Correction to Table 3 in "Determining fault tolerance of XOR-based erasure codes efficiently"

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Abstract

*There are errors in Table 3 of "Determining fault tolerance of XOR-based erasure codes efficiently" by Jay J. Wylie and Ram Swaminathan published in the proceedings of DSN 2007, The 37th Annual IEEE/IFIP International Conference on Dependable Systems and Networks, June 25 - June 28, 2007, Edinburgh, UK. Thanks to Dave Walker who recognized that the parity submatrices and minimal erasure vectors (*MEV*) listed in Table 3 of our DSN paper for $k = 2$ and $m > 2$ were incorrect. An error in the implementation of the Minimal Erasures (ME) Algorithm lead to an incorrect measure of the* MEV *for all (k,m)-code corpi evaluated in Table 3 with $m > k$. For each row that corresponds to a (k,m)-code corpus with $m > k$, the entries in the following columns may be incorrect: "# w* MEV**", "# w d^*", "A parity submatrix w* MEV**", "*MEV**", "d^*", and "$\frac{|\text{EL}|}{|\text{MEL}|}$". To correct our error we are making available a correct listing of all best codes for each (k,m)-code corpus in this short report. Table 1 of this short report lists correct results for all (k,m)-code corpi with $1 \leq k, m \leq 7$. For each code corpus, the best* MEV *(*MEV**) and Hamming distance (d^*) is listed. The parity submatrix for* **every** *non-isomorphic XOR-based erasure code that shares the best* MEV *(*MEV**) is listed on a separate row; this is more information than we had room to include in our DSN paper. The error in our implementation of the ME Algorithm does not affect the correctness of the pseudo-code for the ME Algorithm and proof of its correctness given in our DSN paper. Our conclusions that the empirical evidence supports the claim that the ME Algorithm is an efficient, effective manner to characterize the fault tolerance of an XOR-based erasure code stands, based solely on the empirical results for (k,m)-code corpi with $m \leq k$; we intend to redo the calculations of $\frac{|\text{EL}|}{|\text{MEL}|}$ for (k,m)-code corpi with $m > k$ for an extended technical report or journal submission of this work in the future.*

Table 1: Parity submatrices for all best codes from (k, m)-code corpi for $1 \leq k, m \leq 7$.

k	m	MEV*	d^*	Parity submatrices
1	1	(0)	2	1
1	2	(0, 0)	3	1, 1
1	3	(0, 0, 0)	4	1, 1, 1
1	4	(0, 0, 0, 0)	5	1, 1, 1, 1
1	5	(0, 0, 0, 0, 0)	6	1, 1, 1, 1, 1
1	6	(0, 0, 0, 0, 0, 0)	7	1, 1, 1, 1, 1, 1
1	7	(0, 0, 0, 0, 0, 0, 0)	8	1, 1, 1, 1, 1, 1, 1
2	1	(0)	2	3
2	2	(0, 1)	2	1, 3
				3, 3
2	3	(0, 0, 2)	3	1, 2, 3
				1, 3, 3
2	4	(0, 0, 0, 3)	4	1, 2, 3, 3
2	5	(0, 0, 0, 1, 2)	4	1, 1, 2, 3, 3
				1, 2, 3, 3, 3
2	6	(0, 0, 0, 0, 2, 1)	5	1, 1, 2, 2, 3, 3
				1, 1, 2, 3, 3, 3
2	7	(0, 0, 0, 0, 0, 3, 0)	6	1, 1, 2, 2, 3, 3, 3
3	1	(0)	2	7
3	2	(0, 2)	2	3, 5
				3, 7
3	3	(0, 0, 4)	3	3, 5, 6
				3, 5, 7
3	4	(0, 0, 0, 7)	4	3, 5, 6, 7
3	5	(0, 0, 0, 3, 4)	4	1, 3, 5, 6, 7
				3, 3, 5, 6, 7
				3, 5, 6, 7, 7
3	6	(0, 0, 0, 1, 4, 2)	4	1, 2, 3, 5, 6, 7
				1, 3, 3, 5, 6, 7
				1, 3, 5, 6, 6, 7
				1, 3, 5, 6, 7, 7
				3, 3, 5, 5, 6, 7
				3, 3, 5, 6, 7, 7
3	7	(0, 0, 0, 0, 3, 3, 1)	5	3, 3, 5, 5, 6, 7, 7
				1, 2, 3, 4, 5, 6, 7
				1, 2, 3, 5, 5, 6, 7
				1, 2, 3, 5, 6, 7, 7
				1, 3, 3, 5, 5, 6, 7
				1, 3, 3, 5, 6, 7, 7
				1, 3, 3, 5, 6, 6, 7
4	1	(0)	2	15
4	2	(0, 3)	2	7, 11
4	3	(0, 0, 7)	3	7, 11, 13
4	4	(0, 0, 0, 14)	4	7, 11, 13, 14

