is a deterministic interference channel: the outputs are deterministic functions of the inputs. However, this channel does not belong to the class of deterministic IFCs studied in [9], because given x_1, y_1 does not completely determine x_2 (i.e., when $x_1 = 0$, y_1 does not provide any information about x_2).

Define

$$\begin{aligned} \mathcal{R}'_{\mathrm{BE}} = \bigcup_{0 \leq p_1, p_2 \leq 1} &\{(R_1, R_2) : \\ &R_1 \leq h_2(p_1 p_2) - p_1 h_2(p_2), R_2 \leq h_2(p_2)\} \quad (12) \end{aligned}$$

and

$$\mathcal{R}_{\mathrm{BE}} = \text{Convex Hull}(\mathcal{R}'_{\mathrm{BE}}). \quad (13)$$

We apply Theorem 1 to prove the following.

Theorem 2: The capacity region of the binary erasure interference channel is given by the closure of the set $\mathcal{R}_{\mathrm{BE}}$.

Remark 2: As shown in Appendix A, $\mathcal{R}'_{\mathrm{BE}}$ is not a convex set, with a slight nonconvexity occurring in the vicinity of $(R_1, R_2) \to (1, 0)$. The convex hull operation of (13) thus strictly enlarges the region.

Remark 3: In [14], we erroneously claimed $\mathcal{R}'_{\mathrm{BE}}$ to be the capacity region of the binary erasure interference channel. The statement made in the paragraph preceeding equation (4) in [14] is incorrect.

The capacity region of the binary erasure interference channel is depicted in Fig. 2. We note that the nonconvexity of $\mathcal{R}'_{\mathrm{BE}}$ is so slight that at the resolution of this figure it is not possible to distinguish between $\mathcal{R}'_{\mathrm{BE}}$ and $\mathcal{R}_{\mathrm{BE}}$.

Proof: Since for $X_1 \sim \mathcal{B}(p_1)$ and $X_2 \sim \mathcal{B}(p_2)$

$$\begin{aligned} I(X_1; Y_1) &= h_2(p_1 \cdot p_2) - p_1 h_2(p_2) \\ I(X_2; Y_2) &= h_2(p_2) \end{aligned}$$

it is easy to see that the closure of $\mathcal{R}_{\mathrm{BE}}$ belongs to the Han-Kobayashi region [10] and is therefore achievable.

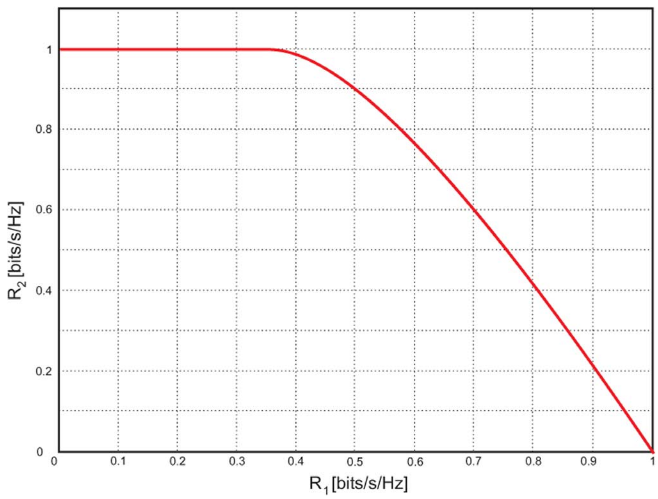

Fig. 2. Capacity region of the binary erasure interference channel.

