noise has become the major part of remaining noise after filtering, it is important to prevent this increase. The RDS is replaced by the IRDS filter which corrects the thermal noise increase.

Total variances, including sensing-node amplifier noise and reset noise, can give us a good idea of the interest in using IRDS filter. Their variance calculation enables us to conclude that the IRDS filter brings a SNR enhancement of 1, 6 dB compared to the RDS filter.

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A New Transform for the Stabilization and Stability Testing of Multidimensional Recursive Digital Filters

Niranjan Damera-Venkata, Mahalakshmi Venkataraman, M. S. Hrishikesh, and P. S. Reddy

***Abstract*—We present a new transform which, when applied to the denominator polynomial of the transfer function of an unstable multidimensional recursive digital filter (of a special class) will yield a stable polynomial with good preservation of the magnitude spectrum. In fact, the discrete Hilbert transform (DHT), used to stabilize 2-D and 1-D recursive digital filters, is a special case of the general multidimensional transform we present here. We also address the problem of stability testing of a multidimensional recursive digital filter and show that the new transform may be used to implement a straightforward test for stability of any causal multidimensional recursive digital filter, having no nonessential singularities of the second kind.**

***Index Terms*—Multidimensional digital filters, stability, stabilization.**

I. INTRODUCTION

The discrete Hilbert transform (DHT) has been used for the stabilization of 1-D and 2-D recursive digital filters. In this brief, we introduce a transform which is the generalization of the well-known DHT to multidimensions. It will be shown that such a transform can also serve as a stability testing tool for any general causal multidimensional filter, having no nonessential singularities of the second kind [1]. In Sections IV, VI, and VII, we present some properties of this transformation, which are valid for 1- and 2-D DHTs but hitherto unknown in the literature. Possible use of such a transformation for the processing of three and higher dimensional signals are in the areas of geophysics [2], [3] and biomedical applications [4], [5]. Design of 3-D recursive digital filters is discussed in [6].

We assume that the z-transform is defined with positive powers of z. In other words, we are concerned with the stabilization and stability testing of multidimensional quarter-plane recursive digital filters with first quadrant support only. This does not impose any restriction on the stability testing of general multidimensional filters with nonsymmetric half plane (NSHP) support, because the necessary and sufficient conditions for stability of these filters are equivalent to the necessary and sufficient conditions for the stability of a mapping of the NSHP coefficients into the first quadrant [1].

II. NECESSARY AND SUFFICIENT CONDITIONS FOR STABILITY

In this section, we review the necessary and sufficient conditions for the stability of a first-quadrant multidimensional recursive digital filter [1]. Let $B(\vec{z})$ be the denominator polynomial of the transfer function of an N-D recursive digital filter, which has no nonessential singularities of the second kind [1].

Theorem 1: The filter $H(\vec{z})$, with denominator polynomial $B(\vec{z})$, is stable if and only if the following hold.

1) $B(\vec{z}) \neq 0$ on T^n, where $\vec{z} = \{z_1, z_2 \dots z_N\}$ and T^n are defined as

$$T^n = \{|z_1| = 1, |z_2| = 1, \dots |z_N| = 1\}.$$

2)

$$B(z, z, \dots z) \neq 0, \qquad |z| \leq 1.$$

A polynomial $B(\vec{z})$ satisfying conditions 1 and 2 is called a minimum phase polynomial.

III. THE NEW TRANSFORM

In this section, we present a new transform, which when applied to a class of N-D unstable polynomials will yield stable polynomials. The new transform reduces to the DHT in the case of 1-D and 2-D polynomials. We desire a transform relation between the magnitude and phase of the discrete Fourier transform of a finite discrete multidimensional causal sequence. A multidimensional N-D discrete sequence $x(\vec{\boldsymbol{i}})$ is causal if

$$x(\vec{\boldsymbol{i}}) = 0 \qquad \forall i_k \geq \frac{M_k}{2} \tag{1}$$

where i_k varies over the set $\{0, 1, \dots, M_k - 1\}$, where M_k is the size along dimension k.

