Channel Coding LP Decoding and Compressed Sensing LP Decoding: Further Connections*

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Abstract—Channel coding linear programming decoding (CC-LPD) and compressed sensing linear programming decoding (CS-LPD) are two setups that are *formally* tightly related. Recently, a connection between CC-LPD and CS-LPD was exhibited that goes beyond this formal relationship. The main ingredient was a lemma that allowed one to map vectors in the nullspace of some zero-one measurement matrix into vectors of the fundamental cone defined by that matrix.

The aim of the present paper is to extend this connection along several directions. In particular, the above-mentioned lemma is extended from real measurement matrices where every entry is equal to either zero or one to complex measurement matrices where the absolute value of every entry is a non-negative integer. Moreover, this lemma and its generalizations are used to translate performance guarantees from CC-LPD to CS-LPD.

In addition, the present paper extends the formal relationship between CC-LPD and CS-LPD with the help of graph covers. First, this graph-cover viewpoint is used to obtain new connections between, on the one hand, CC-LPD for binary parity-check matrices, and, on the other hand, CS-LPD for complex measurement matrices. Secondly, this graph-cover viewpoint is used to see CS-LPD not only as a well-known relaxation of some zero-norm minimization problem but (at least in the case of real measurement matrices with only zeros, ones, and minus ones) also as a relaxation of a problem we call the zero-infinity operator minimization problem.

I. Introduction

This paper is a direct extension of a line of work that was started in [1] and that connects channel coding linear programming decoding [2], [3] and compressed sensing linear programming decoding [4]. Because the motivation and the aim for the results presented here are very much the same as they were in [1], we refer to that paper for an introduction. We remind the reader that **CC-MLD**, **CC-LPD**, **CS-OPT**, and **CS-LPD** stand for "channel coding maximum likelihood decoding," "channel coding linear programming decoding," "compressed sensing (sparsity) optimal decoding," and "compressed sensing linear programming decoding," respectively. Moreover, all vectors are column vectors.

The present paper is structured as follows. Section II presents three generalizations of [1, Lemma 11], which was the key result in [1]. First, this lemma is generalized from real

measurement matrices where every entry is equal to either zero or one to complex measurement matrices where the absolute value of every entry is equal to either zero or one. In that process we also generalize the mapping that is applied to the vectors in the nullspace of the measurement matrix. Secondly, this lemma is generalized to hold also for complex measurement matrices where the absolute value of every entry is a non-negative integer. Finally, the third generalization of this lemma extends the types of mappings that can be applied to the vectors in the nullspace of the measurement matrix. With this, Section III translates performance guarantees from **CC-LPD** to **CS-LPD**. Afterwards, Section IV tightens the already close formal relationship between **CC-LPD** and **CS-LPD** with the help of graph covers, a line of results that is continued in Section V, which presents **CS-LPD** for certain measurement matrices not only as the well-known relaxation of some zero-norm minimization problem but also as the relaxation of some other minimization problem. Finally, some conclusions are presented in Section VI.

Besides the notation defined in [1], we will also use the following conventions and extensions of notions previously introduced. For any $M \in \mathbb{Z}_{>0}$, we let $[M] \triangleq \{1, \ldots, M\}$. We remind the reader that in [1] we extended the use of the absolute value operator $|\cdot|$ from scalars to vectors. Namely, if $\boldsymbol{a} = (a_i)_i$ is a complex vector then we define $|\boldsymbol{a}|$ to be the complex vector $\boldsymbol{a}' = (a_i')_i$ of the same length as $\boldsymbol{a}$ with entries $a_i' = |a_i|$ for all i. Similarly, in this paper we extend the use of the absolute value operator $|\cdot|$ from scalars to matrices.

We let $|\cdot|_*$ be an arbitrary norm for the complex numbers. As such, $|\cdot|_*$ satisfies for any $a, b, c \in \mathbb{C}$ the triangle inequality $|a + b|_* \leqslant |a|_* + |b|_*$ and the equality $|c \cdot a|_* = |c| \cdot |a|_*$. In the same way the absolute value operator $|\cdot|$ was extended from scalars to vectors and matrices, we extend the norm operator $|\cdot|_*$ from scalars to vectors and matrices.