Next, we prove the converse. If (R_1, R_2) is an achievable rate pair, Theorem 1 implies the existence of a joint distribution of the form (1) and ϵ arbitrarily small satisfying

$$\begin{aligned} R_1 - \epsilon & \\ &\leq I(Q_2, X_1; Y_1 \,|\, U) \qquad (14)\\ &= \sum_{u=1}^{6} p(u)\left[H(Y_1\,|\,u) - \sum_{x_1=0}^{1} p(x_1\,|\,u) H(Y_1\,|\,x_1, Q_2, u)\right]\\ &= \sum_{u=1}^{6} p(u)\left[H(Y_1\,|\,u) - p(X_1 = 1\,|\,u) H(X_2\,|\,Q_2, u)\right]\\ &= \sum_{u=1}^{6} p(u)\left[h_2(p_{1,u}\cdot p_{2,u}) - p_{1,u} r_u\right] \qquad (15)\end{aligned}$$

and

$$\begin{aligned} R_2 - \epsilon \leq H(X_2\,|\,Q_2, U) &\\ \leq \sum_{u=1}^{6} p(u) H(X_2\,|\,Q_2, u)&\\ = \sum_{u=1}^{6} p(u) r_u & \qquad (16)\end{aligned}$$

where we have used $p_{i,u}$ to denote $Pr(X_i = 1\,|\,U = u)$ and r_u to denote $H(X_2\,|\,Q_2, u)$. Note that $r_u \leq h_2(p_{2,u})$ for each u.

For $x, y, z \in [0, 1]$ satisfying $z \leq h_2(y)$, consider the function

$$f(x, y, z) = h_2(x \cdot y) - x \cdot z$$

Note that each term from the summation in (15) corresponds to $p_u f(p_{1,u}, p_{2,u}, r_u)$. We next establish that, for fixed z

$$\max_{\substack{\{x,y:0\leq x\leq 1,\\ z\leq h_2(y)\}}} f(x, y, z) = \log_2\left(1 + 2^{z/h^{-1}(z)}\right) - \frac{z}{h^{-1}(z)} \qquad (17)$$

where $h^{-1}(z)$ is the inverse of the binary entropy function and satisfies

$$h^{-1}(z) \geq 1/2, \quad h_2(h^{-1}(z)) = z. \qquad (18)$$

To obtain (17), note first that for fixed y, z, $f(x, y, z)$ is concave in x so that it can be maximized over x (for fixed y, z) by setting the derivative with respect to x to 0. Specifically

$$\frac{df(x, y, z)}{dx} = y \log_2\left(\frac{1 - xy}{xy}\right) - z = 0$$

which can be solved for x to give the maximizing x^*

$$x^* = \frac{1}{y + y 2^{z/y}}. \qquad (19)$$

Substituting this into $f(x, y, z)$ and simplifying yields

$$\begin{aligned}\max_{0\leq x\leq 1} f(x, y, z) &\leq \max_{0\leq x} f(x, y, z)\\ &= \log_2\left(1 + 2^{z/y}\right) - \frac{z}{y}. \qquad (20)\end{aligned}$$

Next, consider the function $g(w) = \log_2(1+2^w) - w$ and note that (20) corresponds to $g(z/y)$. Differentiating $g(w)$ yields

$$g'(w) = \frac{2^w}{1 + 2^w} - 1 \leq 0$$

implying that $g(w)$ is decreasing in w, and, therefore, that (20) is maximized over y (for fixed z) by taking y as large as possible. Thus, the maximizing y is

$$y^* = h^{-1}(z) \qquad (21)$$

with $h^{-1}(z)$ as defined above.

Note that for $y = y^*$, the maximizing x, as given by (19), satisfies

$$\begin{aligned} x^* &= \frac{1}{y^* + y^* 2^{h_2(y^*)/y^*}}\\ &= \frac{1}{y^* + 2^{-((1-y^*)/y^*)\log_2(1-y^*)}}\\ &\leq 1. \qquad (22)\end{aligned}$$

Equation (17) thus follows, with the maximum being attained at (x^*, y^*) as given by (21) and (22).