If $x(\vec{\boldsymbol{i}})$ is a minimum-phase sequence, then the associated complex cepstrum is real and causal [1], and the minimum phase response $\Phi(\vec{\boldsymbol{f}}) = \angle X(\vec{\boldsymbol{f}})$ is related to the magnitude response $|X(\vec{\boldsymbol{f}})|$ by the relation below (the derivation is similar to that described in [7] and is omitted)

$$\Phi(\vec{\boldsymbol{f}}) = -j \times \text{DFT}\left(\left[t(\vec{\boldsymbol{i}})\right] \text{IDFT}\left\{\log|X(\vec{\boldsymbol{f}})|\right\})\right) \tag{2}$$

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where

$$t(\vec{\boldsymbol{i}}) = t_1(\vec{\boldsymbol{i}}) + t_2(\vec{\boldsymbol{i}}) \tag{3}$$

where

$$t_1(\vec{\boldsymbol{i}}) = \begin{cases} 1, & 0 < i_k < \dfrac{M_k}{2}, \quad \vec{\boldsymbol{i}} = 0 \text{ except } i_k, \\ & \quad \text{for } k = 1, 2, \ldots, N \\ -1, & \dfrac{M_k}{2} < i_k < M_k, \quad \vec{\boldsymbol{i}} = 0 \text{ except } i_k, \\ & \quad \text{for } k = 1, 2, \ldots, N \\ 0, & \quad \text{for all other values of } \vec{\boldsymbol{i}} \end{cases} \tag{4}$$

and

$$t_2(\vec{\boldsymbol{i}}) = \begin{cases} 1, & 0 < i_k < \dfrac{M_k}{2}, \quad 0 < i_m < \dfrac{M_m}{2} \\ & \vec{\boldsymbol{i}} = 0,\ i_k,\ i_m \neq 0, \ \text{ for } k \neq m \text{ and} \\ & k = 1, 2, \ldots, N, \quad m = 1, 2, \ldots, N \\ -1, & \dfrac{M_k}{2} < i_k < M_k, \quad \dfrac{M_m}{2} < i_m < M_m \\ & \vec{\boldsymbol{i}} = 0,\ i_k,\ i_m \neq 0, \ \text{ for } k \neq m \text{ and} \\ & k = 1, 2, \ldots, N, \quad m = 1, 2, \ldots, N \\ 0, & \text{for all other values of } \vec{\boldsymbol{i}} \end{cases} \tag{5}$$

where $\vec{\boldsymbol{f}}$ is the discrete frequency vector. For a causal minimum-phase polynomial $X(\vec{z})$ satisfying the conditions of Section II, the DFT of the associated sequence $x(\vec{\boldsymbol{i}})$ satisfies (2). The key to the generalization to multidimensions from two dimensions is (5), which is a nontrivial extension of the 2-D "boundary function" [7] to N dimensions. Using (2), we can construct multidimensional minimum phase polynomials from nonminimum phase polynomials, while closely preserving the magnitude spectrum of the nonminimum phase polynomial . The flowchart for the computer implementation of the proposed procedure is provided in Fig. 1. As discussed in [8], in order that $B_{NT}(\vec{z})$ has almost the same magnitude spectrum as $B(\vec{z})$, it is necessary that the size of the fast Fourier transform (FFT) used to compute the transform be very large, so that the autocorrelation coefficients of $B_{NT}(\vec{z})$ are almost the same as those of $B(\vec{z})$. This also ensures that the array elements which were truncated to get $B_{NT}(\vec{z})$ from $B'_{NT}(\vec{z})$ are negligibly small.

