

Gaussian Interference Channel Capacity to Within One Bit

Raul H. Etkin, *Member, IEEE*, David N. C. Tse, *Member, IEEE*, and Hua Wang

Abstract—The capacity of the two-user Gaussian interference channel has been open for 30 years. The understanding on this problem has been limited. The best known achievable region is due to Han and Kobayashi but its characterization is very complicated. It is also not known how tight the existing outer bounds are. In this work, we show that the existing outer bounds can in fact be arbitrarily loose in some parameter ranges, and by deriving new outer bounds, we show that a very simple and explicit Han–Kobayashi type scheme can achieve to within a single bit per second per hertz (bit/s/Hz) of the capacity for all values of the channel parameters. We also show that the scheme is asymptotically optimal at certain high signal-to-noise ratio (SNR) regimes. Using our results, we provide a natural generalization of the point-to-point classical notion of degrees of freedom to interference-limited scenarios.

Index Terms—Capacity region, Gaussian interference channel, generalized degrees of freedom.

I. INTRODUCTION

INTERFERENCE is a central phenomenon in wireless communication when multiple uncoordinated links share a common communication medium. Most state-of-the-art wireless systems deal with interference in one of two ways:

- orthogonalize the communication links in time or frequency, so that they do not interfere with each other at all;
- allow the communication links to share the same degrees of freedom, but treat each other's interference as adding to the noise floor.

It is clear that both approaches can be suboptimal. The first approach entails an *a priori* loss of degrees of freedom in both links, no matter how weak the potential interference is. The second approach treats interference as pure noise while it actually carries information and has structure that can potentially be exploited in mitigating its effect.

These considerations lead to the natural question of what is the best performance one can achieve without making any *a priori* assumptions on how the common resource is shared. A

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R. Etkin is with Hewlett-Packard Laboratories, Palo Alto, CA 94304 USA (e-mail: raul.etkin@hp.com).

D. N. C. Tse is with the Wireless Foundations, University of California, Berkeley, Berkeley, CA 94720 USA (e-mail: dtse@eecs.berkeley.edu).

H. Wang is with Qualcomm Flarion Technologies, Bridgewater, NJ 08807 USA (e-mail: huawang10@gmail.com).

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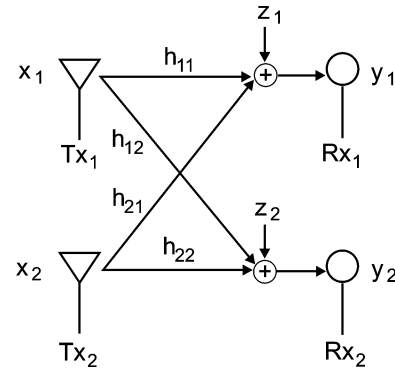


Fig. 1. Two-user Gaussian interference channel.

basic information theory model to study this question is the two-user Gaussian interference channel, where two point-to-point links with additive white Gaussian noise (AWGN) interfere with each other (Fig. 1).

The capacity region of this channel is the set of all simultaneously achievable rate pairs (R_1, R_2) in the two interfering links, and characterizes the fundamental tradeoff between the performance achievable in the two links in the face of interference. Unfortunately, the problem of characterizing this region has been open for over 30 years. The only case in which the capacity is known is in the *strong* interference case, where each receiver has a better reception of the other user's signal than the intended receiver [1], [2]. The best known strategy for the other cases is due to Han and Kobayashi [1]. This strategy is a natural one and involves splitting the transmitted information of both users into two parts: private information to be decoded only at the intended receiver and common information that can be decoded at both receivers. By decoding the common information, part of the interference can be canceled off, while the remaining private information from the other user is treated as noise. The Han–Kobayashi strategy allows arbitrary splits of each user's transmit power into the private and common information portions as well as time sharing between multiple such splits. Unfortunately, the optimization among such myriads of possibilities is not well-understood, so while it is clear that it will be no worse than the above-mentioned strategies as it includes them as special cases, it is not very clear how much improvement can be obtained and in which parameter regime would one get significant improvement. More importantly, it is also not clear how close to capacity can such a scheme get and whether there will be other strategies that can do significantly better.

In this paper, we make progress on this state of affairs by showing that a very simple Han–Kobayashi type scheme can in fact achieve rates within 1 bit/s/Hz of the capacity of the channel

for *all* values of the channel parameters. That is, this scheme can achieve the rate pair $(R_1 - 1, R_2 - 1)$ for any (R_1, R_2) in the interference channel capacity region. This result is particularly relevant in the high signal-to-noise ratio (SNR) regime, where the achievable rates are high and grow unbounded as the noise level goes to zero. In fact, in some high-SNR regimes, we can strengthen our results to show that our scheme is asymptotically optimal. The high-SNR regime is the interference-limited scenario: when the noise is small, interference from one link will have a significant impact on the performance of the other. The low-SNR regime is less interesting since here the performance of each link is primarily noise-limited and interference is not having a significant effect.

The key feature of the scheme is that the power of the private information of each user should be set such that it is received at the level of the Gaussian noise at the other receiver. In this way, the interference caused by the private information has a small effect on the other link as compared to the impairments already caused by the noise. At the same time, quite a lot of private information can be conveyed in the own link if the direct gain is appreciably larger than the cross gain. Moreover, the scheme involves only a single private–common split and no time-sharing is needed.

A key step in obtaining our result is the introduction of the new concept of *generalized degrees of freedom*, a natural generalization of the classical notion of degrees of freedom in point-to-point communication to multiuser scenarios. This notion provides a useful tool to approximate interference-limited performance in the high-SNR regime. Using the generalized degrees of freedom framework as guidance, we show that the gap between the performance of our proposed scheme and existing outer bounds in the literature is small in some parameter regimes but arbitrarily loose in others. We derive new outer bounds to cover for the other parameter ranges. Like the existing bounds, the new outer bounds are also based on giving extra side information to the receivers. But unlike the existing bounds, the side information in the new outer bounds is *not* sufficient for any of the receivers to decode the message from the other user.

The rest of the paper is structured as follows. In Section II, we describe the model. Section III focuses on the symmetric rate point in the symmetric interference channel, where the results can be described in the simplest form. Results on the entire capacity region for the general two-user channel are explained in Section IV. Section V investigates how the generalized degrees of freedom in the general case depend on the various channel parameters. Section VI provides intuition about the simple Han–Kobayashi scheme used in this paper. Section VII explores some analogies between our results and those of El Gamal and Costa on a deterministic interference channel [3].

Regarding notation, we will use lowercase or uppercase letters for scalars, lowercase boldface letters for vectors, and calligraphic letters for sets. For example, we write h or P for scalars, \mathbf{x} for a vector, and \mathcal{R} for a set. We use $H(\cdot)$ to denote binary entropy of a discrete random variable or vector, $h(\cdot)$ to denote binary differential entropy of a continuous random variable or vector, and $I(\cdot; \cdot)$ to denote mutual information. In addition, unless otherwise stated, all logarithms are to the base 2.

II. MODEL

In this section, we describe the model to be used in the rest of this paper. We consider a two-user complex Gaussian interference channel. In this model, there are two transmitter–receiver pairs, where each transmitter wants to communicate with its corresponding receiver (cf. Fig. 1).

This channel is represented by the equations

$$\begin{aligned} y_1 &= h_{11}x_1 + h_{21}x_2 + z_1 \\ y_2 &= h_{12}x_1 + h_{22}x_2 + z_2 \end{aligned} \quad (1)$$

where for $i = 1, 2$, $x_i \in \mathbb{C}$ is subject to a power constraint P_i , i.e., $E[|x_i|^2] \leq P_i$, and the noise processes $z_i \sim \mathcal{CN}(0, N_0)$ are independent and identically distributed (i.i.d.) over time.

While the capacity region of the complex Gaussian interference channel may depend on the phases of the channel gains $\{h_{i,j}\}$, the inner and outer bounds that we present in this paper only depend on the magnitudes $\{|h_{i,j}|\}$. As a result, we can use for our bounds a parameterization in terms of the signal-to-noise and interference-to-noise ratios. For $i = 1, 2$, let $\text{SNR}_i = |h_{ii}|^2 P_i / N_0$ be the SNR of user i , and $\text{INR}_1 = |h_{21}|^2 P_2 / N_0$ ($\text{INR}_2 = |h_{12}|^2 P_1 / N_0$) be the interference to noise ratio of user 1 (2). As will become apparent from our analysis, this parameterization in terms of SNR and INR is more natural for the interference channel, because it puts in evidence the main factors that determine the channel capacity.

For a given block length n , user i communicates a message $m_i \in \{1, \dots, 2^{nR_i}\}$ by choosing a codeword from a codebook $\mathcal{C}_{i,n}$, with $|\mathcal{C}_{i,n}| = 2^{nR_i}$. The codewords $\{\mathbf{c}_i(m_i)\}$ of this codebook must satisfy the average power constraint

$$\frac{1}{n} \sum_{t=1}^n |c_i(m_i)[t]|^2 \leq P_i.$$

Receiver i observes the channel outputs $\{y_i[t] : t = 1, \dots, n\}$ and uses a decoding function $f_{i,n} : \mathbb{C}^n \rightarrow \mathbb{N}$ to get the estimate \hat{m}_i of the transmitted message m_i . The receiver is in error whenever $\hat{m}_i \neq m_i$. The average probability of error for user i is given by

$$\epsilon_{i,n} = E[P(\hat{m}_i \neq m_i)]$$

where the expectation is taken with respect to the random choice of the transmitted messages m_1 and m_2 .

A rate pair (R_1, R_2) is achievable if there exists a family of codebook pairs $\{(\mathcal{C}_{1,n}, \mathcal{C}_{2,n})\}_n$ with codewords satisfying the power constraints P_1 and P_2 , respectively, and decoding functions $\{(f_{1,n}(\cdot), f_{2,n}(\cdot))\}_n$, such that the average decoding error probabilities $\epsilon_{1,n}, \epsilon_{2,n}$ go to zero as the block length n goes to infinity.

The capacity region \mathcal{R} of the interference channel is the closure of the set of achievable rate pairs.

III. SYMMETRIC GAUSSIAN INTERFERENCE CHANNEL

A. Symmetric Channel and Symmetric Rate Point

In order to introduce the main ideas and results in the simplest possible setting, we start our analysis of the interference channel capacity region by considering a symmetric interference channel and the symmetric rate point.

In the symmetric interference channel we have $|h_{11}|^2 = |h_{22}|^2 = |h_d|^2$, $|h_{12}|^2 = |h_{21}|^2 = |h_c|^2$ and $P_1 = P_2 = P$, or equivalently, $\text{SNR}_1 = \text{SNR}_2$ and $\text{INR}_1 = \text{INR}_2$. In addition, the symmetric capacity is the solution to the following optimization problem:

$$C_{\text{sym}} = \begin{cases} \text{Maximize:} & \min\{R_1, R_2\} \\ \text{Subject to:} & (R_1, R_2) \in \mathcal{R} \end{cases}$$

where \mathcal{R} is the capacity region of the interference channel.

Due to the convexity and symmetry of the capacity region of the symmetric channel, the symmetric capacity is attained at the point of the capacity region that maximizes the sum rate $R_1 + R_2$. Since the capacity region is known in the strong interference case when $\text{INR}/\text{SNR} \geq 1$, we will focus on the case where $0 < \text{INR}/\text{SNR} < 1$. In addition, we will concentrate on the situation where $\text{INR} \geq 1$, i.e., the interfering signal power is at least as large as the noise power. The case $\text{INR} < 1$ is not so interesting because the communication is essentially limited by noise. We will address this case briefly later.

B. A Simple Communication Scheme

We will use a simple communication scheme that is a special case of the general type of schemes introduced by Han and Kobayashi in [1]. For a given block length n , user i chooses a private message from codebook $\mathcal{C}_{i,n}^u$ and a common message from codebook $\mathcal{C}_{i,n}^w$. These codebooks satisfy the power constraints P_u and P_w with $P_u + P_w = P$. The sizes of these codebooks are such that $|\mathcal{C}_{i,n}^u| \cdot |\mathcal{C}_{i,n}^w| = 2^{nR_i}$. After selecting the corresponding codewords, user i transmits the signal $\mathbf{x}_i = \mathbf{c}_i^u + \mathbf{c}_i^w$ by adding the private and common codewords. The private codewords are meant to be decoded by receiver i , while the common codewords must be decoded by both receivers.

The general Han and Kobayashi scheme allows to generate the codebooks using arbitrary input distributions, and allows to do time sharing between multiple strategies. We will consider a simple scheme where the codebooks are generated by using i.i.d. random Gaussian variables with the appropriate variances. Let $\text{INR}_p = |h_c|^2 P_u / N_0$, that is, INR_p is the interference-to-noise ratio created onto the nonintending receiver by the private message. We choose $\text{INR}_p = 1$, i.e., the interference created by the private message has the same power as the Gaussian noise.¹ In addition, we use a fixed strategy, i.e., we do not do time sharing.

Why do we choose $\text{INR}_p = 1$? From the point of view of a single user, that is, if we do not take interference into account, one should make the private message power as large as possible (i.e., set $\text{INR}_p = \text{INR}$). However, due to interference, it may be convenient to reduce the private message power, so that part of the interfering signal (the common message) can be decoded and subtracted at the other receiver. We see that there is a tradeoff between achieving a large rate at one's link and minimizing the interference caused at the other user's link. In Fig. 2 we plot the single-user rate as a function of the interference power created by the private message of the other user. We

¹Note that this is possible with the available power under the assumption $\text{INR} \geq 1$. If $\text{INR} < 1$ one can choose $\text{INR}_p = \text{INR}$, but will not consider this case in this section.

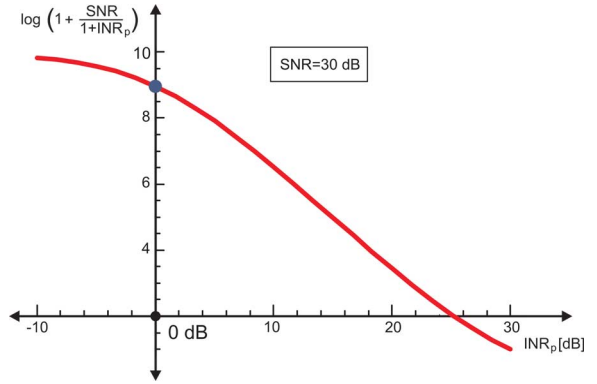


Fig. 2. Rate versus interference power level. The choice $\text{INR}_p = 1$ (0 dB) does not create too much interference and it achieves a large private message rate.

can see that if we choose $\text{INR}_p = 1$, the effect of the interference caused by the private message is small. At the same time, $\text{INR}_p = 1$ allows to obtain a relatively large private message rate in the direct link. We will give a deeper explanation later on in Section VI.

We will show that this simple scheme allows us to achieve a symmetric rate close to the symmetric rate capacity of the channel. In order to determine the symmetric rate that we can achieve with this scheme, it is useful to think of each user as being split into two virtual users: private user U_i and common user W_i . Let MAC_1 be the multiple-access channel formed by virtual users U_1, W_1 , and W_2 , and receiver 1, with the signal from virtual user U_2 being treated as noise. In a similar way, let MAC_2 be the multiple-access channel formed by virtual users U_2, W_1 , and W_2 , and receiver 2, with the signal from virtual user U_1 being treated as noise. Since the common messages must be decoded by both receivers, while the private messages must be decoded only by the intended receiver, we see that the rates achievable by the Han and Kobayashi scheme correspond to the intersection of the capacity regions of MAC_1 and MAC_2 .

Among all the possible rate assignments for the private and common messages of this scheme, we choose the private rates of both users, as well as the common rate of both users to be equal, i.e., $R_{u,1} = R_{u,2}$ and $R_{w,1} = R_{w,2}$. We also fix a decoding order at each receiver, so that the common messages are decoded first, while the private message is decoded last. This choice of rates and decoding order allows for an easy analysis of the scheme and, as will be shown later, also achieves a symmetric rate close to capacity.

Since the private message is decoded last, while the private message of the other user is treated as noise, the private rate of each user is given by

$$R_u = \log \left(1 + \frac{\text{SNR} \cdot \text{INR}_p}{\text{INR}(1 + \text{INR}_p)} \right) = \log \left(1 + \frac{\text{SNR}}{2\text{INR}} \right).$$

Since each receiver decodes the common messages first, both private messages are treated as noise when decoding the common messages. With this decoding order, the sum rate of the common messages must satisfy two constraints

$$R_{w,1} + R_{u,2} \leq \log \left(1 + \frac{(\text{INR} - 1)(\text{SNR} + \text{INR})}{\text{SNR} + 2\text{INR}} \right) \quad (2)$$

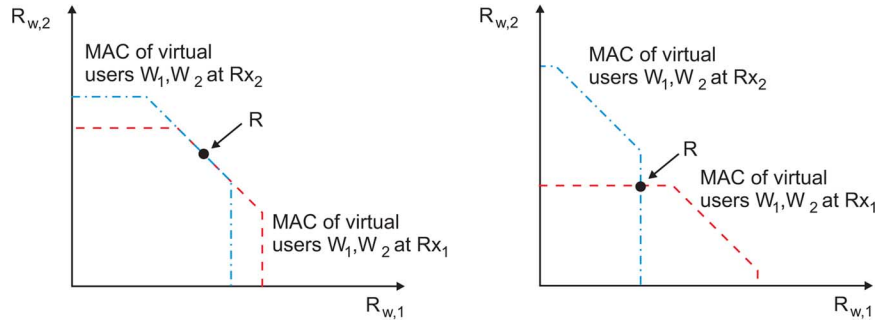


Fig. 3. Intersection of the MAC regions corresponding to virtual users W_1 and W_2 at receivers 1 and 2, when the signals from virtual users U_1 and U_2 are treated as noise. The left graph corresponds to the case in which the sum-rate constraint (2) is active, while the right graph corresponds to the case in which the sum-rate constraint (3) is active. In both cases, the symmetric rate point is indicated. Note that due to the symmetry in the channel and power allocations, the multiple-access regions at receivers 1 and 2 are mirror images of each other with respect to the line $R_{w,1} = R_{w,2}$.

and

$$R_{w,1} + R_{w,2} \leq 2 \log \left(1 + \frac{\text{INR}(\text{INR} - 1)}{\text{SNR} + 2\text{INR}} \right) \quad (3)$$

where (2) arises from the sum rate constraint of the multiple-access channel (MAC) formed by virtual users W_1 and W_2 at receiver 1 (or receiver 2) when the messages from virtual users U_1 and U_2 are treated as noise, and (3) arises from the individual rate constraint of decoding the message of virtual user W_1 at receiver 2 and virtual user W_2 at receiver 1, treating the messages from virtual users U_1 and U_2 as noise (see Fig. 3).

Therefore, with the simple Han and Kobayashi scheme we obtain a symmetric rate

$$\begin{aligned} R_{\text{HK}} &= \log \left(1 + \frac{\text{SNR}}{2\text{INR}} \right) \\ &+ \min \left\{ \frac{1}{2} \log \left(1 + \frac{(\text{INR}-1)(\text{SNR}+\text{INR})}{\text{SNR}+2\text{INR}} \right), \right. \\ &\quad \left. \log \left(1 + \frac{\text{INR}(\text{INR}-1)}{\text{SNR}+2\text{INR}} \right) \right\} \\ &= \min \left\{ \frac{1}{2} \log (1 + \text{SNR} + \text{INR}) + \frac{1}{2} \log \left(2 + \frac{\text{SNR}}{\text{INR}} \right) - 1, \right. \\ &\quad \left. \log \left(1 + \text{INR} + \frac{\text{SNR}}{\text{INR}} \right) - 1 \right\}. \end{aligned} \quad (4)$$

By comparing (2) and (3) we can determine the parameter ranges in which each of the terms of the $\min\{\cdot, \cdot\}$ in (4) is active. Define

$$\begin{aligned} \mathcal{B}_1 &= \{(\text{SNR}, \text{INR}) : \text{INR} \geq 1 \text{ and } \text{SNR}(\text{SNR} + \text{INR}) < \text{INR}^2(\text{INR} + 1)\} \\ \mathcal{B}_2 &= \{(\text{SNR}, \text{INR}) : \text{INR} \geq 1 \text{ and } \text{SNR}(\text{SNR} + \text{INR}) \geq \text{INR}^2(\text{INR} + 1)\}. \end{aligned} \quad (5)$$

Then, the first (second) term of the $\min\{\cdot, \cdot\}$ is active in \mathcal{B}_1 (\mathcal{B}_2). We denote by R_{HK_1} (R_{HK_2}) the symmetric rate expression that results in \mathcal{B}_1 (\mathcal{B}_2).

We can gain further insight into the achievable rate (4) and the different parameter regimes $\mathcal{B}_1, \mathcal{B}_2$, by considering how the ratio $R_{\text{HK}}/C_{\text{awgn}}$ varies for different interference levels. Dividing (4) by $\log(1 + \text{SNR})$, we obtain for $\text{SNR} \gg 1$, $\text{INR} \gg 1$, and $\text{SNR} \gg \text{INR}$ the expression in (6) at the bottom of the page. We define the interference level α as the ratio of INR and SNR in decibels, that is,

$$\alpha := \frac{\log \text{INR}}{\log \text{SNR}}$$

and rewrite (6) as a function of α

$$\frac{R_{\text{HK}}(\alpha)}{C_{\text{awgn}}} \approx \min \left\{ 1 - \frac{\alpha}{2}, \max \{ \alpha, 1 - \alpha \} \right\} \quad (7)$$

By inspecting (7) we can readily identify three different regimes. The first term of the $\min\{\cdot, \cdot\}$ is active in (7) when $2/3 < \alpha < 1$. This corresponds to the parameter range \mathcal{B}_1 , in which the MAC constraint (2) is active. For $0 < \alpha < 2/3$, the second term of the $\min\{\cdot, \cdot\}$ is active in (7), which corresponds to the parameter range \mathcal{B}_2 . In this range, the MAC constraint (3) is active. In addition, we can further identify two subregimes, depending on whether $1/2 < \alpha < 2/3$ (the first term of the $\max\{\cdot, \cdot\}$ is active) or $0 < \alpha < 1/2$ (the second term of the $\max\{\cdot, \cdot\}$ is active). Fig. 4 shows how $R_{\text{HK}}/C_{\text{awgn}}$ varies with α in the different parameter regimes.