Incorporating (17), (21), and (22) into (15) and (16), yields

$$R_1 - \epsilon \leq \sum_{u=1}^{6} p(u)\left[h_2(p_{1,u}^* \cdot p_{2,u}^*) - p_{1,u}^* h_2(p_{2,u}^*)\right] \qquad (23)$$

$$R_2 - \epsilon \leq \sum_{u=1}^{6} p(u) h_2(p_{2,u}^*) \qquad (24)$$

with $(p_{1,u}^*, p_{2,u}^*)$ satisfying (given r_u above)

$$p_{2,u}^* = h^{-1}(r_u) \in [1/2, 1] \qquad (25)$$

and

$$p_{1,u}^* = \frac{1}{p_{2,u}^* + 2^{-((1-p_{2,u}^*)/p_{2,u}^*)\log_2(1-p_{2,u}^*)}} \in [0, 1]. \qquad (26)$$

It follows from (25) and (26) that

$$(R_1^*, R_2^*) \triangleq \left(\sum_{u=1}^{6} p(u) \left[h_2(p_{1,u}^* \cdot p_{2,u}^*) - p_{1,u}^* h_2(p_{2,u}^*) \right], \sum_{u=1}^{6} p(u) h_2(p_{2,u}^*) \right) \quad (27)$$

belongs to $\mathcal{R}_{\rm BE}$, the convex hull of $\mathcal{R}'_{\rm BE}$, and hence, from (23) and (24), and the fact that ϵ can be taken to be arbitrarily small, that (R_1, R_2) belongs to the closure of $\mathcal{R}_{\rm BE}$. ∎

V. Other Examples

In order to illustrate how the bound of Theorem 1 improves upon the existing outer bounds, but also to understand its limitations we present some additional examples.

A. Other Deterministic Two-User Binary IFCs

A deterministic binary IFC is one for which $\mathcal{X}_1 = \mathcal{X}_2 = \{0,1\}$ and $Y_i = f_i(X_1, X_2) \in \{0,1\}$ are deterministic binary valued functions of the inputs. The binary erasure IFC is a special case whose capacity region was heretofore unknown. The capacity regions of most nondegenerate deterministic binary IFCs (those in which both outputs depend on their corresponding inputs) are readily seen, using known methods, to be the time-sharing region $\{(R_1, R_2) : R_1 + R_2 \leq 1\}$. This is obviously the case, by the MAC bound [11], [12], for channels for which $f_1 = f_2$. It is also the case for channels for which, for at least one $i \in \{1,2\}$, f_i is a one-to-one function in x_j, $j \neq i$ for all values of x_i (this is the El Gamal–Costa condition from [9] for these IFCs). In such a case, the i-th receiver, upon decoding its own signal can also decode the interfering signal, thereby implying a MAC constraint of 1 on the rate sum at that receiver.

A useful observation concerning deterministic binary IFCs is that if $\mathcal{C}_{f_1,f_2}$ is the capacity region for a particular choice of (f_1, f_2) then the capacity regions $\mathcal{C}_{f'_1,f'_2}$ for (f'_1, f'_2) given by $f'_1(x_1,x_2) = o_1 \oplus f_1(i_1 \oplus x_1, i_2 \oplus x_2)$ and $f'_2(x_1,x_2) = o_2 \oplus f_2(i_1 \oplus x_1, i_2 \oplus x_2)$ coincide with $\mathcal{C}_{f_1,f_2}$ for all choices of $o_j \in \{0,1\}$ and $i_j \in \{0,1\}$, where $\oplus$ denotes addition modulo 2 (or the "exclusive-or" Boolean operation). Another obvious isomorphism given $\mathcal{C}_{f_1,f_2}$ is that if $(R_1,R_2) \in \mathcal{C}_{f_1,f_2}$ then $(R_2, R_1) \in \mathcal{C}_{f'_1,f'_2}$ where $f'_1(x_1,x_2) = f_2(x_2,x_1)$ and $f'_2(x_1,x_2) = f_1(x_2,x_1)$.