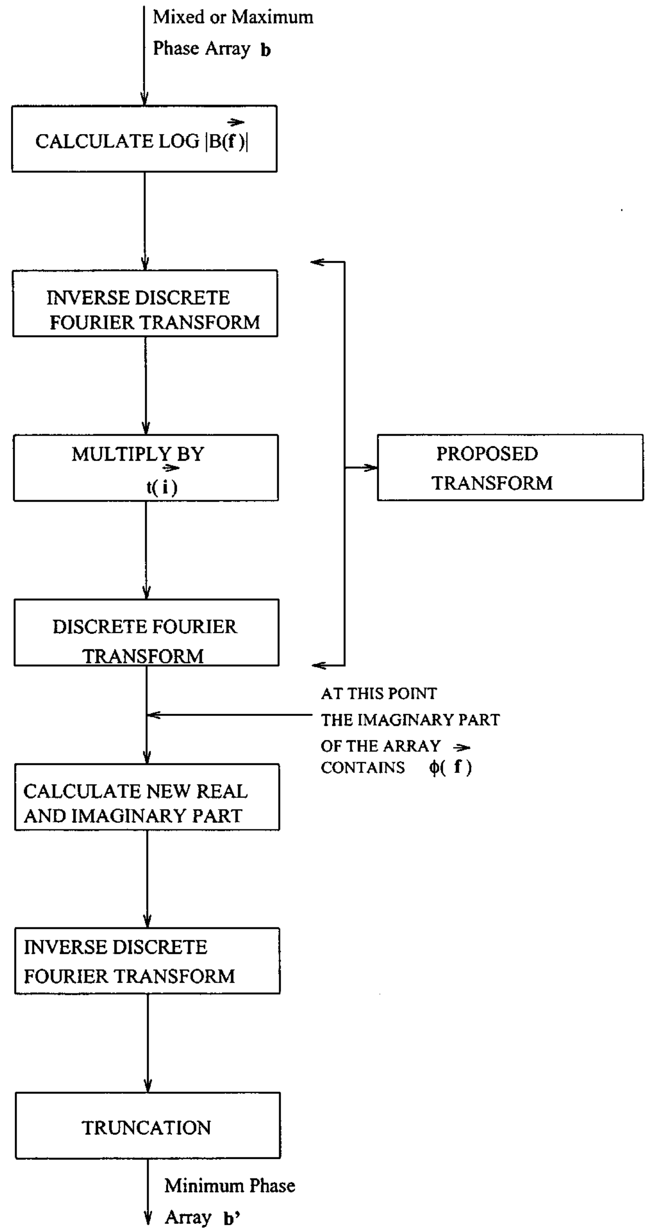

Fig. 1. Procedure to obtain a minimum-phase version of a nonminimum phase multidimensional polynomial.

IV. Factorizability of the Transformed Polynomials

In this section, we show that if the first-quadrant polynomial $B(\vec{z})$ is factorizable as $B(\vec{z}) = B_1(\vec{z})B_2(\vec{z})$, then the transformed version of $B(\vec{z})$, namely $B_{NT}(\vec{z})$, is also factorizable. Here we assume that the same order FFT is used along each direction while implementing the transformation. This does not impose any constraints, as the original array has to be zero padded to satisfy the causality condition of (1). A bound on the error due to using a finite-size FFT was established in [9] for 1-D sequences. A large-order FFT forces the complex cepstrum of the unstable sequence to satisfy (1).

We know that the transform procedure outlined in Section III yields a multidimensional matrix equation relating the phase function $\Phi(\vec{\boldsymbol{f}})$ and the log magnitude function $\log(|B(\vec{\boldsymbol{f}})|)$ for different sliced values of the discrete angular frequencies $\vec{\boldsymbol{f}}$. The proposed transform is a linear transformation which maps samples of a log magnitude function to samples of its minimum phase response. We can express this as

$$\boldsymbol{Y} = \boldsymbol{T}\boldsymbol{X}. \tag{6}$$

Here, $\boldsymbol{Y}$ is an $N^3 \times 1$ column vector of the new phase samples of the given polynomial $B(\vec{z})$. $\boldsymbol{T}$ is the transformation matrix which is square of order $N^3 \times N^3$, where N is the size of the FFT along each direction used to implement the DHT. $\boldsymbol{X}$ is an $N^3 \times 1$ column vector of log magnitude samples of the given polynomial $B(\vec{z})$.

Let $B(\vec{z})$ be factorizable as the product of two N-D polynomials

$$B(\vec{z}) = B_1(\vec{z})B_2(\vec{z}). \tag{7}$$

Then

$$\boldsymbol{X} = \log\left(B_1(\vec{z})B_2(\vec{z})\right) = \log\left(B_1(\vec{z})\right) + \log\left(B_2(\vec{z})\right) = \boldsymbol{X_1} + \boldsymbol{X_2} \tag{8}$$

and from (17) and (19), we have

$$\boldsymbol{Y} = \boldsymbol{T}\left(\boldsymbol{X_1} + \boldsymbol{X_2}\right) = \boldsymbol{T}\boldsymbol{X_1} + \boldsymbol{T}\boldsymbol{X_2} = \boldsymbol{Y_1} + \boldsymbol{Y_2}. \tag{9}$$