C. Known Upper Bounds

In order to evaluate the performance of our communication scheme, we can compare the symmetric rate achieved with an upper bound. We can obtain this upper bound by considering

$$\begin{aligned} \frac{R_{\text{HK}}}{C_{\text{awgn}}} &= \frac{\min \left\{ \frac{1}{2} \log (1 + \text{SNR} + \text{INR}) + \frac{1}{2} \log \left(2 + \frac{\text{SNR}}{\text{INR}} \right) - 1, \log \left(1 + \text{INR} + \frac{\text{SNR}}{\text{INR}} \right) - 1 \right\}}{\log(1 + \text{SNR})} \\ &\approx \frac{\min \left\{ \frac{1}{2} \log (\text{SNR}) + \frac{1}{2} [\log (\text{SNR}) - \log (\text{INR})], \max \{ \log (\text{INR}), \log (\text{SNR}) - \log (\text{INR}) \} \right\}}{\log(\text{SNR})} \\ &\approx \min \left\{ 1 - \frac{1}{2} \frac{\log \text{INR}}{\log \text{SNR}}, \max \left\{ \frac{\log \text{INR}}{\log \text{SNR}}, 1 - \frac{\log \text{INR}}{\log \text{SNR}} \right\} \right\}. \end{aligned} \quad (6)$$

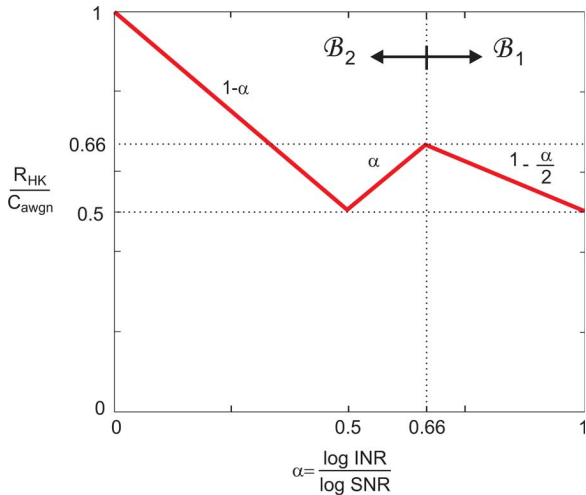


Fig. 4. Achievable rate as a fraction of single-user capacity versus interference level.

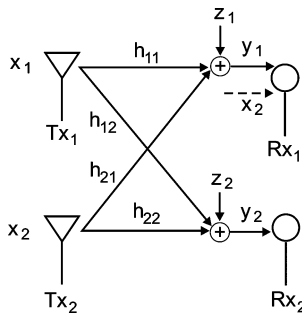


Fig. 5. Genie-aided two-user interference channel. A genie provides signal x_2 to receiver 1.

any outer bound to the interference channel capacity region evaluated at $R_1 = R_2$. The best known outer bound to the interference channel capacity region is that given in [6, Theorem 2]. We analyze this bound in Appendix A, and provide in this subsection an alternative bound that has similar performance and is easier to obtain and analyze.

We will consider a general interference channel so that the upper bounds that we derive are not restricted to the symmetric interference channel. Consider a modified interference channel, where a genie provides the side information x_2 to receiver 1 (see Fig. 5).

Since \mathbf{x}_1^n is independent of \mathbf{x}_2^n we can write for any block of length n

$$\begin{aligned} I(\mathbf{x}_1^n; \mathbf{y}_1^n, \mathbf{x}_2^n) &= I(\mathbf{x}_1^n; \mathbf{x}_2^n) + I(\mathbf{x}_1^n; \mathbf{y}_1^n | \mathbf{x}_2^n) \\ &= I(\mathbf{x}_1^n; h_{11}\mathbf{x}_1^n + \mathbf{z}_1^n) \end{aligned}$$

and it follows that receiver 1 can get an interference-free signal by subtracting the interference $h_{21}x_2$ using the side information provided by the genie. Therefore, we obtain that the genie-aided channel is equivalent to the one-sided interference channel depicted in Fig. 6.

The sum-rate capacity of a one-sided interference channel for the case of $\text{INR}_2 < \text{SNR}_1$ is known from previous results [8] and will be explicitly derived in Section IV-A. It is given by

$$\begin{aligned} R_{\text{sum}}(\text{one-sided IC}) &= \log(1 + \text{SNR}_1) \\ &+ \log\left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_2}\right) \end{aligned} \quad (8)$$

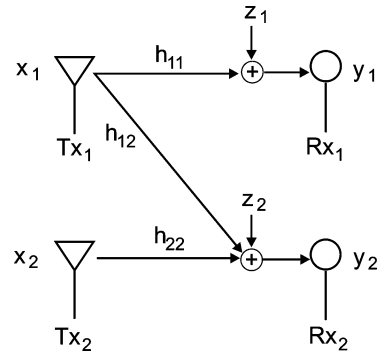


Fig. 6. One-sided interference channel.

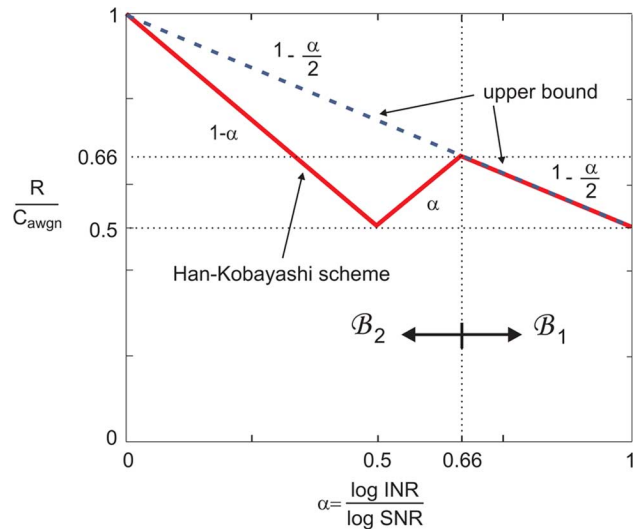


Fig. 7. Upper bound and achievable Han-Kobayashi rate (relative to single-user capacity) as a function of the interference level α .

and since the aid of the genie can only increase the capacity region of the interference channel, we obtain the upper bound for the symmetric rate

$$R_{\text{UB}} = \frac{1}{2} \log(1 + \text{SNR}) + \frac{1}{2} \log\left(1 + \frac{\text{SNR}}{1 + \text{INR}}\right). \quad (9)$$

In order to compare this bound with the rate obtained with our simple Han-Kobayashi scheme we approximately compute the ratio $R_{\text{UB}}/C_{\text{awgn}}$ for $\text{SNR} \gg 1$, $\text{INR} \gg 1$ and $\text{SNR} \gg \text{INR}$

$$\frac{R_{\text{UB}}}{C_{\text{awgn}}} \approx 1 - \frac{1}{2} \frac{\log \text{INR}}{\log \text{SNR}} = 1 - \frac{\alpha}{2}. \quad (10)$$

We see that (10) coincides with (7) when $2/3 < \alpha < 1$, but (10) and (7) differ when $0 < \alpha < 2/3$ (see Fig. 7).

Fig. 7 suggests that the bound (9) is reasonably tight in the parameter range \mathcal{B}_1 . It turns out that the upper bound (9) and the lower bound (4) differ by at most 1 bit/s/Hz in the parameter range \mathcal{B}_1 . This can be checked by writing for the parameter range \mathcal{B}_1

$$\begin{aligned} R_{\text{UB}} - R_{\text{HK}_1} &= \frac{1}{2} \log(1 + \text{SNR}) + \frac{1}{2} \log\left(1 + \frac{\text{SNR}}{1 + \text{INR}}\right) \\ &- \frac{1}{2} \log(1 + \text{SNR} + \text{INR}) \end{aligned}$$

$$\begin{aligned}
 & -\frac{1}{2} \log \left(2 + \frac{\text{SNR}}{\text{INR}} \right) + 1 \\
 &= \frac{1}{2} \log \left(\frac{1 + \text{SNR}}{1 + \text{INR}} \right) - \frac{1}{2} \log \left(\frac{2\text{INR} + \text{SNR}}{\text{INR}} \right) + 1 \\
 &< \frac{1}{2} \log \left(\frac{1 + \text{SNR}}{2\text{INR} + \text{SNR}} \right) + 1 \\
 &\leq 1
 \end{aligned} \tag{11}$$

where we used the assumption $\text{INR} \geq 1$ in the last inequality.

We also observe in Fig. 7 that the gap between the upper bound and the achievable rate can be arbitrarily large in the parameter range \mathcal{B}_2 .² This large gap could be due to a very suboptimal scheme, a loose upper bound, or both. It turns out that the large gap is due to the looseness of the upper bound.

Even though the bound (8) is not as good as the bounds presented in [6], all these bounds have the same worst case 1-bit/s/Hz gap with respect to our simple communication strategy in the parameter range \mathcal{B}_1 . Also, in the parameter range \mathcal{B}_2 all these bounds are arbitrarily loose.

Why are all these bounds loose in \mathcal{B}_2 ? The problem is that they rely, in one way or another, on giving side information to receiver 1 so that he can eventually cancel the interfering signal from user 2. We can gain some intuition about why these bounds are loose in \mathcal{B}_2 by considering our simple communication scheme in the genie-aided channel of Fig. 5. The side information provided by the genie allows receiver 1 to subtract the interference generated by transmitter 2. The rates of the virtual private users U_1 and U_2 are in this case

$$\begin{aligned}
 R_{u_1} &= \log \left(1 + \frac{\text{SNR} \cdot \text{INR}_p}{\text{INR}} \right) = \log \left(1 + \frac{\text{SNR}}{\text{INR}} \right) \\
 R_{u_2} &= \log \left(1 + \frac{\text{SNR} \cdot \text{INR}_p}{\text{INR}(1 + \text{INR}_p)} \right) = \log \left(1 + \frac{\text{SNR}}{2\text{INR}} \right)
 \end{aligned}$$

and we see that virtual user U_1 gains at most 1 bit/s/Hz due to the help of the genie. The sum rate of the MAC formed by virtual users W_1 and W_2 at receiver 2 does not change due to the aid of the genie. Therefore, the sum rate constraint (2) still holds. However, due to the aid of the genie receiver 1 can decode the message of virtual user W_2 and the sum rate constraint (3) does not appear in this case.

In \mathcal{B}_1 , the sum rate constraint (2) is active in the original channel, and the aid of the genie does not allow to increase the sum rate by a large amount. In this regime, the bound obtained from the genie-aided channel is good. In contrast, in \mathcal{B}_2 , the sum rate constraint (3) is active in the original channel, and the genie effectively releases this constraint by providing enough information to receiver 1 to decode the message of virtual user W_2 . Since in \mathcal{B}_2 the constraint (2) is larger than (3) (and the gap between the two constraints can be made arbitrarily large), the bound obtained from the genie-aided channel is loose.

D. A New Upper Bound

In order to derive a tighter sum rate bound for the parameter range \mathcal{B}_2 we will make use of the help of genies, but will

²Note that in Fig. 7 the rates are plotted relative to C_{awgn} and any nonzero gap in the figure translates into an unbounded gap in the rates as $\text{SNR} \rightarrow \infty$.

avoid giving too much information to either receiver. The information that we will provide will not allow either receiver to completely decode the message of the interfering transmitter. The new sum-rate upper bound is given in the following theorem, which we state for a general (not necessarily symmetric) Gaussian interference channel.

Theorem 1: For a Gaussian interference channel as defined in Section II, (1), the sum-rate is upper-bounded by

$$\begin{aligned}
 R_1 + R_2 &\leq \log \left(1 + \text{INR}_1 + \frac{\text{SNR}_1}{1 + \text{INR}_2} \right) \\
 &\quad + \log \left(1 + \text{INR}_2 + \frac{\text{SNR}_2}{1 + \text{INR}_1} \right). \tag{12}
 \end{aligned}$$

Proof: Define

$$\begin{aligned}
 s_1 &= h_{12}x_1 + z_2 \\
 s_2 &= h_{21}x_2 + z_1
 \end{aligned}$$

and consider the genie-aided channel where a genie provides s_1 to receiver 1 and s_2 to receiver 2 (see Fig. 8). Clearly, the capacity region of this genie-aided channel is an outer bound to the capacity region of the original interference channel. Therefore, we can obtain an upper bound for the sum-rate of the original channel by computing an upper bound on the sum-rate of the genie-aided channel. For a block of length n we can bound the sum-rate of the genie-aided channel in the following way:

$$\begin{aligned}
 n(R_1 + R_2) &\leq I(\mathbf{x}_1^n; \mathbf{y}_1^n, \mathbf{s}_1^n) + I(\mathbf{x}_2^n; \mathbf{y}_2^n, \mathbf{s}_2^n) + n\epsilon_n \\
 &= I(\mathbf{x}_1^n; \mathbf{s}_1^n) + I(\mathbf{x}_1^n; \mathbf{y}_1^n | \mathbf{s}_1^n) \\
 &\quad + I(\mathbf{x}_2^n; \mathbf{s}_2^n) + I(\mathbf{x}_2^n; \mathbf{y}_2^n | \mathbf{s}_2^n) + n\epsilon_n \\
 &= h(\mathbf{s}_1^n) - h(\mathbf{s}_1^n | \mathbf{x}_1^n) + h(\mathbf{y}_1^n | \mathbf{s}_1^n) \\
 &\quad - h(\mathbf{y}_1^n | \mathbf{x}_1^n, \mathbf{s}_1^n) \\
 &\quad + h(\mathbf{s}_2^n) - h(\mathbf{s}_2^n | \mathbf{x}_2^n) + h(\mathbf{y}_2^n | \mathbf{s}_2^n) \\
 &\quad - h(\mathbf{y}_2^n | \mathbf{x}_2^n, \mathbf{s}_2^n) + n\epsilon_n \\
 &= h(\mathbf{s}_1^n) - h(\mathbf{z}_2^n) + h(\mathbf{y}_1^n | \mathbf{s}_1^n) - h(\mathbf{s}_2^n) \\
 &\quad + h(\mathbf{s}_2^n) - h(\mathbf{z}_1^n) + h(\mathbf{y}_2^n | \mathbf{s}_2^n) - h(\mathbf{s}_1^n) + n\epsilon_n \\
 &= h(\mathbf{y}_1^n | \mathbf{s}_1^n) + h(\mathbf{y}_2^n | \mathbf{s}_2^n) - h(\mathbf{z}_1^n) - h(\mathbf{z}_2^n) + n\epsilon_n \\
 &\leq \sum_{i=1}^n [h(y_{1i} | s_{1i}) + h(y_{2i} | s_{2i}) \\
 &\quad - h(z_{1i}) - h(z_{2i})] + n\epsilon_n \tag{13}
 \end{aligned}$$

where the last inequality follows by the fact that removing conditioning cannot reduce differential entropy, and $\epsilon_n \rightarrow 0$ as $n \rightarrow \infty$.

Let $\mathbb{E}[x_{1i}^2] = P_{1i}$ and $\mathbb{E}[x_{2i}^2] = P_{2i}$, we have

$$\begin{aligned}
 &\frac{1}{n} \sum_{i=1}^n h(y_{1i} | s_{1i}) \\
 &\stackrel{(a)}{\leq} \frac{1}{n} \sum_{i=1}^n \log \left[\pi e \left(N_0 + |h_{21}|^2 P_{2i} + \frac{|h_{11}|^2 P_{1i} N_0}{N_0 + |h_{12}|^2 P_{1i}} \right) \right] \\
 &\stackrel{(b)}{\leq} \log \left[\pi e \left(N_0 + |h_{21}|^2 \left(\frac{1}{n} \sum_{i=1}^n P_{2i} \right) \right) \right]
 \end{aligned}$$

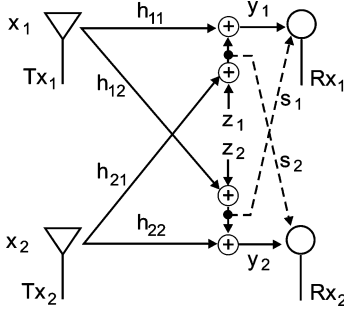


Fig. 8. Genie-aided two-user Gaussian interference channel. A genie provides signals s_1 to receiver 1 and s_2 to receiver 2.

$$\begin{aligned} & + \frac{|h_{11}|^2 \left(\frac{1}{n} \sum_{i=1}^n P_{1i} \right) N_0}{N_0 + |h_{12}|^2 \left(\frac{1}{n} \sum_{i=1}^n P_{1i} \right)} \Bigg] \\ & \stackrel{(c)}{\leq} \log \left[\pi e \left(N_0 + |h_{21}|^2 P_2 + \frac{|h_{11}|^2 P_1 N_0}{N_0 + |h_{12}|^2 P_1} \right) \right] \end{aligned} \quad (14)$$

where in step (a) we use the fact that the circularly symmetric complex Gaussian distribution maximizes conditional differential entropy for a given covariance constraint, in step (b) we use Jensen's inequality applied to a function that, as can be easily checked, is concave, and in step (c) we used the fact that the function is increasing on P_1 and P_2 . Similarly, we have

$$\frac{1}{n} \sum_{i=1}^n h(y_{2i}|s_{2i}) \leq \log \left[\pi e \left(N_0 + |h_{12}|^2 P_1 + \frac{|h_{22}|^2 P_2 N_0}{N_0 + |h_{21}|^2 P_2} \right) \right]. \quad (15)$$

Thus, we have

$$\begin{aligned} R_1 + R_2 & \leq \frac{1}{n} \sum_{i=1}^n [h(y_{1i}|s_{1i}) + h(y_{2i}|s_{2i}) \\ & \quad - h(z_{1i}) - h(z_{2i})] + \epsilon_n \\ & \leq \log \left[\pi e \left(N_0 + |h_{21}|^2 P_2 + \frac{|h_{11}|^2 P_1 N_0}{N_0 + |h_{12}|^2 P_1} \right) \right] - \log(\pi e N_0) \\ & \quad + \log \left[\pi e \left(N_0 + |h_{12}|^2 P_1 + \frac{|h_{22}|^2 P_2 N_0}{N_0 + |h_{21}|^2 P_2} \right) \right] - \log(\pi e N_0) + \epsilon_n \\ & = \log \left(1 + \frac{|h_{21}|^2 P_2}{N_0} + \frac{|h_{11}|^2 P_1}{N_0 + |h_{12}|^2 P_1} \right) \\ & \quad + \log \left(1 + \frac{|h_{12}|^2 P_1}{N_0} + \frac{|h_{22}|^2 P_2}{N_0 + |h_{21}|^2 P_2} \right) + \epsilon_n. \end{aligned}$$

Letting $n \rightarrow \infty$, $\epsilon_n \rightarrow 0$ and we get the desired upper bound. \square

It is interesting to note that the upper bound of Theorem 1 can be achieved with a communication scheme where each receiver treats interference as noise. In the genie-aided channel used to derive the upper bound, the side information provided by the genie compensates for the harm that interference produces on the other link by giving a boost in the own rate in the direct

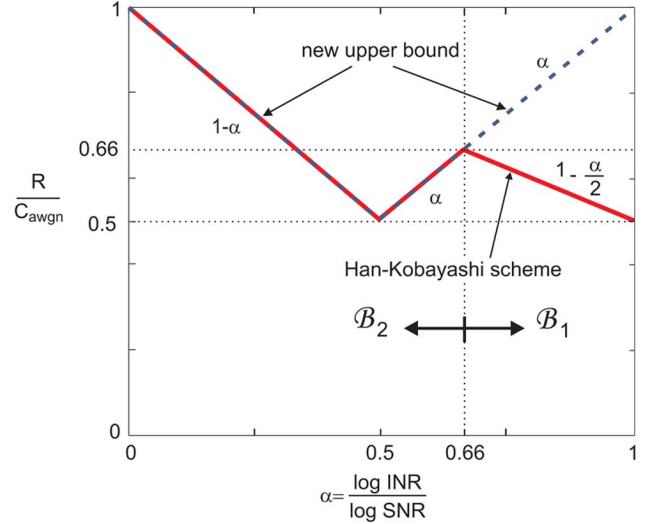


Fig. 9. Upper bound of Theorem 1 and achievable Han-Kobayashi rate (relative to single-user capacity) as a function of the interference level α .

link. Thus, making the signal more random by not sending any common information results in an overall improvement in the sum rate.

We now specialize the bound of Theorem 1 to the symmetric interference channel to obtain the following upper bound on the symmetric rate:

$$R_{\text{UB}_{\text{new}}} = \log \left(1 + \text{INR} + \frac{\text{SNR}}{1 + \text{INR}} \right). \quad (16)$$

To see how this bound performs in the different regimes we compute the ratio of $R_{\text{UB}_{\text{new}}}$ and C_{awgn} for $\text{SNR} \gg 1$ and $\text{INR} \gg 1$

$$\frac{R_{\text{UB}_{\text{new}}}}{C_{\text{awgn}}} \approx \max \left\{ \frac{\log \text{INR}}{\log \text{SNR}}, 1 - \frac{\log \text{INR}}{\log \text{SNR}} \right\} = \max\{\alpha, 1 - \alpha\} \quad (17)$$

which we plot in Fig. 9.

Observing Fig. 9, the new upper bound seems to match the Han-Kobayashi achievable rate in the regime \mathcal{B}_2 , where the upper bound (9) is loose. In fact, this new bound has a finite gap with respect to the achievable symmetric rate with our simple scheme in the parameter range \mathcal{B}_2 . To verify this we compute

$$\begin{aligned} R_{\text{UB}_{\text{new}}} - R_{\text{HK}_2} & = \log \left(1 + \text{INR} + \frac{\text{SNR}}{1 + \text{INR}} \right) \\ & \quad - \log \left(1 + \text{INR} + \frac{\text{SNR}}{\text{INR}} \right) + 1 \\ & < \log \left(1 + \text{INR} + \frac{\text{SNR}}{\text{INR}} \right) \\ & \quad - \log \left(1 + \text{INR} + \frac{\text{SNR}}{\text{INR}} \right) + 1 \\ & = 1 \end{aligned} \quad (18)$$

and we find that the gap in the symmetric rate with respect to the new upper bound is at most 1 bit/s/Hz in \mathcal{B}_2 .

Using (11) and (18) we see that when $\text{INR} \geq 1$ our simple scheme is at most 1 bit/s/Hz away from the symmetric rate channel capacity. Proving that the simple scheme is at most

1 bit/s/Hz away from capacity when $\text{INR} < 1$ is straightforward. We can set $\text{INR}_p = \text{INR}$ and use as a symmetric rate upper bound the single-user capacity. The difference between the achievable rate and the upper bound is

$$\begin{aligned} & \log(1 + \text{SNR}) - \log\left(1 + \frac{\text{SNR}}{1 + \text{INR}}\right) \\ & \leq \log(1 + \text{SNR}) - \log\left(1 + \frac{\text{SNR}}{2}\right) \\ & \leq 1. \end{aligned} \quad (19)$$

E. Small Gap Between Lower and Upper Bounds

In Sections III-C and III-D we showed that the Han–Kobayashi scheme that sets the private message power so that the interference created is at noise level achieves a symmetric rate within 1 bit/s/Hz of the upper bounds. Therefore, we obtained a characterization of the symmetric capacity to within 1 bit/s/Hz. The finite and small gap between the lower and upper bounds on the symmetric capacity was obtained by direct calculation of the difference between the bounds. In this subsection, we present a more intuitive explanation for the tightness of the bounds.