A little reflection based on the above conditions for optimality of the time-sharing region, together with the above isomorphisms, leads to the conclusion, that, again up to the above isomorphisms, there are three deterministic binary IFCs whose capacity regions are not immediate based on previously known bounds. These are:

Binary erasure IFC

$$f_1(x_1,x_2) = x_1 \cdot x_2, \quad f_2(x_1,x_2) = x_2 \quad (28)$$

IFC A

$$f_1(x_1,x_2) = x_1 \cdot x_2, \quad f_2(x_1,x_2) = \overline{x_1} \cdot x_2 \quad (29)$$

and

IFC B

$$f_1(x_1,x_2) = x_1 \cdot x_2, \quad f_2(x_1,x_2) = \overline{x_1} \cdot \overline{x_2} \quad (30)$$

where $\bar{x}$ denotes $1 \oplus x$.

We have shown in the previous section that the capacity region of the binary erasure IFC can be derived using Theorem 1. This capacity region is larger than the time-sharing region. For IFC's A and B, on the other hand, it is natural to conjecture that their capacity regions coincide with the time-sharing region. It turns out, however, that Theorem 1 is not sufficient to prove this conjecture in either case. For IFC A, we are able to prove the conjecture via an alternative genie aided approach which leads to a fortuitous cancellation of conditional entropy terms. For IFC B, this alternative approach fails. In this case, we demonstrate that Theorem 1 gives the tightest bound on the maximum achievable sum-rate among all bounding techniques we are aware of.

1) Deterministic Binary IFC A: We prove the following about this IFC.

Theorem 3: The capacity region of IFC A, defined by (29), is the time-sharing region $\{(R_1, R_2) : R_1 + R_2 \leq 1\}$.

Proof: Achievability is obvious, since each transmitter can create a noiseless channel for the other transmitter with an appropriate constant signal. Time-sharing then achieves the full region.

For the converse, given a sequence of block-length n codes of rates R_1 and R_2 with vanishing error probability, using Fano's inequality, we can write

$$\begin{aligned} n(R_1+R_2) &\leq I(\mathbf{X}_1^n; \mathbf{Y}_1^n) + I(\mathbf{X}_2^n; \mathbf{Y}_2^n) + n\epsilon_n \\ &\leq I(\mathbf{X}_1^n; \mathbf{Y}_1^n, \mathbf{X}_2^n) + I(\mathbf{X}_2^n; \mathbf{Y}_2^n) + n\epsilon_n \\ &= H(\mathbf{Y}_1^n | \mathbf{X}_2^n) + H(\mathbf{Y}_2^n) \\ &\quad - H(\mathbf{Y}_2^n | \mathbf{X}_2^n) + n\epsilon_n \end{aligned} \quad (31)$$

where (31) follows from the deterministic nature of the channel and the independence of $\mathbf{X}_1^n$ and $\mathbf{X}_2^n$.

Given a binary sequence $x^n \in \{0,1\}^n$, let $\mathbf{X}_1^n \cdot x^n$ denote the random variables $X_{1,1} \cdot x_1, X_{1,2} \cdot x_2, \ldots, X_{1,n} \cdot x_n$ (i.e., the component-wise product of $\mathbf{X}_1^n$ and x^n). It is then easy to see that for IFC A

$$\begin{aligned} H(\mathbf{Y}_1^n \,|\, \mathbf{X}_2^n) &= \sum_{x^n \in \{0,1\}^n} H(\mathbf{X}_1^n \cdot x^n) \, Pr(\mathbf{X}_2^n = x^n) \\ &= \sum_{x^n \in \{0,1\}^n} H\left(\overline{\mathbf{X}_1^n} \cdot x^n\right) Pr(\mathbf{X}_2^n = x^n) \\ &= H(\mathbf{Y}_2^n \,|\, \mathbf{X}_2^n) \end{aligned} \quad (32)$$

where $\overline{\mathbf{X}_1^n}$ denotes $\overline{X_{1,1}}, \ldots, \overline{X_{1,n}}$. It then follows from (31) and (32) that

$$\begin{aligned} n(R_1+R_2) &\leq H(\mathbf{Y}_2^n) + n\epsilon_n \\ &\leq n + n\epsilon_n \end{aligned}$$