Since we reconstruct the polynomial by taking $\boldsymbol{Y}$ as the phase (called minimum phase) samples vector and the magnitude samples $|B_1(\vec{z})||B_2(\vec{z})|$, we have the transformed polynomial $B_{NT}(\vec{z})$ as

$$\begin{aligned} B_{NT}(\vec{z}) &= |B_1(\vec{z})||B_2(\vec{z})| \exp^{(\boldsymbol{Y_1}+\boldsymbol{Y_2})} \\ &= |B_1(\vec{z})|e^{(\boldsymbol{Y_1})}|B_2(\vec{z})|e^{(\boldsymbol{Y_2})} \\ &= B_{1NT}(\vec{z})B_{2NT}(\vec{z}). \end{aligned} \tag{10}$$

In (21), $B_{1NT}(\vec{z})$ and $B_{2NT}(\vec{z})$ are the transformed polynomials of $B_1(\vec{z})$ and $B_2(\vec{z})$. Thus, we have proven the following theorem.

Theorem 2: If the given N-D polynomial $B(\vec{z})$ is factorizable, then the transformed polynomial $B_{NT}(\vec{z})$ is also factorizable, and the factors of the transformed polynomial are transformed versions of the factors of the given N-D polynomial.

V. Stabilization Using the New Transform

The following theorem establishes the class of polynomials that may be stabilized by the new transform.

Theorem 3: The N-D polynomial $B_{NT}(\vec{z})$ of any causal N-D polynomial $B(\vec{z})$, not having zeros on the unit hypercircle is stable.

Proof: In our new transform, as stated earlier in Section III, the autocorrelation coefficients of $B(\vec{z})$ and $B_{NT}(\vec{z})$ are almost the same, if we use a large size FFT. When we ensure that the autocorrelation coefficients are approximately the same for both $B(\vec{z})$ and $B_{NT}(\vec{z})$ we can be certain that if $B(\vec{z})$ has no zeros on the unit hypercircle then $B_{NT}(\vec{z})$ will also not have such zeros [8]. Thus, the stability condition 1 of Section II will be satisfied. We can easily extend the results of [10] and show that $B_{NT}(1, 1, \ldots, z_k, \ldots, 1) \neq 0$ when $|z_k| \leq 1$, $k = 1, 2, \ldots, N$ if $B(1, 1, \ldots, z_k, \ldots, 1)$ does not have zeros on the unit circle [8]. $B(1, 1, \ldots, z_k, \ldots, 1)$ will not have zeros on the unit circle since we assume that $B(\vec{z})$ is free of zeros on the unit hyperdisc. So we conclude that the transformed polynomial $B_{NT}(\vec{z})$ will satisfy the stability condition 2 of Section II. Since both conditions 1 and 2 of Section II are satisfied, $B_{NT}(\vec{z})$ will be stable. Hence the proof of Theorem 3.

Example 1: We consider an unstable nonseparable polynomial $B(z_1, z_2, z_3)$ given by

$$\begin{aligned} B(z_1, z_2, z_3) = {}& 0.95z_1z_2z_3 - 0.7z_1z_2 - 0.5z_2z_3 + 2z_3z_1 \\ & - 1.5z_1 + 0.375z_2 - z_3 + 0.75 \end{aligned}$$

which is a (2,2,2) array. The stable polynomial $B_{NT}(z_1, z_2, z_3)$ obtained by processing through the proposed algorithm using (64,64,64) size FFT is

$$\begin{aligned} & B_{NT}(z_1, z_2, z_3) \\ &\quad = 0.3563z_1z_2z3 - 0.4727z_1z_2 - 0.7172z_2z_3 + 0.7545z_3z_1 \\ &\qquad - 1.0059z_1 + .9527z_2 - 1.4971z_3 + 2.0001. \end{aligned}$$

The polynomial $B_{NT}(z_1, z_2, z_3)$ is stable. This can be shown by using some of the standard stability tests in the literature [1]. In the following sections, we will develop a stability test based on the new transform, and revisit example 1 in Section VII.