We can decompose the total gap between the lower and upper bounds in two components Δ_1 and Δ_2 , arising from the following two steps.

- 1) Fix Han–Kobayashi strategy (i.e., set $\text{INR}_p = 1$, decode first the common messages (w_1, w_2) , and then the private message u_1 or u_2 ³) and see how the symmetric rate changes when varying the channel from the given interference channel to the genie-aided interference channel used in the bounds. The gap Δ_1 quantifies the rate change due to the side information for the fixed strategy.
- 2) Fix the channel to the genie-aided interference channel, and change the Han–Kobayashi strategy by varying INR_p from 1 to INR . The gap Δ_2 quantifies the rate change in the genie-aided channel when INR_p is varied.

Referring to Fig. 10, Δ_1 corresponds to the difference in the rates between points A and B. Δ_2 corresponds to the difference in the rates between points B and C.

Since $\text{INR}_p = \text{INR}$ achieves the capacity of the genie-aided channel (one can show that the sum rate upper bounds can be achieved by generating the codewords \mathbf{x}_1^n and \mathbf{x}_2^n with i.i.d. circularly symmetric complex Gaussian components of variance P , and treating interference as noise at the decoder), the sum $\Delta_1 + \Delta_2$ quantifies the rate change from the initial Han–Kobayashi strategy in the original channel (lower bound), to the capacity achieving strategy in the genie-aided channel (upper bound). It follows that a small gap between the lower and upper bounds can only occur if both Δ_1 and Δ_2 are small.

To achieve a small value of Δ_1 , the help of the genie should not change the relevant rate constraints for the initial Han–Kobayashi strategy. Fig. 3 and the discussion at the end of Section III-C describe the active constraints for the different weak interference regimes.

³With some abuse of notation, we use $u_i, w_i, i = 1, 2$ to denote the private and common messages, and also to denote the symbols of the codewords actually sent over the channel.

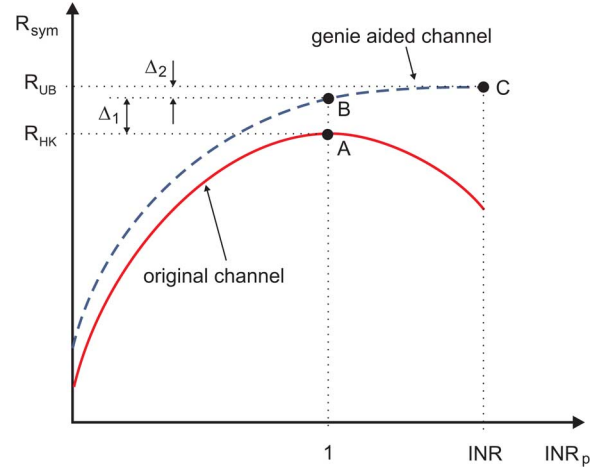


Fig. 10. Gap between the achievable rate with the Han–Kobayashi scheme that sets $\text{INR}_p = 1$ and the symmetric capacity upper bound decomposed into two components: Δ_1 and Δ_2 . Δ_1 results from fixing INR_p and changing the channel; Δ_2 results from increasing INR_p to INR in the genie-aided channel.

In addition to achieving a small value for Δ_1 , the help of the genie should result in a small value of Δ_2 . Δ_2 arises when we vary the communication strategy from the initial Han–Kobayashi strategy to the capacity achieving strategy in the genie-aided channel. The sum capacity of the genie-aided channels that we used can be explicitly computed by treating interference as noise. Unfortunately, it is hard to compute bounds for the interference channel when the interference is not treated as noise. In the original channel, setting, $\text{INR}_p = 1$ achieves good performance, but in general, setting $\text{INR}_p = \text{INR}$ (treating interference as noise) may result in very small rates. The role of the genie in the genie-aided channel is to compensate for the loss in the sum rate when INR_p is increased from 1 to INR . Increasing INR_p beyond 1 in the original channel may produce a loss in the rate of common message due to increased interference. However, the genie provides just enough side information to compensate for this loss while making $\text{INR}_p = \text{INR}$ optimal.

F. Generalized Degrees of Freedom

In the preceding analysis, we see the utility of the approximations like (7), (10), (17) both in identifying the different regimes of interest as well as in developing the relevant upper bounds for the different regimes. We can formalize the approximations of this nature through the following type of definition. Define, for a fixed $\alpha \geq 0$

$$d_{\text{sym}}(\alpha) := \lim_{\text{SNR}, \text{INR} \rightarrow \infty; \frac{\log \text{INR}}{\log \text{SNR}} = \alpha} \frac{C_{\text{sym}}(\text{INR}, \text{SNR})}{C_{\text{awgn}}(\text{SNR})}. \quad (20)$$

If there were no interference between the two links (i.e., $\alpha = 0$), then the capacity per link is just the AWGN capacity $\log(1 + \text{SNR})$. Hence, $d_{\text{sym}}(0) = 1$. This can be interpreted as each link having the full degree of freedom to itself. Since interference cannot help in communicating each user's message, it follows that $C_{\text{sym}} \leq C_{\text{awgn}}$ and, therefore, $d_{\text{sym}}(\alpha) \leq 1$ for $\alpha > 0$. We can think of interference as effectively reducing the degrees of freedom of the channel, and thus it is natural to think of $d_{\text{sym}}(\alpha)$ as a *generalized degree of freedom*. The approximations we

made can be thought of as computing analogous limits for the achievable rates and upper bounds. Since the lower and upper bounds on the symmetric capacity we derived earlier differ by at most one bit, they allow us to precisely characterize $d_{\text{sym}}(\alpha)$. For $0 \leq \alpha \leq 1$, this is plotted in Fig. 9. $\alpha \geq 1$ corresponds to the strong and very strong interference regimes, and since the capacity is known in these regimes, we can compute $d_{\text{sym}}(\alpha)$ in a straightforward way.

In the very strong interference case, each user can decode the interfering message before decoding his own message [2]. After decoding the interference and subtracting it from the received signal, the user effectively gets an AWGN channel for communicating his own message. It follows that the symmetric capacity in the very strong interference case is

$$C_{\text{sym}} = \log(1 + \text{SNR}) \approx \log(\text{SNR}). \quad (21)$$

The channel is in the very strong interference situation whenever $\text{INR} \geq \text{SNR}^2 + \text{SNR}$. Taking logs and assuming $\text{SNR}, \text{INR} \gg 1$, the very strong interference condition becomes $\log \text{INR} \geq 2 \log \text{SNR}$. In this regime, we obtain $d_{\text{sym}} = 1$, and therefore, interference does not reduce the available degrees of freedom of the channel.

In the strong interference regime, each receiver is able to decode both messages. The capacity region of the interference channel is given by the intersection of the capacity regions of the two MACs formed by the two transmitters and each of the receivers. In the symmetric case, the sum capacity of both MACs is the same and the corresponding symmetric capacity is given by

$$C_{\text{sym}} = \frac{1}{2} \log(1 + \text{SNR} + \text{INR}). \quad (22)$$

The symmetric channel is in the strong interference situation whenever it is not in very strong interference and $\text{INR} \geq \text{SNR}$, which after taking logs becomes

$$\log \text{SNR} \leq \log \text{INR} < 2 \log \text{SNR}.$$

This condition and (22) together with the assumption $\text{SNR}, \text{INR} \gg 1$ imply that $C_{\text{sym}} \approx \frac{1}{2} \log \text{INR}$. It follows that under strong interference the generalized degrees of freedom are

$$d_{\text{sym}} = \frac{\log \sqrt{\text{INR}}}{\log \text{SNR}} = \frac{\alpha}{2}. \quad (23)$$

We now have the complete picture shown in (24) at the bottom of the page, and

$$d_{\text{sym}} = \begin{cases} 1 - \alpha, & 0 \leq \alpha < \frac{1}{2} \\ \alpha, & \frac{1}{2} \leq \alpha < \frac{2}{3} \\ 1 - \frac{\alpha}{2}, & \frac{2}{3} < \alpha \leq 1 \\ \frac{\alpha}{2}, & 1 \leq \alpha < 2 \\ 1, & \alpha \geq 2. \end{cases} \quad (25)$$

The generalized degrees of freedom are plotted in Fig. 11, together with the performance of the baseline strategies of orthogonalizing the users (in frequency or time) and treating interference as noise. Note that orthogonalizing between the links, in which each link achieves half the degrees of freedom, is strictly suboptimal except when $\alpha = \frac{1}{2}$ and $\alpha = 1$. Treating interference as noise, on the other hand, is strictly suboptimal except for $\alpha \leq \frac{1}{2}$.

Note that there are five regimes in which the qualitative behaviors of the capacity are different. The first three regimes fall into the weak interference regime, and the characterization of the symmetric capacity in these regimes is a consequence of the new results that we obtained. In these regimes, the interference is not strong enough to be decoded in its entirety. In fact, in regime 1, where the interference is very weak, treating interference as noise is optimal. In regimes 2 and 3, where the interference is not very weak, decoding it partially can significantly improve performance.

Interestingly, the capacity is *not* monotonically decreasing with INR in the weak interference regime. Increasing INR has two opposing effects: more common information can be decoded and canceled but less private information can be sent under the constraint $\text{INR}_p = 1$. Depending on which of these two effects dominates, the capacity increases or decreases with INR .

In regime 1, where treating interference as noise is optimal, the common messages carry negligible information. In this regime, the loss in the private rate due to the increase in INR makes the capacity decrease with INR . However, once interference becomes strong enough to reach regime 2, the users can start using common information to partially cancel interference. As the interference level increases, more and more of this common information can be decoded and partially canceled, and this effects dominates the behavior of capacity with INR . Therefore, capacity increases with INR in regime 2. However, as INR increases further to reach regime 3, the gains obtained by partially canceling interference through the common messages are not enough to offset the loss of rate in the private information. In this regime, capacity decreases with INR until

$$C_{\text{sym}} \approx \begin{cases} \log \left(\frac{\text{SNR}}{\text{INR}} \right) & \log \text{INR} < \frac{1}{2} \log \text{SNR} \\ \log \text{INR} & \frac{1}{2} \log \text{SNR} < \log \text{INR} < \frac{2}{3} \log \text{SNR} \\ \log \frac{\text{SNR}}{\sqrt{\text{INR}}} & \frac{2}{3} \log \text{SNR} < \log \text{INR} < \log \text{SNR} \\ \log \sqrt{\text{INR}} & \log \text{SNR} < \log \text{INR} < 2 \log \text{SNR} \\ \log \text{SNR} & \log \text{INR} > 2 \log \text{SNR} \end{cases} \quad (24)$$

the strong interference regime is reached. Since in the strong interference regime all the information is common information, increasing INR increases capacity. Finally, in the very strong interference regime all the interference can be canceled before decoding the useful information, and interference does not have any effect on capacity.

It is worthwhile to note that the communication strategies presented in [8] also showed nonmonotonic behavior under weak interference. However, it was not clear whether this was the actual behavior of capacity or just an artifact of possibly loose capacity inner-bounds. What we demonstrated here is that in fact it is the former.

G. Tight Characterization of Symmetric Capacity

Our simple Han and Kobayashi type scheme, together with the symmetric capacity upper bounds (9) and (16) allowed us to characterize the symmetric capacity to within 1 bit/s/Hz. We will now show that in some parameter ranges, the gap between the upper bound (16) and the rates achievable with some improved communication schemes vanishes for SNR, INR $\rightarrow \infty$.

The communication scheme that sets the private message power so that the interference generated onto the other receiver is at noise level (i.e., $\text{INR}_p = 1$) is “universal” in the sense that the same scheme can be used to achieve a symmetric rate within 1 bit/s/Hz of capacity in the weak interference regime, regardless of the values of the parameters.

However, we can further improve the achievable symmetric rate by modifying the communication scheme for different parameter ranges. In regime 1, when $\log \text{INR} < \frac{1}{2} \log \text{SNR}$, we can simply assign $\text{INR}_p = \text{INR}$ and not use common messages at all. As stated in the previous subsection, this scheme achieves a symmetric rate

$$R = \log \left(1 + \frac{\text{SNR}}{1 + \text{INR}} \right) \quad (26)$$

and the gap between this rate and the upper bound (16) is

$$\begin{aligned} R_{\text{UB}_{\text{new}}} - R &= \log \left(1 + \text{INR} + \frac{\text{SNR}}{1 + \text{INR}} \right) - \log \left(1 + \frac{\text{SNR}}{1 + \text{INR}} \right) \\ &= \log \left(1 + \frac{\text{INR}(1 + \text{INR})}{1 + \text{INR} + \text{SNR}} \right) \\ &\approx \log \left(1 + \frac{\text{INR}^2}{\text{SNR}} \right). \end{aligned} \quad (27)$$

Note that the same gap would be obtained with any scheme that uses INR_p such that $\text{INR}_p \rightarrow \infty$ as SNR, INR $\rightarrow \infty$.

Recall that $\alpha = \frac{\log \text{INR}}{\log \text{SNR}}$. Regime 1 corresponds to $0 < \alpha < \frac{1}{2}$. In this regime, (27) implies that for fixed α , $C_{\text{sym}} - R \rightarrow 0$ as SNR, INR $\rightarrow \infty$. Therefore, we have that for $0 < \alpha < 1/2$ the symmetric capacity is tightly characterized by

$$C_{\text{sym}} = \log \left(\frac{\text{SNR}}{\text{INR}} \right). \quad (28)$$

In regime 2, where $\frac{1}{2} < \alpha < \frac{2}{3}$, we can choose $\text{INR}_p = (\text{INR}/\text{SNR})^{1-\gamma}$, where

$$0 < \frac{2\alpha - 1}{1 - \alpha} < \gamma < 1 \quad (29)$$

is fixed but arbitrary. This choice of INR_p makes the received interference power corresponding to the private message to go to zero as SNR, INR $\rightarrow \infty$. Note that in regime 2 we have $\frac{1}{2} \log \text{SNR} < \log \text{INR} < \frac{2}{3} \log \text{SNR}$ and, therefore, $\text{INR}/\text{SNR} \rightarrow 0$ as SNR, INR $\rightarrow \infty$.

Fixing the decoding order so that the private messages are decoded last, this scheme achieves a symmetric rate

$$\begin{aligned} R &= \log \left[1 + \frac{\text{SNR} \cdot \text{INR}_p}{\text{INR}(1 + \text{INR}_p)} \right] \\ &\quad + \min \left\{ \frac{1}{2} \log \left[1 + \frac{(\text{SNR} + \text{INR})(\text{INR} - \text{INR}_p)}{\text{INR} + (\text{SNR} + \text{INR})\text{INR}_p} \right], \right. \\ &\quad \left. \log \left[1 + \frac{\text{INR}(\text{INR} - \text{INR}_p)}{\text{INR} + (\text{SNR} + \text{INR})\text{INR}_p} \right] \right\} \\ &\stackrel{(a)}{\approx} \log \left[1 + \left(\frac{\text{SNR}}{\text{INR}} \right)^\gamma \right] \\ &\quad + \min \left\{ \frac{1}{2} \log \left[1 + \frac{\text{SNR}}{(\text{SNR}/\text{INR})^\gamma} \right], \right. \\ &\quad \left. \log \left[1 + \frac{\text{INR}}{(\text{SNR}/\text{INR})^\gamma} \right] \right\} \\ &\approx \gamma \log \left(\frac{\text{SNR}}{\text{INR}} \right) \\ &\quad + \min \left\{ \frac{1}{2} \log [\text{SNR}^{1-\gamma} \text{INR}^\gamma], \log \left[\frac{\text{INR}^{1+\gamma}}{\text{SNR}^\gamma} \right] \right\} \\ &= \gamma(1 - \alpha) \log(\text{SNR}) \\ &\quad + \min \left\{ \frac{1 - \gamma + \alpha\gamma}{2}, \alpha(1 + \gamma) - \gamma \right\} \log(\text{SNR}) \\ &\stackrel{(b)}{\approx} \alpha \log(\text{SNR}) \end{aligned} \quad (30)$$

where \approx means that the difference between the left- and right-hand sides goes to zero as SNR, INR $\rightarrow \infty$, (a) follows from the fact that $\text{INR}_p \rightarrow 0$ and $\text{INR}/\text{SNR} \rightarrow 0$ as SNR, INR $\rightarrow \infty$, and (b) follows because the second term of the $\min\{\cdot, \cdot\}$ dominates due to (29).

From the upper bound (16) we obtain

$$\begin{aligned} R_{\text{UB}_{\text{new}}} &= \log \left(1 + \text{INR} + \frac{\text{SNR}}{1 + \text{INR}} \right) \\ &\approx \log(\text{SNR}^\alpha + \text{SNR}^{1-\alpha}) \\ &\approx \max\{\alpha, 1 - \alpha\} \log(\text{SNR}) \\ &= \alpha \log(\text{SNR}). \end{aligned} \quad (31)$$

Comparing (30) with (31) we see that the difference $R_{\text{UB}_{\text{new}}} - R \rightarrow 0$ as SNR, INR $\rightarrow \infty$ and therefore in regime 2 the symmetric capacity is given by

$$C_{\text{sym}} = \log(\text{INR}). \quad (32)$$

We note that both in regimes 1 and 2 we have some flexibility in setting the private message power to asymptotically achieve the symmetric capacity. In regime 1 we can choose any private message power as long as $\text{INR}_p \rightarrow \infty$ when SNR, INR $\rightarrow \infty$. In a similar way, in regime 2 we can use any private message power that satisfies $\text{INR}_p \rightarrow 0$ as SNR, INR $\rightarrow \infty$. In both cases, setting $\text{INR}_p = 1$ does not asymptotically achieve the symmetric capacity, but results in a symmetric rate no smaller than 1 bit/s/Hz from it. Unfortunately, in regime 3 the only

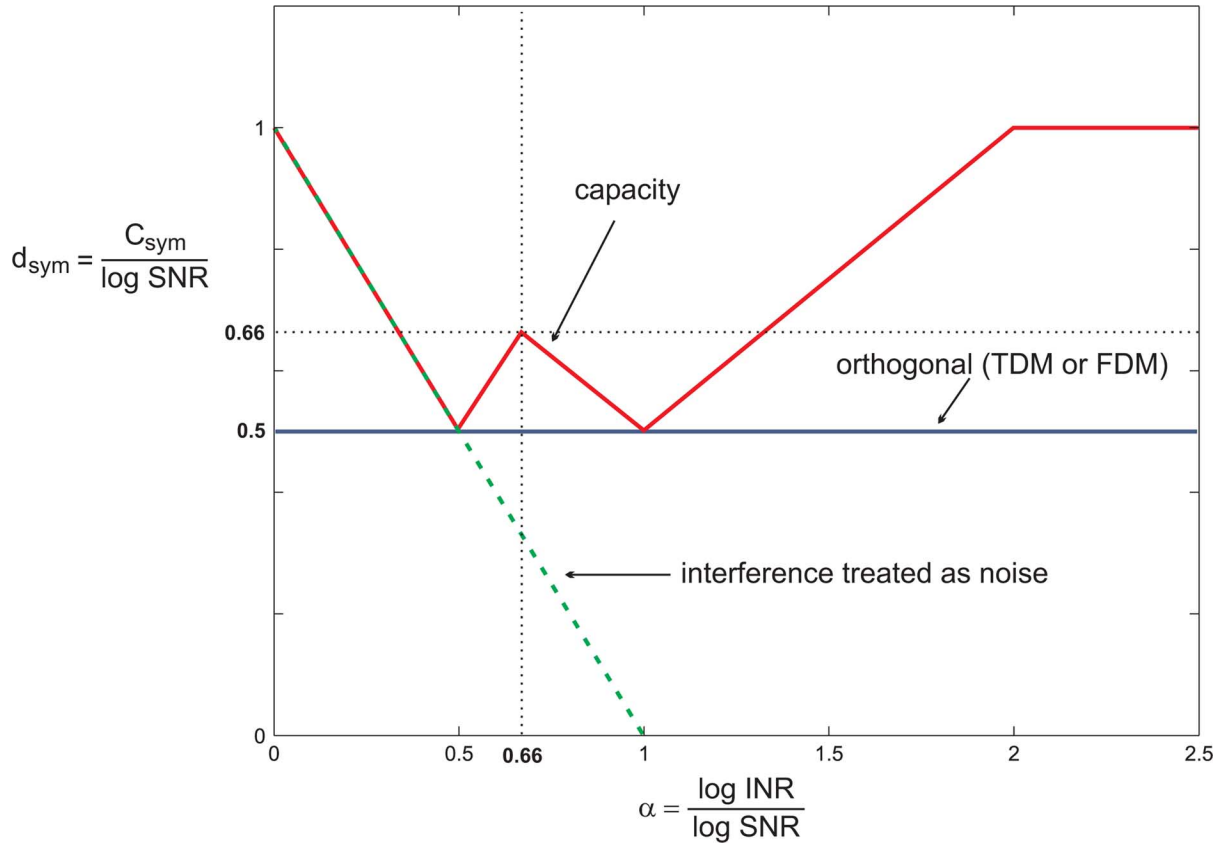


Fig. 11. Generalized degrees of freedom for two suboptimal schemes versus capacity. These suboptimal schemes are treating interference as noise and orthogonalizing the users over time or frequency.

choice of private message power that achieves a symmetric rate with bounded difference from the upper bound (9) is $\text{INR}_p = \text{constant}$, and this choice of private message power does not result in a gap that vanishes as $\text{SNR}, \text{INR} \rightarrow \infty$.

In the strong interference regime, the symmetric capacity is given by

$$C_{\text{sym}} = \frac{1}{2} \log(1 + \text{INR} + \text{SNR}) \quad (33)$$

which asymptotically approaches $\log(\sqrt{\text{INR}})$ for $1 < \alpha \leq 2$ as $\text{SNR}, \text{INR} \rightarrow \infty$.

Finally, in the very strong interference regime the symmetric capacity is given by

$$C_{\text{sym}} = \log(1 + \text{SNR}) \quad (34)$$

which asymptotically approaches $\log(\text{SNR})$ for $\text{SNR} \rightarrow \infty$.

We summarize the results of this subsection in the following theorem.

Theorem 2: Let $\alpha = (\log \text{INR} / \log \text{SNR})$. For $0 < \alpha < 1/2$, $1/2 < \alpha < 2/3$ and $\alpha > 1$, the approximation given in (35), shown at the bottom of the page, is asymptotically tight in the sense that the difference between C_{sym} and the approximation goes to zero as SNR, INR go to infinity with α fixed.

IV. WITHIN ONE BIT OF THE GENERAL CAPACITY REGION

In the previous section, we showed that a simple Han-Kobayashi scheme can achieve to within one bit of the symmetric rate of the symmetric Gaussian interference channel. We will show that this is also true for the whole capacity region of the general two-user Gaussian interference channel (not necessarily symmetric). Depending on the parameters of the Gaussian interference channel ($\text{SNR}_1, \text{SNR}_2, \text{INR}_1$, and INR_2), we can divide the analysis of the Gaussian interference channel into the following three cases.