which, since $\epsilon_n \to 0$, in turn, implies $R_1 + R_2 \leq 1$. ∎

Remark 4: The first step in the the converse (i.e., bounding by $I(\mathbf{X}_1^n; \mathbf{Y}_1^n, \mathbf{X}_2^n)$) corresponds to a genie aided IFC in which the interference $\mathbf{X}_2^n$ is supplied to receiver 1. We note that the alternative genie aided IFC obtained by instead providing the interference to receiver 2 (i.e., bounding $I(\mathbf{X}_2^n;\mathbf{Y}_2^n)$ by $I(\mathbf{X}_2^n;\mathbf{Y}_2^n,\mathbf{X}_1^n)$) can only lead to a strictly inferior bound on the sum-rate. The analogue of (31) in this case is

$$n(R_1+R_2) \le H(\mathbf{Y}_2^n|\mathbf{X}_1^n)+H(\mathbf{Y}_1^n) - H(\mathbf{Y}_1^n|\mathbf{X}_1^n)+n\epsilon_n \quad (33)$$

and, unfortunately, the terms $H(\mathbf{Y}_2^n\,|\,\mathbf{X}_1^n)$ and $H(\mathbf{Y}_1^n\,|\,\mathbf{X}_1^n)$ do not cancel as above. This can be seen from the definition of IFC A (29), which indicates that conditioned on $\mathbf{X}_1^n = x^n$, $\mathbf{Y}_1^n$ and $\mathbf{Y}_2^n$ now correspond to *complementary* subsets of the components of $\mathbf{X}_2^n$. In fact, optimizing the multiletter bound (33) just over i.i.d. channel inputs $\mathbf{X}_1^n$ and $\mathbf{X}_2^n$ already leads to a bound on the sum-rate that is greater than the time sharing bound.

2) Deterministic Binary IFC B: The technique used to bound the sum-rate for IFC A does not yield the conjectured $R_1+R_2 \le 1$ for IFC B. The reason is rooted in Remark 4 in that in this case, both genie aided IFCs suggested by the analysis of IFC A, are similar to the suboptimal genie mentioned in the remark with respect to the difference in the multiletter conditional entropies appearing in the corresponding step (31) in their analysis.

We would like to apply the bound of Theorem 1 to this deterministic channel. However, a direct numerical calculation of the outer bound is infeasible due to the relatively large number of variables over which we need to optimize. We derive a sum-rate bound from the bound of Theorem 1 that can be evaluated numerically with reasonable complexity. Using (2)–(4), we can write [see (34)–(36), shown at the bottom of the page], where in (a) we specialized the bound to the channel considered in this subsection and we defined $p_i = Pr(X_i=1)$, $q_{ij} = Pr(Q_i = j)$, $p_{ij} = Pr(X_i = 1\,|\,Q_i = j)$, and $\bar{p}_i = 1 - p_i$ for $i = 1, 2$. Using Carathéodory's theorem [5] and the procedure of [13] one can show that it is enough to take $|\mathcal{Q}_i| = 2$, $i = 1, 2$.

We did a numerical search to find "good" values for the variables in the outer minimization, and obtained $\lambda_1 = 0.14$, $\lambda_2 = 0.64$, $\phi_1 = 0.12$, and $\phi_2 = 0.66$. Note that any choice of these variables leads to a valid sum-rate upper bound. With these choices, we computed the inner maximization by doing an exhaustive search over a grid with diminishing step sizes in three passes. The steps used were $3.33\cdot 10^{-2}$, $3.33\cdot10^{-3}$, and $3.33\cdot10^{-4}$, searching in each pass around the optimal solution found in the previous pass. The final result that we obtained was $R_1 + R_2 \le 1.189$.

