the domain in which the signal parameters can be estimated unambiguously. This can be used in applications to avoid aliasing. Moreover, it has been shown that the mean squared error of PPS parameter estimates can be significantly decreased when a nonuniform sampling scheme is used [1]. Hence, from an accuracy and ambiguity point of view, a properly chosen nonuniform sampling scheme is preferred before a uniform scheme when estimating PPS parameters.

REFERENCES

[1] J. Ängeby, "Statistical analysis of polynomial phase signal parameter estimates based on structured auto-regressive modeling," in *Proc. 8th IEEE Signal Process. Workshop Stat. Signal Array Process.*, vol. 3, June 1996, pp. 444–447.
[2] S. Barbarossa and V. Petrone, "Analysis of polynomial-phase signals by the integrated generalized amibiguity function," *IEEE Trans. Signal Processing*, vol. 45, pp. 316–327, Feb. 1997.
[3] P. M. Djuric and M. Kay, "Parameter estimation of chirp signals," *IEEE Trans. Signal Processing*, vol. 38, pp. 2118–2126, Dec. 1990.
[4] S. Peleg and B. Porat, "Estimation and classification of polynomial-phase signals," *IEEE Trans. Inform. Theory*, vol. 37, pp. 422–430, Mar. 1991.
[5] ——, "The Cramér–Rao lower bound for signals with constant amplitude and polynomial phase," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 39, pp. 749–752, Mar. 1991.
[6] G. Zhou, G. B. Giannakis, and A. Swami, "On polynomial phase signals with time-varying amplitudes," *IEEE Trans. Signal Processing*, vol. 44, pp. 848–861, Apr. 1996.

Design of Optimal Minimum-Phase Digital FIR Filters Using Discrete Hilbert Transforms

Niranjan Damera-Venkata, Brian L. Evans, and Shawn R. McCaslin

***Abstract*—We present a robust noniterative algorithm to design optimal minimum-phase digital FIR filters with real or complex coefficients. We derive: 1) the discrete Hilbert transform (DHT) of the complex cepstrum of a causal complex minimum-phase sequence and 2) the minimum fast Fourier transform length for computing the DHT to achieve a desired coefficient accuracy.**

***Index Terms*—Cepstrum, complex FIR filters, fast Fourier transform, low-delay filters.**

I. INTRODUCTION

The coefficients of a linear-phase digital FIR filter are symmetric or antisymmetric about the midpoint [1]. Therefore, a nonlinear-phase digital FIR filter of the same length would have twice as many free parameters. A linear-phase FIR filter has zeros inside, on, and outside the unit circle, whereas a minimum-phase FIR filter has all of its zeros on or within the unit circle. Given optimal minimum-phase and linear-phase digital FIR filters that meet the same magnitude specification, the minimum-phase filter would have a *reduced filter length* of typically one half to three fourths of the linear-phase filter length [2], and *minimum group delay*, where energy would be concentrated in the low-delay instead of the medium-delay coefficients [3]. Minimum-phase filters can simultaneously meet constraints on delay and magnitude response while generally requiring fewer computations and less memory than linear-phase filters.

Manuscript received March 24,1998; revised November 10, 1999. This work was supported by a United States National Science Foundation CAREER Award under Grant MIP-9702707. The associate editor coordinating the review of this paper and approving it for publication was Prof. P. C. Ching.

N. Damera-Venkata and B. Evans are with the Embedded Signal Processing Laboratory, Department of Electrical and Computer Engineering, The University of Texas, Austin, TX 78712-1084 USA (e-mail: damera-v@ece.utexas.edu; bevans@ece.utexas.edu).

S. McCaslin is with Cicada Semiconductor, Austin, TX 78746 USA (e-mail: srm@cicada-semi.com).

Publisher Item Identifier S 1053-587X(00)03310-9.

Previous algorithms for designing minimum-phase digital FIR filters have been limited to real filters and may be divided into two classes. One class [4]–[8] designs an optimal linear-phase FIR filter for a power spectrum computed by squaring an ideal piecewise constant magnitude response. The process of factoring the linear-phase polynomial transfer function (a.k.a. polynomial deflation) and reconstructing the minimum-phase filter coefficients (a.k.a. polynomial inflation) may introduce catastrophic numerical errors in the coefficients.