We let $\|\cdot\|_*$ be an arbitrary vector norm for complex vectors that reduces to $|\cdot|_*$ for vectors of length one. As such, $\|\cdot\|_*$ satisfies for any $c \in \mathbb{C}$ and any complex vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ of equal length the triangle inequality $\|\boldsymbol{a} + \boldsymbol{b}\|_* \leqslant \|\boldsymbol{a}\|_* + \|\boldsymbol{b}\|_*$ and the equality $\|c \cdot \boldsymbol{a}\|_* = |c| \cdot \|\boldsymbol{a}\|_*$.

For any complex vector $\boldsymbol{a}$ we define the zero-infinity operator to be $\|\boldsymbol{a}\|_{0,\infty} \triangleq \|\boldsymbol{a}\|_0 \cdot \|\boldsymbol{a}\|_\infty$, i.e., the product of the zero norm $\|\boldsymbol{a}\|_0 = \# \operatorname{supp}(\boldsymbol{a})$ of $\boldsymbol{a}$ and of the infinity

norm $\|\boldsymbol{a}\|_\infty = \max_i |a_i|$ of $\boldsymbol{a}$. Note that for any $c \in \mathbb{C}$ and any complex vector $\boldsymbol{a}$ it holds that $\|c \cdot \boldsymbol{a}\|_{0,\infty} = |c| \cdot \|\boldsymbol{a}\|_{0,\infty}$.

Finally, for any $n, M \in \mathbb{Z}_{>0}$ and any length-n vector $\boldsymbol{a}$ we define the M-fold lifting of $\boldsymbol{a}$ to be the vector $\boldsymbol{a}^{\uparrow M} = (a_{(i,m)}^{\uparrow M})_{(i,m)} \in \mathbb{C}^{Mn}$ with components given by

$$a_{(i,m)}^{\uparrow M} \triangleq a_i, \quad (i,m) \in [n] \times [M].$$

Moreover, for any vector $\tilde{\boldsymbol{a}} = (\tilde{a}_{(i,m)})_{(i,m)}$ of length $M \cdot n$ over $\mathbb{C}$ or $\mathbb{F}_2$ we define the projection of $\tilde{\boldsymbol{a}}$ to the space $\mathbb{C}^n$ to be the vector $\boldsymbol{a} \triangleq \boldsymbol{\varphi}_M(\tilde{\boldsymbol{a}})$ with components given by

$$a_i \triangleq \frac{1}{M} \sum_{m \in [M]} \tilde{a}_{(i,m)}, \quad i \in [n].$$

(In the case where $\tilde{\boldsymbol{a}}$ is over $\mathbb{F}_2$, the summation is over $\mathbb{C}$ and we use the the standard embedding of $\{0,1\}$ into $\mathbb{C}$.)

II. BEYOND MEASUREMENT MATRICES WITH ZEROS AND ONES

The aim of this section is to extend [1, Lemma 11], which is a reformulation of [5, Lemma 6], to matrices beyond zero-one measurement matrices. In that vein we will present three generalizations in Lemmas 2, 5, and 6. For ease of reference, let us restate [1, Lemma 11].

Lemma 1 ([1, Lemma 11]) *Let $\boldsymbol{H}_{\mathrm{CS}}$ be a measurement matrix that contains only zeros and ones. Then*

$$\boldsymbol{\nu} \in \mathrm{nullspace}_{\mathbb{R}}(\boldsymbol{H}_{\mathrm{CS}}) \quad \Rightarrow \quad |\boldsymbol{\nu}| \in \mathcal{K}(\boldsymbol{H}_{\mathrm{CS}}).$$

∎

Because in the proofs of the upcoming lemmas we will have to show that certain vectors lie in the fundamental cone $\mathcal{K} \triangleq \mathcal{K}(\boldsymbol{H}_{\mathrm{CC}})$ [2], [3], [6], [7] of the parity-check matrix $\boldsymbol{H}_{\mathrm{CC}}$ of some binary linear code, for convenience let us list here a set of inequalities that characterize $\mathcal{K}$. Namely, $\mathcal{K}$ is the set of all vectors $\boldsymbol{\omega} \in \mathbb{R}^n$ that satisfy

$$\omega_i \geqslant 0 \qquad \text{(for all } i \in \mathcal{I}) , \tag{1}$$
$$\omega_i \leqslant \sum_{i' \in \mathcal{I}_j \setminus i} \omega_{i'} \quad \text{(for all } j \in \mathcal{J}, \text{ for all } i \in \mathcal{I}_j) . \tag{2}$$

