A Factor-Graph Approach to the Context-Tree Weighting Method

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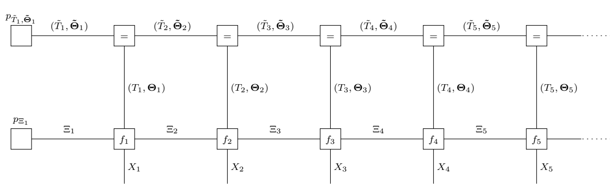
Factor graphs [1, 2] (FGs) are graphical models with origins in coding theory. The sum-product and the max-product algorithms (SPA/MPA), which operate by message passing in an FG, subsume a great variety of algorithms in coding, signal processing, and artificial intelligence. E.g., FGs together with the SPA/MPA have been used successfully to formulate the decoding of capacity-approaching codes like turbo and LDPC codes. This paper aims at showing that it is possible to give an FG/SPA interpretation of one of the best data compression algorithms, namely the context-tree weighting (CTW) method by Willems, Shtarkov, and Tjalkens [3, 4].

An arithmetic encoder/decoder needs the conditional probabilities of the next symbol based on the past. These probabilities can be obtained by performing the SPA on the FG (see figure) which represents the joint pmf/pdf of all occuring random variables. (This FG is actually a normal graph as defined in [2].) Note that in order to obtain SPA update rules that can be computed efficiently, the use of a special weighting of all context trees as proposed in [3] is absolutely crucial.

Once the CTW algorithm is formulated in the FG/SPA-framework, new extensions of the algorithm become readily apparent, e.g. compression of two-dimensional data or embedding the data compression algorithm within a larger system. More information can be found under [5].



References

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[4] http://www.data-compression.info/Algorithms/CTW/.

[5] http://www.ifp.uiuc.edu/~vontobel.

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