1) Weak interference channel:

In this case, the parameters of the Gaussian interference channel satisfy $\text{INR}_1 < \text{SNR}_2$ and $\text{INR}_2 < \text{SNR}_1$.

$$C_{\text{sym}} \approx \begin{cases} \log\left(\frac{\text{SNR}}{\text{INR}}\right), & \log \text{INR} < \frac{1}{2} \log \text{SNR} \leftrightarrow 0 < \alpha < \frac{1}{2} \\ \log \text{INR}, & \frac{1}{2} \log \text{SNR} < \log \text{INR} < \frac{2}{3} \log \text{SNR} \leftrightarrow \frac{1}{2} < \alpha < \frac{2}{3} \\ \log \sqrt{\text{INR}}, & \log \text{SNR} < \log \text{INR} < 2 \log \text{SNR} \leftrightarrow 1 < \alpha < 2 \\ \log \text{SNR}, & \log \text{INR} \geq 2 \log \text{SNR} \leftrightarrow \alpha \geq 2 \end{cases} \quad (35)$$

2) Mixed interference channel:

In this case, the parameters of the Gaussian interference channel satisfy $\text{INR}_1 \geq \text{SNR}_2$ and $\text{INR}_2 < \text{SNR}_1$, or $\text{INR}_1 < \text{SNR}_2$ and $\text{INR}_2 \geq \text{SNR}_1$.

3) Strong interference channel:

In this case, the parameters of the Gaussian interference channel satisfy $\text{INR}_1 \geq \text{SNR}_2$ and $\text{INR}_2 \geq \text{SNR}_1$.

The capacity region of the strong interference channel is already known [1], [7]. In the following, we will show that we can get to within one bit of the capacity region of the Gaussian interference channel for both the weak interference channel and the mixed interference channel.

A. Outer Bound on the Capacity Region of the Gaussian Interference Channel

Since the existing bounds on the capacity region of the Gaussian interference channel can be arbitrarily loose, we need a new outer bound. In the following subsections, we provide a new outer bound on capacity region of the weak and mixed interference channels.

1) *Outer Bound for Weak Interference Channel:* For the weak interference channel, i.e., $\text{INR}_1 < \text{SNR}_2$ and $\text{INR}_2 < \text{SNR}_1$, we have the following outer bound on the capacity region.

Theorem 3: The capacity region of the weak interference channel is contained within the set of rate pairs (R_1, R_2) satisfying

$$\begin{aligned}
 R_1 &\leq \log(1 + \text{SNR}_1) \\
 R_2 &\leq \log(1 + \text{SNR}_2) \\
 R_1 + R_2 &\leq \log(1 + \text{SNR}_1) + \log\left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_2}\right) \\
 R_1 + R_2 &\leq \log(1 + \text{SNR}_2) + \log\left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_1}\right) \\
 R_1 + R_2 &\leq \log\left(1 + \text{INR}_1 + \frac{\text{SNR}_1}{1 + \text{INR}_2}\right) \\
 &\quad + \log\left(1 + \text{INR}_2 + \frac{\text{SNR}_2}{1 + \text{INR}_1}\right) \\
 2R_1 + R_2 &\leq \log(1 + \text{SNR}_1 + \text{INR}_1) \\
 &\quad + \log\left(1 + \text{INR}_2 + \frac{\text{SNR}_2}{1 + \text{INR}_1}\right) + \log\left(\frac{1 + \text{SNR}_1}{1 + \text{INR}_2}\right) \\
 R_1 + 2R_2 &\leq \log(1 + \text{SNR}_2 + \text{INR}_2) \\
 &\quad + \log\left(1 + \text{INR}_1 + \frac{\text{SNR}_1}{1 + \text{INR}_2}\right) + \log\left(\frac{1 + \text{SNR}_2}{1 + \text{INR}_1}\right). \tag{36}
 \end{aligned}$$

Proof: We prove the bounds in (36) one by one.

- 1) The bounds on R_1 and R_2 are just the point-to-point capacity of the AWGN channel obtained by removing the interference from the other user.
- 2) The first bound on $R_1 + R_2$ is just the capacity of the one-sided interference channel resulting from the genie-aided channel in which a genie gives x_2 to receiver 1 (see Fig. 5). We already used this bound in the symmetric case. This bound is known from previous results [8] but we provide here an alternative derivation since the

same bounding techniques will be useful for obtaining the bounds for $2R_1 + R_2$ and $R_1 + 2R_2$. Using Fano's inequality we can write for any codebook of block length n

$$\begin{aligned}
 &n(R_1 + R_2) \\
 &\leq I(\mathbf{x}_1^n; \mathbf{y}_1^n) + I(\mathbf{x}_2^n; \mathbf{y}_2^n) + n\epsilon \\
 &\leq I(\mathbf{x}_1^n; \mathbf{y}_1^n, \mathbf{x}_2^n) + I(\mathbf{x}_2^n; \mathbf{y}_2^n) + n\epsilon \\
 &= I(\mathbf{x}_1^n; \mathbf{y}_1^n | \mathbf{x}_2^n) + I(\mathbf{x}_2^n; \mathbf{y}_2^n) + n\epsilon \\
 &= h(\mathbf{y}_1^n | \mathbf{x}_2^n) - h(\mathbf{y}_1^n | \mathbf{x}_1^n, \mathbf{x}_2^n) \\
 &\quad + h(\mathbf{y}_2^n) - h(\mathbf{y}_2^n | \mathbf{x}_2^n) + n\epsilon \\
 &= h(h_{11}\mathbf{x}_1^n + \mathbf{z}_1^n) - h(\mathbf{z}_1^n) + h(\mathbf{y}_2^n) \\
 &\quad - h(h_{12}\mathbf{x}_1^n + \mathbf{z}_2^n) + n\epsilon \\
 &= h(\mathbf{y}_2^n) - h(\mathbf{z}_1^n) + h(h_{11}h_{12}\mathbf{x}_1^n + h_{12}\mathbf{z}_1^n) \\
 &\quad - h(h_{11}h_{12}\mathbf{x}_1^n + h_{11}\mathbf{z}_2^n) \\
 &\quad - n\log|h_{12}|^2 + n\log|h_{11}|^2 + n\epsilon \\
 &= h(\mathbf{y}_2^n) - h(\mathbf{z}_1^n) + h(\tilde{\mathbf{x}}_1^n + \tilde{\mathbf{z}}_1^n) - h(\tilde{\mathbf{x}}_1^n + \tilde{\mathbf{z}}_2^n) \\
 &\quad - n\log|h_{12}|^2 + n\log|h_{11}|^2 + n\epsilon, \tag{37}
 \end{aligned}$$

where we defined

$$\begin{aligned}
 \tilde{x}_{1i} &= h_{11}h_{12}x_{2i} \\
 \tilde{z}_{1i} &= h_{12}z_{1i} \\
 \tilde{z}_{2i} &= h_{11}z_{2i}. \tag{38}
 \end{aligned}$$

In the weak interference case we have $\text{INR}_2 < \text{SNR}_1$, which implies

$$\mathbb{E}|\tilde{z}_{1i}^2| < \mathbb{E}|\tilde{z}_{2i}^2|. \tag{39}$$

Since the capacity region of Gaussian interference channel only depends on the marginal distribution of z_1 and z_2 , we can assume that there exists an i.i.d Gaussian random vector \mathbf{z}^n , with $z_i \sim \mathcal{CN}(0, \mathbb{E}|\tilde{z}_{2i}^2| - \mathbb{E}|\tilde{z}_{1i}^2|)$, such that

$$\tilde{z}_{2i} = \tilde{z}_{1i} + z_i. \tag{40}$$

Thus, we have

$$\begin{aligned}
 n(R_1 + R_2) &\leq h(\mathbf{y}_2^n) - h(\mathbf{z}_1^n) + h(\tilde{\mathbf{x}}_1^n + \tilde{\mathbf{z}}_1^n) \\
 &\quad - h(\tilde{\mathbf{x}}_1^n + \tilde{\mathbf{z}}_1^n + \mathbf{z}^n) \\
 &\quad - n\log|h_{12}|^2 + n\log|h_{11}|^2 + n\epsilon \\
 &= h(\mathbf{y}_2^n) - h(\mathbf{z}_1^n) - I(\mathbf{z}^n, \mathbf{z}^n + \tilde{\mathbf{x}}_1^n + \tilde{\mathbf{z}}_1^n) \\
 &\quad - n\log|h_{12}|^2 + n\log|h_{11}|^2 + n\epsilon. \tag{41}
 \end{aligned}$$

Using the worst case noise result [9], we can see that $-I(\mathbf{z}^n, \mathbf{z}^n + \tilde{\mathbf{x}}_1^n + \tilde{\mathbf{z}}_1^n)$ is maximized when \mathbf{x}_1^n is i.i.d Gaussian random vector with $x_{1i} \sim \mathcal{CN}(0, P_1)$. Note that $h(\mathbf{y}_2^n)$ is maximized when \mathbf{x}_1^n is a Gaussian random vector with i.i.d. components $x_{1i} \sim \mathcal{CN}(0, P_1)$ and \mathbf{x}_2^n is a Gaussian random vector with i.i.d. components $x_{2i} \sim \mathcal{CN}(0, P_2)$. A simple calculation leads to

$$\begin{aligned}
 R_1 + R_2 &\leq \log\left(1 + \frac{|h_{11}|^2 P_1}{N_0}\right) \\
 &\quad + \log\left(1 + \frac{|h_{22}|^2 P_2}{N_0 + |h_{12}|^2 P_1}\right) + \epsilon \\
 &= \log(1 + \text{SNR}_1) + \log\left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_2}\right) + \epsilon. \tag{42}
 \end{aligned}$$

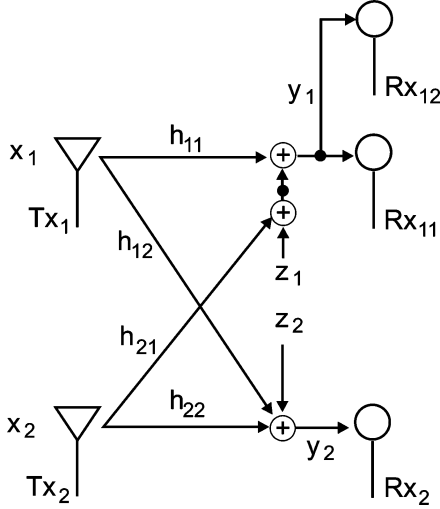
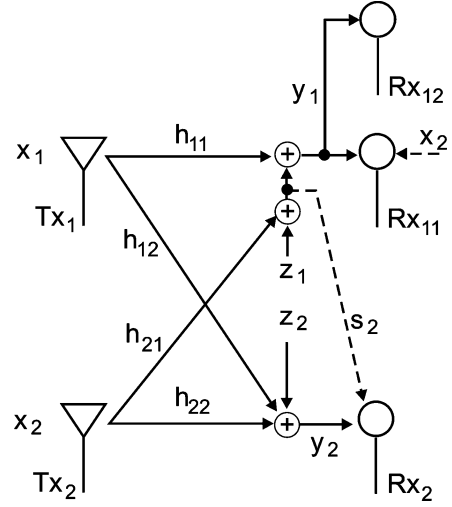


Fig. 12. Gaussian interference channel with three receivers.

- 3) The second bound on $R_1 + R_2$ can be derived similarly by using the genie-aided channel in which a genie gives x_1 to receiver 2.
- 4) The third bound on $R_1 + R_2$ has been proved in Section III-D.
- 5) Next we bound $2R_1 + R_2$. We consider the interference channel drawn in Fig. 12, in which there are two identical receivers (Rx_{11} and Rx_{12}) for user 1's message, and one receiver for user 2's message. We can think of $2R_1 + R_2$ as the sum of the rates at the three receivers. To derive an upper bound, we consider the genie-aided channel where a genie provides s_1 to receiver Rx_{11} and s_2 to receiver Rx_2 (see Fig. 13). For any codebook of block length n we can write

$$\begin{aligned}
n(2R_1 + R_2) &\leq I(\mathbf{x}_1^n; \mathbf{y}_1^n) + I(\mathbf{x}_2^n; \mathbf{y}_2^n) + I(\mathbf{x}_1^n; \mathbf{y}_1^n) + n\epsilon \\
&\leq I(\mathbf{x}_1^n; \mathbf{y}_1^n) + I(\mathbf{x}_2^n; \mathbf{y}_2^n s_2^n) + I(\mathbf{x}_1^n; \mathbf{y}_1^n | \mathbf{x}_2^n) + n\epsilon \\
&= h(\mathbf{y}_1^n) - h(\mathbf{y}_1^n | \mathbf{x}_1^n) + h(\mathbf{y}_2^n s_2^n) \\
&\quad - h(\mathbf{y}_2^n s_2^n | \mathbf{x}_2^n) + h(\mathbf{y}_1^n | \mathbf{x}_2^n) - h(\mathbf{y}_1^n | \mathbf{x}_1^n \mathbf{x}_2^n) + n\epsilon \\
&= h(\mathbf{y}_1^n) - h(\mathbf{s}_2^n) + h(\mathbf{s}_2^n) + h(\mathbf{y}_2^n | \mathbf{s}_2^n) \\
&\quad - h(\mathbf{y}_2^n | \mathbf{x}_2^n) - h(\mathbf{s}_2^n | \mathbf{y}_2^n \mathbf{x}_2^n) \\
&\quad + h(\mathbf{y}_1^n | \mathbf{x}_2^n) - h(\mathbf{y}_1^n | \mathbf{x}_1^n \mathbf{x}_2^n) + n\epsilon \\
&= h(\mathbf{y}_1^n) + h(\mathbf{y}_2^n | \mathbf{s}_2^n) - h(\mathbf{y}_2^n | \mathbf{x}_2^n) \\
&\quad - h(\mathbf{z}_1^n) + h(\mathbf{y}_1^n | \mathbf{x}_2^n) - h(\mathbf{z}_1^n) + n\epsilon \\
&\leq \sum_{i=1}^n [h(y_{1i}) + h(y_{2i} | s_{2i}) - 2h(z_{1i})] \\
&\quad - h(\mathbf{y}_2^n | \mathbf{x}_2^n) + h(\mathbf{y}_1^n | \mathbf{x}_2^n) + n\epsilon \\
&= \sum_{i=1}^n [h(y_{1i}) + h(y_{2i} | s_{2i}) - 2h(z_{1i})] \\
&\quad - h(h_{12}\mathbf{x}_1^n + \mathbf{z}_2^n) + h(h_{11}\mathbf{x}_1^n + \mathbf{z}_1^n) + n\epsilon. \quad (43)
\end{aligned}$$

Fig. 13. Genie-aided three-receiver Gaussian interference channel. A genie provides x_2 to receiver Rx_{11} and s_2 to receiver Rx_2 .

Using similar reasons as those in the proof of the third bound on $R_1 + R_2$, we have

$$\begin{aligned}
&\frac{1}{n} \sum_{i=1}^n h(y_{1i}) \\
&\leq \frac{1}{n} \log \pi e (|h_{11}|^2 P_{1i} + |h_{21}|^2 P_{2i} + N_0) \\
&\leq \log \pi e \left(|h_{11}|^2 \frac{1}{n} \sum_{i=1}^n P_{1i} + |h_{21}|^2 \frac{1}{n} \sum_{i=1}^n P_{2i} + N_0 \right) \\
&\leq \log \pi e (|h_{11}|^2 P_1 + |h_{21}|^2 P_2 + N_0), \quad (44)
\end{aligned}$$

and

$$\begin{aligned}
&\frac{1}{n} \sum_{i=1}^n h(y_{2i} | s_{2i}) \\
&\leq \log \pi e \left(N_0 + |h_{12}|^2 P_1 + \frac{|h_{22}|^2 P_2 N_0}{N_0 + |h_{21}|^2 P_2} \right). \quad (45)
\end{aligned}$$

Since $\text{INR}_2 < \text{SNR}_1$, we can use the worst case noise result to bound $-h(h_{12}\mathbf{x}_1^n + \mathbf{z}_2^n) + h(h_{11}\mathbf{x}_1^n + \mathbf{z}_1^n)$ as

$$\begin{aligned}
&-h(h_{12}\mathbf{x}_1^n + \mathbf{z}_2^n) + h(h_{11}\mathbf{x}_1^n + \mathbf{z}_1^n) \\
&\leq \log \left(\frac{1 + \text{SNR}_1}{1 + \text{INR}_2} \right). \quad (46)
\end{aligned}$$

Combining all the above, we have

$$\begin{aligned}
2R_1 + R_2 &\leq \log(1 + \text{SNR}_1 + \text{INR}_1) \\
&\quad + \log \left(1 + \text{INR}_2 + \frac{\text{SNR}_2}{1 + \text{INR}_1} \right) \\
&\quad + \log \left(\frac{1 + \text{SNR}_1}{1 + \text{INR}_2} \right) + \epsilon. \quad (47)
\end{aligned}$$

- 6) Similarly we can derive the bound for $R_1 + 2R_2$. \square

Remark 1: As mentioned, our first bound on $R_1 + R_2$ is also an upper bound on the sum rate of the one-sided interference channel (also known as Z-channel) [4], [5], [8] generated by removing the link from transmitter 2 to receiver 1. In the one-sided

channel, this upper bound can actually be achieved by both users when they communicate using codebooks generated from i.i.d. samples of a Gaussian distribution at full power and receiver 2 treats the signal from transmitter 1 as noise. Hence, we have a simple derivation of the sum capacity of the one-sided interference channel. Note that the proof of the sum capacity of the one-sided interference channel of [4], [5], [8] is quite indirect. In [5], the degraded Gaussian interference channel is introduced, and the capacity region of the degraded Gaussian interference channel is shown to be included in the capacity region of a degraded Gaussian broadcast channel. Moreover, the boundaries of the two regions are shown to touch at one point A . Later it was shown in [4] that the one-sided interference channel is equivalent to the degraded Gaussian interference channel. Recently, the author of [8] has pointed out that through a slope calculation in [5], the sum capacity of the degraded Gaussian interference channel is achieved at point A , thus establishing the sum capacity of the Gaussian one-sided interference channel.

Note that in our derivation of the first and second outer bounds on the sum rate, the outer bound on $2R_1 + R_2$, and the outer bound on $R_1 + 2R_2$, we used the conditions $\text{INR}_2 < \text{SNR}_1$ and $\text{INR}_1 < \text{SNR}_2$. For this reason, this outer bound only holds for the weak interference channel. Next we present an outer bound on the capacity region of the mixed interference channel.

2) *Outer Bound for Mixed Interference Channel:* For the mixed interference channel, i.e., $\text{INR}_1 \geq \text{SNR}_2$ and $\text{INR}_2 < \text{SNR}_1$, we have the following outer bound on the capacity region.

Theorem 4: For the Gaussian mixed interference channel, the capacity region is contained within the set of rate pairs (R_1, R_2) satisfying

$$\begin{aligned}
 R_1 &\leq \log(1 + \text{SNR}_1) \\
 R_2 &\leq \log(1 + \text{SNR}_2) \\
 R_1 + R_2 &\leq \log(1 + \text{SNR}_1) + \log\left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_2}\right) \\
 R_1 + R_2 &\leq \log(1 + \text{SNR}_1 + \text{INR}_1) \\
 R_1 + 2R_2 &\leq \log(1 + \text{SNR}_2 + \text{INR}_2) \\
 &\quad + \log\left(1 + \text{INR}_1 + \frac{\text{SNR}_1}{1 + \text{INR}_2}\right) \\
 &\quad + \log\left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_1}\right). \tag{48}
 \end{aligned}$$

Proof: We prove this outer bound by examining the proof of the bounds in (36).

- 1) The bounds on R_1 and R_2 still hold.
- 2) The proof of the first upper bound on the sum rate in (36) needs the condition $\text{INR}_2 < \text{SNR}_1$, which still holds in this mixed interference channel. So we have the same bound.
- 3) The proof of the second upper bound on the sum rate in (36) needs the condition $\text{INR}_1 < \text{SNR}_2$, which does not hold. Note that this bound is actually the sum capacity of the one-sided interference channel in which the link from transmitter 1 to receiver 2 is removed. When $\text{INR}_1 \geq \text{SNR}_2$, we are dealing with a one-sided interference channel with strong interference. It is shown in [4] that a sum-rate outer

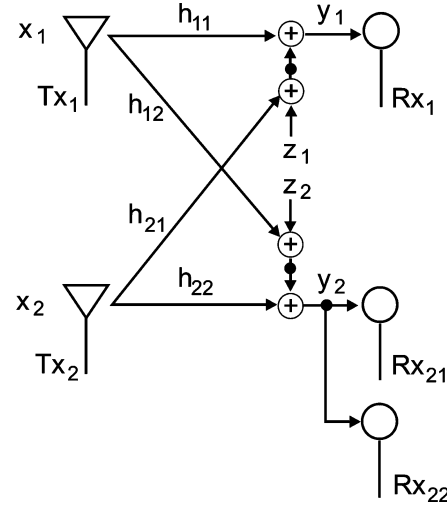


Fig. 14. Gaussian interference channel with three receivers.

bound for this channel is the sum-rate of the MAC at receiver 1, i.e.,

$$R_1 + R_2 \leq \log(1 + \text{SNR}_1 + \text{INR}_1) \tag{49}$$

and it is obviously an upper bound on sum-rate of the mixed interference channel.