With this, we are ready to discuss our first generalization of [1, Lemma 11], which generalizes [1, Lemma 11] from real measurement matrices where every entry is equal to either zero or one to complex measurement matrices where the absolute value of every entry is equal to either zero or one. Note that the upcoming lemma also generalizes the mapping that is applied to the vectors in the nullspace of the measurement matrix.

Lemma 2 *Let $\boldsymbol{H}_{\mathrm{CS}} = (h_{j,i})_{j \in \mathcal{J}, i \in \mathcal{I}}$ be some measurement matrix over $\mathbb{C}$ such that $|h_{j,i}| \in \{0,1\}$ for all $(j,i) \in \mathcal{J} \times \mathcal{I}$, and let $|\cdot|_*$ be an arbitrary norm for complex numbers. Then*

$$\boldsymbol{\nu} \in \mathrm{nullspace}_{\mathbb{C}}(\boldsymbol{H}_{\mathrm{CS}}) \quad \Rightarrow \quad |\boldsymbol{\nu}|_* \in \mathcal{K}(|\boldsymbol{H}_{\mathrm{CS}}|).$$

Remark: Note that $\mathrm{supp}(\boldsymbol{\nu}) = \mathrm{supp}(|\boldsymbol{\nu}|_*)$.
Proof: Omitted. ∎

The second generalization of [1, Lemma 11] generalizes that lemma to hold also for complex measurement matrices where the absolute value of every entry is an integer. In order to present this lemma, we need the following definition, which will be illustrated by Example 4.

Definition 3 *Let $\boldsymbol{H}_{\mathrm{CS}} = (h_{j,i})_{j \in \mathcal{J}, i \in \mathcal{I}}$ be some measurement matrix over $\mathbb{C}$ such that $|h_{j,i}| \in \mathbb{Z}_{\geqslant 0}$ for all $(j,i) \in \mathcal{J} \times \mathcal{I}$, and let $M \in \mathbb{Z}_{>0}$ be such that $M \geqslant \max_{(j,i) \in \mathcal{J} \times \mathcal{I}} |h_{j,i}|$. We define an M-fold cover of $\tilde{\boldsymbol{H}}_{\mathrm{CS}}$ of $\boldsymbol{H}_{\mathrm{CS}}$ as follows: for $(j,i) \in \mathcal{J} \times \mathcal{I}$, $h_{j,i}$ is replaced by $h_{j,i}/|h_{j,i}|$ times the sum of $|h_{j,i}|$ arbitrary $M \times M$ permutation matrices with non-overlapping support.* □

Note that the entries of the matrix $\tilde{\boldsymbol{H}}_{\mathrm{CS}}$ in Definition 3 all have absolute value equal to either zero or one.

Example 4 Let

$$\boldsymbol{H}_{\mathrm{CS}} \triangleq \begin{pmatrix} 1 & 0 & \sqrt{2}(1+i) \\ -2 & i & 3 \end{pmatrix}.$$

Clearly

$$|\boldsymbol{H}_{\mathrm{CS}}| \triangleq \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \end{pmatrix},$$

and so, choosing $M = 3$,

$$\tilde{\boldsymbol{H}}_{\mathrm{CS}} \triangleq \left(\begin{array}{ccc|ccc|ccc} 0 & 1 & 0 & 0 & 0 & 0 & \frac{1+i}{\sqrt{2}} & \frac{1+i}{\sqrt{2}} & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & \frac{1+i}{\sqrt{2}} & 0 & \frac{1+i}{\sqrt{2}} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{1+i}{\sqrt{2}} & \frac{1+i}{\sqrt{2}} \\ \hline 0 & -1 & -1 & i & 0 & 0 & 1 & 1 & 1 \\ -1 & -1 & 0 & 0 & i & 0 & 1 & 1 & 1 \\ -1 & 0 & -1 & 0 & 0 & i & 1 & 1 & 1 \end{array} \right).$$