k	m	MEV*	d^*	Parity submatrices
4	5	(0, 0, 0, 6, 8)	4	7, 11, 13, 14, 15
				3, 5, 9, 14, 15
				3, 7, 11, 13, 14
4	6	(0, 0, 0, 2, 8, 4)	4	3, 5, 6, 9, 14, 15
				3, 5, 7, 9, 14, 15
				3, 5, 7, 11, 13, 14
				3, 5, 7, 10, 12, 14
				3, 7, 11, 12, 13, 14
				3, 7, 11, 13, 14, 15
4	7	(0, 0, 0, 0, 6, 6, 2)	5	3, 5, 7, 11, 13, 14, 15
				3, 5, 6, 10, 11, 12, 13
				3, 5, 6, 10, 11, 12, 15
				3, 5, 7, 10, 11, 12, 14
				3, 5, 7, 10, 12, 14, 15
				3, 5, 7, 11, 12, 13, 14
				3, 5, 6, 9, 11, 14, 15
				3, 5, 7, 9, 11, 13, 14
				3, 5, 7, 9, 11, 13, 15
				3, 5, 7, 9, 11, 14, 15
5	1	(0)	2	31
5	2	(0, 5)	2	7, 27
				15, 23
5	3	(0, 1, 10)	2	7, 11, 29
				7, 27, 29
				15, 23, 27
5	4	(0, 0, 4, 14)	3	15, 23, 27, 29
				7, 11, 19, 29
				7, 11, 29, 30
5	5	(0, 0, 0, 10, 16)	4	15, 23, 27, 29, 30
				7, 11, 19, 29, 30
5	6	(0, 0, 0, 4, 14, 8)	4	7, 11, 13, 19, 29, 30
				7, 11, 15, 19, 29, 30
				7, 11, 19, 29, 30, 31
				3, 13, 14, 23, 27, 28
				7, 14, 22, 25, 27, 29
				7, 15, 23, 27, 29, 30
				3, 15, 23, 27, 29, 30
				3, 13, 21, 25, 30, 31
				3, 5, 14, 22, 25, 31
				3, 5, 15, 23, 25, 30
				3, 13, 21, 26, 27, 28
				3, 7, 13, 21, 27, 30
5	7	(0, 0, 0, 1, 8, 12, 8)	4	7, 11, 13, 14, 19, 21, 25
				7, 11, 13, 19, 21, 25, 31
				7, 11, 13, 14, 21, 25, 31
				7, 11, 14, 21, 25, 28, 31
				7, 11, 14, 19, 25, 28, 31