Next we compare this bound to some other bounds that we can derive for this channel. A simple bound is obtained by allowing the receivers to cooperate [11]. The resulting channel is a multiple access channel, whose sum capacity is given by

$$\begin{aligned} R_1 + R_2 &\le \max_{p(X_1)p(X_2)} I(X_1, X_2; Y_1, Y_2) \\ &= \max_{p(X_1)p(X_2)} H(Y_1, Y_2) \\ &\le \max_{0\le p_1,p_2\le 1} h_3(p_1\cdot p_2, (1-p_1)(1-p_2)) \\ &= 1.5. \end{aligned}$$

A different bound is obtained by noting that removing one of the interfering links at either receiver 1 or receiver 2 results in a channel whose capacity region includes the capacity region of the original channel. These channels are $y_1 = x_1\cdot x_2, y_2 = \bar{x}_2$,

$$\begin{aligned}
&R_1 + R_2 \\
&\le \max_{\substack{p_U, p_{Q_1|U},\\ p_{Q_2|U}, p_{X_1|Q_1,U},\\ p_{X_2|Q_2,U}}} \{r_1 + r_2\} && (34)\\
&= \max_{\substack{p_U, p_{Q_1|U},\\ p_{Q_2|U}, p_{X_1|Q_1,U},\\ p_{X_2|Q_2,U}}} \{\min\{I(X_1;Y_1\,|\,X_2,U),\ \ I(Q_2,X_1;Y_1\,|\,U), H(X_1\,|\,Q_1,U)\}\\
&\quad + \min\{I(X_2;Y_2\,|\,X_1,U), I(Q_1,X_2;Y_2\,|\,U), H(X_2\,|\,Q_2,U)\}\}\\
&= \max_{\substack{p_U, p_{Q_1|U},\\ p_{Q_2|U}, p_{X_1|Q_1,U},\\ p_{X_2|Q_2,U}}} \min_{\substack{\lambda_1,\lambda_2,\phi_1,\phi_2:\\ \lambda_1\ge0,\lambda_2\ge0,\lambda_1+\lambda_2\le1,\\ \phi_1\ge0,\phi_2\ge0,\phi_1+\phi_2\le1}} \{\lambda_1 I(X_1;Y_1\,|\,X_2,U) + \lambda_2 I(Q_2,X_1;Y_1\,|\,U) + (1-\lambda_1-\lambda_2)H(X_1\,|\,Q_1,U)\\
&\quad +\phi_1 I(X_2;Y_2\,|\,X_1,U) + \phi_2 I(Q_1,X_2;Y_2\,|\,U) + (1-\phi_1-\phi_2)H(X_2\,|\,Q_2,U)\}\\
&\le \min_{\substack{\lambda_1,\lambda_2,\phi_1,\phi_2:\\ \lambda_1\ge0,\lambda_2\ge0,\lambda_1+\lambda_2\le1,\\ \phi_1\ge0,\phi_2\ge0,\phi_1+\phi_2\le1}} \max_{\substack{p_{Q_1},p_{Q_2}\\ p_{X_1|Q_1},p_{X_2|Q_2}}} \{\lambda_1 I(X_1;Y_1\,|\,X_2) + \lambda_2 I(Q_2,X_1;Y_1) + (1-\lambda_1-\lambda_2)H(X_1\,|\,Q_1)\\
&\quad +\phi_1 I(X_2;Y_2\,|\,X_1) + \phi_2 I(Q_1,X_2;Y_2) + (1-\phi_1-\phi_2)H(X_2\,|\,Q_2)\} && (35)\\
&\overset{(a)}{=} \min_{\substack{\lambda_1,\lambda_2,\phi_1,\phi_2:\\ \lambda_1\ge0,\lambda_2\ge0,\lambda_1+\lambda_2\le1,\\ \phi_1\ge0,\phi_2\ge0,\phi_1+\phi_2\le1}} \max_{0\le p_1,p_2\le1} \max_{\substack{0\le q_{1i},q_{2i},p_{1i},p_{2i}\le1\\ \sum_{i=1}^{|\mathcal{Q}_1|} q_{1i}p_{1i}=p_1, \sum_{i=1}^{|\mathcal{Q}_1|} q_{1i}=1\\ \sum_{i=1}^{|\mathcal{Q}_2|} q_{2i}p_{2i}=p_2, \sum_{i=1}^{|\mathcal{Q}_2|} q_{2i}=1}} \Bigg\{\lambda_1 p_2 h_2(p_1) + (1-\lambda_1-\lambda_2)\sum_{i=1}^{|\mathcal{Q}_1|} q_{1i}h_2(p_{1i})\\
&\quad +\lambda_2\left[h_2(p_1p_2) - p_1\sum_{i=1}^{|\mathcal{Q}_2|} q_{2i}h_2(p_{2i})\right] + \phi_1\bar{p}_1 h_2(p_2) + (1-\phi_1-\phi_2)\sum_{i=1}^{|\mathcal{Q}_2|} q_{2i}h_2(p_{2i}) + \phi_2\left[h_2(\bar{p}_1\bar{p}_2) - \bar{p}_2\sum_{i=1}^{|\mathcal{Q}_1|} q_{1i}h_2(p_{1i})\right]\Bigg\} && (36)
\end{aligned}$$