- 4) The third upper bound on the sum-rate in (36) still holds; however, it is straightforward to show that the third bound on $R_1 + R_2$ is larger than the second bound on $R_1 + R_2$ in (48), so we do not need to include this bound.
- 5) The upper bound on $2R_1 + R_2$ in (36) still holds since $\text{INR}_2 < \text{SNR}_1$ still holds. However, it is straightforward to show that the bound on $2R_1 + R_2$ is larger than the sum of the bound on R_1 and the second bound on $R_1 + R_2$ in (48), so we do not need to include this bound.
- 6) The proof of the bound on $R_1 + 2R_2$ in (36) needs the condition of $\text{INR}_1 < \text{SNR}_2$, which is no longer true. To obtain a bound that does not require the condition $\text{INR}_1 < \text{SNR}_2$, we consider the interference channel drawn in Fig. 14, in which there are two identical receivers (Rx_{21} and Rx_{22}) for user 2's message, and one receiver for user 1's message. We can think of $R_1 + 2R_2$ as the sum-rate at the three receivers of this new interference channel. To derive an upper bound, we consider the genie-aided channel where a genie provides both s_2 and x_1 to receiver Rx_{21} and s_1 to receiver Rx_1 (see Fig. 15)

$$\begin{aligned}
 &n(R_1 + 2R_2) \\
 &\leq I(\mathbf{x}_1^n; \mathbf{y}_1^n) + I(\mathbf{x}_2^n; \mathbf{y}_2^n) + I(\mathbf{x}_2^n; \mathbf{y}_2^n) + n\epsilon \\
 &\leq I(\mathbf{x}_1^n; \mathbf{y}_1^n, \mathbf{s}_1^n) + I(\mathbf{x}_2^n; \mathbf{y}_2^n) \\
 &\quad + I(\mathbf{x}_2^n; \mathbf{y}_2^n, \mathbf{s}_2^n | \mathbf{x}_1^n) + n\epsilon \\
 &= I(\mathbf{x}_1^n; \mathbf{s}_1^n) + I(\mathbf{x}_1^n; \mathbf{y}_1^n | \mathbf{s}_1^n) + I(\mathbf{x}_2^n; \mathbf{y}_2^n) \\
 &\quad + I(\mathbf{x}_2^n; \mathbf{s}_2^n | \mathbf{x}_1^n) + I(\mathbf{x}_2^n; \mathbf{y}_2^n | \mathbf{s}_2^n, \mathbf{x}_1^n) + n\epsilon \\
 &= h(\mathbf{s}_1^n) - h(\mathbf{s}_1^n | \mathbf{x}_1^n) + h(\mathbf{y}_1^n | \mathbf{s}_1^n) \\
 &\quad - h(\mathbf{y}_1^n | \mathbf{x}_1^n, \mathbf{s}_1^n) + h(\mathbf{y}_2^n) - h(\mathbf{y}_2^n | \mathbf{x}_2^n) \\
 &\quad + h(\mathbf{s}_2^n | \mathbf{x}_1^n) - h(\mathbf{s}_2^n | \mathbf{x}_1^n, \mathbf{x}_2^n) \\
 &\quad + h(\mathbf{y}_2^n | \mathbf{s}_2^n, \mathbf{x}_1^n) - h(\mathbf{y}_2^n | \mathbf{s}_2^n, \mathbf{x}_1^n, \mathbf{x}_2^n) + n\epsilon
 \end{aligned}$$

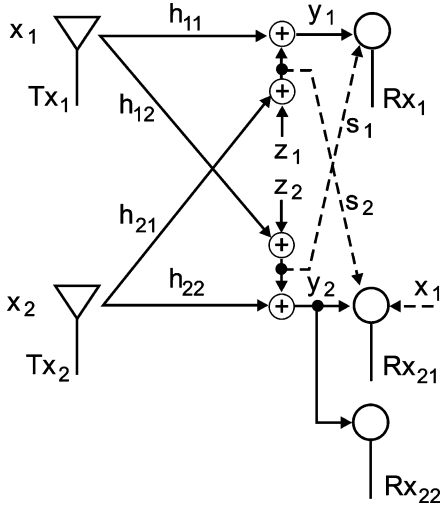


Fig. 15. Genie-aided three-receiver Gaussian interference channel. A genie provides signals s_2 and x_1 to receiver R_{x21} , and s_1 to R_{x1} .

$$\begin{aligned}
&= h(\mathbf{s}_1^n) - h(\mathbf{z}_2^n) + h(\mathbf{y}_1^n | \mathbf{s}_1^n) \\
&\quad - h(\mathbf{s}_2^n) + h(\mathbf{y}_2^n) - h(\mathbf{s}_1^n) \\
&\quad + h(\mathbf{s}_2^n) - h(\mathbf{z}_1^n) + h(\mathbf{y}_2^n | \mathbf{s}_2^n \mathbf{x}_1^n) - h(\mathbf{z}_2^n) + n\epsilon \\
&\leq h(\mathbf{y}_1^n | \mathbf{s}_1^n) + h(\mathbf{y}_2^n) + h(\mathbf{y}_2^n | \mathbf{s}_2^n \mathbf{x}_1^n) \\
&\quad - h(\mathbf{z}_1^n) - 2h(\mathbf{z}_2^n) + n\epsilon \\
&\leq \sum_{i=1}^n [h(y_{1i} | s_{1i}) + h(y_{2i}) + h(y_{2i} | s_{2i} x_{1i}) \\
&\quad - h(z_{1i}) - 2h(z_{2i})] + n\epsilon.
\end{aligned}$$

Using similar reasons as those in the proof of Theorem 1 in Section III-D we can write

$$\begin{aligned}
&\frac{1}{n} \sum_{i=1}^n h(y_{1i} | s_{1i}) \\
&\leq \frac{1}{n} \log \left[\pi e \left(N_0 + |h_{21}|^2 P_{2i} + \frac{|h_{11}|^2 P_{1i} N_0}{N_0 + |h_{12}|^2 P_{1i}} \right) \right] \\
&\leq \log \left[\pi e \left(N_0 + |h_{21}|^2 \left(\frac{1}{n} \sum_{i=1}^n P_{2i} \right) \right. \right. \\
&\quad \left. \left. + \frac{|h_{11}|^2 \left(\frac{1}{n} \sum_{i=1}^n P_{1i} \right) N_0}{N_0 + |h_{12}|^2 \left(\frac{1}{n} \sum_{i=1}^n P_{1i} \right)} \right) \right] \\
&\leq \log \left[\pi e \left(N_0 + |h_{21}|^2 P_2 + \frac{|h_{11}|^2 P_1 N_0}{N_0 + |h_{12}|^2 P_1} \right) \right] \quad (50)
\end{aligned}$$

together with

$$\begin{aligned}
&\frac{1}{n} \sum_{i=1}^n h(y_{2i}) \\
&\leq \frac{1}{n} \log \pi e (|h_{22}|^2 P_{2i} + |h_{12}|^2 P_{1i} + N_0) \\
&\leq \log \pi e \left(|h_{22}|^2 \frac{1}{n} \sum_{i=1}^n P_{2i} + |h_{12}|^2 \frac{1}{n} \sum_{i=1}^n P_{1i} + N_0 \right) \\
&\leq \log \pi e (|h_{22}|^2 P_2 + |h_{12}|^2 P_1 + N_0) \quad (51)
\end{aligned}$$

$$\begin{aligned}
&\text{and} \\
&\frac{1}{n} \sum_{i=1}^n h(y_{2i} | s_{2i} x_{1i}) \\
&= \frac{1}{n} \sum_{i=1}^n h(h_{22} x_{2i} + z_{2i} | s_{2i}) \\
&\leq \frac{1}{n} \sum_{i=1}^n \log \pi e \left(N_0 + \frac{|h_{22}|^2 N_0 P_{2i}}{N_0 + |h_{21}|^2 P_{2i}} \right) \\
&\leq \log \pi e \left(N_0 + \frac{|h_{22}|^2 N_0 \left(\frac{1}{n} \sum_{i=1}^n P_{2i} \right)}{N_0 + |h_{21}|^2 \left(\frac{1}{n} \sum_{i=1}^n P_{2i} \right)} \right) \\
&\leq \log \pi e \left(N_0 + \frac{|h_{22}|^2 N_0 P_2}{N_0 + |h_{21}|^2 P_2} \right). \quad (52)
\end{aligned}$$

Thus, we have

$$\begin{aligned}
&R_1 + 2R_2 \\
&\leq \log \pi e (|h_{22}|^2 P_2 + |h_{12}|^2 P_1 + N_0) - \log \pi e N_0 \\
&\quad + \log \pi e \left(N_0 + |h_{21}|^2 P_2 + \frac{|h_{11}|^2 P_1 N_0}{N_0 + |h_{12}|^2 P_1} \right) \\
&\quad - \log \pi e N_0 \\
&\quad + \log \pi e \left(N_0 + \frac{|h_{22}|^2 N_0 P_2}{N_0 + |h_{21}|^2 P_2} \right) \\
&\quad - \log \pi e N_0 + \epsilon \\
&= \log \left(1 + \frac{|h_{22}|^2 P_2}{N_0} + \frac{|h_{12}|^2 P_1}{N_0} \right) \\
&\quad + \log \left(1 + \frac{|h_{21}|^2 P_2}{N_0} + \frac{|h_{11}|^2 P_1 / N_0}{1 + |h_{12}|^2 P_1 / N_0} \right) \\
&\quad + \log \left(1 + \frac{|h_{22}|^2 P_2 / N_0}{1 + |h_{21}|^2 P_2 / N_0} \right) + \epsilon \\
&= \log (1 + \text{SNR}_2 + \text{INR}_2) \\
&\quad + \log \left(1 + \text{INR}_1 + \frac{\text{SNR}_1}{1 + \text{INR}_2} \right) \\
&\quad + \log \left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_1} \right) + \epsilon. \quad (53)
\end{aligned}$$

□

B. Achievable Scheme

The Han–Kobayashi scheme [1] is the best known achievable scheme for the interference channel. Recently, a simplified yet equivalent Han–Kobayashi achievable region was given in [10] which we state next in Lemma 1. The achievability of this simplified region was later proved with a direct coding theorem in [11, Theorem 1] (see [11, Sec. VI-A] for a specialization of [11, Theorem 1] to a noncompound interference channel, noting that the region of Lemma 1 is obtained by applying the Fourier–Motzkin elimination procedure to the inequalities that define $\mathcal{R}_{\text{in}}^{(4)}$ in [11], together with $R_1 = S_1 + T_1$ and $R_2 = S_2 + T_2$).

Lemma 1: Let \mathcal{P}^* be the set of joint probability distributions $P^*(\cdot)$ that factor as

$$\begin{aligned}
&P^*(q, w_1, w_2, x_1, x_2, y_1, y_2) \\
&= P(q) \cdot P(w_1, x_1 | q) \cdot P(w_2, x_2 | q) \cdot P(y_1, y_2 | x_1, x_2). \quad (54)
\end{aligned}$$

For a fixed $P^* \in \mathcal{P}^*$, let $\mathcal{R}(P^*)$ be the set of (R_1, R_2) satisfying

$$\begin{aligned}
 R_1 &\leq I(x_1; y_1 | w_2 q) \\
 R_2 &\leq I(x_2; y_2 | w_1 q) \\
 R_1 + R_2 &\leq I(x_2 w_1; y_2 | q) + I(x_1; y_1 | w_1 w_2 q) \\
 R_1 + R_2 &\leq I(x_1 w_2; y_1 | q) + I(x_2; y_2 | w_1 w_2 q) \\
 R_1 + R_2 &\leq I(x_1 w_2; y_1 | w_1 q) + I(x_2 w_1; y_2 | w_2 q) \\
 2R_1 + R_2 &\leq I(x_1 w_2; y_1 | q) + I(x_1; y_1 | w_1 w_2 q) \\
 &\quad + I(x_2 w_1; y_2 | w_2 q) \\
 R_1 + 2R_2 &\leq I(x_2 w_1; y_2 | q) + I(x_2; y_2 | w_1 w_2 q) \\
 &\quad + I(x_1 w_2; y_1 | w_1 q).
 \end{aligned} \tag{55}$$

Then the Han–Kobayashi achievable region is given by $\mathcal{R} = \bigcup_{P^* \in \mathcal{P}^*} \mathcal{R}(P^*)$.

In (55), $w_1(w_2)$ is the common information of user 1 (user 2) that can be decoded at both receivers, and q is the time-sharing parameter. For the Gaussian interference channel, if we use Gaussian codebooks, and use u_1 and u_2 to denote the private information of user 1 and user 2 respectively, we can write

$$\begin{aligned}
 x_1 &= u_1 + w_1 \\
 x_2 &= u_2 + w_2
 \end{aligned} \tag{56}$$

where u_1, u_2, w_1 , and w_2 are independent complex Gaussian random variables. Different $P^* \in \mathcal{P}^*$ correspond to different power splits between common and private messages, and different time-sharing strategies between the power splits.

Consider a fixed power splitting (i.e., we do not do time sharing) between private information and common information of the two users. Let P_{u1} and P_{u2} be the power of user 1 and user 2’s private messages, respectively. We define INR_{p2} to be the interference-to-noise ratio of user 1’s private message at receiver 2 and INR_{p1} to be the interference to noise ratio of user 2’s private message at receiver 1, i.e.,

$$\begin{aligned}
 \text{INR}_{p2} &= \frac{|h_{12}|^2 P_{u1}}{N_0} \\
 \text{INR}_{p1} &= \frac{|h_{21}|^2 P_{u2}}{N_0}.
 \end{aligned} \tag{57}$$

It is clear that $0 \leq \text{INR}_{p2} \leq \text{INR}_2$ and $0 \leq \text{INR}_{p1} \leq \text{INR}_1$. With this definition, the SNR of user 1’s private message at receiver 1 is $\text{SNR}_{p1} = \text{INR}_{p2} \frac{\text{SNR}_1}{\text{INR}_2}$ and the SNR of user 2’s private message at receiver 2 is $\text{SNR}_{p2} = \text{INR}_{p1} \frac{\text{SNR}_2}{\text{INR}_1}$. We can parameterize a Han–Kobayashi achievable scheme with a fixed power splitting by using INR_{p2} and INR_{p1} . We denote the Han–Kobayashi scheme with parameters $\text{INR}_{p2}, \text{INR}_{p1}$ as $\text{HK}(\text{INR}_{p2}, \text{INR}_{p1})$, and the corresponding achievable region as $\mathfrak{R}(\text{INR}_{p2}, \text{INR}_{p1})$.

Note that $\text{HK}(\text{INR}_{p2}, \text{INR}_{p1})$ and $\mathfrak{R}(\text{INR}_{p2}, \text{INR}_{p1})$ correspond to a Han–Kobayashi scheme where there is a fixed private and common message power split and there is no time sharing (i.e., the time-sharing random variable q is a constant). Therefore, $\mathfrak{R}(\text{INR}_{p2}, \text{INR}_{p1}) \subset \mathcal{R}$, where \mathcal{R} is the general Han–Kobayashi achievable region given in Lemma 1. In general, the inclusion is strict, that is, varying the power allocations

and time sharing between multiple private and common message power splits allows to achieve a larger rate region. However, we will see that the region achievable with a clever choice of a fixed private and common message power split and without time sharing is close to the capacity region of the channel.

To evaluate the Han–Kobayashi region (55) for the Gaussian interference channel, even if we restrict ourselves to use only Gaussian codebooks, we need to consider all possible power splits and different time-sharing strategies among them. This is in general very complicated and a calculation of a subset of the Han–Kobayashi achievable region using some special choices of power splitting and time sharing strategies can be found in [8]. However, from the intuition we built in Section III, we know that a good power splitting should have the property that $\text{INR}_{p2} = 1$ and $\text{INR}_{p1} = 1$, i.e., the interference-to-noise ratio of each user’s private message at the other user’s receiver is one. We also showed that this power splitting can achieve to within one bit the symmetric rate capacity of the symmetric Gaussian interference channel. In the next section, we will show that this is also a good splitting for the entire capacity region. More specifically, we will show that by choosing $\text{INR}_{p2}, \text{INR}_{p1}$ as close to 1 as possible, we can achieve rates within one bit of the whole capacity region.

C. Within One Bit of the Capacity Region

Equipped with the new outer bound derived in Section IV-A1 and the intuition of a good power splitting in Section III, we are now ready to prove our main result: a simple Han–Kobayashi scheme can achieve to within one bit of the capacity region of the Gaussian interference channel. First we provide a formal definition of the *within one bit* notion.

Definition 1: An achievable region is said to be *within one bit of the capacity region* if for any rate pair (R_1, R_2) on the boundary of the achievable region, the rate pair $(R_1 + 1, R_2 + 1)$ is not achievable. Equivalently, $(R_1 - 1, R_2 - 1)$ is in the achievable region for any rate pair (R_1, R_2) in the capacity region.

Since the outer bound of the weak interference channel and the outer bound of the mixed interference channel are different, we treat these two channels separately in the following two subsections.

1) *Weak Interference Channel:* Our main result is stated in the following theorem.

Theorem 5: The achievable region

$$\mathfrak{R}(\min(1, \text{INR}_2), \min(1, \text{INR}_1)) \tag{58}$$

is within one bit of the capacity region of the Gaussian weak interference channel.

Remark 2: The reason to consider the region $\mathfrak{R}(\min(1, \text{INR}_2), \min(1, \text{INR}_1))$ is because when $\text{INR}_1 < 1$ or $\text{INR}_2 < 1$, we cannot use the Han–Kobayashi scheme $\text{HK}(1, 1)$. However, say in the case of $\text{INR}_1 < 1$, the interference caused by user 2 at receiver 1 is even weaker than the additive Gaussian noise. Thus, we shall not lose much of the optimality by simply treating all of user 2’s signal as noise at receiver 1, i.e., letting $\text{INR}_{p1} = \text{INR}_1$.

Proof: It can be seen from (36) and (55) that both the outer bound to the capacity region and the achievable region $\mathfrak{R}(\min(1, \text{INR}_2), \min(1, \text{INR}_1))$ are piecewise linear, and only consist of straight lines with slopes 0, $-1/2$, -1 , -2 , and ∞ . We define δ_{R_1} to be the difference between the outer bound on R_1 in (36) (first constraint) and achievable R_1 in $\mathfrak{R}(\min(1, \text{INR}_2), \min(1, \text{INR}_1))$, and similarly define $\delta_{R_2}, \delta_{R_1+R_2}, \delta_{2R_1+R_2}$, and $\delta_{R_1+2R_2}$. Note that if the rate pair (R_1, R_2) is on the boundary of the achievable region $\mathfrak{R}(\min(1, \text{INR}_2), \min(1, \text{INR}_1))$, it must be on one of the bounding straight lines. Thus, if the following holds

$$\begin{aligned} \delta_{R_1} &< 1 \\ \delta_{R_2} &< 1 \\ \delta_{R_1+R_2} &< 2 \\ \delta_{2R_1+R_2} &< 3 \\ \delta_{R_1+2R_2} &< 3 \end{aligned} \quad (59)$$

then the rate pair $(R_1 + 1, R_2 + 1)$ would be outside the outer bound (36), and hence $\mathfrak{R}(\min(1, \text{INR}_2), \min(1, \text{INR}_1))$ is within one bit of the capacity region of the Gaussian weak interference channel. We now show that (59) holds for the different parameter ranges.

1) $\text{INR}_1 \geq 1$ and $\text{INR}_2 \geq 1$

In this case, we have

$$\mathfrak{R}(\min(1, \text{INR}_2), \min(1, \text{INR}_1)) = \mathfrak{R}(1, 1). \quad (60)$$

It is straightforward to evaluate $\mathfrak{R}(1, 1)$ from Lemma 1. The result is provided in the following corollary.

Corollary 1: The achievable rate region $\mathfrak{R}(1, 1)$ contains all the rate pairs (R_1, R_2) satisfying

$$\begin{aligned} R_1 &\leq \log(2 + \text{SNR}_1) - 1 \\ R_2 &\leq \log(2 + \text{SNR}_2) - 1 \\ R_1 + R_2 &\leq \log(2\text{INR}_2 + \text{SNR}_1) \\ &\quad + \log\left(1 + \frac{1 + \text{SNR}_2}{\text{INR}_2}\right) - 2 \\ R_1 + R_2 &\leq \log(2\text{INR}_1 + \text{SNR}_2) \\ &\quad + \log\left(1 + \frac{1 + \text{SNR}_1}{\text{INR}_1}\right) - 2 \\ R_1 + R_2 &\leq \log\left(1 + \text{INR}_1 + \frac{\text{SNR}_1}{\text{INR}_2}\right) \\ &\quad + \log\left(1 + \text{INR}_2 + \frac{\text{SNR}_2}{\text{INR}_1}\right) - 2 \\ 2R_1 + R_2 &\leq \log(1 + \text{SNR}_1 + \text{INR}_1) \\ &\quad + \log\left(1 + \text{INR}_2 + \frac{\text{SNR}_2}{\text{INR}_1}\right) \\ &\quad + \log\left(2 + \frac{\text{SNR}_1}{\text{INR}_2}\right) - 3 \\ R_1 + 2R_2 &\leq \log(1 + \text{SNR}_2 + \text{INR}_2) \\ &\quad + \log\left(1 + \text{INR}_1 + \frac{\text{SNR}_1}{\text{INR}_2}\right) \\ &\quad + \log\left(2 + \frac{\text{SNR}_2}{\text{INR}_1}\right) - 3. \end{aligned} \quad (61)$$

If we denote the three bounds on the sum rate in the outer bound (36) by a_1, a_2, a_3 , respectively, and the three bounds on the sum rate in the inner bound (61) by b_1, b_2, b_3 , respectively, we have

$$\begin{aligned} \delta_{R_1+R_2} &= \min\{a_1, a_2, a_3\} - \min\{b_1, b_2, b_3\} \\ &\leq \max\{a_1 - b_1, a_2 - b_2, a_3 - b_3\} \end{aligned} \quad (62)$$

and hence we can simply upper-bound $\delta_{R_1+R_2}$ by the maximum of the differences between the i th bound on $R_1 + R_2$ in the outer bound (36) and the corresponding i th bound on $R_1 + R_2$ in the inner bound (61) for $i = 1, 2, 3$. We can readily compare (36) and (61) term by term, and see that (59) is true. (See Fig. 16). For example

$$\begin{aligned} \delta_{2R_1+R_2} &= \log(1 + \text{SNR}_1 + \text{INR}_1) \\ &\quad + \log\left(1 + \text{INR}_2 + \frac{\text{SNR}_2}{1 + \text{INR}_1}\right) \\ &\quad + \log\left(\frac{1 + \text{SNR}_1}{1 + \text{INR}_2}\right) \\ &\quad - \left[\log(1 + \text{SNR}_1 + \text{INR}_1) \right. \\ &\quad \left. + \log\left(1 + \text{INR}_2 + \frac{\text{SNR}_2}{\text{INR}_1}\right) \right. \\ &\quad \left. + \log\left(2 + \frac{\text{SNR}_1}{\text{INR}_2}\right) - 3 \right] \\ &= \left[\log\left(1 + \text{INR}_2 + \frac{\text{SNR}_2}{1 + \text{INR}_1}\right) \right. \\ &\quad \left. - \log\left(1 + \text{INR}_2 + \frac{\text{SNR}_2}{\text{INR}_1}\right) \right] \\ &\quad + \left[\log\left(\frac{1 + \text{SNR}_1}{1 + \text{INR}_2}\right) - \log\left(2 + \frac{\text{SNR}_1}{\text{INR}_2}\right) \right] + 3 \\ &< 3. \end{aligned} \quad (63)$$

2) $\text{INR}_1 < 1$ and $\text{INR}_2 \geq 1$

In this case, we have

$$\mathfrak{R}(\min(1, \text{INR}_2), \min(1, \text{INR}_1)) = \mathfrak{R}(1, \text{INR}_1). \quad (64)$$