The other class of design algorithms [9]–[11] uses the complex cepstrum of the minimum-phase filter and is less error prone than methods based on polynomial deflation or inflation. Two algorithms [9], [10] deconvolve the complex cepstrum. The algorithm in [9] uses spectral factorization, whereas the algorithm in [10] requires time-domain recursion. The algorithms in [11] and this correspondence are based on the discrete Hilbert transform (DHT) relationship between the magnitude spectrum of a causal real sequence and its minimum-phase delay phase spectrum [1], [12], [13]. As the fast Fourier transform (FFT) length used to compute the DHT increases [11], [13], we more accurately approximate the continuous Hilbert transform and improve the accuracy of the minimum-phase filter coefficients.

By extending the DHT approach to the complex case, we present an algorithm to design optimal, complex, minimum-phase digital FIR filters. For the same constraints, complex digital FIR filters sometimes have lower computational complexity than real digital FIR filters, e.g., in seismic processing [14]. For piecewise magnitude constraints *and* a linear-phase constraint over the passband, one algorithm [14] designs optimal, complex, nonlinear-phase digital FIR filters. By relaxing the phase constraint, we can design an optimal minimum-phase complex filter that meets the same magnitude constraints but is up to 50% shorter than [14].

Section II extends the DHT relationship to complex sequences. Section III describes the new design algorithm. Section IV gives two design examples. Section V offers conclusions. The Appendix gives the FFT length to obtain a desired coefficient accuracy for a given filter order.

II. DHT RELATION FOR COMPLEX SEQUENCES

This section derives a DHT relation between the magnitude spectrum of a causal complex sequence and its minimum group delay phase spectrum. Any sequence can be represented as a sum of conjugate symmetric and antisymmetric parts

$$x[n] = x_e[n] + x_o[n] \tag{1}$$

where $x_e[n] = 1/2\,(x[n]+x^*[-n])$ and $x_o[n] = 1/2\,(x[n]-x^*[-n])$ such that x^* represents the complex conjugate of x. Using Fourier transform properties [1]

$$X(e^{j\omega}) = X_R(e^{j\omega}) + jX_I(e^{j\omega}) \tag{2}$$

where $X_R(e^{j\omega})$ and $jX_I(e^{j\omega})$ are the Fourier transforms of $x_e[n]$ and $x_o[n]$, respectively. Furthermore, $X_R(e^{j\omega})$ and $X_I(e^{j\omega})$ are the real and imaginary parts of $X(e^{j\omega})$, respectively. If $x[n]$ is causal, i.e.,

$x[n] = 0$ for $n < 0$, then it is possible to recover $x[n]$ from $x_e[n]$ by using

$$x[n] = 2x_e[n]u[n] - x^*[0]\delta[n]. \tag{3}$$

By taking the Fourier transform of (3), we obtain

$$X(e^{j\omega}) = \frac{1}{\pi}\int_{-\pi}^{\pi} X_R(e^{j\theta})U(e^{j(\omega-\theta)})\,d\theta - x^*[0] \tag{4}$$

where $U(e^{j\omega})$ is the Fourier transform of the unit step sequence [1]. By using

$$U(e^{j\omega}) = \frac{1}{2} - \frac{j}{2}\cot\left(\frac{\omega}{2}\right) + \sum_{k=-\infty}^{\infty} \pi\delta(\omega - 2\pi k) \tag{5}$$

we can express (4) as

$$\begin{aligned} X(e^{j\omega}) &= X_R(e^{j\omega}) + jX_I(e^{j\omega}) \\ &= X_R(e^{j\omega}) + \frac{1}{2\pi}\int_{-\pi}^{\pi} X_R(e^{j\theta})\,d\theta \\ &\quad - \frac{j}{2\pi}\int_{-\pi}^{\pi} X_R(e^{j\theta})\cot\left(\frac{\omega-\theta}{2}\right)d\theta \\ &\quad - \Re e\{x^*[0]\} - j\,\Im m\{x^*[0]\}. \end{aligned} \tag{6}$$

Equating real and imaginary parts in (6) and noting that

$$\Re e\{x^*[0]\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_R(e^{j\theta})\,d\theta \tag{7}$$

we obtain the relationship

$$X_I(e^{j\omega}) = -\frac{1}{2\pi}\int_{-\pi}^{\pi} X_R(e^{j\theta})\cot\left(\frac{\omega-\theta}{2}\right)d\theta - \Im m\{x^*[0]\}. \tag{8}$$