k	m	MEV*	d^*	Parity submatrices
				7, 11, 14, 19, 21, 22, 25
				7, 11, 14, 19, 21, 25, 28
				7, 11, 14, 19, 21, 25, 31
6	1	(0)	2	63
6	2	(0, 7)	2	15, 51
				15, 55
6	3	(0, 2, 14)	2	7, 27, 45
				7, 27, 61
				15, 23, 59
				15, 29, 51
				15, 51, 61
				15, 55, 59
6	4	(0, 0, 8, 18)	3	7, 27, 43, 53
				7, 27, 43, 61
				7, 27, 45, 62
				15, 23, 59, 61
				15, 29, 51, 54
				7, 27, 45, 56
6	5	(0, 0, 0, 25, 0)	4	15, 29, 39, 51, 57
				7, 25, 42, 52, 63
				7, 25, 42, 53, 62
				7, 27, 45, 55, 57
				7, 25, 43, 53, 63
				7, 25, 43, 54, 60
				7, 25, 43, 55, 61
6	6	(0, 0, 0, 6, 24, 16)	4	7, 27, 43, 51, 61, 62
				7, 27, 30, 45, 53, 56
				15, 23, 39, 59, 61, 62
				15, 29, 45, 51, 54, 58
				7, 11, 29, 45, 51, 62
6	7	(0, 0, 0, 2, 16, 18, 10)	4	15, 23, 27, 43, 51, 61, 62
				7, 11, 29, 46, 49, 50, 60
				7, 11, 29, 46, 49, 50, 63
				7, 11, 29, 46, 53, 54, 59
				7, 11, 30, 46, 51, 53, 57
				7, 11, 31, 47, 51, 53, 57
				7, 11, 21, 26, 45, 51, 62
				7, 11, 21, 31, 45, 51, 62
				7, 11, 19, 29, 45, 53, 59
				7, 11, 19, 30, 45, 53, 59
				7, 29, 30, 43, 51, 57, 63
				7, 27, 29, 43, 45, 51, 62
				7, 11, 29, 45, 55, 56, 62
				7, 29, 30, 43, 51, 57, 60
				7, 11, 29, 46, 53, 54, 56
				7, 11, 29, 45, 49, 52, 62
				7, 11, 30, 46, 49, 52, 61

k	m	MEV*	d^*	Parity submatrices
				7, 14, 25, 47, 51, 53, 62
				7, 27, 30, 43, 45, 55, 56
				15, 23, 29, 30, 39, 43, 51
				7, 11, 29, 45, 51, 53, 57
				7, 11, 29, 30, 46, 53, 56
				7, 11, 30, 45, 53, 56, 59
				7, 11, 31, 47, 53, 57, 60
				7, 11, 21, 25, 45, 51, 62
				7, 11, 21, 25, 45, 54, 59
				7, 11, 21, 45, 46, 49, 55
				7, 11, 29, 45, 49, 55, 62
				7, 11, 29, 30, 45, 49, 63
				7, 11, 29, 30, 46, 49, 60
				7, 11, 21, 31, 46, 51, 61
				7, 11, 30, 45, 49, 60, 63
				7, 11, 30, 46, 49, 55, 61
				7, 11, 22, 31, 45, 49, 60
				7, 11, 21, 35, 45, 57, 63
				7, 11, 21, 35, 45, 58, 60
				7, 11, 21, 35, 46, 57, 63
				7, 11, 21, 35, 46, 58, 60
				7, 25, 31, 43, 45, 55, 62
				7, 11, 22, 28, 46, 49, 63
				7, 27, 29, 47, 55, 57, 58
				7, 11, 21, 26, 46, 54, 56
				7, 11, 19, 29, 44, 53, 58
				7, 11, 21, 25, 42, 54, 60
				7, 11, 22, 25, 42, 53, 63
				7, 11, 29, 30, 45, 53, 59
				7, 11, 29, 30, 47, 52, 56
				7, 11, 29, 30, 47, 55, 59
				7, 11, 25, 42, 53, 54, 59
				7, 11, 21, 31, 46, 54, 56
				7, 11, 29, 45, 51, 53, 58
				7, 11, 29, 45, 51, 53, 63
				7, 11, 29, 45, 51, 54, 63
				7, 11, 29, 45, 51, 53, 60
				7, 11, 29, 45, 51, 54, 60
				7, 11, 29, 45, 55, 59, 62
				7, 11, 21, 26, 45, 54, 59
7	1	(0)	2	127
7	2	(0, 9)	2	31, 103
7	3	(0, 3, 19)	2	15, 51, 85
				15, 51, 93
				15, 51, 125
				15, 55, 91
				15, 55, 121