Evaluating the achievable region (55) with $\text{INR}_{p_2} = 1$ and $\text{INR}_{p_1} = \text{INR}_1$, we have the following achievable region $\mathfrak{R}(1, \text{INR}_1)$:

$$\begin{aligned} R_1 &\leq \log\left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_1}\right) \\ R_2 &\leq \log(2 + \text{SNR}_2) - 1 \\ R_1 + R_2 &\leq \log\left(\text{INR}_2 + \frac{\text{SNR}_1}{1 + \text{INR}_1}\right) \\ &\quad + \log\left(1 + \frac{1 + \text{SNR}_2}{\text{INR}_2}\right) - 1 \\ R_1 + R_2 &\leq \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_1}\right) \\ &\quad + \log(2 + \text{SNR}_2) - 1 \\ R_1 + R_2 &\leq \log\left(\text{INR}_2 + \frac{\text{SNR}_1}{1 + \text{INR}_1}\right) \\ &\quad + \log\left(1 + \frac{1 + \text{SNR}_2}{\text{INR}_2}\right) - 1 \end{aligned}$$

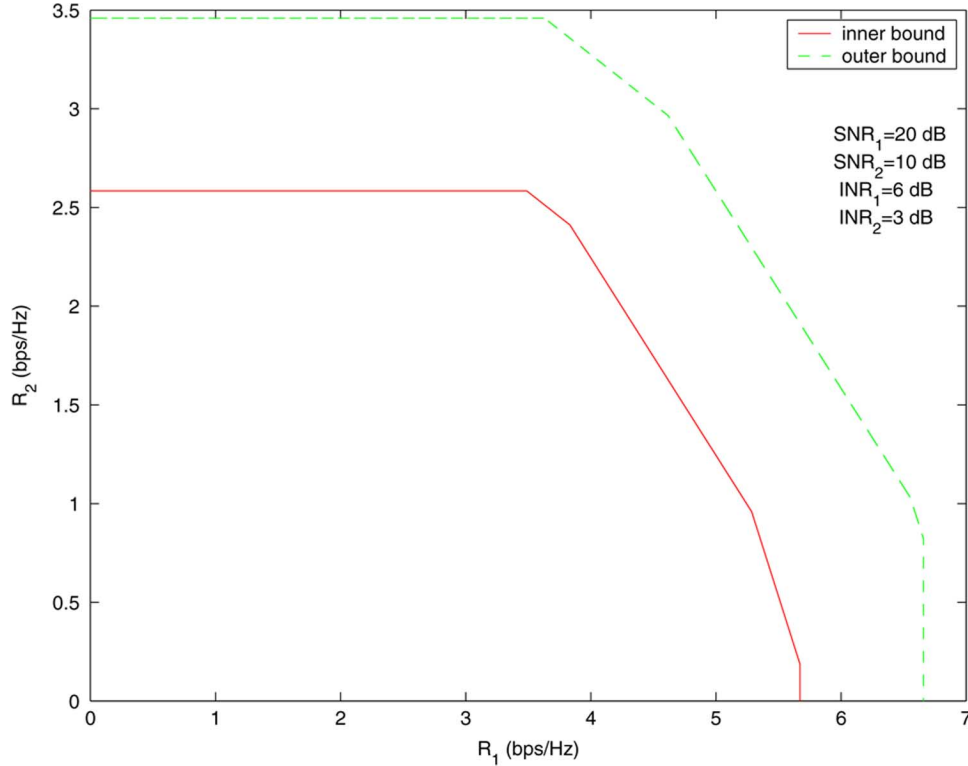


Fig. 16. Comparison of Han–Kobayashi achievable region $\mathfrak{R}(1, 1)$ and outer bound (36).

$$\begin{aligned}
 2R_1 + R_2 &\leq \log(1 + \text{SNR}_1 + \text{INR}_1) \\
 &\quad + \log(1 + \text{INR}_2 + \text{SNR}_2) \\
 &\quad + \log\left(1 + \text{INR}_1 + \frac{\text{SNR}_1}{\text{INR}_2}\right) \\
 &\quad - \log 2(1 + \text{INR}_1)^2 \\
 R_1 + 2R_2 &\leq \log(2 + \text{SNR}_2) \\
 &\quad + \log\left(\text{INR}_2 + \frac{\text{SNR}_1}{1 + \text{INR}_1}\right) \\
 &\quad + \log\left(1 + \frac{1 + \text{SNR}_2}{\text{INR}_2}\right) - 2. \quad (65)
 \end{aligned}$$

Since the second bound on $R_1 + R_2$ is the sum of the bound on R_1 and the bound on R_2 , the first and the third bounds on $R_1 + R_2$ are the same, and the bound on $R_1 + 2R_2$ is the sum of the bound on R_2 and the first bound on $R_1 + R_2$, these three bounds are redundant and we have the following simplified achievable region $\mathfrak{R}(1, \text{INR}_1)$:

$$\begin{aligned}
 R_1 &\leq \log\left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_1}\right) \\
 R_2 &\leq \log(2 + \text{SNR}_2) - 1 \\
 R_1 + R_2 &\leq \log\left(\text{INR}_2 + \frac{\text{SNR}_1}{1 + \text{INR}_1}\right) \\
 &\quad + \log\left(1 + \frac{1 + \text{SNR}_2}{\text{INR}_2}\right) - 1 \\
 2R_1 + R_2 &\leq \log(1 + \text{SNR}_1 + \text{INR}_1) \\
 &\quad + \log(1 + \text{INR}_2 + \text{SNR}_2) \\
 &\quad + \log\left(1 + \text{INR}_1 + \frac{\text{SNR}_1}{\text{INR}_2}\right) \\
 &\quad - \log 2(1 + \text{INR}_1)^2. \quad (66)
 \end{aligned}$$

Comparing $\mathfrak{R}(1, \text{INR}_1)$ with the corresponding bounds in (36) (use the first bound on $R_1 + R_2$ in (36)), and using the fact that $\text{INR}_1 < 1$, we can see that (59) is true. (See Fig. 17).

3) $\text{INR}_1 \geq 1$ and $\text{INR}_2 < 1$

In this case, we have

$$\mathfrak{R}(\min(1, \text{INR}_2), \min(1, \text{INR}_1)) = \mathfrak{R}(\text{INR}_2, 1). \quad (67)$$

This case is similar to the previous case and we can show that (59) is true. (See Fig. 18).

4) $\text{INR}_1 < 1$ and $\text{INR}_2 < 1$

In this case, we have

$$\mathfrak{R}(\min(1, \text{INR}_2), \min(1, \text{INR}_1)) = \mathfrak{R}(\text{INR}_2, \text{INR}_1). \quad (68)$$

Evaluating the achievable region (55) with $\text{INR}_{p_2} = \text{INR}_2$ and $\text{INR}_{p_1} = \text{INR}_1$ and getting rid of redundant bounds, we have the following region $\mathfrak{R}(\text{INR}_2, \text{INR}_1)$:

$$\begin{aligned}
 R_1 &\leq \log\left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_1}\right) \\
 R_2 &\leq \log\left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_2}\right). \quad (69)
 \end{aligned}$$

Note that $\mathfrak{R}(\text{INR}_2, \text{INR}_1)$ is the achievable region obtained by each user treating the other user's signal as noise. Comparing this region with the outer bound (36), we can see that (59) is true. (See Fig. 19)

Combining the above four cases, we have shown that (59) is true for all values of SNR_1 , SNR_2 , INR_1 , and INR_2 , given that

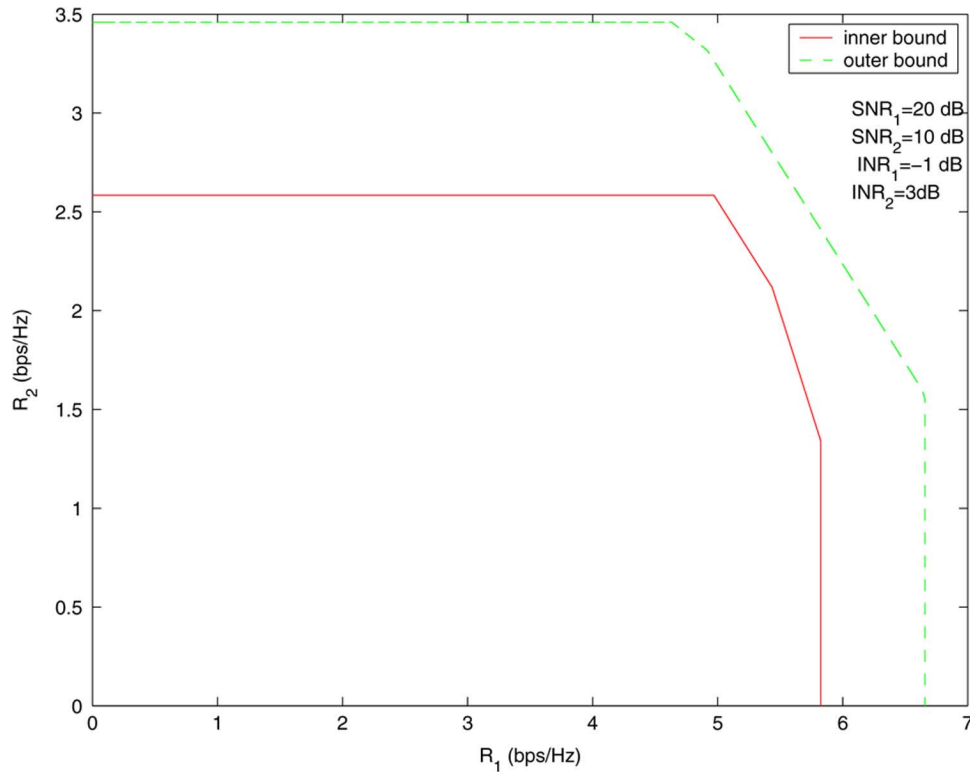


Fig. 17. Comparison of Han-Kobayashi achievable region $\mathfrak{R}(1, \text{INR}_1)$ and outer bound (36).

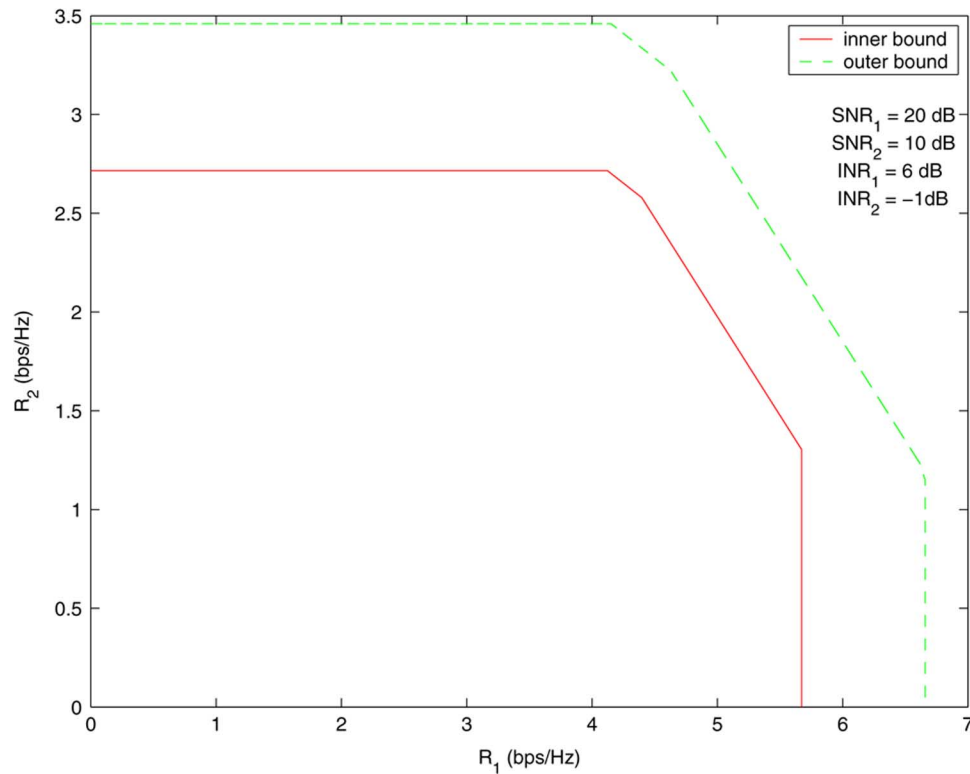


Fig. 18. Comparison of Han-Kobayashi achievable region $\mathfrak{R}(\text{INR}_2, 1)$ and outer bound (36).

$\text{INR}_1 < \text{SNR}_2, \text{INR}_2 < \text{SNR}_1$. Thus, we have proved that the achievable region

$$\mathfrak{R}(\min(1, \text{INR}_2), \min(1, \text{INR}_1))$$

is within one bit of the capacity region of the Gaussian weak interference channel. \square

2) *Mixed Interference Channel:* We assume that $\text{INR}_1 \geq \text{SNR}_2$ and $\text{INR}_2 < \text{SNR}_1$ in the mixed interference channel. A remarkable feature of this channel is that user 2's message can be fully decoded at receiver 1. Using this fact, a natural scheme for user 2 is to use all of his power on the common message, i.e., set $\text{INR}_{p1} = 0$. We also let INR_{p2} to be as close to 1 as possible

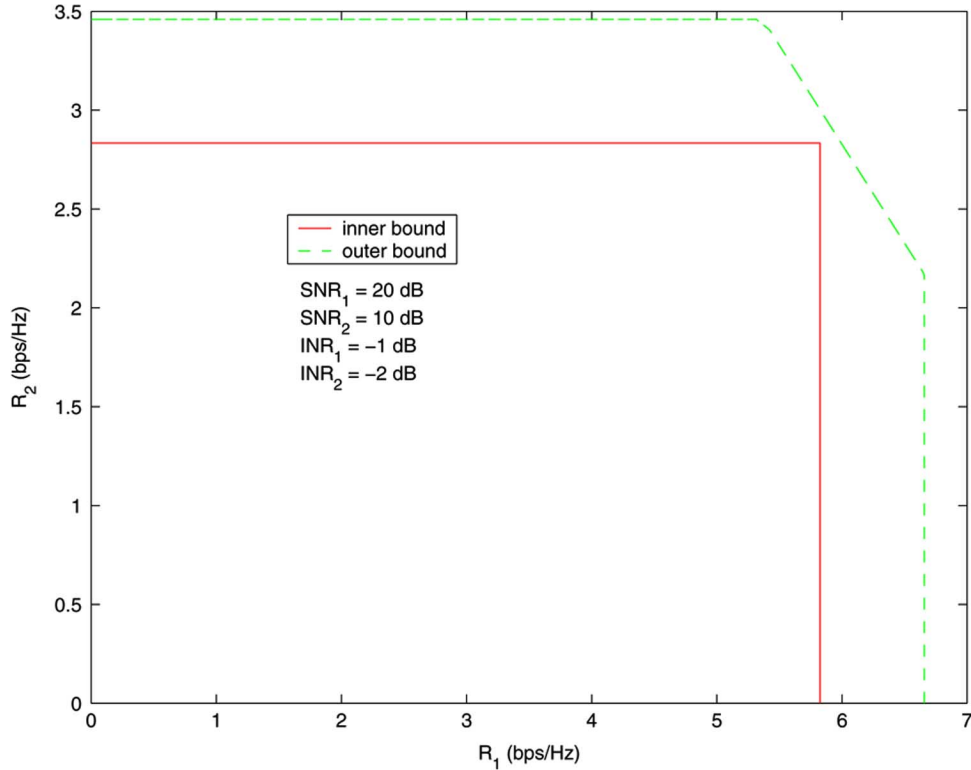


Fig. 19. Comparison of Han–Kobayashi achievable region $\mathfrak{R}(\text{INR}_2, \text{INR}_1)$ and outer bound (36).

using the intuition derived from the weak interference channel. We have the following result.

Theorem 6: The achievable region

$$\mathfrak{R}(\min(1, \text{INR}_2), 0)$$

is within one bit of the capacity region of the Gaussian interference channel when $\text{INR}_1 \geq \text{SNR}_2$, $\text{INR}_2 < \text{SNR}_1$.

Proof: We only need to prove that (59) is true. There are two cases to consider:

1) $\text{INR}_2 \geq 1$

In this case we use the Han–Kobayashi scheme $\text{HK}(1, 0)$.

By evaluating (55), we have the following result.

Corollary 2: The achievable rate region $\mathfrak{R}(1, 0)$ contains all the rate pairs (R_1, R_2) satisfying

$$\begin{aligned} R_1 &\leq \log(1 + \text{SNR}_1) \\ R_2 &\leq \log(2 + \text{SNR}_2) - 1 \\ R_1 + R_2 &\leq \log(\text{INR}_2 + \text{SNR}_1) + \log\left(1 + \frac{1 + \text{SNR}_2}{\text{INR}_2}\right) - 1 \\ R_1 + R_2 &\leq \log(1 + \text{INR}_1 + \text{SNR}_1) \\ R_1 + R_2 &\leq \log\left(1 + \text{INR}_1 + \frac{\text{SNR}_1}{\text{INR}_2}\right) + \log(1 + \text{INR}_2) - 1 \\ 2R_1 + R_2 &\leq \log(1 + \text{INR}_2) + \log(1 + \text{SNR}_1 + \text{INR}_1) \\ &\quad + \log\left(1 + \frac{\text{SNR}_1}{\text{INR}_2}\right) - 1 \\ R_1 + 2R_2 &\leq \log(1 + \text{SNR}_2 + \text{INR}_2) \\ &\quad + \log\left(1 + \text{INR}_1 + \frac{\text{SNR}_1}{\text{INR}_2}\right) - 1. \end{aligned} \quad (70)$$

Comparing $\mathfrak{R}(1, 0)$ in (70) with the outer bound (48), we can see that (59) is true. (See Fig. 20).

2) $\text{INR}_2 < 1$

We use the Han–Kobayashi scheme $\text{HK}(\text{INR}_2, 0)$, and get the following Han–Kobayashi achievable region $\mathfrak{R}(\text{INR}_2, 0)$:

$$\begin{aligned} R_1 &\leq \log(1 + \text{SNR}_1) \\ R_2 &\leq \log\left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_2}\right) \\ R_1 + R_2 &\leq \log(1 + \text{SNR}_1) + \log\left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_2}\right) \\ R_1 + R_2 &\leq \log(1 + \text{SNR}_1 + \text{INR}_1) \\ 2R_1 + R_2 &\leq (1 + \text{SNR}_1) + \log(1 + \text{SNR}_1 + \text{INR}_1) \\ R_1 + 2R_2 &\leq \log\left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_2}\right) + \log(1 + \text{SNR}_1 + \text{INR}_1). \end{aligned} \quad (71)$$

We can see that the bounds on $2R_1 + R_2$ and $R_1 + 2R_2$ are redundant. Comparing this region with the outer bound (48), we can see that (59) is true. (See Fig. 21).

Thus we have shown that (59) is true and we have proved theorem 6. \square

D. Discussion on One-Bit Result

The achievable region discussed in the previous section is not the largest possible. In fact, we can easily improve the achievable region by using other private–common message power splits. For example, Costa [4] pointed out that if we require receiver 2 to fully decode user 1’s message before

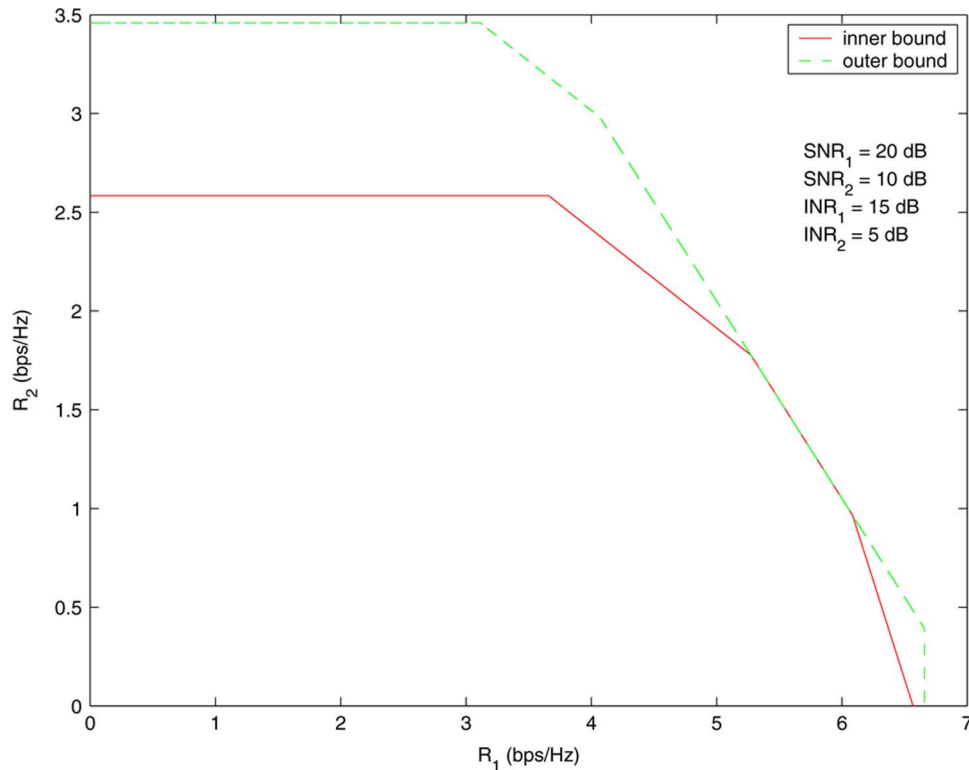


Fig. 20. Comparison of Han-Kobayashi achievable region $\mathfrak{R}(1, 0)$ and outer bound (36).

decoding his own message, and we require receiver 1 to treat user 2's signal as noise, then the rate pair

$$\begin{aligned} R_1 &= \log \left(1 + \frac{\text{INR}_2}{1 + \text{SNR}_2} \right) \\ R_2 &= \log(1 + \text{SNR}_2) \end{aligned} \quad (72)$$

is achievable. This rate pair is not inside $\mathfrak{R}(1, 1)$. However, we can achieve this rate pair by using the scheme $\text{HK}(\text{INR}_2, 0)$, i.e., user 1 has only common message, and user 2 has only private message. We do not intend to optimize over all possible Han-Kobayashi strategies to get the largest achievable region, which can be a very complicated task. In fact, the most important point that we want to make with our one-bit result is that we do not lose much by using a simple Han-Kobayashi strategy.