By using the DHT relation for the complex cepstrum [1] of a complex sequence $\hat{x}[n]$, we obtain

$$\arg X(e^{j\omega}) = -\frac{1}{2\pi}\int_{-\pi}^{\pi} \log|X(e^{j\theta})|\cot\left(\frac{\omega-\theta}{2}\right)d\theta - K \tag{9}$$

where $K = \Im m\{\hat{x}^*[0]\}$. The minimum-phase spectrum of a complex sequence has the same form as the minimum-phase spectrum of a real sequence plus a constant. If we minimize group delay, which is the derivative of the phase response with respect to frequency ω, then the constant term K will vanish, and the same DHT relation would hold for complex and real sequences.

III. Optimal Minimum-Phase FIR Design Algorithm

The optimal minimum-phase filter is designed by transforming an optimal linear-phase FIR filter into an optimal minimum-phase filter. The design is decoupled into two steps:

1) Obtain the squared magnitude response of the desired optimal minimum-phase FIR filter.
2) Use the DHT to produce a minimum-phase FIR filter by directly supplying the square root of the magnitude response of step 1.

Several methods achieve the goal of the first step. We modify the two-level ripple specification in [8] in the following subsections. In order to handle multiple ripple levels over different bands, we replace step 1 with the method in [9], which uses a modified Parks–McClellan algorithm [15] to allow the error to oscillate between 0 and $2\delta_k$ instead of $-\delta_k$ to $+\delta_k$, where δ_k is the ripple in the kth band [16]. Thus, optimal minimum-phase filters with the least complexity may be obtained for arbitrary magnitude specifications over the passband and stopband using the DHT.

A. Obtaining the Squared Magnitude Response of Optimal Magnitude Minimum-Phase FIR Filter

First, we design a symmetric lowpass filter of length $2N-1$ using the Parks–McClellan algorithm. Given the desired passband and stopband ripples of the optimal minimum-phase filter as δ_1' and δ_2', respectively, we compute the passband and stopband ripples of the linear-phase filter, which are, respectively, denoted as δ_1 and δ_2. The order of the minimum-phase filter that is finally obtained is N, and this filter is not only minimum-phase but also has optimal magnitude characteristics in the Chebyshev sense. The length $2N-1$ linear-phase filter is typically designed with the smallest number of taps that will meet the computed linear-phase specifications [17]:

$$\delta_1 = \frac{4\delta_1'}{2 + 2\delta_1'^2 - \delta_2'^2} \tag{10}$$

$$\delta_2 = \frac{\delta_2'^2}{2 + 2\delta_1'^2 - \delta_2'^2}. \tag{11}$$

The transfer function of the linear-phase filter designed in this way is

$$H_{\text{linear}}(z) = z^{-(N-1)}H_0(z) \tag{12}$$

where $H_0(z)$ is the zero-phase transfer function

$$H_0(z) = \sum_{k=0}^{N-1} h(k)(z^k + z^{-k}). \tag{13}$$

Second, we shift the transfer function by $\delta_2 + \epsilon_1$. Shifting the transfer function by δ_2 makes the magnitude spectrum non-negative. Since the DHT does not exist on the unit circle [18], we add ϵ_1 to ensure that the DHT exists. The ϵ_1 term may be chosen to be arbitrarily small so as not to affect the magnitude spectrum significantly. We typically use $\epsilon_1 = 10^{-10}$.

Third, we normalize $H_0(z) + \delta_2$ by using $H(z) = (H_0(z) + \delta_2)$ SCAL, where [8]

$$\text{SCAL} = \frac{4}{\left(\sqrt{1+\delta_1+\delta_2} + \sqrt{1-\delta_1+\delta_2}\right)^2}. \tag{14}$$

This causes the passband ripple to oscillate between $1+\delta_1'$ and $1-\delta_1'$. $H(z)$ corresponds to the magnitude squared response of the minimum-phase filter $H_{\min}(z)$

$$H(z) = (H_0(z) + \delta_2)\text{SCAL} = H_{\min}(z)H_{\min}(z^{-1}). \tag{15}$$

$H_{\min}(z)$ has the required magnitude response of the minimum-phase spectral factor.