is a possible matrix obtained by the procedure defined in Definition 3. □

Lemma 5 *Let $\boldsymbol{H}_{\mathrm{CS}} = (h_{j,i})_{j \in \mathcal{J}, i \in \mathcal{I}}$ be some measurement matrix over $\mathbb{C}$ such that $|h_{j,i}| \in \mathbb{Z}_{\geqslant 0}$ for all $(j,i) \in \mathcal{J} \times \mathcal{I}$. With this, let $M \in \mathbb{Z}_{>0}$ be such that $M \geqslant \max_{(j,i) \in \mathcal{J} \times \mathcal{I}} |h_{j,i}|$, and let $\tilde{\boldsymbol{H}}_{\mathrm{CS}}$ be a matrix obtained by the procedure in Definition 3. Moreover, let $|\cdot|_*$ be an arbitrary norm for complex numbers. Then*

$$\boldsymbol{\nu} \in \mathrm{nullspace}_{\mathbb{C}}(\boldsymbol{H}_{\mathrm{CS}}) \quad \Rightarrow \quad \boldsymbol{\nu}^{\uparrow M} \in \mathrm{nullspace}_{\mathbb{C}}(\tilde{\boldsymbol{H}}_{\mathrm{CS}})$$
$$\Rightarrow \quad |\boldsymbol{\nu}^{\uparrow M}|_* \in \mathcal{K}(|\tilde{\boldsymbol{H}}_{\mathrm{CS}}|).$$

Additionally, with respect to the first implication sign we have the following converse: for any $\tilde{\boldsymbol{\nu}} \in \mathbb{C}^{Mn}$ we have

$$\boldsymbol{\varphi}_M(\tilde{\boldsymbol{\nu}}) \in \mathrm{nullspace}_{\mathbb{C}}(\boldsymbol{H}_{\mathrm{CS}}) \quad \Leftarrow \quad \tilde{\boldsymbol{\nu}} \in \mathrm{nullspace}_{\mathbb{C}}(\tilde{\boldsymbol{H}}_{\mathrm{CS}}).$$

Proof: Omitted. ∎

Finally, we present our third generalization of [1, Lemma 11], which generalizes the mapping that is applied to the vectors in the nullspace of the measurement matrix.

Lemma 6 *Let $\boldsymbol{H}_{\mathrm{CS}} = (h_{j,i})_{j \in \mathcal{J}, i \in \mathcal{I}}$ be some measurement matrix over $\mathbb{C}$ such that $|h_{j,i}| \in \{0,1\}$ for all $(j,i) \in \mathcal{J} \times \mathcal{I}$. Moreover, let $L \in \mathbb{Z}_{>0}$, and let $\|\cdot\|_*$ be an arbitrary vector norm for complex numbers. Then*

$$\boldsymbol{\nu}^{(1)}, \ldots, \boldsymbol{\nu}^{(L)} \in \mathrm{nullspace}_{\mathbb{C}}(\boldsymbol{H}_{\mathrm{CS}}) \quad \Rightarrow \quad \boldsymbol{\omega} \in \mathcal{K}(|\boldsymbol{H}_{\mathrm{CS}}|),$$

where $\boldsymbol{\omega} \in \mathbb{R}^n$ is defined such that for all $i \in \mathcal{I}$,

$$\omega_i = \left\| \left(\nu_i^{(1)}, \ldots, \nu_i^{(L)} \right) \right\|_*.$$

Proof: Omitted. ∎

We conclude this section with two remarks. First, it is clear that Lemma 6 can be extended in the same way as Lemma 5 extends Lemma 2. Secondly, similarly to the approach in [1] where [1, Lemma 11] was used to translate "positive results" about **CC-LPD** to "positive results" about **CS-LPD**, the new Lemmas 2, 5, and 6 can be the basis for translating results from **CC-LPD** to **CS-LPD**.

III. Translating Performance Guarantees

In this section we use [1, Lemma 11] to transfer "positive performance results" for **CC-LPD** of low-density parity-check (LDPC) codes to "positive performance results" for **CS-LPD** of zero-one measurement matrices. In particular, three positive threshold results for **CC-LPD** are used to obtain three results that are, to the best of our knowledge, novel for compressed sensing. At the end of the section we will also use Lemma 2 with $|\cdot|_* = |\cdot|$ to study dense measurement matrices with entries in $\{-1, 0, +1\}$.