Our one-bit result shows that $\mathfrak{R}(1, 1)$ is a good approximation to the capacity region in the high-SNR, INR regime, since one bit is a relatively small number compared to the rates of the users. The high-SNR, INR regime corresponds to the interference-limited situation where interference plays a major role in communication. The low-SNR, INR regime ($\text{SNR} \ll 1$ and $\text{INR} \ll 1$) is not very interesting since the effect of interference is smaller than that of the additive Gaussian noise. Nevertheless, a loss of one bit in this regime may be large compared to the rates of the users. However, in the low-SNR, INR regime we can achieve the following region by simply treating interference as noise:

$$\begin{aligned} R_1 &\leq \log \left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_1} \right) \\ R_2 &\leq \log \left(1 + \frac{\text{SNR}_2}{1 + \text{INR}_2} \right). \end{aligned} \quad (73)$$

Comparing this region with the simple point-to-point outer bound

$$\begin{aligned} R_1 &\leq \log(1 + \text{SNR}_1) \\ R_2 &\leq \log(1 + \text{SNR}_2), \end{aligned} \quad (74)$$

we can see that if (R_1, R_2) is on the boundary of the achievable region (73), then $(2R_1, 2R_2)$ is outside of the capacity region. We say that region (73) is *within half of the capacity region*. This is a complementary result to our one-bit result. Note that treating interference as noise is one special case of our Han-Kobayashi scheme. In fact, we have similar results for all parameter values for our scheme.

Theorem 7: The achievable region

$$\mathfrak{R}(\min(1, \text{INR}_2), \min(1, \text{INR}_1)) \quad (75)$$

is within half of the capacity region of the Gaussian weak interference channel.

Theorem 8: The achievable region

$$\mathfrak{R}(\min(1, \text{INR}_2), 0)$$

is within half of the capacity region of the Gaussian interference channel when $\text{INR}_1 \geq \text{SNR}_2$, $\text{INR}_2 < \text{SNR}_1$.

Proof: By comparing the achievable region with the corresponding outer bound, we can prove these results after some algebraic manipulation. The proofs of these two theorems are similar to the proofs of Theorems 5 and 6, and hence are omitted. \square

We conclude this section two additional remarks.

- 1) We can achieve fairly good performance by using a simple choice of the Han-Kobayashi scheme where INR_p is

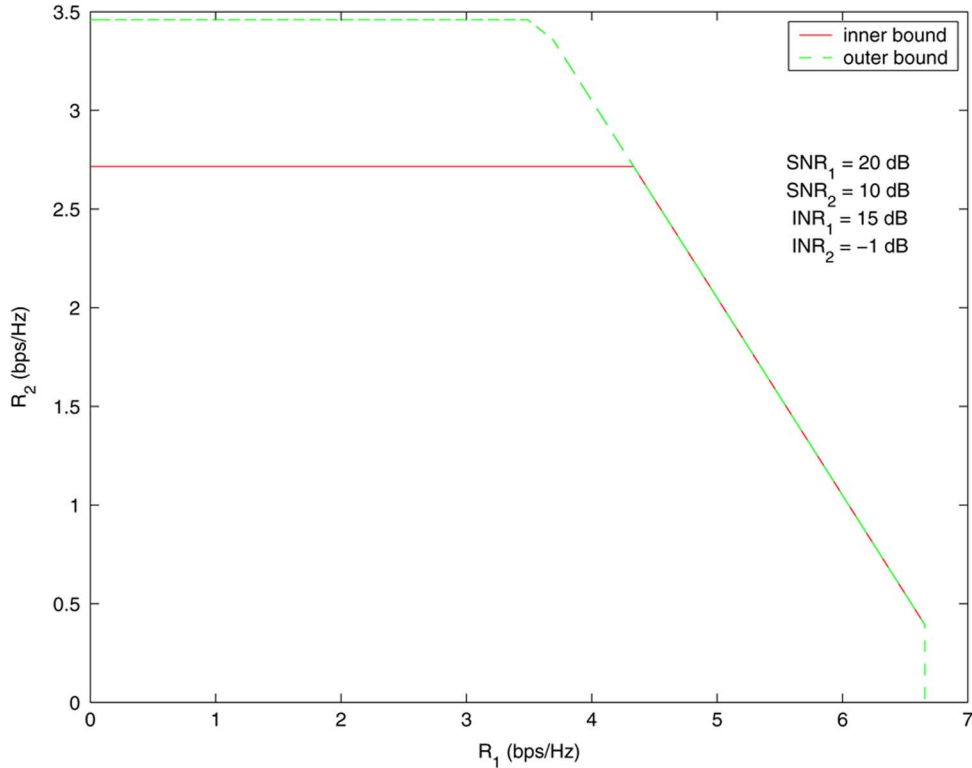


Fig. 21. Comparison of Han–Kobayashi achievable region $\mathfrak{R}(\text{INR}_2, 0)$ and outer bound (36).

chosen as close to 1 as possible. We will provide additional insights about why this choice is a natural one in Section VI.

- 2) We derived a new outer bound on the capacity region of the Gaussian interference channel. The one-bit result shows that this outer bound is quite good in the high-SNR, INR regime. The new bound is motivated by the results for a certain class of deterministic interference channels of [3]. We will investigate the connection between our results and those of [3] in Section VII.

V. GENERALIZED DEGREES OF FREEDOM REGION

At high SNR and INR, we can generalize the notion of degrees of freedom for the symmetric capacity of the symmetric Gaussian interference channel to the entire region for all values of parameters by focusing only on the first-order terms in $\log \text{SNR}_1$, $\log \text{SNR}_2$, $\log \text{INR}_1$, and $\log \text{INR}_2$. More precisely, we use approximations such as

$$\begin{aligned} \log(1 + \text{SNR}_1 + \text{INR}_1) &\approx \max(\log \text{SNR}_1, \log \text{INR}_1) \\ \log\left(1 + \frac{\text{SNR}_1}{1 + \text{INR}_1}\right) &\approx \left(\log \frac{\text{SNR}_1}{\text{INR}_1}\right)^+ \end{aligned} \quad (76)$$

to provide an expansion of the capacity region of the Gaussian interference channel which is accurate to first order. These first-order approximations satisfy the property that the higher order terms are $O(1)$. Therefore, the approximation error relative to $\log \text{SNR}_1$, etc., vanishes as $\text{SNR}_1 \rightarrow \infty$. This property will be useful in the derivation of the generalized degrees of freedom region to be considered next.

Let $\mathcal{C}(\text{SNR}_1, \text{SNR}_2, \text{INR}_1, \text{INR}_2)$ denote the capacity region of the interference channel with parameters $\text{SNR}_1, \text{SNR}_2,$

$\text{INR}_1, \text{INR}_2$. Let $\tilde{\mathcal{D}}$ be a scaled version of $\mathcal{C}(\text{SNR}_1, \text{SNR}_2, \text{INR}_1, \text{INR}_2)$ given by

$$\begin{aligned} \tilde{\mathcal{D}}(\text{SNR}_1, \text{SNR}_2, \text{INR}_1, \text{INR}_2) &= \left\{ \left(\frac{R_1}{\log \text{SNR}_1}, \frac{R_2}{\log \text{SNR}_2} \right) : (R_1, R_2) \right. \\ &\quad \left. \in \mathcal{C}(\text{SNR}_1, \text{SNR}_2, \text{INR}_1, \text{INR}_2) \right\} \end{aligned}$$

and let

$$\begin{aligned} \alpha_1 &= \frac{\log \text{SNR}_2}{\log \text{SNR}_1} \\ \alpha_2 &= \frac{\log \text{INR}_1}{\log \text{SNR}_1} \\ \alpha_3 &= \frac{\log \text{INR}_2}{\log \text{SNR}_1}. \end{aligned}$$

We define the generalized degrees of freedom region as

$$\mathcal{D}(\alpha_1, \alpha_2, \alpha_3) = \lim_{\substack{\text{SNR}_1, \text{SNR}_2, \text{INR}_1, \text{INR}_2 \rightarrow \infty \\ \alpha_1, \alpha_2, \alpha_3 \text{ fixed}}} \tilde{\mathcal{D}}(\text{SNR}_1, \text{SNR}_2, \text{INR}_1, \text{INR}_2).$$

With this definition, the capacity region can be approximately expressed as the set of rate pairs (R_1, R_2) such that

$$\begin{aligned} R_1 &= d_1 \log \text{SNR}_1 \\ R_2 &= d_2 \log \text{SNR}_2 \end{aligned}$$

for $(d_1, d_2) \in \mathcal{D}$.

The generalized degrees of freedom d_1, d_2 give a sense of how interference affects communication. In the absence of interference, each user can achieve a rate $R_i \approx \log \text{SNR}_i$. Due to interference, the single user capacity is scaled by a factor d_i .

In the following subsections we first compute the generalized degrees of freedom region \mathcal{D} for the interference channels, then present the generalized degrees of freedom region for two examples: the symmetric channel and the one-sided interference channel.

A. Generalized Degrees of Freedom Region of Interference Channel

1) *Weak Interference Channel:* For the weak interference channel, by applying approximations like the ones of (76) to the outer bound (36) and the achievable region $\mathfrak{R}(1, 1)$ (61), we can easily see that the first-order expansions of the corresponding bounds are equal, and the resulting first-order expansion of the capacity region has the following form:

$$\begin{aligned}
 R_1 &\leq \log \text{SNR}_1 \\
 R_2 &\leq \log \text{SNR}_2 \\
 R_1 + R_2 &\leq \log \text{SNR}_1 + \left(\log \left(\frac{\text{SNR}_2}{\text{INR}_2} \right) \right)^+ \\
 R_1 + R_2 &\leq \log \text{SNR}_2 + \left(\log \left(\frac{\text{SNR}_1}{\text{INR}_1} \right) \right)^+ \\
 R_1 + R_2 &\leq \max \left(\log \text{INR}_1, \log \left(\frac{\text{SNR}_1}{\text{INR}_2} \right) \right) \\
 &\quad + \max \left(\log \text{INR}_2, \log \left(\frac{\text{SNR}_2}{\text{INR}_1} \right) \right) \\
 2R_1 + R_2 &\leq \max(\log \text{SNR}_1, \log \text{INR}_1) \\
 &\quad + \max \left(\log \text{INR}_2, \log \frac{\text{SNR}_2}{\text{INR}_1} \right) + \log \frac{\text{SNR}_1}{\text{INR}_2} \\
 R_1 + 2R_2 &\leq \max(\log \text{SNR}_2, \log \text{INR}_2) \\
 &\quad + \max \left(\log \text{INR}_1, \log \frac{\text{SNR}_1}{\text{INR}_2} \right) + \log \frac{\text{SNR}_2}{\text{INR}_1}.
 \end{aligned} \tag{77}$$

From (77), we have that the generalized degrees of freedom region is given by

$$\begin{aligned}
 d_1 &\leq 1 \\
 d_2 &\leq 1 \\
 d_1 + \alpha_1 d_2 &\leq 1 + (\alpha_1 - \alpha_3)^+ \\
 d_1 + \alpha_1 d_2 &\leq \alpha_1 + (1 - \alpha_2)^+ \\
 d_1 + \alpha_1 d_2 &\leq \max(\alpha_2, 1 - \alpha_3) + \max(\alpha_3, \alpha_1 - \alpha_2) \\
 2d_1 + \alpha_1 d_2 &\leq \max(1, \alpha_2) + \max(\alpha_3, \alpha_1 - \alpha_2) + 1 - \alpha_3 \\
 d_1 + 2\alpha_1 d_2 &\leq \max(\alpha_1, \alpha_3) + \max(\alpha_2, 1 - \alpha_3) + \alpha_1 - \alpha_2.
 \end{aligned} \tag{78}$$

2) *Mixed Interference Channel:* For the mixed interference channel, by applying approximations like the ones of (76) to the outer bound (48) and the inner bound (70), we can easily see that the first-order expansions of the corresponding bounds are equal, and the resulting first-order expansion of the capacity region has the following form:

$$\begin{aligned}
 R_1 &\leq \log \text{SNR}_1 \\
 R_2 &\leq \log \text{SNR}_2 \\
 R_1 + R_2 &\leq \log \text{SNR}_1 + \left(\log \left(\frac{\text{SNR}_2}{\text{INR}_2} \right) \right)^+ \\
 R_1 + R_2 &\leq \max(\log \text{SNR}_1, \log \text{INR}_1)
 \end{aligned}$$

$$\begin{aligned}
 R_1 + R_2 &\leq \max \left(\log \text{INR}_1, \log \left(\frac{\text{SNR}_1}{\text{INR}_2} \right) \right) + \log \text{INR}_2 \\
 2R_1 + R_2 &\leq \max(\log \text{SNR}_1 + \log \text{INR}_1, 2 \log \text{SNR}_1) \\
 R_1 + 2R_2 &\leq \max(\log \text{SNR}_2, \log \text{INR}_2) \\
 &\quad + \max \left(\log \text{INR}_1, \log \frac{\text{SNR}_1}{\text{INR}_2} \right).
 \end{aligned} \tag{79}$$

Note that the third bound on $R_1 + R_2$ can be written as

$$\begin{aligned}
 R_1 + R_2 &\leq \max \left(\log \text{INR}_1, \log \left(\frac{\text{SNR}_1}{\text{INR}_2} \right) \right) + \log \text{INR}_2 \\
 &= \max(\log \text{INR}_1 + \log \text{INR}_2, \log \text{SNR}_1)
 \end{aligned} \tag{80}$$

which is larger than the second bound on $R_1 + R_2$. Hence, the third bound on $R_1 + R_2$ is redundant. The bound on $2R_1 + R_2$ is the same as the sum of the bound on R_1 and the second bound on $R_1 + R_2$, so it is also redundant. So we end up with a first-order expansion of the capacity region of the form

$$\begin{aligned}
 R_1 &\leq \log \text{SNR}_1 \\
 R_2 &\leq \log \text{SNR}_2 \\
 R_1 + R_2 &\leq \log \text{SNR}_1 + \left(\log \left(\frac{\text{SNR}_2}{\text{INR}_2} \right) \right)^+ \\
 R_1 + R_2 &\leq \max(\log \text{SNR}_1, \log \text{INR}_1) \\
 R_1 + 2R_2 &\leq \max(\log \text{SNR}_2, \log \text{INR}_2) \\
 &\quad + \max \left(\log \text{INR}_1, \log \left(\frac{\text{SNR}_1}{\text{INR}_2} \right) \right).
 \end{aligned} \tag{81}$$

Using (81) we obtain the generalized degrees of freedom region for the mixed interference channel

$$\begin{aligned}
 d_1 &\leq 1 \\
 d_2 &\leq 1 \\
 d_1 + \alpha_1 d_2 &\leq 1 + (\alpha_1 - \alpha_3)^+ \\
 d_1 + \alpha_1 d_2 &\leq \max(1, \alpha_2) \\
 d_1 + 2\alpha_1 d_2 &\leq \max(\alpha_1, \alpha_3) + \max(\alpha_2, 1 - \alpha_3).
 \end{aligned} \tag{82}$$

3) *Strong Interference Channel:* The capacity region of the strong interference channel is shown to be the intersection of that of two MACs, and is given by

$$\begin{aligned}
 R_1 &\leq \log(1 + \text{SNR}_1) \\
 R_2 &\leq \log(1 + \text{SNR}_2) \\
 R_1 + R_2 &\leq \log(1 + \text{SNR}_1 + \text{INR}_1) \\
 R_1 + R_2 &\leq \log(1 + \text{SNR}_2 + \text{INR}_2).
 \end{aligned} \tag{83}$$

By applying approximations like the ones of (76), we obtain the generalized degrees of freedom region for the strong interference channel

$$\begin{aligned}
 d_1 &\leq 1 \\
 d_2 &\leq 1 \\
 d_1 + \alpha_1 d_2 &\leq \max(1, \alpha_2) \\
 d_1 + \alpha_1 d_2 &\leq \max(\alpha_1, \alpha_3).
 \end{aligned} \tag{84}$$

B. Example 1: The Symmetric Channel

For the symmetric Gaussian interference the channel with $\text{SNR}_1 = \text{SNR}_2 = \text{SNR}$, $\text{INR}_1 = \text{INR}_2 = \text{INR}$, and $\text{SNR} \gg \text{INR}$,

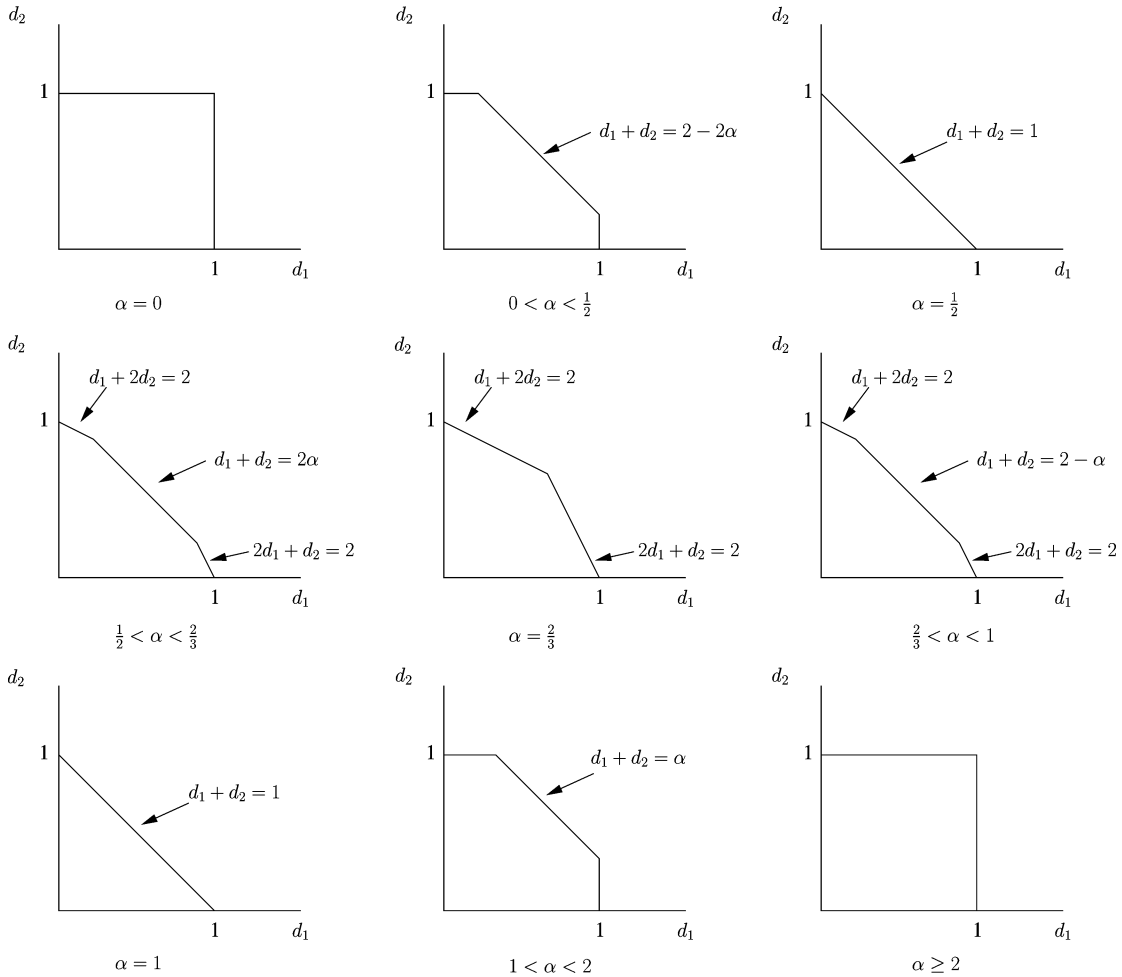


Fig. 22. Generalized degrees of freedom region for the symmetric Gaussian interference channel.

we have $\alpha_1 = 1$ and $\alpha_2 = \alpha_3 = \alpha$. In this case, for weak interference channel in which $0 < \alpha < 1$, we have the following generalized degrees of freedom region:

$$\begin{aligned}
 d_1 &\leq 1 \\
 d_2 &\leq 1 \\
 d_1 + d_2 &\leq \min\{2 - \alpha, 2 \max(\alpha, 1 - \alpha)\} \\
 2d_1 + d_2 &\leq 2 - \alpha + \max(\alpha, 1 - \alpha) \\
 d_1 + 2d_2 &\leq 2 - \alpha + \max(\alpha, 1 - \alpha).
 \end{aligned} \tag{85}$$

For a strong interference channel in which $\alpha \geq 1$, we have the following generalized degrees of freedom region:

$$\begin{aligned}
 d_1 &\leq 1 \\
 d_2 &\leq 1 \\
 d_1 + d_2 &\leq \alpha.
 \end{aligned} \tag{86}$$

The generalized degrees of freedom region of the symmetric channel is plotted in Fig. 22. Note that the diagram for $\alpha \geq 2$ corresponds to the very strong interference case, in which interference does not reduce the available degrees of freedom of the channel. We see that the degrees of freedom region is *not* monotonically decreasing with INR in the weak interference regime. As in the case of the symmetric rate, discussed

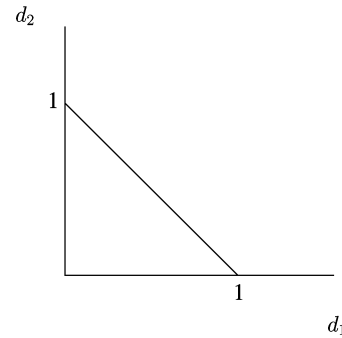


Fig. 23. Generalized degrees of freedom region for the symmetric Gaussian weak interference channel using orthogonalizing scheme.

in Section III, there are three regimes in which the degrees of freedom region shows different qualitative behaviors, namely, $0 \leq \alpha < 1/2$, $1/2 \leq \alpha < 2/3$, and $2/3 \leq \alpha < 1$.

If we use an orthogonalizing strategy, the generalized degrees of freedom region that we can achieve is shown in Fig. 23. If we treat interference as noise, the generalized degrees of freedom region that we can achieve is shown in Fig. 24. So an orthogonalizing strategy is strictly suboptimal except when $\alpha = 1/2$ and $\alpha = 1$, and treating interference as noise is strictly suboptimal except for $\alpha \leq 1/2$, as was already shown in Section III-F for the symmetric rate.

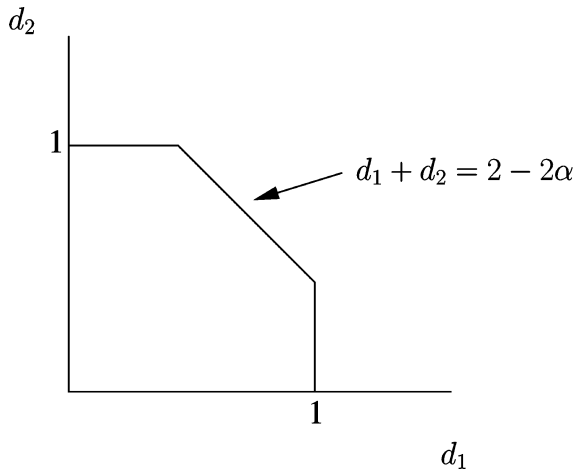


Fig. 24. Generalized degrees of freedom region for the symmetric Gaussian weak interference channel when treating interference as noise.