B. Application of the Discrete Hilbert Transform

We apply the DHT to $H_{\min}(z)$, which is the square-root response of $H(z)$, reconstruct the minimum-phase polynomial by combining the desired magnitude and phase responses, and take the inverse FFT. For any magnitude response, the minimum-phase filter of a given order is unique. The algorithm is idempotent for a given ϵ_1, within the limits of arithmetic precision.

The discrete version of the integral transform in (9) uses a discrete Fourier transform (DFT) [13]. Given a sampled magnitude spectrum $|X[i]|$ for $i = 0, \cdots, M-1$, where M is the DFT length, we compute the corresponding minimum-phase spectrum in two steps. First, we compute the sampled phase spectrum

$$\theta = -j\text{DFT}\{\boldsymbol{s} \bullet \text{IDFT}\{\boldsymbol{a}\}\} \tag{16}$$

where • represents pointwise vector multiplication, and

$$\theta = [\theta[0],\, \theta[1],\, \cdots,\, \theta[M-1]]$$
$$\boldsymbol{s} = [\mathrm{sgn}[0],\, \mathrm{sgn},\, \cdots,\, \mathrm{sgn}[M-1]]$$
$$\boldsymbol{a} = [\log|X[0]|,\, \log|X[1]|,\, \cdots,\, \log|X[M-1]|]$$

such that

$$\mathrm{sgn}[i] = \begin{cases} 0 & i = 0,\, \dfrac{M}{2} \\ 1 & 0 < i < \dfrac{M}{2} \\ -1 & \dfrac{M}{2} < j < M. \end{cases} \tag{17}$$

Second, we form $|X[i]|e^{j\theta[i]}$ for $i = 0,\, 1,\, \cdots,\, M-1$. We can make the discrete approximation in (16) arbitrarily close to the continuous integral transform in (9) by choosing a large-enough FFT length, as quantified in the Appendix.

The minimum-phase filter with the same magnitude response as $x[n]$ may be constructed by

1) computing $|X[i]|$ for $i = 0,\, \cdots,\, M-1$, which is the sampled magnitude spectrum of $x[n]$;
2) calculating $\theta[i]$ for $i = 0,\, \cdots,\, M-1$ by using (16);
3) constructing the FFT of length M of the minimum-phase sequence as $|X[i]|e^{j\theta[i]}$;
4) taking the inverse FFT transform of length M to obtain a minimum-phase sequence;
5) truncating the resulting sequence to the desired filter impulse response length N.

The formula for choosing M is given in the Appendix. By using a long FFT, the magnitude spectrum of the truncated minimum-phase sequence closely matches the original magnitude spectrum, and the truncated terms may be made negligibly small, e.g., on the order of 10^{-7}.

IV. Example Optimal Minimum–Phase Filter Designs

We design two optimal minimum-phase filters using the DHT-based algorithm in Section III. Example 1 is designed according to the specifications of an example in [8]. Example 2 presents the design of a complex-tap optimal minimum-phase FIR filter. In the examples, all frequency values are normalized in the range from 0–1, where 1 represents half of the sampling frequency. For each example, we report the FFT length used and the time that the DHT-based algorithm took to run in MATLAB [19] version 5.0 on a 167 Hz Sun Ultra workstation running Solaris 2.5.1.

A. Example 1—Real 325-Tap Lowpass Filter

To show that the algorithm can handle very long filters, we design a real lowpass FIR filter using a set of specifications from [8]:

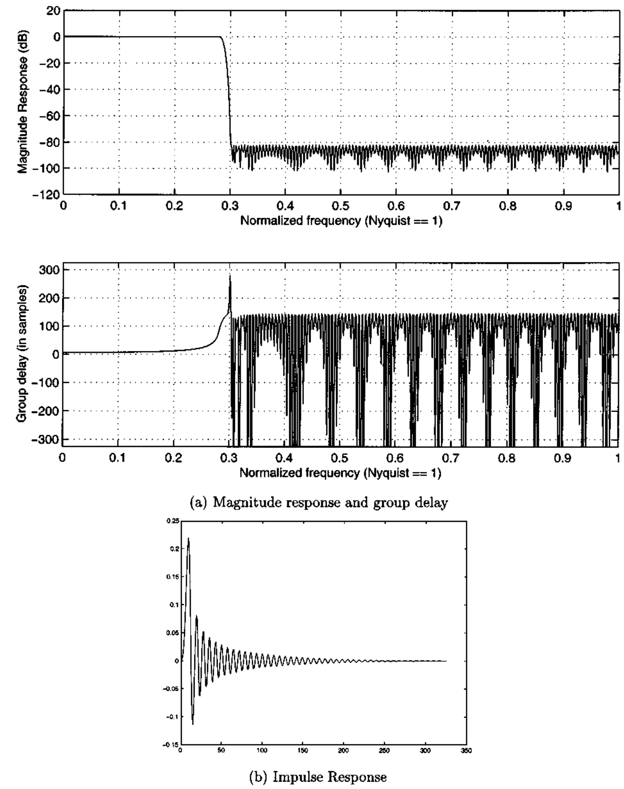
- passband edge $f_p = 0.28$;
- stopband edge $f_s = 0.3$;
- weighting function $1 : 5 \times 10^5$;
- passband ripple $\delta_1' = 0.000\,830$;
- stopband ripple $\delta_2' = 8.2008 \times 10^{-5}$.

Using (10) and (11), we calculate $\delta_1 = 0.001\,660$ and $\delta_2 = 3.3627 \times 10^{-9}$ for the optimal linear-phase filter. The optimal linear-phase filter has a length of $L = 649$, and the optimal minimum-phase FIR filter has a length of $N = 325$. Fig. 1(a) shows the magnitude response and group delay of the optimal minimum-phase filter. Fig. 1(b) shows the plot of the impulse response coefficients. We used an FFT length of 524 288 (2^{19}), and the algorithm took 60 s to run.

In the DHT-based optimal design, the actual ripple parameter values were $\delta_1' = 0.000\,828$ and $\delta_2' = 8.1684 \times 10^{-5}$, which meet the specifications. The relative percentage error was 0.24% in the passband and 0.39% in the stopband, which are comparable with the figures of 0.29 and 0.21%, respectively, in [8]. While the ripples in both bands of the DHT-based design are lower than the specified value, the ripples in the design in [8] are greater than the specified values.

B. Example 2—Complex 26-Tap Lowpass Filter

We design a lowpass complex-tap minimum-phase filter. The optimal linear-phase filter is an equiripple filter having linear-phase over



(a) Magnitude response and group delay

(b) Impulse Response

Fig. 1. Magnitude response and group delay for an optimal real 325-tap minimum-phase digital FIR filter designed by the new algorithm based on the discrete Hilbert transform.

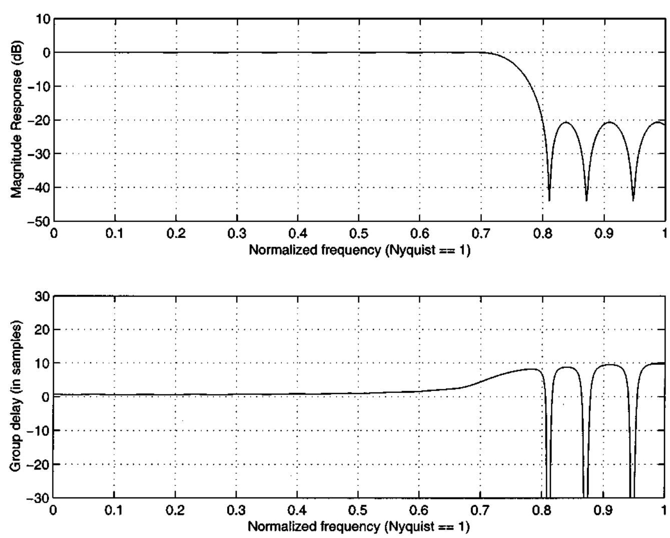


Fig. 2. Magnitude response and group delay for an optimal complex 26-tap minimum-phase digital FIR filter designed by the new algorithm based on the DHT. The filter coefficients are given in Table I.

the passband only. Hence, it has complex coefficients and may be designed using the algorithm in [14]. The specifications are $f_p = 0.7$, $f_s = 0.8$, $\delta_1' = 0.002\,125$, $\delta_2' = 0.092\,510$, and a weighting function of $1:1$. Using (10) and (11), we calculate $\delta_1 = 0.004\,268$ and $\delta_2 = 0.004\,297$. The optimal complex-tap filter with linear phase over the passband has 50 taps. Using the DHT-based algorithm, we design a minimum-phase complex FIR filter with 26 taps to meet the specifications. Fig. 2 shows the magnitude response and group delay of the optimal complex filter, and Table I lists its coefficients. We used an FFT length of $32\,678$, and the algorithm took 6 s to run.