VI. Uniqueness Theorem

In this section, we present an N-D to 1-D transformation by which any N-D polynomial $B(\vec{z})$ can be transformed into a 1-D polynomial $B(z)$, such that both $B(\vec{z})$ and $B(z)$ have the same autocorrelation coefficients [3]. We then prove a theorem that says that the minimum phase polynomial of a nonminimum phase polynomial, if it exists, is unique. This has been an open problem in the literature [7]. We do not restrict ourselves to N-D polynomials not having zeros on the unit hypercircle T^n.

Let

$$P(\vec{z}) = \sum_{j_1=0}^{M_1}\sum_{j_2=0}^{M_2}\cdots\sum_{j_N=0}^{M_N} p(j_1, j_2, \ldots, j_n) z_1^{j_1} z_2^{j_2} \ldots z_N^{j_N}$$

be an N-D polynomial. The 1-D polynomial $P(z)$ having the same auto-correlation coefficients as those of polynomial $P(\vec{z})$ can be obtained by the transformation [3], $z_1 = z, z_i = z^{L_i}$, $(i = 2, 3 \ldots N)$, where $L_i = (2M_{i-1} + 1)L_{i-1}$ for $(i = 2, 3 \ldots N)$. Also, it was shown that if the N-D polynomial was stable, then the 1-D polynomial obtained by such a transformation would also be stable [3].

Theorem 4 (Uniqueness Theorem): If an N-D minimum phase polynomial exists, such that it has the same magnitude response as a given general N-D causal multidimensional polynomial of the same order, then that polynomial is the unique minimum-phase polynomial corresponding to the given magnitude response.

Proof: Let us assume that there are two multidimensional minimum-phase polynomials of the same order denoted by $B_1(\vec{z})$ and $B_2(\vec{z})$, both having the same autocorrelation coefficients as the corresponding nonminimum phase multidimensional polynomial $B(\vec{z})$.

On applying the form preserving transformation outlined in this section, we get two 1-D polynomials which are stable and also have the same autocorrelation coefficients as follows:

$$B_1(\vec{z}) \xrightarrow{z_1=z,\ z_i=z^{L_i}} B_1(z) \tag{11}$$

$$B_2(\vec{z}) \xrightarrow{z_1=z,\ z_i=z^{L_i}} B_2(z) \tag{12}$$

$L_i = (2M_{i-1} + 1)L_{i-1}$ for $(i = 2, 3 \ldots N)$. But from 1-D polynomial theory, we know that the two 1-D polynomials $B_1(z)$ and $B_2(z)$ cannot have the same autocorrelation coefficients and be stable simultaneously. Therefore, our assumption is not valid. So only one of $B_1(\vec{z})$ and $B_2(\vec{z})$ is a stable polynomial. Thus, Theorem 4 is proven.

The fact that a minimum phase may not exist at all for certain 2-D polynomials has been brought out by means of a counter-example [11]. Such counter examples are to be expected in the multidimensional case also. Using the results of Theorem 4, we have the following.

Corrolary 1: Minimum-phase polynomials are fixed points of the new transform, with the order of the FFT chosen as discussed in Section V.

VII. The Stability Test

In this section, a test procedure is presented on an array $\boldsymbol{B}$ based on the discussion in Section VI. It is required to ascertain whether array $\boldsymbol{B}$ is stable or not. It may be noted that the polynomial corresponding to $\boldsymbol{B}$, namely $B(\vec{z})$, can be allowed to have zeros on the unit hypercircle T^n

1) Apply the new transform as outlined in Section III to obtain array $\boldsymbol{A}$.
2) Compare arrays $\boldsymbol{B}$ and $\boldsymbol{A}$.

a) If $\boldsymbol{B} \equiv \boldsymbol{A}$,then $\boldsymbol{B}$ is a stable array.
b) If $\boldsymbol{B} \not\equiv \boldsymbol{A}$, then $\boldsymbol{B}$ is unstable.

Let us now check the stability of the polynomial of Example 1.