We will need the notion of an *expander graph*.

Definition 7 *Let G be a bipartite graph where the nodes in the two node classes are called left-nodes and right-nodes, respectively. If $\mathcal{S}$ is some subset of left-nodes, we let $\mathcal{N}(\mathcal{S})$ be the subset of right-nodes that are adjacent to $\mathcal{S}$. Then, given parameters $d_{\mathrm{v}} \in \mathbb{Z}_{>0}$, $\gamma \in (0,1)$, $\delta \in (0,1)$, we say that G is a $(d_{\mathrm{v}}, \gamma, \delta)$-expander if all left-nodes of G have degree d_{v} and if for all left-node subsets $\mathcal{S}$ with $\#\mathcal{S} \leqslant \gamma n$ it holds that $\#\mathcal{N}(\mathcal{S}) \geqslant \delta d_{\mathrm{v}} \cdot \#\mathcal{S}$.* □

Expander graphs have been studied extensively in past work on channel coding and compressed sensing (see, e.g., [8], [9]). It is well-known that randomly constructed left-regular bipartite graphs are expanders with high probability (see, e.g., [10]).

In the following, similar to the way a Tanner graph is associated with a parity-check matrix [11], we will associate a Tanner graph with a measurement matrix. Note that the variable and constraint nodes of a Tanner graph will be called left-nodes and right-nodes, respectively.

Corollary 8 *Let $d_{\mathrm{v}} \in \mathbb{Z}_{>0}$, let $\gamma \in (0,1)$, and let $\boldsymbol{H}_{\mathrm{CS}} \in \{0,1\}^{n' \times n}$ be a measurement matrix. Moreover, assume that the Tanner graph of $\boldsymbol{H}_{\mathrm{CS}}$ is a $(d_{\mathrm{v}}, \gamma, \delta)$-expander with sufficient expansion, more precisely, with*

$$\delta > \frac{2}{3} + \frac{1}{3d_{\mathrm{v}}}$$

*(along with the technical condition $\delta d_{\mathrm{v}} \in \mathbb{Z}_{>0}$). Then **CS-LPD** based on the measurement matrix $\boldsymbol{H}_{\mathrm{CS}}$ can recover all k-sparse vectors, i.e., all vectors whose support size is at most k, for*

$$k < \frac{3\delta - 2}{2\delta - 1} \cdot (\gamma n - 1).$$

Proof: This result is obtained by combining the results in [1] with [10, Theorem 1]. ∎

Interestingly, for $\delta = 3/4$ the recoverable sparsity k matches exactly the performance of the fast compressed sensing algorithm in [9] and the performance of the simple bit-flipping channel decoder of Sipser an Spielman [8], but our result holds for the basis pursuit LP relaxation **CS-LPD**. Expansion has been shown to suffice for **CS-LPD** in [12] but with a different proof and yielding different constants. For $n'/n = 1/2$ and $d_{\mathrm{v}} = 32$, the result of [10] establishes that sparse expander-based zero-one measurement matrices will recover all $k = \alpha n$ sparse vectors for $\alpha \leqslant 0.000175$.

Whereas the above result gave a deterministic guarantee, the following result is based on a so-called weak bound for **CC-LPD** and gives a probabilistic guarantee.

Corollary 9 *Let $d_{\mathrm{v}} \in \mathbb{Z}_{>0}$. Consider a random measurement matrix $\boldsymbol{H}_{\mathrm{CS}} \in \{0,1\}^{n' \times n}$ that is formed by placing d_{v} random ones in each column, and zeros elsewhere. This measurement matrix succeeds to recover a randomly supported $k = \alpha n$ sparse vector with probability $1 - o(1)$ if α is below some threshold function $\alpha_{n'}(d_{\mathrm{v}}, n'/n)$.*

Proof: The result is obtained by combining the results in [1] with [13, Theorem 1]. The latter paper also contains a way to compute achievable threshold values $\alpha_{n'}(d_{\mathrm{v}}, n'/n)$. ∎

For $n'/n = 1/2$ and $d_{\mathrm{v}} = 8$, a random measurement matrix will recover a $k = \alpha n$ sparse vector with random support with high probability if $\alpha \leqslant 0.002$. This is, of course, a much higher threshold compared to the one presented above but it only holds with high probability over the vector support (therefore it is a so-called weak bound). To the best of our knowledge, this is the first weak bound obtained for random sparse measurement matrices.