k	m	MEV*	d^*	Parity submatrices
				15, 55, 123
				31, 103, 123
7	4	(0, 0, 12, 26)	3	15, 51, 85, 107
				15, 51, 85, 126
				15, 51, 85, 127
				15, 51, 87, 109
				15, 51, 87, 125
				15, 51, 93, 118
				15, 54, 90, 113
				15, 54, 91, 113
				15, 55, 91, 113
				15, 55, 91, 125
7	5	(0, 0, 0, 38, 0)	4	15, 55, 91, 109, 113
				7, 57, 91, 109, 119
				7, 57, 90, 108, 119
				15, 51, 85, 105, 127
				15, 51, 85, 107, 125
				7, 57, 90, 109, 118
				15, 54, 90, 109, 113
				15, 51, 87, 109, 121
7	6	(0, 0, 0, 14, 28, 24)	4	15, 29, 51, 94, 103, 121
				15, 29, 51, 94, 103, 107
				15, 29, 51, 87, 109, 123
				15, 29, 51, 87, 109, 118
				15, 23, 59, 91, 109, 113
				7, 27, 43, 93, 110, 120
				15, 23, 57, 92, 99, 126
				15, 23, 57, 93, 99, 120
				15, 23, 57, 91, 99, 126
				15, 23, 57, 93, 99, 127
				15, 23, 60, 93, 99, 122
				15, 23, 59, 93, 99, 101
				15, 46, 51, 85, 109, 118
				15, 23, 61, 91, 99, 122
				15, 51, 61, 85, 107, 126
				15, 51, 85, 94, 107, 121
				7, 29, 56, 75, 109, 126
				15, 30, 55, 91, 113, 125
				7, 46, 56, 75, 85, 118
				7, 27, 45, 85, 99, 126
				7, 27, 45, 93, 99, 117
				7, 27, 45, 94, 99, 117
				15, 45, 54, 90, 102, 113
				15, 45, 54, 90, 107, 113
				15, 54, 91, 93, 110, 113
				15, 23, 59, 93, 103, 126
				15, 51, 61, 87, 91, 126

k	m	MEV*	d^*	Parity submatrices
				15, 55, 61, 91, 111, 113
				15, 51, 87, 107, 125, 126
				15, 51, 87, 109, 110, 123
				7, 27, 53, 86, 105, 122
				7, 27, 53, 86, 105, 127
				7, 27, 43, 77, 113, 127
				7, 27, 43, 77, 116, 122
				7, 27, 43, 77, 119, 121
				7, 27, 43, 85, 109, 115
				7, 27, 61, 94, 105, 113
				7, 27, 61, 94, 105, 116
				7, 27, 62, 94, 105, 116
				7, 57, 89, 102, 107, 117
				7, 27, 61, 93, 105, 119
				7, 27, 62, 93, 105, 119
				7, 27, 61, 94, 111, 119
				7, 27, 45, 78, 119, 121
				7, 27, 45, 86, 99, 125
				7, 27, 45, 86, 101, 120
				7, 27, 45, 86, 107, 117
				7, 27, 53, 85, 108, 122
				7, 57, 90, 105, 116, 127
				7, 57, 90, 105, 119, 124
				7, 27, 45, 93, 110, 120
				7, 27, 45, 94, 110, 120
				15, 29, 51, 86, 109, 122
				15, 29, 51, 86, 122, 127
				7, 27, 45, 93, 107, 126
				7, 27, 45, 93, 110, 123
				7, 57, 90, 111, 119, 124
7	7	(0, 0, 0, 3, 24, 36, 16)	4	7, 27, 45, 56, 93, 110, 115
				7, 27, 61, 93, 107, 115, 126
				7, 27, 62, 94, 109, 117, 120
				15, 30, 61, 93, 102, 106, 113
				15, 29, 51, 54, 87, 109, 122
				15, 30, 59, 61, 87, 102, 113
				15, 23, 57, 89, 99, 101, 126
				15, 23, 57, 89, 99, 101, 127
				7, 27, 45, 62, 86, 99, 117
				15, 23, 57, 58, 92, 99, 127
				7, 27, 45, 51, 86, 110, 120
				15, 29, 51, 87, 91, 109, 126
				15, 23, 59, 91, 99, 125, 126
				7, 27, 61, 93, 101, 107, 115
				7, 27, 43, 53, 77, 115, 126
				7, 27, 61, 93, 99, 110, 118
				15, 23, 59, 91, 103, 125, 126

k	m	MEV*	d^*	Parity submatrices
				7, 27, 43, 62, 77, 117, 120
				7, 27, 43, 61, 77, 115, 126
				15, 23, 62, 94, 99, 101, 121