In the DHT-based optimal design, the actual ripple parameter values were $\delta_1' = 0.002\,125$ and $\delta_2' = 0.092\,359$, which meet the specifications. The relative errors are 0% in the passband and 0.16% in the stopband. The approach also gives good results for higher-order complex FIR filters.

V. Conclusion

We present a robust noniterative algorithm to design optimal minimum-phase digital FIR filters with real or complex coefficients given an arbitrary magnitude specification. We derive the DHT of the complex cepstrum of a causal complex minimum-phase sequence, which is the DHT of the real cepstrum plus a constant. The Appendix derives the minimum fast Fourier transform length for computing the DHT to achieve a desired coefficient accuracy for a given filter order.

Appendix

We relate the FFT length for the DHT-based design algorithm to the filter order and the filter coefficient accuracy ϵ. We express the normalized transfer function of the minimum-phase filter $H_{\min}(z)$ in (15) as a product of two shorter transfer functions $H_1(z)$ and $H_2(z)$:

$$H_1(z) = \prod_{k_1=1}^{N_1} (1 - z_{k_1} z^{-1})$$
$$H_2(z) = \prod_{k_2=1}^{N_2} (1 - e^{j\theta_{k_2}} z^{-1}). \tag{18}$$

Here, z_{k_1} is the k_1st zero such that $|z_{k_1}| < 1$ for $k_1 = 1, 2, \cdots, N_1$. $H_1(z)$ corresponds to the passband and transition band zeros, and $H_2(z)$ corresponds to the stopband zeros on the unit circle. The logarithm of $H_{\min}(z)$ can be expressed as

$$\log(H_{\min}(z)) = \log(H_1(z)) + \log(H_2(z)). \tag{19}$$

The terms may be expanded using Taylor series for $\log(1+z)$ about $z = 0$

$$\log(H_1(z)) = -\sum_{k_1=1}^{N_1} \left(\sum_{m=1}^{\infty} \frac{z_{k_1}^m}{m} z^{-m} \right) \tag{20}$$
$$\log(H_2(z)) = -\sum_{k_2=1}^{N_2} \left(\sum_{m=1}^{\infty} \frac{e^{jm\theta_{k_2}}}{m} z^{-m} \right). \tag{21}$$

Since both infinite summations converge, the terms in the infinite summations decay faster than $1/m$. In addition, the infinite summation terms in $\log(H_1(z))$ decay faster than those in $\log(H_2(z))$ since the zeros of $H_1(z)$ are inside the unit circle, and the zeros of $H_2(z)$ are on the unit circle.

TABLE I
Coefficients of an Optimal Complex 26-Tap Minimum-Phase Digital FIR Filter. Magnitude Response and Group Delay Are Plotted in Fig. 2

Real part	Imaginary part
0.327129781635836	$1.10104503387227 \times 10^{-8}$
0.512001406284635	0.262431899245427
0.163947468371105	0.227163058898302
− 0.0257167097493542	0.169356927781137
0.0355816232408139	0.107986540639453
− 0.093077273032793	0.0915095753055589
− 0.0273636983641238	0.00902835562046581
0.100091962583298	0.0167244203666966
− 0.0190331892220469	0.0145608794832695
− 0.0286050656554984	0.0569396810712755
− 0.000707266912356274	0.0495568548051081
− 0.0104438457279702	0.0208216969515995
0.0411664224502276	0.0293187012314285
− 0.00914800135573782	0.00101446776486144
− 0.032619881411354	0.0107003684593396
0.0183176695129369	0.0188941052638534
0.00391196683007148	0.011337275264548
0.00443371597075111	0.0270242535593892
− 0.00386068199999262	0.00485717178612143
− 0.0162004814698202	0.00834426154063499
0.0154867955225919	0.00016176926155688
0.00680124235641706	0.00318781993786719
− 0.0140452818544805	0.0190124777686621
0.00327648746946536	0.0200572861433658
0.00238954698448227	0.00749923742815284
0.000915786507148754	0.000874039273345126