Example 2: Here

$$\begin{aligned} B(z_1, z_2, z_3) = {}& 0.95z_1z_2z_3 - 0.7z_1z_2 - 0.5z_2z_3 + 2z_3z_1 \\ & - 1.5z_1 + 0.375z_2 - z_3 + 0.75. \end{aligned}$$

Application of the new transform using (64,64,64) size FFT, we get

$$A(z_1,\, z_2,\, z_3) \\ = 0.3563 z_1 z_2 z3 - 0.4727 z_1 z_2 - 0.7172 z_2 z_3 + 0.7545 z_3 z_1 \\ - 1.0059 z_1 + .9527 z_2 - 1.4971 z_3 + 2.0001.$$

Clearly $\boldsymbol{B} \not\equiv \boldsymbol{A}$, therefore array $\boldsymbol{B}$ represents an unstable polynomial $B(z_1,\, z_2,\, z_3)$. To test the stability of $A(z_1,\, z_2,\, z_3)$ obtained after the application of the new transform, we apply the transform again to the coefficient array $\boldsymbol{A}$ to yield a new coefficient array $\boldsymbol{A}'$

$$A'(z_1,\, z_2,\, z_3) \\ = 0.3564 z_1 z_2 z3 - 0.4727 z_1 z_2 - 0.7172 z_2 z_3 + 0.7545 z_3 z_1 \\ - 1.0059 z_1 + .9527 z_2 - 1.4971 z_3 + 2.0001.$$

Clearly, $\boldsymbol{A} \equiv \boldsymbol{A}'$. Therefore, $\boldsymbol{A}$ represents a stable polynomial. Since we base our test on the validity of Theorem 4, for all practical purposes, we would like to state that a higher order FFT would result in closer preservation of the autocorrelation coefficients and hence yield a more accurate test for the equivalence condition in step 2 of the test.

VIII. Conclusion

We have proposed a new transform for the stabilization and stability testing of multidimensional recursive digital filters. The ease with which the transform may be applied using the FFT makes it an efficient tool in the stabilization and stability testing of these filters. Also, the order of the FFT needs to be as high as possible. It may be stressed that though only a special class of N-D polynomials, not having zeros on T^n can be stabilized by this method, testing for stability need not be restricted to such polynomials only. Our stability test is based on the uniqueness theorem for multidimensional minimum phase polynomials presented in this paper. We also need to use a high-order FFT to stabilize the class of multidimensional polynomials not having zeros on the unit hypercircle. In this case a large order FFT, forces the autocorrelation coefficients to be preserved by the new transformation as in [8].

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Unbiased LMS Filtering in the Presence of White Measurement Noise with Unknown Power

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***Abstract*—This paper presents a new modified least mean squares (LMS) adaptive filtering algorithm for autoregressive (AR) modeling in the presence of white measurement noise with unknown power. In the proposed algorithm, a first-order filter is used to filter the noise-corrupted signal. In this way, the AR model is augmented to have a known pole which can, based on asymptotic analysis, be used to extract and eliminate the noise-induced bias in the standard LMS filtering result, and thus the unbiased parameter estimation can be achieved.**

I. Introduction

The parameter estimation of an autoregressive (AR) model plays a very important role in a wide range of applications such as spectrum estimation, speech analysis, noise cancelation, and digital communications[1]. In all these applications, the observed signal is usually corrupted by a white measurement noise. It has been shown that the standard least-mean-square (LMS) filter, in this case, cannot yield unbiased parameter estimates of the coefficients in the AR model. To overcome the bias problem, a γ-LMS filter was suggested in [3] and its second-order statistical property was analyzed later in [4], where a ρ-LMS filter was proposed. It is shown that when the noise variance is known, both the γ-LMS and the ρ-LMS filters can give unbiased estimates of the AR coefficients, and the ρ-LMS filter has better second-order statistics performance than the γ-LMS filter. However, the noise variance is hardly available in many practical circumstances. Thus, it is of theoretical and practical interest to study how one can obtain unbiased parameter estimation for the AR model in the presence of white measurement noise with unknown power.

In this brief, we propose a modified LMS adaptive filter to remedy the bias in LMS parameter estimation for AR models in the presence of white measurement noise with unknown variance. In the design, a first-order filter is used to filter the noise polluted signal. As a result, a known pole is artificially inserted into the AR model to be estimated. Using this known pole, an equation relating the unknown noise variance to the unknown AR parameters is derived from asymptotical analysis. By this equation, the estimate of the bias item in the standard LMS filter can be expressed in terms of the estimates of the AR parameters.

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