and $y_1 = x_1, y_2 = \bar{x}_1 \cdot \bar{x}_2$. Both of these channels are isomorphic to the binary erasure interference channel whose capacity region was derived in Section IV. Computing the intersection of these capacity regions and maximizing the sum-rate we get the bound $R_1 + R_2 \leq 1.314$.

Another bound results from using genies that provide the interfering signals to the receivers

$$\begin{aligned} R_1 + R_2 &\overset{(a)}{\leq} \frac{1}{n}\left(I\left(\mathbf{X}_1^n;\mathbf{Y}_1^n,\mathbf{X}_2^n\right) + I\left(\mathbf{X}_2^n;\mathbf{Y}_2^n,\mathbf{X}_1^n\right)\right) + \epsilon_n \\ &\overset{(b)}{=} \frac{1}{n}\left(I\left(\mathbf{X}_1^n;\mathbf{Y}_1^n \,|\, \mathbf{X}_2^n\right) + I\left(\mathbf{X}_2^n;\mathbf{Y}_2^n \,|\, \mathbf{X}_1^n\right)\right) + \epsilon_n \\ &\overset{(c)}{=} \frac{1}{n}\left(H\left(\mathbf{Y}_1^n \,|\, \mathbf{X}_2^n\right) + H\left(\mathbf{Y}_2^n \,|\, \mathbf{X}_1^n\right)\right) + \epsilon_n \\ &\overset{(d)}{\leq} \frac{1}{n}\sum_{i=1}^{n}\left(H\left(Y_{1,i} \,|\, X_{2,i}\right) + H\left(Y_{2,i} \,|\, X_{1,i}\right)\right) + \epsilon_n \\ &\overset{(e)}{\leq} \max_{0\leq p_1,p_2\leq 1} p_2 h_2(p_1) + (1-p_1)h_2(1-p_2) \\ &= 1.2338 \end{aligned}$$

where (a) follows from Fano's inequality and the non-negativity of mutual information, (b) from the chain rule for mutual information and the independence between $\mathbf{X}_1^n$ and $\mathbf{X}_2^n$, (c) from the deterministic nature of the channel, (d) by dropping conditioning, which does not reduce entropy, and (e) by letting $n \to \infty$ so that $\epsilon_n \to 0$ and using $p_i = Pr(X_i = 1)$, $i = 1, 2$.

We see that the sum-rate bound derived from Theorem 1 (i.e., $R_1 + R_2 \leq 1.189$) is tighter than the last three bounds.

The sum-rate bound (34) is applicable to a general channel. This bound can be further strengthened by adding, for example, the bound of [11]

$$R_1 + R_2 \leq I(X_1, X_2; Y_1, Y_2 \,|\, U) = r_s$$

and taking $\min\{r_1 + r_2, r_s\}$ as the function to be maximized in (34). In this case, in (35), we can introduce a new variable $\beta \in [0,1]$ in the minimization and replace the objective of the optimization $\{\cdots\}$ by $\beta\{\cdots\} + (1-\beta)I(X_1, X_2; Y_1, Y_2)$. For the channel considered in this section, the resulting bound does not seem better than (36).