The complex cepstrum is $h_1(n) + h_2(n)$. Each component may be obtained by taking the inverse z transform of $\log(H_1(z))$ and $\log(H_2(z))$, respectively

$$h_1(n) = -\sum_{k_1=1}^{N_1} \mathcal{Z}^{-1} \left\{ \sum_{m=1}^{\infty} \frac{z_{k_1}^m}{m} z^{-m} \right\} \tag{22}$$
$$h_2(n) = -\sum_{k_2=1}^{N_2} \mathcal{Z}^{-1} \left\{ \sum_{m=1}^{\infty} \frac{e^{jm\theta_{k_2}}}{m} z^{-m} \right\}. \tag{23}$$

Since $h_2(n)$ decays at a much slower rate, we consider $h_2(n)$. When $n > 0$

$$h_2(n) = -\sum_{k_2=1}^{N_2} \frac{e^{jn\theta_{k_2}}}{n}. \tag{24}$$

For the complex filter case, $|h_2(n)| < N_2/n$. For causality, $|h_2(n)| < \epsilon$ for $n > M_2/2$, where M_2 is the length of the sequence or the order of the FFT used:

$$\left| h_2\left(\frac{M_2}{2}\right) \right| < \epsilon = 2\,\frac{N_2}{M_2}. \tag{25}$$

For real filters, we obtain a better bound because we know that the stopband zeros occur in complex conjugate pairs and are spaced uniformly around the unit circle

$$|h_2(n)| = -\frac{2}{n} \sum_{i=0}^{l} \cos(2\pi f_i) \tag{26}$$

where $l = N_2/2 - 1$ when N_2 is even and $l = (N_2 - 1/2)$ when N_2 is odd.

Using the fact that $\theta_i = 2\pi f_i$ and $f_0 = f_s$, where f_s is the stopband frequency, the stopband frequency response specification (26) becomes

$$|h_2(n)| = -\frac{2}{n} \left[\sum_{i=0}^{l} c(f_s,\, i,\, N_2)\right] \tag{27}$$

where

$$c(f_s,\, i,\, N_2) = \cos\left(2\pi \left\{f_s + i\left(\frac{1-2f_s}{N_2 - 1}\right)\right\}\right). \tag{28}$$

Therefore, the error bound for the real case is

$$\left|h_2\left(\frac{M_2}{2}\right)\right| < \epsilon = \frac{4}{M_2}\left|\sum_{i=1}^{l} c(f_s,\, i,\, N_2)\right|. \tag{29}$$

We can make ϵ arbitrarily small by using a large-enough FFT length. If the FFT length is restricted to be a power of two, then we may compute the required FFT length M_2 from the specified cepstral error ϵ for the complex and real cases as

$$m_{\text{complex}} = \lceil 1 + \log_2(N_2) - \log_2(\epsilon) \rceil \tag{30}$$

$$m_{\text{real}} = \left\lceil 2 + \log_2\left|\sum_{i=0}^{l} c(f_s,\, i,\, N_2)\right| - \log_2(\epsilon)\right\rceil \tag{31}$$

$$M_2 = 2^{m_2} \tag{32}$$

where m_2 is m_{complex} for the complex case and m_{real} for the real case. For a filter with 100 stopband zeros, $f_s = 0.3$, and $\epsilon = 0.001$, we obtain $m_2 = 18$ in both cases. Although (30) is more conservative than (31), requiring a power-of-two FFT may yield the same FFT length.

ACKNOWLEDGMENT

The authors would like to thank, for their suggestions, Prof. R. Baldick, S. Gummadi, and D. Wei at UT Austin; Prof. J. H. McClellan at Georgia Tech; and Prof. P. S. Reddy at the University of Madras, India.

REFERENCES

[1] A. V. Oppenheim and R. W. Schafer, *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1989.

[2] P. Leistner and T. W. Parks, "On the design of FIR digital filters with optimum magnitude and minimum-phase," *Arch. Elekt. Uebertragungstechnik*, vol. 29, no. 6, pp. 270–274, June 1975.

[3] E. A. Lee and G. David, *Digital Communication*, 2nd ed. Messerschmitt, Germany: Kluwer, 1994.