C. Example 2: The One-Sided Interference Channel

For the one-sided interference channel shown in Fig. 6, we have $\text{INR}_1 = 0$. In the weak interference case ($\text{SNR}_1 > \text{INR}_2$), by applying approximations like the ones of (76) to the achievable region $\mathfrak{R}(1, \text{INR}_1)$ given in (66), we get the following first-order expansion of the achievable region:

$$\begin{aligned} R_1 &\leq \log(\text{SNR}_1) \\ R_2 &\leq \log(\text{SNR}_2) \\ R_1 + R_2 &\leq \log(\text{SNR}_1) + \left(\log\left(\frac{\text{SNR}_2}{\text{INR}_2}\right) \right)^+. \end{aligned} \quad (87)$$

By examining the proof of the outer bound (36) we can easily see that the previous first-order expansion of the achievable region is actually tight. Thus, we get the following generalized degrees of freedom region for the one-sided interference channel:

$$\begin{aligned} d_1 &\leq 1 \\ d_2 &\leq 1 \\ d_1 + \alpha_1 d_2 &\leq \max(1, 1 + \alpha_1 - \alpha_3). \end{aligned} \quad (88)$$

In Fig. 25 we plot the generalized degrees of freedom of the weak one-sided interference channel. There are two different cases.

1) $\alpha_1 > \alpha_3$

In this case, the generalized degrees of freedom region is

$$\begin{aligned} d_1 &\leq 1 \\ d_2 &\leq 1 \\ d_1 + \alpha_1 d_2 &\leq 1 + \alpha_1 - \alpha_3. \end{aligned} \quad (89)$$

The entire region can be achieved by adjusting the power of the long-range link and treating its interference as noise. In particular, to achieve the corner point $A = (1, 1 - \frac{\alpha_3}{\alpha_1})$, user 1 transmits at full power and user two treats user 1's interference as noise. To achieve the corner point $B = (1 - \alpha_3, 1)$, user 1's transmitting power needs to be reduced so that the received signal to noise ratio at receiver 1 is $\frac{\text{SNR}_1}{\text{INR}_2}$, and user 2 treats user 1's interference as noise.

2) $\alpha_1 < \alpha_3$

In this case, the generalized degrees of freedom region is

$$\begin{aligned} d_1 &\leq 1 \\ d_2 &\leq 1 \\ d_1 + \alpha_1 d_2 &\leq 1. \end{aligned} \quad (90)$$

The corner point $A = (1 - \alpha_1, 1)$ has to be achieved by a private common split in user 1's message. Treating interference as noise will only get to the point $B = (1 - \alpha_3, 1)$ which is strictly smaller.

We also draw the performance of the orthogonalizing scheme in Fig. 25, which is suboptimal in both cases.

In the strong interference case ($\text{SNR}_1 \leq \text{INR}_2$), the capacity of the one-sided interference channel is known. For completeness, we present the corresponding generalized degrees of freedom region in the following:

$$\begin{aligned} d_1 &\leq 1 \\ d_2 &\leq 1 \\ d_1 + \alpha_1 d_2 &\leq \max(\alpha_1, \alpha_3). \end{aligned} \quad (91)$$

VI. PRIVATE VERSUS COMMON INFORMATION

In Section IV, we have shown that the simple Han-Kobayashi scheme that sets the private message power so that $\text{INR}_{p1} = 1$ and $\text{INR}_{p2} = 1$ achieves to within 1 bit/s/Hz of the capacity region. We also argued that setting $\text{INR}_{p1} = 1$ and $\text{INR}_{p2} = 1$ achieves a good tradeoff between obtaining a good direct link rate and not causing excessive interference to the other link. In this section, we will provide an alternative analysis that justifies the choice $\text{INR}_{p1} = 1$ and $\text{INR}_{p2} = 1$. In brief, we will argue that the information received at the nonintending receiver that is above the noise level should essentially be decodable, and hence can be thought of as common information.

Consider a communication scheme that splits the message to be sent into many submessages of small rate and power. The transmitted message is the superposition of these submessages, and has total power P . Receiver 2 is able to decode the message transmitted from its own transmitter, and subtract it from the received signal y_2 , obtaining the signal \tilde{y}_2 . We further make the optimistic assumption that receiver 1 can also decode and subtract the interference received from transmitter 2, obtaining a signal \tilde{y}_1 . This interference cancellation may not always be possible, and therefore we will obtain an upper bound on the rate of user 1. The resulting channels are

$$\begin{aligned} \tilde{y}_1 &= \sqrt{\text{SNR}_1} \tilde{x}_1 + z_1 \\ \tilde{y}_2 &= \sqrt{\text{INR}_2} \tilde{x}_1 + z_2 \end{aligned} \quad (92)$$

where $z_1, z_2 \sim \mathcal{CN}(0, 1)$ and we normalized \tilde{x}_1 so that $\mathbb{E}[|\tilde{x}_1|^2] \leq 1$.

We define the differential rates

$$\begin{aligned} r_1(z) &= \frac{\text{SNR}_1}{1 + \text{SNR}_1 z} \\ r_2(z) &= \frac{\text{INR}_2}{1 + \text{INR}_2 z} \end{aligned} \quad (93)$$

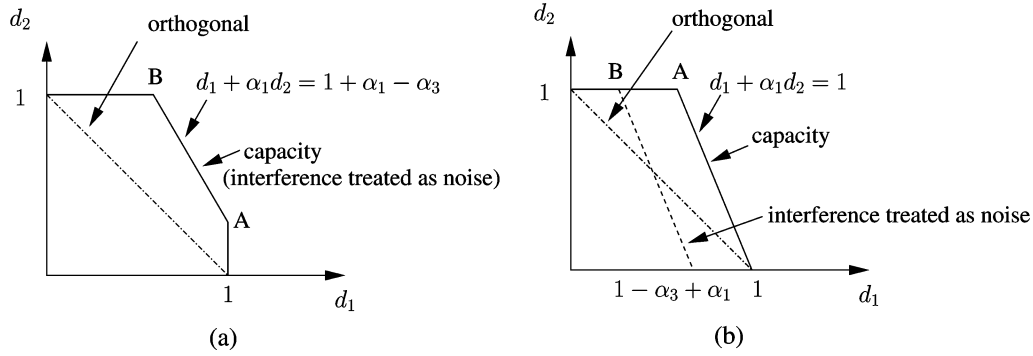


Fig. 25. Generalized degrees of freedom region of the weak one-sided interference channel for treating interference as noise, orthogonalizing, and optimal power splitting.

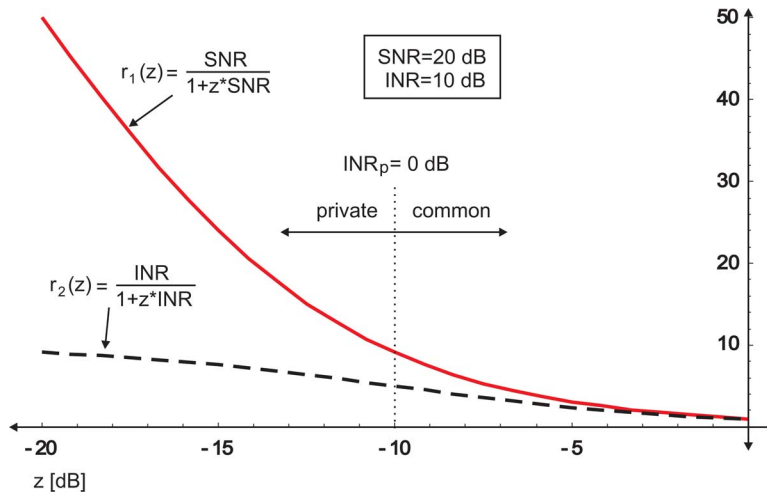


Fig. 26. Differential rates $r_1(z)$ and $r_2(z)$ for a symmetric channel where $\text{SNR} = 20 \text{ dB}$ and $\text{INR} = 10 \text{ dB}$.

which can be interpreted in the following way: $r_1(z) \cdot dz$ ($r_2(z) \cdot dz$) is the rate that can be achieved in a submessage of power dz facing an interference of power $z \cdot \text{SNR}_1$ ($z \cdot \text{INR}_2$) in channel 1 (2). (See [12, p. 2802], where this concept is introduced). These functions can also be interpreted as the marginal increase in rate at interference level $z \cdot \text{SNR}_1$ (or $z \cdot \text{INR}_2$).

Imagine we plot these two functions and let us see what happens as z goes from 0 to 1. For $\text{INR}_2 \cdot z \ll 1$, $r_1(z) \gg r_2(z)$. The marginal increase in rate in the direct link is much larger than in the indirect link, and therefore the other receiver has no hope of decoding this submessage if information is sent at this rate. Thus, at this signal level, information should be private, only decodable by receiver 1. When $\text{INR}_2 \cdot z \gg 1$, $r_2(z) \approx r_1(z) \approx 1/z$ and any information sent in the direct link at this rate can also be decoded by the other receiver. At this signal level, we should therefore be sending common information. Fig. 26 shows how the differential rates $r_1(z)$ and $r_2(z)$ vary as a function of z for a symmetric channel where $\text{SNR}_1 = \text{SNR}_2 = 20 \text{ dB}$ and $\text{INR}_1 = \text{INR}_2 = 10 \text{ dB}$. We see that when $z \approx -10 \text{ dB}$, which corresponds to $\text{INR}_p = 0 \text{ dB}$, the differential rates are approximately equal.

The above argument shows that the submessages that are decoded first and that face an interference level $z \cdot \text{INR}_2 > 1$ at

receiver 2 can be decoded by both receivers, and are therefore common information.

We will now analyze in what situations there is a gain in sending common information. Suppose we start with a nominal strategy of sending all private information at full power on both links. Each receiver treats the interference as noise.

How can we improve this strategy? From the preceding discussion, without loss of optimality we can convert the part of the signals in both links above the other receiver’s noise level into common information. We focus on the part of user 1’s signal above receiver 2’s noise level, which we call c_1 . Assume that this signal is getting a rate R in the nominal strategy of treating interference as noise. This can be viewed as common information, decoded in the following ways by the two receivers: receiver 1 decodes c_1 first, treating the component sent by transmitter 1 received at receiver 2 below noise level plus the interference from transmitter 2 as noise. From receiver 2’s point of view, c_1 acts as interference to its own signal and therefore we can view receiver 2 as decoding c_1 last, after decoding its own information. Note that since the decoding order is different at the two receivers, only receiver 1 is limiting the rate of c_1 . In receiver 2, there is still slack: even if we increased the rate of c_1 beyond R , receiver 2 would still be able to decode. By the same

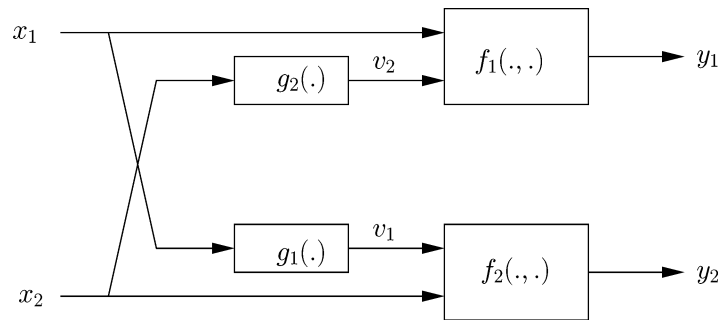


Fig. 27. Deterministic interference channel.

logic, the part of the signal above noise level sent from transmitter 2 (call it c_2) also has slack at receiver 1.

This suggests that we can improve the performance by changing the decoding order of c_1 in receiver 1. If we decode an ϵ part of c_1 after decoding an ϵ part of c_2 (ϵ swap of ordering in receiver 1), then the rate assigned to c_1 can be improved from R to $R + \delta$. Note that c_1 can still be decoded by receiver 2, since there was slack in the first place. Also, the mutual information achieved for c_2 at receiver 1 has decreased because its ordering is slightly less favorable, but because there was slack for c_2 at receiver 1, c_2 can still be decoded.

Thus, we have improved the rate of user 1 while keeping the rate of user 2 invariant. Therein lies the power of viewing the signal above noise level as common information: there is flexibility in changing the decoding order. When viewed as private information, the decoding order is fixed and there is slack in one of the two receivers that cannot be exploited. That is in essence the “structure” in the interfering signals that is not exploited in treating interference as noise. By changing the cancellation ordering, we are reducing the slack in one of the receivers of the common information.

VII. CONNECTION TO A DETERMINISTIC INTERFERENCE CHANNEL

In Section VI, we argued that the portion of the received interfering signal above the noise level should be common information and that hidden below the noise level should be private. In other words, the part of the received interfering signal that is most visible to the other link is made common while the rest is made private. This argument is only approximate, as the part of the interfering signal below the noise level still has some visibility to the other link. Therefore, the proposed strategy still has up to one-bit gap to capacity.

There is in fact a channel in which part of the interfering signal is *completely* invisible to the other link. This channel, introduced by El Gamal and Costa in [3], is a special type of a deterministic interference channel. Because of the complete invisibility of part of the signal, they can show that a Han–Kobayashi strategy of assigning common information to the visible part and private information to the invisible part is *exactly* optimal. Our approach to the Gaussian interference channel is in fact based on drawing analogies to this deterministic channel.

The deterministic interference channel of [3] is shown in Fig. 27. In this channel, x_1 and x_2 are the inputs, $v_1 = g_1(x_1)$

is the interference caused by x_1 at receiver 2, and $v_2 = g_2(x_2)$ is the interference caused by x_2 at receiver 1. y_1 and y_2 are the outputs, and they are deterministic functions of (x_1, v_2) and (x_2, v_1) , respectively

$$\begin{aligned} y_1 &= f_1(x_1, v_2) \\ y_2 &= f_2(x_2, v_1). \end{aligned} \quad (94)$$

In addition, there is an important assumption about the interference signals v_1 and v_2 given by the following conditions:

$$\begin{aligned} H(y_1|x_1) &= H(v_2) \\ H(y_2|x_2) &= H(v_1). \end{aligned} \quad (95)$$

The previous conditions are equivalent to the existence of functions $h_1(\cdot, \cdot)$ and $h_2(\cdot, \cdot)$ such that

$$\begin{aligned} v_1 &= h_2(x_2, y_2) \\ v_2 &= h_1(x_1, y_1). \end{aligned} \quad (96)$$

These conditions guarantee that each receiver can observe a clean version of the interfering signal after decoding its own message. This assumption is key for deriving the capacity region of the channel. One can argue that regardless of the communication strategy, the signals v_1 and v_2 are common information, since they can be cleanly observed after decoding the own message. In addition, due to the functions $g_1(\cdot)$ and $g_2(\cdot)$ part of the transmitted message is completely invisible to the nonintending receiver. This part of the message becomes private information.

In the deterministic channel, conditions (95) seem artificial, but in the Gaussian channel analogous conditions arise more naturally. For the Gaussian interference channel, receiver 2 can observe $s_1 = h_{12}x_1 + z_2$ after decoding x_2 . Similarly, we can define $s_2 = h_{21}x_2 + z_1$, which is the signal that receiver 1 can observe after decoding the message x_1 . As we argued in the case of the deterministic channel, the signals s_1 and s_2 can be thought of as the common information that can be observed after decoding the own message. The role of z_1 (z_2) in s_2 (s_1) can be compared to the role of the function $g_2(\cdot)$ ($g_1(\cdot)$) in the deterministic channel, that is, hiding the private information to the nonintending receiver.

The outer bounds derived in [3] to establish the capacity region of the deterministic channel can be interpreted in terms of genie-aided channels, with various combinations of x_1, x_2, v_1, v_2 given to the receivers. Analogous genie-aided channels, with appropriate modifications of the side information, were used in Sec-

tion IV to derive the outer bounds for the Gaussian interference channel with weak and mixed interference.

The connection between Gaussian and deterministic channel models is strengthened in several subsequent works since the present work was first done. In [13], a class of noisy interference channels is considered, which subsumes the Gaussian interference channel and the El Gamal and Costa deterministic interference channel as special cases. In [14], a deterministic channel model is proposed to approximate general Gaussian networks. When specialized to the two-user interference channel, this model is a special case of the El Gamal and Costa model and it is shown in [15] that in fact the capacity region of this channel matches *exactly* the generalized degrees of freedom region of the Gaussian interference channel we derived here.

APPENDIX A
ANALYSIS OF UPPER BOUND OF [6, THEOREM 2]

In this appendix, we will show that the upper bound of [6, Theorem 2] achieves a worst case gap of 1 bits/s/Hz in the parameter range \mathcal{B}_1 with respect to the symmetric rate of our simple Han–Kobayashi scheme of Section III-B. In addition, the gap between this bound and the Han–Kobayashi scheme can be

unbounded in the parameter range \mathcal{B}_2 . Therefore, the bound of [6, Theorem 2] does not give better performance than the bound (9) in terms of characterizing the symmetric capacity of the symmetric channel to within 1 bit/s/Hz.

The results of [6] are derived for the normalized interference channel, i.e., $|h_d|^2 = 1, N_0 = 1$, so in order to use the results, we need to replace: $|h_c|^2 \rightarrow |h_c|^2/|h_d|^2, P \rightarrow |h_d|^2 P/N_0$. Specializing [6, Theorem 2] to the complex symmetric interference channel with $0 < |h_c|^2/|h_d|^2 < 1$ ($0 < \text{INR} < \text{SNR}$) one obtains the symmetric rate upper bound given in (97) at the bottom of the page.

Consider the case in which the first term of the $\min\{\cdot, \cdot\}$ of (4) is active, that is, the parameter range \mathcal{B}_1 . Then we can write (98), also at the bottom of the page, where we used $\sqrt{1+x} \geq 1$ for $x \geq 0$ in the first inequality and the assumption $\text{INR} \leq \text{SNR}$ in the second inequality. It is easy to check that, for example, if $\text{INR} = \text{SNR}^{3/4}, R_{\text{UB}_1} - R_{\text{HK}_1} \rightarrow 1$ as $\text{SNR} \rightarrow \infty$. Therefore, the worst case difference of 1 bit/s/Hz in (98) can actually occur.

We see that in the parameter range \mathcal{B}_1 , the achievable strategy and the upper bound differ for at most 1 bit/s/Hz. Therefore, in this parameter range our simple scheme gives a bounded (and small) gap with respect to the upper bound.

$$\begin{aligned}
 R_{\text{UB}_K} &= \log \left[1 + \frac{-(|h_c|^2 + |h_d|^2) + \sqrt{(|h_c|^2 + |h_d|^2)^2 + 4|h_c|^2|h_d|^2(P/N_0)(|h_c|^2 + |h_d|^2)}}{2|h_c|^2} \right] \\
 &= \log \left[2 - \left(1 + \frac{\text{SNR}}{\text{INR}} \right) + \sqrt{\left(1 + \frac{\text{SNR}}{\text{INR}} \right)^2 + 4 \cdot \text{SNR} \left(1 + \frac{\text{SNR}}{\text{INR}} \right)} \right] - 1
 \end{aligned} \tag{97}$$

$$\begin{aligned}
 R_{\text{UB}_K} - R_{\text{HK}_1} &= \log \left[2 - \left(1 + \frac{\text{SNR}}{\text{INR}} \right) + \sqrt{\left(1 + \frac{\text{SNR}}{\text{INR}} \right)^2 + 4 \cdot \text{SNR} \left(1 + \frac{\text{SNR}}{\text{INR}} \right)} \right] - 1 \\
 &\quad - \frac{1}{2} \log(1 + \text{SNR} + \text{INR}) - \frac{1}{2} \log \left(2 + \frac{\text{SNR}}{\text{INR}} \right) + 1 \\
 &= \frac{1}{2} \log \left\{ \left[1 - \frac{\text{SNR}}{\text{INR}} + \sqrt{\left(1 + \frac{\text{SNR}}{\text{INR}} \right)^2 + 4 \cdot \text{SNR} \left(1 + \frac{\text{SNR}}{\text{INR}} \right)} \right]^2 \right\} \\
 &\quad - \frac{1}{2} \log(1 + \text{SNR} + \text{INR}) - \frac{1}{2} \log \left(2 + \frac{\text{SNR}}{\text{INR}} \right) \\
 &\leq \frac{1}{2} \log \left\{ 2 \left[1 + \left(\frac{\text{SNR}}{\text{INR}} \right)^2 \right] + 4 \cdot \text{SNR} \left(1 + \frac{\text{SNR}}{\text{INR}} \right) + 2 \left[1 - \left(\frac{\text{SNR}}{\text{INR}} \right)^2 \right] \right\} \\
 &\quad - \frac{1}{2} \log \left[(1 + \text{SNR} + \text{INR}) \cdot \left(2 + \frac{\text{SNR}}{\text{INR}} \right) \right] \\
 &= \frac{1}{2} \log \left\{ \frac{4}{2 + (\text{SNR}/\text{INR})} \cdot \frac{1 + \text{SNR} \cdot (1 + \text{SNR}/\text{INR})}{1 + \text{SNR} + \text{INR}} \right\} \\
 &\leq \frac{1}{2} \log \left[\frac{4}{2 + (\text{SNR}/\text{INR})} \cdot \frac{\text{SNR}}{\text{INR}} \right] \\
 &= \frac{1}{2} \log \left[\frac{4}{2(\text{INR}/\text{SNR}) + 1} \right] \\
 &< 1
 \end{aligned} \tag{98}$$

$$\begin{aligned}
R_{UB_K} - R_{HK_2} &= \log \left[1 - \frac{\text{SNR}}{\text{INR}} + \sqrt{\left(1 + \frac{\text{SNR}}{\text{INR}}\right)^2 + 4 \cdot \text{SNR} \left(1 + \frac{\text{SNR}}{\text{INR}}\right)} \right] - \log \left(1 + \text{INR} + \frac{\text{SNR}}{\text{INR}} \right) \\
&\geq \log \left[1 - \frac{\text{SNR}}{\text{INR}} + 2\sqrt{\text{SNR} \left(1 + \frac{\text{SNR}}{\text{INR}}\right)} \right] - \log \left(1 + \text{INR} + \frac{\text{SNR}}{\text{INR}} \right)
\end{aligned}$$

We will now show that in the parameter range \mathcal{B}_2 , the gap between our scheme and the upper bound (97) can be arbitrarily large. In this parameter range we can write $R_{UB_K} - R_{HK_2}$ as shown in the expression at the top of the page, where the inequality follows from discarding $(1 + \text{SNR}/\text{INR})^2$ in the square root. To show that this difference can be unbounded, take $\text{INR} = \sqrt{\text{SNR}}$. With this choice of parameters we get

$$R_{UB_K} - R_{HK_2} \geq \log \left[\frac{1 - \sqrt{\text{SNR}} + 2\sqrt{\text{SNR}(1 + \sqrt{\text{SNR}})}}{1 + 2\sqrt{\text{SNR}}} \right]$$

where the right-hand side goes to infinity for $\text{SNR} \rightarrow \infty$.

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