The best thresholds known for LP decoding were recently obtained by Arora, Daskalakis, and Steurer [14] but require matrices that are both left and right regular and also have logarithmically growing girth. A random bipartite matrix will not have this latter property but there are explicit deterministic constructions that achieve this (for example the construction presented in Gallager's thesis [15, Appendix C]). By translating the results from [14] to the compressed sensing setup we obtain the following result.

Corollary 10 *Let $d_\mathrm{v}, d_\mathrm{c} \in \mathbb{Z}_{>0}$. Consider a measurement matrix $\boldsymbol{H}_{\mathrm{CS}} \in \{0,1\}^{n' \times n}$ whose Tanner graph is a $(d_\mathrm{v}, d_\mathrm{c})$-regular bipartite graph with $\Omega(\log n)$ girth. This measurement matrix succeeds to recover a randomly supported $k = \alpha n$ sparse vector with probability $1 - o(1)$ if α is below some threshold function $\alpha'_{n'}(d_\mathrm{v}, d_\mathrm{c}, n'/n)$.*

Proof: The result is obtained by combining the results in [1] with [14, Theorem 1]. The latter paper also contains a way to compute achievable threshold values $\alpha'_{n'}(d_\mathrm{v}, d_\mathrm{c}, n'/n)$. ∎

For $n'/n = 1/2$, an application of the above result to a $(3, 6)$-regular Tanner graph with logarithmic girth (obtained from the Gallager construction) tells us that sparse vectors with sparsity $k = \alpha n$ are recoverable with high probability for $\alpha \leqslant 0.05$. Therefore, measurement matrices based on Gallager's deterministic construction (of low-density parity-check matrices) form the best known class of sparse measurement matrices for the compressed sensing setup considered here.

We conclude this section with some considerations about dense measurement matrices, highlighting our current understanding that the translation of positive performance guarantees from **CC-LPD** to **CS-LPD** displays the following behavior: the denser a measurement matrix is the weaker are the translated performance guarantees.

Remark 11 Consider a randomly generated $n' \times n$ measurement matrix $\boldsymbol{H}_{\mathrm{CS}}$ where every entry is generated i.i.d. according to the distribution

$$\begin{cases} +1 & \text{with probability } 1/6 \\ 0 & \text{with probability } 2/3 \\ -1 & \text{with probability } 1/6 \end{cases}.$$

This matrix, after multiplying it by the scalar $\sqrt{3/n}$, has the restricted isometry property (RIP). (See [16], which proves this property based on results in [17], which in turn proves that this family of matrices has a non-zero threshold.) On the other hand, one can show that the family of parity-check matrices where every entry is generated i.i.d. according to the distribution

$$\begin{cases} 1 & \text{with probability } 1/3 \\ 0 & \text{with probability } 2/3 \end{cases}$$

does *not* have a non-zero threshold under **CC-LPD** for the BSC [18]. ☐

Therefore, we conclude that the connection between **CS-LPD** and **CC-LPD** given by Lemma 2 is not tight for dense matrices in the sense that the performance of **CS-LPD** based on dense measurement matrices can be much better than predicted by the performance of **CC-LPD** based on their parity-check matrix counterpart.

IV. REFORMULATIONS BASED ON GRAPH COVERS

(This section has been omitted.)

V. MINIMIZING THE ZERO-INFINITY OPERATOR

(This section has been omitted.)

VI. CONCLUSIONS AND OUTLOOK

In this paper we have extended the results of [1] along various directions. In particular, we have translated performance guarantees from **CC-LPD** to performance guarantees for the recovery of *exactly sparse* vectors under **CS-LPD**. As part of future work we plan to investigate the translation of performance guarantees from **CC-LPD** to performance guarantees for the recovery of *approximately sparse* vectors under **CS-LPD**.

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