[4] O. Herrmann and H. W. Schüessler, "Design of nonrecursive digital filters with minimum-phase," *Electron. Lett.*, vol. 6, no. 11, pp. 329–330, May 1970.

[5] J. D. Fisher and T. W. Parks, "Optimal Chebyshev magnitude approximation for FIR filters," in *Proc. IEEE Southwest. Conf.*, Houston, TX, Apr. 1973.

[6] Y. Kamp and C. J. Wellekens, "Optimal design of minimum-phase FIR filters," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-31, pp. 922–926, Aug. 1983.

[7] J. K. Liang and R. J. P. de Figueiredo, "A design algorithm for optimal lowpass nonlinear-phase digital filters," *Signal Process.*, vol. 8, pp. 3–21, Feb. 1985.

[8] X. Chen and T. W. Parks, "Design of optimal minimum-phase FIR filters by direct factorization," *Signal Process.*, vol. 10, no. 4, pp. 369–383, June 1986.

[9] R. Boite and H. Leach, "A new procedure for the design of high order minimum-phase FIR digital or CCD filters," *Signal Process.*, vol. 3, no. 2, pp. 101–108, Apr. 1981.

[10] G. A. Mian and A. P. Nainer, "A fast procedure to design equiripple minimum-phase FIR filters," *IEEE Trans. Circuits Syst.*, vol. CAS-29, pp. 327–331, May 1982.

[11] Acoustics, Speech, and Signal Processing Society/Digital Signal Processing Committee, *Programs for Digital Signal Processing*. New York: IEEE, 1979.

[12] A. Papoulis, *The Fourier Integral and its Applications*. New York: McGraw-Hill, 1962.

[13] V. Cizek, "Discrete Hilbert Transform," *IEEE Trans. Audio Electroacoust.*, vol. AE-18, pp. 340–343, Dec. 1970.

[14] L. J. Karam and J. H. McClellan, "Complex Chebyshev approximation for FIR digital filter design," *IEEE Trans. Circuits Syst. II*, vol. 42, pp. 207–216, Mar. 1995.

[15] T. W. Parks and J. H. McClellan, "Chebyshev approximation for nonrecursive digital filters with linear-phase," *IEEE Trans. Circuit Theory*, vol. CT-19, no. 2, pp. 189–194, Mar. 1972.

[16] T. W. Parks and C. S. Burrus, *Digital Filter Design*. New York: Wiley, 1987.

[17] D. Wei, private communication, Mar. 1, 1998.

[18] P. S. Reddy and H. S. N. Murthy, "Stabilization of 2-D digital filters via Hilbert transform," *Electron. Lett.*, vol. 19, no. 2, pp. 68–69, Jan. 20, 1983.

[19] D. Hanselman and B. Littlefield, *Mastering MATLAB*. Englewood Cliffs, NJ: Prentice-Hall, 1996.

Factorable FIR Nyquist Filters with Least Stopband Energy under Sidelobe Level Constraints

Mingyu Liu, Christopher J. Zarowski, and Frederick W. Fairman

***Abstract*—Spectrally factorable Nyquist filters are used in data communications to avoid intersymbol interference. In this correspondence, an approach is developed for obtaining a Nyquist filter that is factorable having the smallest stopband energy for a given sidelobe level. The resulting constrained minimization problem is solved efficiently and reliably using the Goldfarb–Idnani algorithm. Some examples are presented comparing the present method with a previous approach from the literature.**

***Index Terms*—FIR Nyquist filters, sidelobe level constraints.**

I. INTRODUCTION

Mueller [1] proposed a method of designing Nyquist filters (Mueller sequences) with minimum stopband energy (SE). However, [1] does not present a method of imposing a non-negative frequency response, which is essential to achieve a spectrally factorable sequence. One spectral factor is used as the transmit pulse, whereas the other is used as the matched filter in the receiver. Since the work of [1],

Manuscript received September 13, 1999; revised October 20, 1999. The associate editor coordinating the review of this paper and approving it for publication was Dr. Hitoshi Kiya.

The authors are with the Department of Electrical and Computer Engineering, Queen's University, Kingston, Ont., Canada K7L 3N6 (e-mail: czarow@eleceng.ee.queensu.ca).

Publisher Item Identifier S 1053-587X(00)03329-8.