B. *One-Sided El Gamal–Costa IFC*

Consider a deterministic interference channel defined by

$$\begin{aligned} v_1 &= g_1(x_1), \quad v_2 = g_2(x_2) \\ y_1 &= f_1(x_1, v_2), \quad y_2 = f_2(x_2, v_1) \end{aligned}$$

where $g_1(\cdot)$ and $g_2(\cdot)$ are arbitrary functions, while for fixed x_1 (resp. x_2), $f_1(x_1, \cdot)$ (resp. $f_2(x_2, \cdot)$) is injective. The capacity region of this channel has been obtained in [9].

We will show that for this special class of channels, when $y_2 = x_2$ (i.e., $f_2(x_2, v_1) = x_2$) Theorem 1 is tight. Applying Theorem 1 to this channel, we obtain

$$\begin{aligned} r_1 &\leq I(X_1; Y_1 \,|\, X_2, U) = H(Y_1 \,|\, X_2, U) \\ &= H(Y_1 \,|\, V_2, X_2, U) \leq H(Y_1 \,|\, V_2, U) \\ r_2 &\leq I(X_2; Y_2 \,|\, X_1, U) = H(Y_2 \,|\, X_1, U) = H(Y_2 \,|\, U) \\ r_1 + r_2 &\leq I(Q_2, X_1; Y_1 \,|\, U) + H(X_2 \,|\, Q_2, U) \\ &= H(Y_1 \,|\, U) + H(X_2 \,|\, Q_2, U) - H(Y_1 \,|\, Q_2, X_1, U) \\ &= H(Y_1 \,|\, U) + H(X_2 \,|\, Q_2, U) - H(V_2 \,|\, Q_2, U) \\ &= H(Y_1 \,|\, U) + H(X_2 \,|\, V_2, Q_2, U) \\ &\overset{(a)}{\leq} H(Y_1 \,|\, U) + H(Y_2 \,|\, V_2, U) \end{aligned} \tag{37}$$

where (a) follows from the assumption $y_2 = x_2$ and removing conditioning. It is easy to check that the right hand sides of (37) coincide with the nonredundant bounds in [9, Theorem 1]. It follows that $\overline{\mathcal{R}_{\mathrm{OB}}}$ of Theorem 1 is the capacity region of this channel.

APPENDIX A
NONCONVEXITY OF $\mathcal{R}'_{\mathrm{BE}}$.

Equation (17) in the proof of Theorem 2 implies that

$$\begin{aligned} R(r) &\triangleq \max\{R_1 : (R_1, r) \in \mathcal{R}'_{\mathrm{BE}}\} \\ &= \log_2\left(1 + 2^{r/h^{-1}(r)}\right) - \frac{r}{h^{-1}(r)} \end{aligned}$$

where $h^{-1}(r)$ is the inverse of the binary entropy function defined by (18). The nonconvexity of $\mathcal{R}'_{\mathrm{BE}}$ follows from the nonconcavity of $R(r)$ in r. To see the latter, note that from basic calculus

$$\frac{dh^{-1}(r)}{dr} = \left[\log_2\left(\frac{1 - h^{-1}(r)}{h^{-1}(r)}\right)\right]^{-1}$$

and therefore (after some simplification)

$$\begin{aligned} \frac{dR(r)}{dr} = &\left(\frac{1}{h^{-1}(r)} - \frac{r}{(h^{-1}(r))^2}\right. \\ &\left.\cdot\left[\log_2\left(\frac{1 - h^{-1}(r)}{h^{-1}(r)}\right)\right]^{-1}\right)\left(\frac{-1}{1 + 2^{r/h^{-1}(r)}}\right). \end{aligned}$$

Evaluating $dR(r)/dr$ at $r = h_2(1) = 0$, $r = h_2(.984)$, and $r = h_2(.9)$, respectively, gives $-.5$, $-.497$, and $-.531$, indicating a slight nonconcavity.

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